ELECTRIC GENERATION PRODUCTION SCHEDULING USING A QUASI-OPTIMAL SEQUENTIAL TECHNIQUE

J. Gruhl

Report #MIT-EL 73-003 April 1973

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This study was done in association with the Electric Power Systems Engineering Laboratory and the Department of Civil Engineering (Ralph M. Parsons Laboratory for Water Resources and Hydrodynamics and the Civil Engineering Systems Laboratory).

ABSTRACT

A quasi-optimal technique ('quasi' in that the technique discards unreasonable optimums), realized by a dynamically evolving mixed integer program, is used to develop regional electric power maintenance and production schedules for a two to five year planning horizon. This sophisticated, yet computationally feasible, method is used to develop the bulk dispatch schedules required to meet electric power demands at a given reliability level while controlling the associated dollar costs and environmental impacts.

The electric power system considered is a power exchange pool of closely coupled generation facilities supplying a region approximately the size of New England. Associated with a tradeoff between a given cost of production and the relevant ecological factors, an optimum production schedule is formulated which considers fossil, nuclear, hydroelectric, gas turbine and pumped storage generation facilities; power demands, reliabilities, maintenance and nuclear refueling requisites; labor coordination, geographic considerations, as well as various contracts such as interregional power exchanges, interruptible loads, gas contracts and nuclear refueling contracts.

A prerequisite of the model was that it be flexible enough for use in the evaluation of the optimum system performance associated with hypothesized expansion patterns. Another requirement was that the effects of changed scheduling factors could be predicted, and if necessary corrected with a minimum computational effort.

A discussion of other possible optimization techniques is included, however, this study was primarily intended as a development of a static procedure; a dynamic technique counterpart with a more probabilistic approach is being undertaken as a Part II of this study and at its conclusion the two techniques will be compared. Although the inputs are precisely defined, this paper does not deal explicitly with any of the fabrications of the required inputs to the model. Rather, it is meant as a method of incorporating those inputs into the optimum operation schedule process.

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1. Introduction

A great problem to develop from this industrial era is the dilemma between the increasing demands for energy and the increasing demands that environmental qualities not be degraded. As the electric power industry assumes an ever increasing commitment to resolve the energy supply problem it is subjected to escalating societal pressures to:

(1) generate reliably a sufficient amount of electricity to meet any demands,

(2) retain or decrease its price rates, and

(3) minimize the impact of its generation efforts upon the ecosphere.

The solution to this problem will take a long and unremitting effort from all sectors of society. In the long-term (30 years) program of action must be included, among many other things, efforts to develop more efficient means of power generation and more efficient power utilization.[2] There can be no doubt that to reverse the trend of environmental deterioration a tremendous technological effort will be required.

There is, however, another aspect of the solution to the 'electric power-environment' dilemma which should be closely coordinated with (and is definitely not meant to be a replacement for) the technological advances, but is essentially a separate effort. This is the development of methods

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2. A detailed documentation of the course of action required from technological improvements is contained in a report by Philip Sporn, reference (1).

to assure the best possible operation of an imperfect power generation system. That is, until facilities which are perfectly compatible with the ecosystem are producing all of our power there must be a method for insuring that the imperfect plants are utilized in the least damaging manner. This effort breaks essentially into two segments. First, the plants must be sited to take the best advantage of the site options available.[3] Secondly, the operation of existing systems must be directed toward those objectives enumerated in the beginning of this section.

This optimum operation of existing systems is the overall project being undertaken in the author's Ph.D. thesis, of which this study is one portion.

1.1 Problem

For a more thorough description of the part this research effort will assume in the overall study of 'optimum operation of existing systems' the reader is directed to reference (4). However, a basic understanding of the interconnections involved can be gotten from figure 1.1 and the descriptive outline in table 1.1.

Briefly, the problem undertaken in this study is the development of a scheduling and/or simulation tool which prepares, out to an indefinitely far horizon, weekly production

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3. This is a problem receiving a great deal of research effort, see for example reference (2). The author's particular project is also to be used as a simulation technique for the evaluation of specifically hypothesized expansion alternatives, as explained in reference (3).

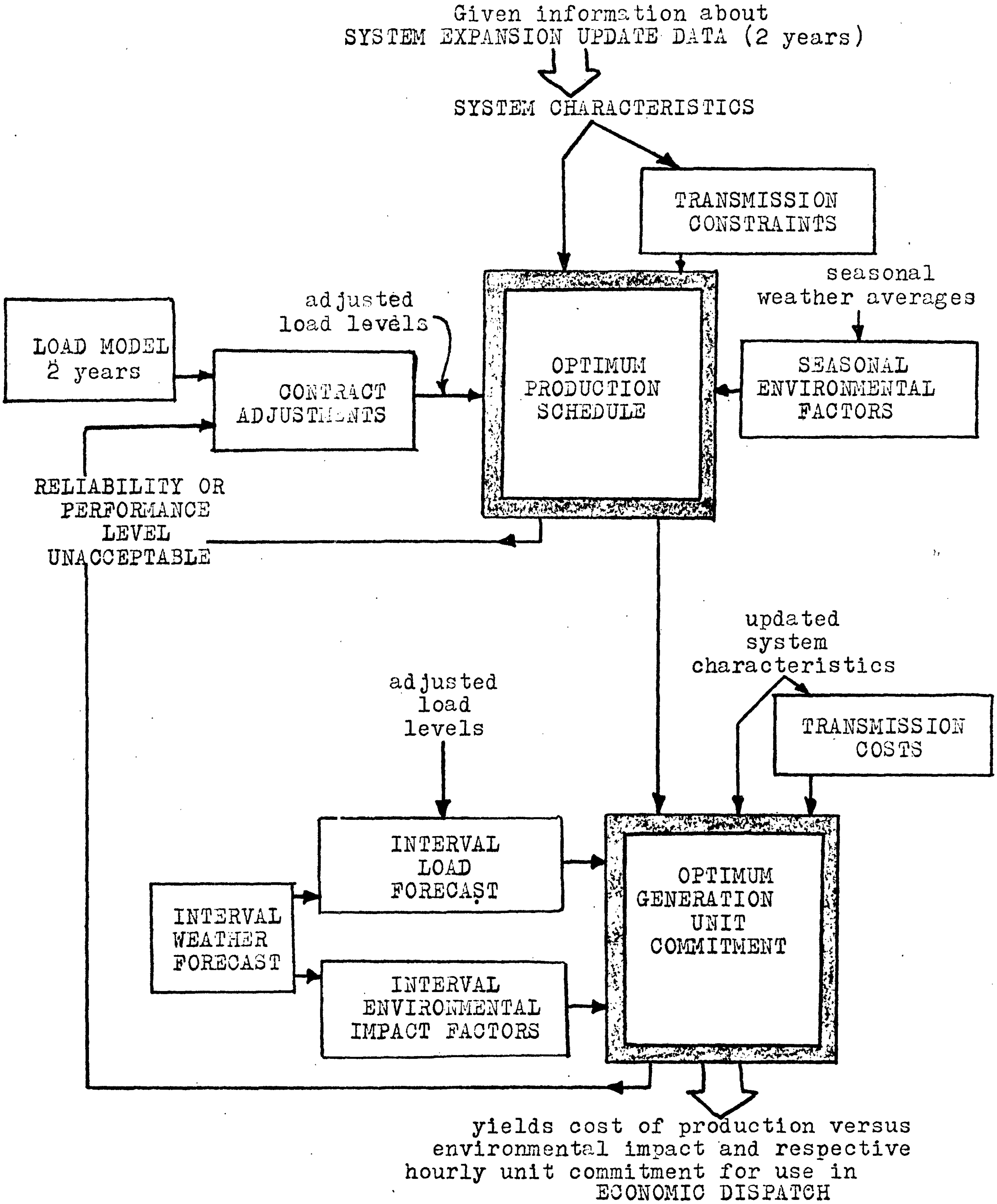

Figure 1.1 Simplified Representation of the Method of Problem Solution in Block Diagram Form

