MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.01T **Fall Term 2004**

In-Class Problems 11-13: Uniform Circular Motion Solutions

Section_________Table and Group Number ______________________

Names ____________________________________

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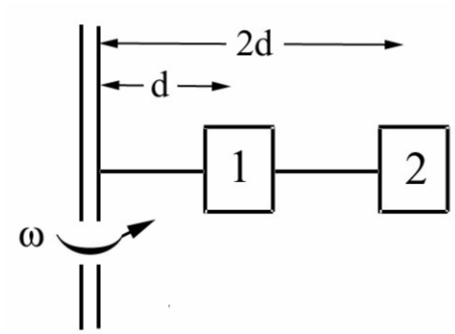
Hand in one solution per group.

We would like each group to apply the problem solving strategy with the four stages (see below) to answer the following two problems.

I. **Understand – get a conceptual grasp of the problem**
II. **Devise a Plan - set up a procedure to obtain the desired solution**
III. **Carry our your plan – solve the problem!**
IV. **Look Back – check your solution and method of solution**

In-Class-Problem 11: Whirling Objects

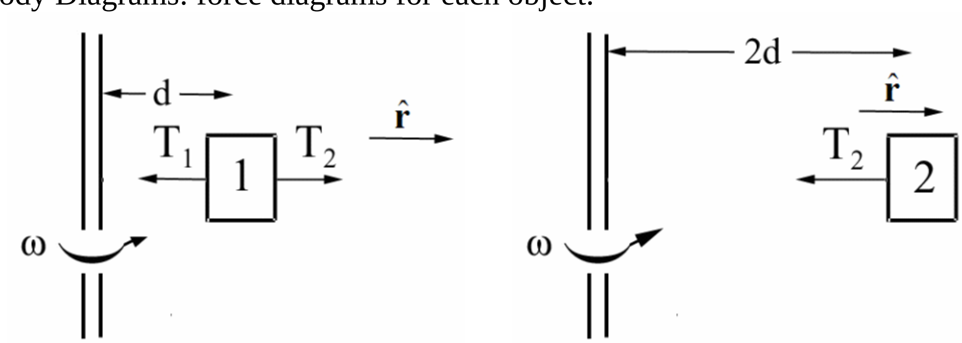
Two objects of equal mass m are whirling around a shaft with a constant angular velocity ω. The first object is a distance d from the central axis, and the second object is a distance $2d$ from the axis. You may assume the strings are massless and inextensible. You may ignore the effect of gravity. Find the tensions in the two strings.



As you begin this problem, consider what types of force diagrams you may need. In particular what body or system of bodies will use for your free body diagram. It never hurts to draw as many free body diagrams as you can.

Solution:

Free Body Diagrams: force diagrams for each object.



Newton's Second Law, $\vec{\mathbf{F}}_1 = m_1\vec{\mathbf{a}}_1$, in the radial direction for the inner object is

$$\hat{\mathbf{r}} : T_2 - T_1 = -md\omega^2 .$$

Newton's Second Law, $\vec{\mathbf{F}}_1 = m_1\vec{\mathbf{a}}_1$, in the radial direction for the outer object is

$$\hat{\mathbf{r}} : -T_2 = -m(2d)\omega^2 .$$

We can now solve for the tension in the string between the inner object and the outer object. From the force equation for outer object, the tension in the string between the inner object and the outer object is

$$T_2 = m(2d)\omega^2$$

The tension in the string between the shaft and the inner object can be found by using the result from part b) in the force equation for the inner object yielding,

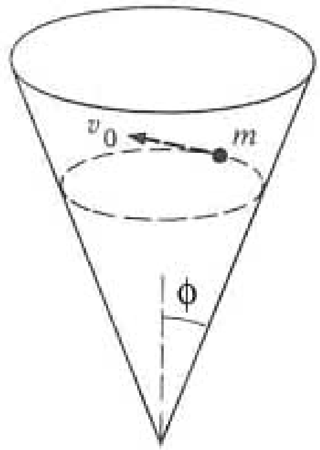
$$m(2d)\omega^2 - T_1 = -md\omega^2 .$$

Solving for the tension in the string between the shaft and the inner object gives

$$T_1 = 3md\omega^2$$

In-Class Problem 12: Object undergoing Circular Motion on the Inside of a Cone

Consider an object of mass m that moves in a circular orbit with constant velocity v_0 along the inside of a cone. Assume the wall of the cone is frictionless. The cone makes an angle ϕ with respect to a vertical axis.



a) Find the radius of the orbit of the object in terms of the given information.

