MASSACHUSETTS INSTITUTE OF TECHNOLOGY

5.73 Quantum Mechanics I
Fall, 2002

Professor Robert W. Field

Problem Set #8

DUE: At the start of Lecture on Friday, November 8.

Reading: Angular Momentum Handouts. C-TDL, pages 643-676

Problems:

1. CTDL, page 765, #1.

2. CTDL, page 765, #2.

3. CTDL, page 768, #6.

4. Consider the spectrum and dynamics of an alkali atom in electric and magnetic fields, both along the z-axis. You must consider three electronic states, the $n^2S_{1/2}$ ground state, the $n^2P_{1/2}$ and $n^2P_{3/2}$ excited states, and the $(n+1)^2S_{1/2}$ excited state. The Zeeman Hamiltonian is

$$\mathbf{H}^{z} = -\gamma B_z\left(\mathbf{L}_z + 2\mathbf{S}_z\right).$$

The Stark Hamiltonian is

$$\mathbf{H}^{S} = \varepsilon_z\,\mathbf{z}$$

where the only nonzero matrix elements of **z** are $\Delta L = 1$, $\Delta M_L = 0$. For both Zeeman and Stark effects, you will want to work in the uncoupled basis set. For the Stark effect, the only relevant nonzero matrix elements are

$$\left\langle n^2P, M_L = 0, M_S \middle| \mathbf{z} \middle| (n+1)^2S, M_L = 0, M_S \right\rangle \equiv \mu_1$$

where μ_1 is independent of M_S. The optical excitation Hamiltonian

$$\mathbf{H}^{\text{opt}} = \vec{\varepsilon}^{\,\text{opt}} \sum\left(\hat{i}\mathbf{x} + \hat{j}\mathbf{y} + \hat{k}\mathbf{z}\right)$$

has the same form as the Stark Hamiltonian, except that its nonzero elements are exclusively

$$\left\langle n^2S, M_L = 0, M_S \right| \mathbf{z} \left| n^2P, M_L = 0, M_S \right\rangle \equiv \mu_0$$
$$\left\langle n^2S, M_L = 0, M_S \right| 2^{-1/2}(\mathbf{x} \pm i\mathbf{y}) \left| n^2P, M_L = \mp 1, M_S \right\rangle = 2^{1/2}\mu_0 \, .$$

Finally, there is the spin-orbit Hamiltonian,

$$H^{SO} = \zeta \, \mathbf{L} \bullet \mathbf{S}$$

which begs to be evaluated in the coupled representation. Let the energies of the relevant zero field states be:

$$E_{n^2S} = 0 \ \text{cm}^{-1}$$
$$E_{(n+1)^2S} = 20{,}000 \ \text{cm}^{-1}$$
$$E_{n^2P_{1/2}} = 19{,}800 \ \text{cm}^{-1}$$
$$E_{n^2P_{3/2}} = 19{,}900 \ \text{cm}^{-1}.$$

A. At zero applied magnetic and electric fields, sketch the optical spectrum in the region between 19,700 and 20,100 cm^{-1}. Be sure to indicate the relative intensities of transitions if more than one resolvable feature occurs in the spectrum.

B. At zero electric field, show how the z-polarized and x-polarized spectra evolve as the magnetic field is increased from 0 to a large enough value so that $|\gamma B_z| >> hc(100 \ \text{cm}^{-1})$. I am asking for two spectra.

C. At zero magnetic field, show how the z and x polarized spectra evolve as the electric field is increased from 0 V/cm to a large enough value so that $\varepsilon_z \mu_1$ is much larger than $hc(200 \ \text{cm}^{-1})$.

D. Now consider the nature of the quantum beats you might observe in the unresolved fluorescence following a 1fs excitation pulse. Under what conditions would the observable quantum beats look like "population" beats (i.e., the z and x polarized fluorescence intensities would oscillate in-phase) and under what conditions would they look like polarization beats (i.e., the fluorescence intensity would appear to be rotating from z to x and back to z polarization)? This is an open-ended question. Be as explicit as you can about the various possibilities. These include either of two excitation polarizations, either of two detection polarizations, and the ($B_z \neq 0$, $\varepsilon_z = 0$) and ($B_z = 0$, $\varepsilon_z \neq 0$) magnetic and electric field possibilities.