Table 1.1 Description of Model Components

System Expansion Update Data
1) New generation facilities and their characteristics
2) New transmission lines
3) Current inadvertent system changes

System Characteristics
1) Generation
   a) Types and location
   b) Cost per megawatt curves
   c) Emissions performance information for different fuel qualities
   d) Forced outage rates (probably with respect to time of last maintenance and maturation of plant)
   e) Maintenance and refueling schedules and current status within those schedules
2) Transmission structure
   a) Power transfer limits per line
   b) Losses
   c) Outage probabilities

Transmission Constraints (might be ignored)
1) Loss of transmission probability due to excessive distances
2) Cost of transmission
3) Line limitations

Load Model (2 years)
1) Geographic load pattern
2) Load demand probabilities

Contract Adjustments
1) Interregional inflexible power purchase and power sale contracts
2) Semifirm exchange contracts
3) Outside support probabilities
4) Interruptible loads

Seasonal Environmental Factors
1) Seasonal probabilities for extent of air pollutant accumulation and impact
2) Seasonal variations in thermal pollution effects

Optimum Production Schedule
1) Produces week to week schedule over two year period
2) Levelizes loss of load (or loss of energy) probability for environmental impact versus production cost levels
3) Checks acceptability of best reliability level

Reliability or Performance Level Unacceptable
1) Changes contracts if reliability too low or unreasonably high
2) Changes contracts if cost much lower than some purchase prices or higher than power sales prices
3) Changes contracts if optimum environmental impact represents improper degradation or overstress

Transmission Costs
1) Modelled as power transfer limits with appropriate losses, OR
2) Aggregated DC load flow network solution

Interval Weather Forecast
1) Water temperature predictions and meteorological condition forecasts at thermal pollution sites
2) Forecasts of atmospheric conditions at air pollution locations
3) Temperature prediction at load demand areas

Interval Load Forecast
(a program exists for using temperature predictions, past data, and estimates of predictable anomalies to forecast loads)

Interval Environmental Impact Factors
1) Impact of thermal pollution levels on the aquasphere
2) Assessment of air pollution impacts given weather predictions

Optimum Generation Unit Commitment
1) Uses weekly production schedule and other inputs to make hourly unit commitments
2) For a specific reliability level produces dollar cost versus environmental impact possibilities
3) For a specified reliability level that is unattainable it returns to change contracts and/or maintenance scheduling

schedules for a regional electric power pool. These schedules are to be schemes which optimize the multiple-objective function including reliability, dollar and environmental considerations. "Optimize" is actually not a correct choice of words in that schedules which may perhaps be the exact optimum may in fact be very undesirable. For example, the mathematical optimum might depend for its slight edge over other schedules upon some very tenuous, unwaverable procedure over a long span of time. Thus, the need developes for the use of the term 'quasi-optimal,' that is, 'in-a-sense optimal,' for, what is really sought is a reasonable schedule (or simulation), respecting the vagaries of the future by offering a number of alternative schemes from each point.

One final consideration must be mentioned. Due to the number of ever changing factors which affect the production schedule it would be very desirable to have a scheduling scheme which would be minimally disrupted by changes of the input factors. To achieve minimal disruption it would be necessary to decide <u>without computational efforts</u>:

(1) which future changing factors will be outside of the concern of the current schedule, and what point in the future they must be included,

(2) which factors will cause only slight schedule variations, and which scheduling decisions and parameters are most sensitive to these changes, and

(3) which future factors will require recomputation

of the schedule, and at what point in time must that recomputation start, and if possible stop,[4] to insure the total inclusion of the changing factor's sphere of influence.

This then is a short encapsulation of all the demands which are made upon an ideal production schedule, and thus, represent the goal for this particular research effort.

1.2 <u>Historical Approaches</u>

With the operation and maintenance costs accounting for between 5 and 10% of the utility's expenditures,[5] the economic advantages of optimum production scheduling have long been recognized. Methods for the effective coordination of reserve requirements, forced outage probabilities and the millions of dollars worth of maintenance have been steadily increasing in complexity.

Very early scheduling efforts, when only a few power plants were considered, consisted of plotting the amount of capacity which could be spared to maintenance and then iteratively scheduling the largest facility in the largest space available. The technique worked well for small systems, using a minimum amount of clerical help, and had the advantage of more or less assuring that the largest facility would

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4. In generating a new schedule due to changing factors it would be desirable to be able to determine at what point in the future (if a point exists) the scheduling process has settled back to the pattern of the old schedule so computation can be stopped.

5. See, for example, reference (5).

not be squeezed out of its slot by small changes in demand. But, there is no economic consideration in this technique, that is to say, leveling the oversupply is not necessarily consistent with any system performance measure except possibly maximum system reliability. And even at leveling the over-supply, this scheduling technique is not necessarily the optimum procedure.[6]

During the World War II hyperintensive energy using period new problems in the maintenance and production scheduling became evident, as explained in a 1942 Electrical World article[7] by Philip Sporn:

> "The object of any program of co-ordination of major unit outage is to maintain the maximum margin feasible between demand on a system and load capability of the various plants serving the system. For an individual system this means careful study and evaluation of the shapes of the annual load and capability curves. The latter involves taking into account not only seasonal variations in hydro capability but seasonal variations in steam-plant capability. However, in wartime, with rapidly growing loads, three other factors have to be taken into consideration. These are the rate of growth of new load, because such growth can overbalance the seasonal trend factor; the rate of bringing in new capacity on the system; and the broad integrated, regional-area picture."

Since World War II, and in fact in general, the hourly unit commitment within a week horizon time has received a great deal more of the research effort than has the annual

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6. Consider, for a trivial example of the non-optimality of this procedure, the very simple system with plants of capacity 4, 3, and 2 to be fit into slots of 5 and 4. This algorithm would place the largest facility, 4, in the largest slot, 5, and would thus fail.

7. Excerpted from reference (6).

maintenance and production schedule. Although many of the problems with which the unit commitment must contend are pertinent only to the hourly schedule, e.g. cost of cold startup, minimum shutdown times of plants, nonlinear loading costs i.e. fuel costs and incremental fuel rates, parameters relating unit restart costs to down times, transmission costs, etc., it is still instructive to consider the different methods of attacking this scheduling problem.

One of the most common dynamic unit commitment scheduling methods has been an extension of the incremental costs used in minute to minute economic dispatch.[8] Other dynamic solution approaches, such as dynamic programming, work well[9] until a large number of plants must be considered. Dynamic approaches with probabilistic load meeting requirements have also been considered.[10] A limited amount of research in the use of the maximum principle is available in print, and, at least for the economic operation of hydroelectric plants seems to enjoy the advantage of greater accuracy than is available with dynamic programming.[11]

Static techniques also have been developed, with varying success, for solving the unit commitment problem. Over a daily interval, the problem of using an interruptible gas

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8. See references (7) and (8)

9. This opinion is contained in reference (9)

10. See reference (10)

11. Refer to either reference (11) or reference (12).

supply has been considered.[12] Integer programming[13] and mixed integer programming[14] have been attempted for the solution to this problem, but because of the dynamic programming nature required to consider probabilistic demand curves and the more or less continuous nature of many of the variables, these techniques fall prey[15] to the same dimensionality and magnitude problems that plague the dynamic programming techniques. Another static technique that has been tried is the gradient search,[16] but this approach does not appear to be promising for use over long time spans with large systems, that is, in the maintenance problem.