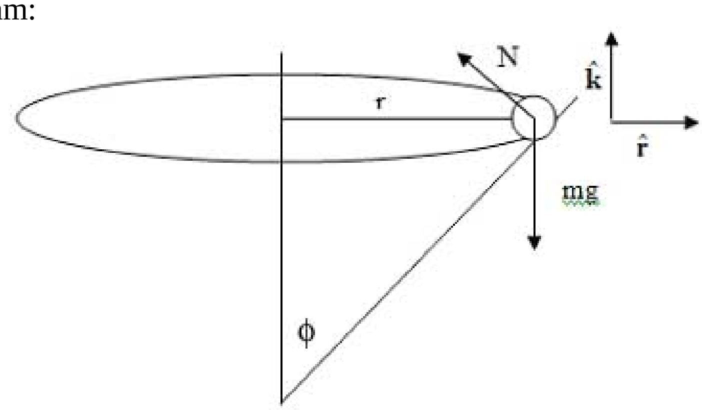
In the ***Look Back – check your solution and method of solution*** part of our methodology answer the following questions.

b) Is the normal force a centripetal force or is a compopnent of the normal force the centripetal force in this problem?

c) Is the radius of the circular orbit independent of the mass of the object? Explain why or why not.

Solution:

Free Body Diagram:



Choose cylindrical coordinates as shown in the above figure. Choose the unit vector $\hat{\mathbf{r}}$ to point in the radial outward direction. Since we are now working in three dimensions, choose the unit vector $\hat{\mathbf{k}}$ to point upwards. (Note that the unit vector $\hat{\boldsymbol{\theta}}$ is now automatically defined to point into the page. Since there are no forces in this direction, we need not worry about it while solving the problem.)

The force diagram on the object is shown in the figure above. The two forces acting on the object are the normal force of the wall on the object and the gravitational force. Then Newton's Second Law becomes:

$$\vec{\mathbf{F}} = m\vec{\mathbf{a}}$$

becomes:

$$\hat{\mathbf{r}} : -N\cos\phi = \frac{-m{v_0}^2}{r}$$
$$\hat{\mathbf{k}} : N\sin\phi - mg = 0$$

These equations become

$$N\cos\phi = \frac{m{v_0}^2}{r}$$
$$N\sin\phi = mg$$

We can divide these two equations,

$$\frac{N\sin\phi}{N\cos\phi} = \frac{mg}{m\frac{{v_0}^2}{r}}$$

yielding

$$\tan\phi = \frac{rg}{{v_0}^2}$$

This can be solved for the radius.

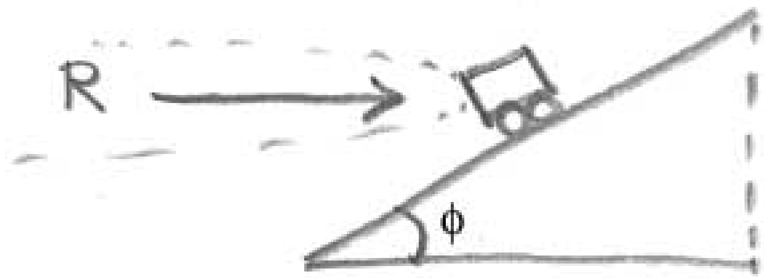
$$r = \frac{{v_0}^2}{g}\tan\phi\,.$$

b) The centripetal force in this problem is the vector component of the contact force that points radially inwards, $F_{cent} = N\cos\phi$.

c) The radius is independent of the mass because the component of the normal force in the vertical direction must balance the gravitational force so the normal force is proportional to the mass. Therefore the radially inward component of the normal force is also proportional to mass. Since this force is causing the object to accelerate inward, by Newton's Second Law, the mass will cancel and the acceleration is independent of the mass. Therefore the radius is also independent of the mass.