Fewer research attempts have been directed toward resolving the problems which arise when preparing the annual maintenance and production schedules.

From the dynamic point of view a technique which may prove promising during the investigations of Part II of this study is a dynamic programming successive approximations technique[17] which might be successful using crew by crew evaluations. Some work has already been done in the area of water reservoir planning using this successive approximations

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12. See reference (13)

13. This application was done in reference (14)

14. See reference (15)

15. See reference (16), page 321 for an authority for, and explanation of this opinion.

16. See reference (17)

17. This technique is explained in reference (18).

technique.[18] Other dynamic programming work has been done[19] including an application which uses a probabilistic approach to the long term expansion problem.[20]

Few static solution techniques have been applied to the annual maintenance and production scheduling problem. One notable exception uses a branch and bound search for one maintenance crew at a time, starting with the crew responsible for the most capacity.[21] This type of search, however, leaves no room for any continuously varying (or economic) considerations, and can be an exhausting, non-optimal search for a large system.

The need for a viable scheduling technique has, thus, been growing steadily.[22] The automated scheduling techniques available today are not good enough to make their usage popular and the problem has become so complex that what developes, as one regional exchange staff officer has told me, is a "horror show."

To demonstrate how little this field has progressed, consider what is done today by the regional power pool NEPEX, New England Power Exchange. They have been a pioneer in the use of sophisticated computation equipment for the purpose

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18. Both in reference (19) and reference (20).

19. See reference (21).

20. See reference (22).

21. This paper was first presented in reference (16).

22. Reference (23) in 1970 outlined the need for a good scheduling algorithm, using a static or dynamic technique, whichever would resolve the problem.

of system operation,[23] and they are responsible for, among other things, the coordination of the maintenance of 25 hydroelectric plants, and some 150 fossil and nuclear fueled generating stations. So, in this case, both the computational ability and the need exist for a viable scheduling technique. However, their maintenance schedule comes from staff members sitting in monthly, sometimes weekly, meetings studying forms such as shown in Appendix A, which they have received from the superintendents of production in charge of the individual plants.

Currently what is needed is a scheduling (and simulation) technique which considers cycling and base loaded potentials, and can give highly refined, but reasonable, figures such as precise end of the week height requirements for reservoirs and nuclear batch allotments, as well as definitive yes or no decisions for various problems such as different types of maintenance options or interregional power exchange contracts.

This unsolved problem is further complicated by the pressing environmental issues. A. H. Aymond, head of the Edison Electric Institute has pointed out that "the days are gone when a utilityman could sit confident that power is an undebatable blessing, accepted without argument or discussion by the people."[24] Thus, where simple maintenance and production scheduling techniques have previously existed

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23. See reference (24).

24. Excerpted from reference (25), page 52.

avoiding even economic performance measures,[25] what is required now is a sophisticated technique which includes both economic and environmental performance measures.

1.3 Results

The results of this research project include:

(1) a modelling of all the components of the scheduling problem,

(2) a solution technique which reaches the desired quasi-optimal schedule and requires minimum readjustment for changed input factors, and

(3) a computer program realization of the solution technique, with a sample problem for the comparison of the quasi-optimum technique to the optimum.

1.3.1 Model Description

The model for the production scheduling problem is set in a linear framework. Although this format is somewhat constricting upon some of the nonlinear scheduling factors, for the most part the nonlinearities approach linear functions before the scheduling decisions are made.

The forecasted demand to be met by the schedule is assumed known, and the necessary reserve requirements are included in the demand which must be met. Adjustments to the demand-to-be-met curve are made for fixed and flexible interregional

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25. Leveling the oversupply beyond reserve requirements can not be considered an economic technique. A linear programming production cost method has been developed, see reference (26), but it is not a scheduling device. Other non-scheduling, but economic, simulations are in refs. (27) and (28).

power exchange contracts, probabilistic emergency support and interruptible loads. The solution technique makes decisions about which contracts to honor, and extent to which variable contracts should be subscribed, as well as indications of when oversupplies of power are available for bulk interregional sale possibilities. Contract possibilities are enumerated even at times when the region has no oversupply of power, with the final schedule yielding a list of all the intervals and the cost of producing more power in those intervals. Also, the cost of meeting extra unexpected demands is produced for each interval, pointing out the times when it might be prudent to overestimate the reserve requirements. The cycling capabilities of the system using the schedule are assured to cover the cycling demands of the load.

The capacities of the generating system in the model are time varying to account for the seasonal variations in output capabilities. The most expensive capacity of the system is shut down over portions of the weeks when it is not needed and it is not economical or possible to sell power to neighboring exchanges. Capacities of the plants are derated to the extent that they incur forced outages, and provisions are made in the model for the further derating and further expenses involved in pushing a plant to its maintenance time limit. Variable extensions of the plant outputs beyond the 'nameplate' capacity are modelled along with the extra costs they produce, both dollar and environmental.

Maintenance decisions are made based on the total system performance. The performance is a function both of the options of maintenance (i.e. longer or shorter sessions) and of the time within the maintenance 'window' over which the sessions for maintenance are scheduled. Constraints are presented which allow for the coordination of the maintenance from one portion of one window to an appropriate portion of the next. The system is appropriately rewarded for leaving the plants in good repair, that is, rewarded according to the position in time beyond the horizon time that the next maintenance window falls. Coordination of the maintenance crews, equipment usage, or individual utility requirements are also modelled.

Geographic constraints, viz. 'must run' plants or minimum capacity requirements within a sector, as well as a certain amount of transmission limitation can also be modelled.

Capacity limitations over time spans are considered for gas contracts; hydroelectric and pumped storage facilities according to the river, pumped input, and reservoir storage capabilities; and the management of the production of nuclear power so that the optimum batch depletion is realized at the time when the schedule plans for refueling.

The time intervals vary in size over the span of time covered by the simulation. As less information is known about the future, for example maintenance windows are larger farther in the future, this changing size interval insures that equal weightings are attached to equal amounts of information. This scheme is also an attempt to reduce the number of variables.

The quality measure of the simulation is measured in both dollar costs and ecological impact units (e.i.u.), and the use of the presented solution techniques results in the determination of all possible optimum pairings of $ to e.i.u. ranging from the minimum cost end to the minimum possible ecological impact for a given reliability level.

1.3.2 Method of Solution

The method for the solution of the proposed model is a dynamically evolving decision process which uses mixed integer programming to make current decisions and linear programs to keep the future system within its restrictions (but not forcing decisions for the future system). This is then a quasi-optimal sequential process which requires operator participation at each iteration (about two months covered per iteration).

A decision field is defined which includes all decisions within a time span (about two months) as well as those outside the span which are directly or importantly coupled to the current decision-making process. Those firmly determined decisions within one field are fixed, and the process passes to the next field (which overlaps the previous field slightly in time).

When used as a scheduling tool it is only necessary to proceed far enough in the sequence to fix the current decisions, usually only two or three iterations. As a simulation tool, the model must be iterated over the entire time span in question, but has the advantage of computation time required being linearly (not exponentially) dependent upon the span of time considered.

Recomputation of a schedule due to changing factors requires a minimal computational effort. The dual solution to both the mixed integer and linear programs presents a sensitivity measure of the decisions to various changing input parameters (such as changes in forecasted demands, river levels, or new or bought capacities becoming unavailable). When it is determined that a recomputation is required, the solution to the decision fields previous to the disturbance can be salvaged intact, and if it happens that the perturbation has a short-lived effect, the old solution can be reclaimed for some of the future decision fields.

A solution to a small (equivalent to scheduling 16 power plant maintenance sessions per year) sample problem is presented, primarily to test the validity of the quasi-optimal technique. The problem was taken to be relatively small so the total overall optimum could be computed for comparative reasons. Even in the worst case, where no intelligent human participation (using dual sensitivities) was used, i.e. strictly a mechanical algorithm, the quasi-optimal technique presented the best three overall schedules. Only three schedules out of the top eight computed by the overall optimization, were missing in the quasi-optimal technique, and these resulted from the algorithm firming a very closely contested decision in the next to last decision field (the other alternative decision accounting for the other three schedules). This 'jittery' decision, however, would have been carried into the next field if any operator participation had been used.

A program is also presented (with a trivial example) which parameterizes the performance quality to determine the full range of different optimum dollar-to-environmental pairings, varying from minimum dollar costs to minimum ecological impact.

1.3.3 Computational Feasibility

Because this problem has been set up in a form for which the integer decisions are all bivalent, the computer time, and thus costs, are small. Besides the fact that with the pseudo-Boolean constraints all integer solutions are on the corners (the linear programming simplex method seeks out only corners) of the space of feasible solutions, the problem setup has a distinct mutual exclusivity, i.e. 'multiple choice,' characteristic which decreases to a small fraction the time required per integer decision. At the MIT Information Processing Center an IBM 370-155 was used with 258K byte memory to solve a decision field involving 46 simultaneous decisions (with a two month decision field and an average of 2½ decisions per plant this is equivalent to 108 generating facilities). The execution time for this job was 37 seconds, with a total cost from card reading to handling of $11.63.

Almost every computation facility has available the linear and mixed integer functions used in the solution technique presented in this project.[26] If, however, the facility to be used does not have sufficient capability there are a number of simplifications in the form of approximations which can be made. For example, the decision field could be cut in size (although not substantially).[27] Nuclear, hydro or gas usage limitations could be dropped, in fact, the maintenance schedule alone could be considered (with no production considerations). Until available, of course, environmental costs must either be eliminated or approximated. However, even with <u>no</u> costs whatsoever included and only yes or no maintenance decisions, this solution technique is better than any presently available.[28]

1.4 Presuppositions

The greatest assumption of this problem is the assumed linearity of the problem form. Any nonlinearities which might have been included would not have had a substantial effect upon the current decision process (and since there are no decisions made in the far future it is felt that dropping the nonlinearities has not substantially affected the validity

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26. It would not be impossible to create a fairly good schedule without the mixed integer subroutine, i.e. with the linear and dual solutions alone.

27. Cycling capability requirements could also be dropped.

28. Reference (16), published in the I.E.E.E. Transactions, PAS-91 of January 1972, considers a problem of this particular type, but has no performance measure beyond leveling oversupply and uses a crew by crew branch and bound search and thus does not have the advantage of the fast initial linear programming optimal continuous solution. This solution greatly reduces the number of decisions which must be considered.

of the model). Exceptions, such as the synergistic ecological effects of operating two plants in close proximity, can be dealt with to a certain extent by overestimating the costs of each plant operating alone, and preserving the linear pattern. In general, the solution of nonlinear problems with the dimensionality considered here, are either not computationally feasible or are prohibitively time consuming procedures. One nonlinear possibility, however, for future considerations in this research area, would involve a linear problem setup with a nonlinear objective function.[29]

In the problem modelling process there have been many assumptions and approximations. For example, the reserve requirement is assumed to be a function of the load and not of the plants in use at that particular time (which would have caused a nonlinearity). Similar linearity assumptions are explained throughout Chapter 2 as they are introduced into the model.

There is in this project no attempt to level the oversupply of power, that is, above and beyond the demand plus reserve requirements. If the reserve is not felt to be adequate it can be pushed up (until it is at a level where there is no feasible schedule in which case the ϵ-optimal solution is found), and in this way any particular desire

29. It is highly unlikely that attempts at problems which are either not quadratic or are inseparable would be fruitful. The most likely candidates for nonlinear objective functions would be those which were convex in nature, although even convex functions are fairly time consuming for linear programs to handle, let alone mixed integer programs.

for leveling the oversupply can be met. Any intervals for which there is particular concern can be granted extra added reserve allotments.

Forced outages have been averaged in as percentage plant capacity deratings, instead of being treated probabilistically.

No attempt has been made to refine the time intervals down beyond one week. Further refinements are possible, though, within the framework of the model.

There is a slight loss in accuracy involved in meeting the optimum nuclear batch sizes. This approximation almost disappears, though, if only a single nuclear refueling window is considered, and does disappear entirely once the positioning of the refueling within that window is fixed. This approximation is also done away with if the production of the nuclear plant can be predicted for the intervals within the window (for example, if the plant is always base loaded). Gas contracts, and hydroelectric productions are treated in manners similar to the nuclear productions, and are thus also subject to slight approximations. The difficulty which necessitates these approximations is caused by the variable production schedules which must meet a variable deadline. These are then coupled variables which must be carefully approximated to preserve the linear framework. The treatment which partially resolves this problem is the fractional addition or subtraction of production quantities to the intervals before the deadline contingent upon earlier or later variations in that deadline.

Further studies have been undertaken by the author so as to refine this particular research project. These studies include a more probabilistically oriented technique using a more dynamic framework, and they include a clarification and further definition of the precise role played by the dual space so as to hopefully allow its inclusion in the rigid, mechanical algorithm.

2. Model

In formulating the model for this scheduling problem it is not possible, and in fact not as instructive, to remain completely impartial to the theoretical and computational feasibilities of the various setup's solutions. The fact that abstract formulations do shed light upon the variety of possible solution techniques is granted, and for this reason is discussed in section 3.1.1. However, when aiming at a clear portrayal of the problem, it is best wherever possible to deal with physical or visualizable quantities. Inevitably implied in such a detailed problem formulation is a solution technique. And that this problem setup seems conducive to a dynamically evolving mixed integer program should not be viewed as a contrival intended to make this seem like the 'obvious' technique, but should be considered a foresight to the results of the survey of possible optimization methods.

2.1 System Requirements

A logical first step in the formulation of a system model is a detailed study of the requirements imposed upon that system from external sources. For this problem, these exogenous demands are in the form of minimum constraints upon the output, such as meeting all requests for energy with good quality (i.e. constant voltage), reliable electricity, and in the form of a minimization of the inputs, that is payments from customers and usage of the environment.

By incorporating within the system, endogenously, the predicted demand levels and the fixed reliability requirements, it is possible to measure the 'performance' of the system in terms of its decision making alternatives alone. Section 2.5 on performance levels deals with the collection and weighting of the various input terms, and the remainder of this section deals with the endogenous incorporation of the 'output' demands.

2.1.1 Power Demands

The term 'power demand' is not a precise term, and thus it is important to define its meaning for the purposes of this study. In actuality, power demand is a stochastic process through time, and represents the sum of all possible power requests made upon the system from outside and from within its franchise area. It is useful, however, to limit this term to encompass only those demands the power pool is definitely obligated to supply. All contracts between regions which are not binding,[30] and any interruptible loads are therefore not included. The refinement required of the 'power demand' to meet these inclusions is outlined in section 2.4.2.