In-Class Problem 13: Car on a Banked turn

A car of mass m is going around a circular turn of radius R with constant speed v, which is banked at an angle ϕ with respect to the ground. The car is held up on the bank by static friction between the wheels and the road with a coefficient of static friction μ_s. Let g be the magnitude of the acceleration due to gravity. Derive an expression for the minimum velocity necessary to keep the car moving in a circle without slipping down the embanked turn. Express your answer in terms of the given quantities.

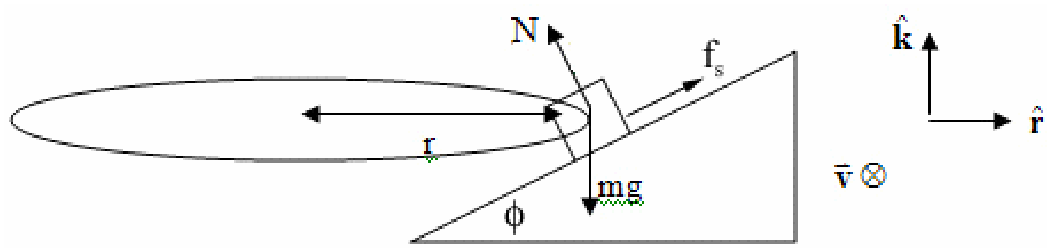


Car on banked turn undergoing circular motion

Solution

As you try to ***Understand – get a conceptual grasp of the problem*** consider the following question. There are all types of resistive forces acting on the car along the direction of motion of the car; for example: air friction and rolling resistance. Do you need to know these in order to model this problem? Explain your reasoning.

Choose cylindrical coordinates. Choose unit vectors $\hat{\mathbf{r}}$ pointing in the radial outward direction and $\hat{\mathbf{k}}$ pointing upwards (figure 6.2.5). (Once again, the unit vector $\hat{\boldsymbol{\theta}}$ is implicitly defined.) The force diagram on the car is shown in figure 6.2.5 when the car is just about to slide down the embanked turn.



Free body diagram for car

When $\vec{\mathbf{v}}_{\min} < \vec{\mathbf{v}} < \vec{\mathbf{v}}_0$, the car would slide down the banked turn if static friction did not hold it up. $\vec{\mathbf{v}}_0$ is the velocity such that $f_{static} = 0$, at $\vec{\mathbf{v}} = \vec{\mathbf{v}}_0$.

Newton's Second law, $\vec{\mathbf{F}} = m\vec{\mathbf{a}}$, becomes (in the radial and vertical directions)

$$\hat{\mathbf{r}}: -N\sin\phi + f_{static}\cos\phi = -\frac{mv^2}{r}$$
$$\hat{\mathbf{k}}: N\cos\phi + f_{static}\sin\phi - mg = ma_z$$

When $v = v_{\min}$, the 'just slipping' condition is that the acceleration in the z-direction is zero, and the static friction has its maximum value:

$$a_z = 0 \text{ and } f_{static} = \mu_s N$$

so the force equations become

$$-N\sin\phi + \mu_s N\cos\phi = -\frac{m{v_{\min}}^2}{r}$$
$$N\cos\phi + \mu_s N\sin\phi = mg$$

Dividing these equations yields

$$\frac{-\sin\phi + \mu_s\cos\phi}{\cos\phi + \mu_s\sin\phi} = -\frac{{v_{\min}}^2}{rg}$$

that can then be solved for the minimum speed, $v_{\min}$, necessary to avoid sliding down the embanked turn.

$$v_{\min} = \left(rg\left(\frac{\sin\phi - \mu_s\cos\phi}{\cos\phi + \mu_s\sin\phi}\right)\right)^{\frac{1}{2}}$$