Ordinarily, only the projected future demand for power is of interest in the scheduling problem. Thus, by 'power demand' will be meant the most general definition, where $P_d(t)$ is the collection of forecasted power demands and their associated probabilities of not being exceeded by the actual demand at each future point in time, t.

The reason for leaving the power demand as a pointwise

30. This usually includes all interregional contracts.

probabilistic model will become apparent when other distribution functions are introduced, viz. the flexible interregional contracts which are uncertain at any future time can take the form of a probability distribution function.

In any event, this probabilistic $P_d(t)$ is the 'real' demand; any attempts to average, i.e. Ex(P(t)), use estimates of high reliability, i.e. for example 99% certainties that demands will be less than $P_{99}(t)$, or dividing $P_d(t)$ into discrete sections with probabilities, are all artificial methods of treating the forecasted distribution.

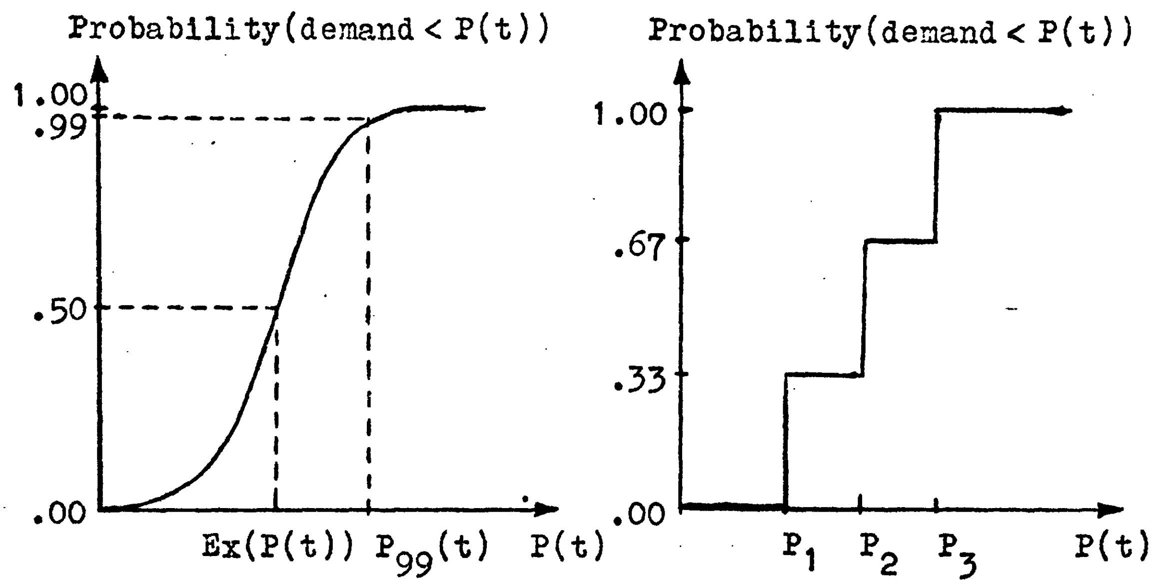

Figure 2.1.1 Forecasted continuous and discrete probability distribution functions of power demanded.

The means are available for calculating the probabilistic models of $P_d(t)$ at various times in the future and much work has been done in this area. This forecasting is not within the scope of this study, and is thus considered an input. Generally, the components of the long term load model will include a constant growth factor (about 8% a year), seasonal adjustments and predictable special effects which would perturb the long range demand figures (such as world fairs, industrial strikes, very large conventions or celebrations, etc.).

The load model may have to be areally discretized, according to the requirements of the transmission or geographic constraints.

2.1.2 Reliability and Reserve Requirements

Reliability[31] and reserve[32] requirements are measures set by the utilities so as to meet the demanded load with a uniformly acceptable degree of certainty. These requirements are necessitated by the uncertainties involved in load forecasting (covered by the reliability requisites) and the inadvertent failure of the system components, chiefly the generation equipment (necessitating spinning reserve requirements).

These requirements expand the load level a utility plans to meet. There are numerous methods of computing these load requirements, but in the maintenance schedule they can usually be predetermined from load forecasts alone. In some cases the reserve requirements are included using an additive term equal to between 1 and 1½ times the largest generator's

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31. Reliability calculations can be found in references (29) and (30).

32. For a more detailed explanation of the computation of reserve requirements consult reference (31).

production.[33] However, in this study the system reliability will be satisfied by a pre-forecasted demand-to-be-met level computed either from an acceptable level of certainty in covering the demand, or in a percentage increase in the expected load forecast (typically 5% to 15%). The spinning reserve requirement will be met by systematically derating the capacities of the generation facilities (between 5% and 20% depending upon type and maturity of plant).

2.2 System Capabilities

From section 2.4.2 can be obtained a number of megawatts $P(k)$ which represents the power level in the k^{th} interval which must be supplied by the system in order to realize the prespecified reliability level (thus $P(k)$ includes reserve requirements).

If $A_i(k)$ represents the capacity of the i^{th} plant in the k^{th} interval after it has been derated to average in the effects of its forced outage rate, and if

$$u_i(k) = \begin{cases} 1 & \text{if the plant } i \text{ is scheduled for maintenance in interval } k \\ 0 & \text{if plant } i \text{ is operating during the } k^{th} \text{ interval} \end{cases} \qquad 22\text{-}1$$

then for the operative system capacity in the k^{th} interval to at least meet the demand level

$$\sum_{\text{all } i} \left(A_i(k) \left[1-u_i(k) \right] \right) \geqslant P(k) \quad \text{for each } k \qquad 22\text{-}2$$

Alternatively, if $S(k)$ is the total updated system capacity derated to account for outages, that is

33. Strictly enforced this condition would introduce a nonlinearity.

$$S(k) = \sum_{\text{all } i} A_i(k) \qquad \text{22-3}$$

then the amount of generation capacity available for maintenance scheduling, M(k) is

$$S(k) - P(k) = M(k) \qquad \text{for each } k \quad \text{22-4}$$

So, equivalent to equation 22-2 is the equation

$$\sum_{\text{all } i} A_i(k)\, u_i(k) \leq M(k) \qquad \text{for each } k \quad \text{22-5}$$

A further very important constraint which must be imposed upon feasible schedules is the flexibility of the generation with respect to meeting load variations, or cycling, within the time periods. Denoting some plants as cycling, or 'lightly-loaded,' and making a minimum megawatt requirement of 'lightly-loaded' equipment is one treatment this problem has been given (in long range simulations). A more plausible method, which has not appeared in the literature, would include a rating of each plant according to its cycling capabilities, $c_{\%,i}(k)$. Thus, a gas turbine might be rated $c_{\%,i}(k) = .90$ and a strictly base loaded plant as $c_{\%,i}(k) = .05$. So if $C_{\%}(k)$ percent of the demand P(k) is of a cycling nature, then

$$\sum_i c_{\%,i}(k)\, A_i(k) - \sum_i c_{\%,i}(k)\, A_i(k)\, u_i(k) \geq C_{\%}(k)\, P(k) \qquad \text{22-6}$$

That is to say, the total cycling capabilities of the system less the amount of cycling capacity out for maintenance must exceed or equal the cycling expectations of the demanded load.[34]

34. Cycling expectations in the load can be estimated from the available peak-to-average ratios forecasted by utilities.

If now instead $C_{\%}(k)\ M(k)$ denotes the amount of cycling capacity which can be maintained in interval k (which can be computed from equation 22-6), then the generation flexibility requirement becomes

$$\sum_{i} c_{\%,i}(k)\ A_i(k)\ u_i(k) \leq C_{\%}(k)\ M(k) \quad \text{for all } k \qquad 22\text{-}7$$

Many special cases must be considered when determining both total system capacity and cycling capabilities. When additional variables are essential, care must be taken to preserve the linearity of the problem. Consider, for example, the overload capabilities of some steam units. As much as 50% additional capacity can be extracted over the nominal rating, but at extra cost. Outside maintenance windows, for this type of plant, the additional capacity may be viewed as a new variable power source, $0 \leq v_{x,j}(k) \leq 1$ percent utilized, with the appropriate extra cost added to the system performance.[35]

$$A_{x,j}(k)\ v_{x,j}(k) = \text{extra capacity from overloads} \qquad 22\text{-}8$$

where $A_{x,j}(k)$ is the maximum additional capacity available beyond nominal. Within maintenance windows, this extra capacity could only be included using an estimate of its usage and making that figure contingent upon the plant operation $1 - u_j(k)$.

2.2.1 Capacity Levels

Generally, maximum capacity ratings for the generating

35. The term $A_{x,j}(k)$ is assumed reduced by the appropriate forced outage derating term, which may be higher here due to the extra-strenuous operation mode.

units are time varying quantities and are to some extent dependent upon predictable quantities such as cooling water temperatures. It would be reasonable, however, to define the maximum i^{th} unit capacity $A_{max,i}(k)$ in the interval k using forecasted values for the pertinent variable factors. Then if it appears to be a worthwhile venture, a post-optimal sensitivity study could be performed with respect to the prominent variables, such as temperature.

Forced outage rates are not easily dealt with explicitly in long range scheduling, nonetheless, a satisfactory technique for their inclusion is known. This method involves a percentage derating, $d_{\%,i}(k)$, of $A_{max,i}(k)$ to accurately reflect that unit's average contribution abilities.[36] Successful methods have been developed enabling the computation of $d_{\%,i}(k)$ from forced outage probabilities and self-imposed loss-of-load-probability (LOLP) standards. This derating percentage will be time varying, chiefly dependent upon the maturation[37] of the facility and the time interval elapsed since the last performed maintenance.

Now $A_i(k)$ is defined in terms of predictable quantities, that is

$$A_i(k) = d_{\%,i}(k)\, A_{max,i}(k) \quad \text{for all } i,k \qquad 221\text{-}1$$

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36. Since the time a unit is forced out of the system is usually small compared to the one week to one month discretized time unit this averaging technique offers a good approximation. More exact approaches to this problem, if the extra complexity involved seems worthwhile, might attempt approaches such as the type presented in references (32) or (33).

37. Reference (34) contains derating versus maturation figures.

As one might imagine, delaying the planned maintenance of a facility within its window will increase the percentage derating $d_{\%,i}(k)$ of the plant in the prece ding window intervals. A method for recognizing this aspect of the capacity problem (which may only be needed in far future intervals) is the use of an estimated additional derated capacity for the i^{th} plant, $A_{d,i,m}(k)$, contingent upon maintenance delayed to the m^{th} interval. Now we may subtract from the total capacity at time k a term $A_{d,i,m}(k)\ u_i(m)$ whenever the increase in forced outages necessitates this capacity derating.

It may also be necessary to derate the capacity of a facility due to the maintenance of support equipment. This term can be handled directly in the $A_i(k)$ term if the support maintenance interval is known. If unknown, then a binary variable $u_{s,j}(k)$ is needed, and the derated capacity subtracted from the k^{th} interval contingent upon $u_{s,j}(k) = 1$. It seems unlikely that the support maintenance window and the total plant maintenance window would intersect and there would be no decision to force them to coincide, however, in this case care must be taken not to derate a plant to a negative capacity.

2.2.1.1 <u>Fossil Fueled Units</u>

The output capabilities of most fossil fueled plants are limited by their derated capacity $A_i(k)$ and by the constraining maintenance requisites. Maintenance requirements for a given plant i typically take the form of options such as:

(1) two weeks of maintenance may be started anytime between 44 and 60 weeks from the present

or (2) four weeks of maintenance may be started anytime between 40 and 56 weeks from the present.

The advantages of option (2) may be realized in fewer future forced outages, possibly a longer interval until the next maintenance outage, preclusion of a necessary four week session at some subsequent maintenance outage, or something as intangible as increased plant longevity.

If the basic unit of discretized time is two weeks and the present is represented as k = 0, then the options take the form:

$$\sum_{k=22}^{30} u_i^1(k) + \sum_{k=20}^{28} u_i^2(k) = 1 \qquad 2211\text{-}1$$

where u^2 represents a maintenance session spanning two intervals. The contribution of any plant to the capacities down for maintenance in interval k is then

$$A_i(k)\ u_i^1(k) + A_i(k)\ u_i^2(k) + A_i(k)\ u_i^2(k-1) \ldots \qquad 2211\text{-}2$$

plus other terms if for example maintenance options longer than four weeks are involved.

For gas turbine generators the problem of determining capacity capabilities is further restricted by the possible gas contracts. For example, a contract may stipulate that the utility is bound to purchase a certain quantity of gas at a fixed price over a fixed interval, with options to purchase more up to a limit at an additional charge per unit. If this contract covers all gas units, say all $j \in G$, where G is the set of gas units, and the contract covers the time interval $k \in [t_1, t_2]$, then this contract amounts to a capability restriction

$$W_j(k) = E_j(k)\, g_j(k) \qquad 2211\text{-}3$$

$$0 \leq W_j(k) \leq A_j(k) \qquad 2211\text{-}4$$

$$0 \leq \sum_{k=t_1}^{t_2} \sum_{j \in G} g_j(k) \leq L \qquad 2211\text{-}5$$

where $W_j(k)$ is the actual output, $E_j(k)$ represents the conversion efficiency, and $g_j(k)$ is the amount of gas used by plant j in interval k, and L represents the maximum amount of gas usage allowed by the contract.[38]

It will be mentioned here, and not again in the nuclear production or hydroelectric production sections, that a simplified approximation can be made for production limitations. This simplification results from setting estimated limitations for production over some smaller portion of the overall production time span. For example, the hydroelectric production capability could be constrained to stay within some limit (predicted from river flows) over the smaller period of time, say two months.

2.2.1.2 <u>Nuclear Energy Relegation</u>

Assuming a refueling scheme has been established,[39]

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38. By making equation 2211-5 an inequality it is assumed that gas which must be purchased could be wasted; the associated cost equation will make this eventuality unlikely. A linearity assumption here is made: we must either know the maintenance plans of these plants over this interval, or we must predict with fair accuracy their gas usage in time intervals during which they might be withdrawn.

39. A method for producing an optimum refueling policy can be found in reference (35). General considerations for fitting nuclear generation into systems is discussed in reference (36).

the questions of interest in the production scheduling problem for systems including reactors are when to perform the refueling (and the coincidental maintenance) and what quantity of the fixed fuel batch should be burnt up in each interval.

A fair assumption is that the exact refueling time should be fairly well known within eight months of the actual procedure. Thus, suppose hypothetically the j^{th} reactor is due for a refueling in the k^{th} interval and there is still left in the core N_j units of energy, that is, before the optimum amount will have been used. Then if $n_j(k)$ is the energy extracted from the fuel batch in the k^{th} interval and $A_j(k)$ is the derated plant capacity, $E_j(k)$ the conversion coefficient and $W_j(k)$ the wattage output then

$$\sum_{k=1}^{k-1} n_j(k) \approx N_j \qquad 2212\text{-}1$$

where

$$W_j(k) = E_j(k)\, n_j(k) \qquad 2212\text{-}2$$

$$0 \leq W_j(k) \leq A_j(k) \qquad 2212\text{-}3$$

It is also important to determine the optimum refueling time under circumstances where some leeway is available. The best way to introduce this variable, without including nonlinearities, appears to require an estimate of fuel usage in intervals around the expected refueling time. For example, if refueling takes one interval, and k-1 and k+1 are the possible refueling time alternatives to k, then with the definition of the following terms the output equations can be written. The expected fuel usage during the $k-1^{st}$ interval if the plant refuels on schedule at time k is $n_{e,j,k}(k-1)$.

And $n_{e,j,k+1}(k)$ is that fuel usage expected in k if refueling is done in k+1. Then equations 2212-1 and 2212-3 still hold, however

$$W_j(i) = E_j(i)\left\{n_j(i) + \frac{n_{e,j,k}(k-1)}{k-2}\,u_j(k-1) - \frac{n_{e,j,k+1}(k)}{k}\,u_j(k+1)\right\} \quad 1=i=k-2 \qquad 2212\text{-}4$$

and

$$W_j(k) = E_j(k)\left\{n_{e,j,k+1}(k)\;u_j(k+1)\right\} \qquad 2212\text{-}5$$

$$W_j(k-1) = E_j(k-1)\left\{n_j(k-1) - n_{e,j,k}(k-1)\;u_j(k-1) - \frac{n_{e,j,k+1}(k)}{k}\,u_j(k+1)\right\} \qquad 2212\text{-}6$$

$$\sum_{i=k-1}^{k+1} u_j(i) = 1 \qquad 2212\text{-}7$$

The intent of these modifications is to force each interval to accept slightly more power if refueling and maintenance are preformed at the sooner time k-1, or to detract from that amount of power in each interval if the batch must last until the delayed k+1 replenishment.[40] The equations for wider refueling windows are straightforward, however, calculations including all ramifications of the planning horizon covering more than one refueling window become more complex.

It is granted that this technique is an approximation, however, it is only meant to give an indication of the tendencies of the system as it strives toward an optimum. These tendencies once noticed can then be used to redetermine the 'expected' refueling time.

40. The assumption is made that the optimum output for any interval would not fall below this incremental adjustment.

If the basic time unit of the simulation at the time of the window is two weeks and the refueling takes four weeks obviously the refueling must be scheduled as an outage in two consecutive intervals. In the most general case, there will be N_{j1} energy until the first refueling at around interval f_{1e}, i.e. the first refueling window is $F_{j1} = [k \; ; \; f_{1f} \leq k \leq f_{1L}]$ with expected value f_{1e}, and N_{j2} energy will be available between the first refueling and the second window defined $F_{j2} = [k \; ; \; f_{2f} \leq k \leq f_{2L}]$ with expected time of f_{2e}. The linear[41] equations for determining the wattage output at each k are now (assuming refueling takes one interval of time[42]) :

$$\sum_{k \in F_{j1}} u_j(k) = 1 \qquad 2212\text{-}8$$

$$\sum_{k \in F_{j2}} u_j(k) = 1 \qquad 2212\text{-}9$$

$$\sum_{k=1}^{f_{1e}-1} n_j(k) \leq N_{j1} \qquad 2212\text{-}10$$

$$\sum_{k=f_{1e}+1}^{f_{2e}-1} n_j(k) \leq N_{j2} \qquad 2212\text{-}11$$

$$0 \leq W_j(k) \leq A_j(k) \qquad \text{for all } k \qquad 2212\text{-}12$$

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41. Note that there is no place within the equations 2212-8 to 2212-17 where unknowns $u_j(k)$ and $n_j(k)$ multiply each other.

42. This is likely to be the case, because with variations in refueling time only in the far future these will probably occur when the simulation interval time has stretched out to include five or six weeks (see section 2.6.1).

$$W_j(k) = E_j(k)\left\{ n_j(k) + \sum_{m=1}^{f_{e1}-f_{ef}} \left[\frac{\sum_{i=1}^{m} n_{e,j,fe1}(f_{e1}-1)}{f_{e1} - (m+1)} u_j(f_{e1}-m) \right] - \sum_{q=1}^{f_{eL}-f_{e1}} \left[\frac{\sum_{i=0}^{q-1} n_{e,j,fe1+q}(f_{e1}+i)}{f_{e1} - 1} u_j(f_{e1}+q) \right] \right\}$$

$$\text{for } 1 \leq k < f_{ef} \qquad \text{2212-13}$$

$$W_j(k) = E_j(k)\left\{ n_j(k) - n_{e,j,fe1}(k)\, u_j(k) + \sum_{m=1}^{f_{e1}-k} \left[\frac{\sum_{i=1}^{m} n_{e,j,fe1}(f_{e1}-1)}{f_{e1} - (m+1)} u_j(f_{e1}-m) \right] + \sum_{i=1}^{f_{e1}-(k-1)} n_{e,j,f_{e1}-i}(k)\, u_j(f_{e1}-i) - \sum_{q=1}^{f_{eL}-f_{e1}} \left[\frac{\sum_{i=0}^{q-1} n_{e,j,fe1+q}(f_{e1}+i)}{f_{e1} - 1} u_j(f_{e1}+q) \right] \right\}$$

$$\text{for } f_{ef} \leq k < f_{e1} \qquad \text{2212-14}$$

During the interval of expected nuclear refueling, the energy output becomes:

$$W_j(f_{e1}) = E_j(f_{e1}) \sum_{\substack{i=f_{ef} \\ i \neq f_{e1}}}^{f_{eL}} n_{e,j,i}(f_{e1})\, u_j(i) \qquad \text{2212-15}$$

The output equations for intervals beyond the expected refueling time are very similar, almost symmetric, to the preceding equations:

$$W_j(k) = E_j(k)\Bigg\{ n_j(k) - n_{e,j,f_{e1}}(k)\, u_j(k) - \sum_{q=1}^{f_{e1}-f_{ef}} \left[\frac{\sum_{i=0}^{q-1} n_{e,j,f_{e1}-q}(f_{e1}-i)}{f_{m,1,2} - (f_{e1}-1)}\, u_j(f_{e1}-q) \right] + \sum_{m=1}^{k-f_{e1}} \left[\frac{\sum_{i=1}^{m} n_{e,j,f_{e1}}(f_{e1}+i)}{f_{m,1,2} - (f_{e1}+m+1)}\, u_j(f_{e1}+m) \right] + \sum_{i=1}^{(k+1)-f_{e1}} n_{e,j,f_{e1}+i}(k)\, u_j(f_{e1}+i) \Bigg\} \quad \text{for } f_{e1} < k \leq f_{eL} \qquad 2212\text{-}16$$

$$W_j(k) = E_j(k)\Bigg\{ n_j(k) + \sum_{m=1}^{f_{eL}-f_{e1}} \left[\frac{\sum_{i=1}^{m} n_{e,j,f_{e1}}(f_{e1}+1)}{f_{m,1,2} - (f_{e1}+m+1)}\, u_j(f_{e1}+m) \right] - \sum_{q=1}^{f_{e1}-f_{ef}} \left[\frac{\sum_{i=0}^{q-1} n_{e,j,f_{e1}-q}(f_{e1}-1)}{f_{m,1,2} - (f_{e1}-1)}\, u_j(f_{e1}-q) \right] \Bigg\} \quad \text{for } f_{eL} < k \leq f_{m,1,2} \qquad 2212\text{-}17$$

where $f_{m,1,2}$ is a midpoint interval between nuclear refueling windows F_{j1} and F_{j2}. The reasons for going to a midpoint are: (1) it reduces the complexity of the equations by making unnecessary the overlapping of effects of changes in window F_{j2} upon those intervals in

F_{j1} (and vice versa), (2) it decouples the effects of maintenance changes and thus increases the accuracy of the equations, because otherwise, further estimates on the exact size of the intersession between refuelings would have been required, increasing the number of approximations.

It would seem desirable to include the nuclear cycling potential in the generation flexibility equation 22-7 only if the cycling usage of the nuclear facility was to be utilized. This inclusion could be accomplished by considering the difference between the actual production in a given interval and some nominal production level, thus measuring the deficiency from its potential capability.

The prospect of 'hot refueling,' if developed commercially, could change the pattern of this nuclear fuel relegation problem, in that maintenance and refueling would not necessarily be coincidental events as they are now.

2.2.1.3 <u>Hydroelectric Capabilities</u>[43]

It may happen that the restrictions upon a hydroelectric facility are so stringent that appreciable long range buildups or depletions in reservoirs are not possible. In this case, the output capabilities can be predicted with fair accuracy, and the maintenance can be scheduled to take place in the time slot which optimizes system performance.

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43. Problems concerning the dispatch of hydroelectric power can be found in, for example, references (37) and (38). An incremental water loss approach is taken in reference (39), while a nonincremental approach is presented in reference (40).

On the other hand, variances in the capabilities of a facility may be attainable. The exactitude of the linear equation format for use in pondage, or forebay, accounting depends upon a knowledge either of the maintenance interval to be used, or estimates of the losses to be incurred for maintenance sessions at any interval within the maintenance windows.

Predictions of the water inflows to the forebays must be forecast for each interval within the planning horizon. Reservoir levels must also be forecast for use in the $E_j(k)$, conversion efficiency coefficients of each plant, which are dependent upon the heads at the reservoirs. This assumption will preserve the linearity of the simulation.

Define $w_j(k)$ as the usable water content of the reservoir associated with the j^{th} plant at the conclusion of the k^{th} interval. Let $r_j(k)$ be the inflow into the forebay, and $h_j(k)$ the volume of water consumed for hydroelectric generation. The conservation of water equation is then:

$$w_j(k-1) + r_j(k) - h_j(k) - s_j(k) = w_j(k) \qquad 2213\text{-}1$$

where $s_j(k)$ is the spillage, intentional or inadvertent, and this spillage will be penalized in the cost equation. It is also necessary to describe power production limitations

$$0 \leq W_j(k) = E_j(k)\, h_j(k) \leq A_j(k) \qquad 2213\text{-}2$$

For known maintenance intervals we can preset $h_j(k)=0$ and this should tend to call for reservoir level drawdowns just before outages, with buildups occuring during them.

Unless a plant has what has been termed a 'run-of-the-river', it will be necessary to respect minimum flow requirements (especially for navigated waterways):

$$h_j(k) + s_j(k) \geq R_j(k) \qquad 2213\text{-}3$$

The physical limitations of the hydro facility necessitate the constraint

$$0 \leq w_j(k) \leq T_j \qquad 2213\text{-}4$$

where T_j is the maximum limit of water volume available to the hydroelectric facility.

Backwater effects to downstream forebays can be included in the form of

$$w_{j+1}(k-1) + r_{j+1}(k) + \left[h_j(k) + s_j(k)\right] - h_{j+1}(k) - s_{j+1}(k) = w_{j+1}(k) \qquad 2213\text{-}5$$

where $r_{j+1}(k)$ is the inflow to forebay j+1 from outside tributaries. An assumption here is made that the delay in the hydraulic coupling (usually about one day) is small compared to the interval time.

2.2.1.4 Pumped Storage Constraints[44]

If the capacity of a pumped storage facility is more or less memoryless, that is, quickly used with respect to the length of a single simulation time unit, then we can estimate its power consumption (a common figure is 2/3 efficiency for pumped storage facilities). Calling this then an added demand to the system, and apportioning the power

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44. Parameters, capabilities and dispatching of power from pumped storage facilities are discribed in references (41), (42) and (43).

created to offset cycling demands accounts for the operation of the pumped storage facility. These system adjustments are obviously contingent upon the maintenance of the facility, $u_j(k)$.

A large storage capacity at a facility would make possible a treatment much like that used for hydroelectric facilities. The pondage accounting equations which result are:

$$w_j(k-1) + r_j(k) + e_j(k) - h_j(k) = w_j(k) \qquad 2214\text{-}1$$

where $e_j(k)$ is the net water volume input from the electric power pumping, $r_j(k)$ is the pondage inflow from other sources, and it is assumed there is no spillage. Again a maximum retention limitation exists

$$0 \leq w_j(k) \leq T_j \qquad 2214\text{-}2$$

The pumping efficiency, $E_{p,j}(k)$ and the conversion efficiency $E_j(k)$ yield the equations:

$$0 \leq W_{in,j}(k) = E_{p,j}(k)\; e_j(k) \leq A_{in,j}(k) \qquad 2214\text{-}3$$

$$0 \leq W_j(k) = E_j(k)\; h_j(k) \leq A_j(k) \qquad 2214\text{-}4$$

where $A_{in,j}(k)$ is the maximum input capability.

If a pumped storage facility is small (as is usually the case) or if the facility runs between its maximum and minimum limitations within an interval (commonly a week long cycle), and the environmental impacts of its operations are considered in the unit commitment problem, then simplifications are possible. Suppose $\bar{v}_j(k)$[45] represents the fractional

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45. Here the bar $-$ is defined such that $\bar{v} = 1 - v$.

extent of usage of the j^{th} facility in the interval k. Then with $A_{out,j}(k)$ the maximum output in the interval, the term

$$A_{out,j}(k)\ \bar{v}_j(k)$$

represents the relevant contribution to the cycling equation 22-7 in the interval k.[46]

One further constraint is now required,

$$\bar{v}_j(k) + u_j(k) \leq 1 \qquad 2214\text{-}5$$

or

$$v_j(k) - u_j(k) \geq 0 \qquad 2214\text{-}6$$

a constraint which will not allow for the maintenance of a plant as well as its usage in the system.[47]

2.3 Coordination Equations

The specification of individual power plant capabilities is a first step in analyzing the systems' abilities to meet the requirements asked of it. However, many restrictions arise due to the coordination of efforts required between facilities, interdependence of a single plant's capabilities at different intervals of time, coordination of the available labor and equipment, and geographic constraints imposed upon the scheduling of the generators due to the peculiarities of the network configuration.

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46. The assumption made here is that the pumped storage facility does not detract from the total bulk power equation 22-2 because the pumping is not done at inopportune times, that is, when the maximum extent of power is needed.

47. This condition now makes possible the exclusion of exactly $A_{out,j}(k)\ u_j(k)$ from a pumped storage facility when maintenance is being performed.

2.3.1 Maintenance Coordination

The most obvious coordination effort required of the system over the one to two year planning horizon of the simulation involves the careful examination of the maintenance of the facilities. In general, the scheduler has available to him the spread of possible interval alternatives for the outage of each plant. The next set of outage possibilities for the j^{th} plant will be denoted F_{j1}, the following window as F_{j2}, and so on. It should be noted that F_{ji} will be restricted by many predictable constraints. For example:

(1) the time between maintenance sessions cannot exceed certain limits,

(2) the window may not overlap any so-called 'inhibited periods' when maintenance is not possible, as during winter months on outdoor facilities,[48]

(3) if it is decided that two plants are so closely coupled that they must be maintained simultaneously then these plants will share the same variable u and the same window,

(4) required parts being unavailable will restrict the window,

(5) outage of a large plant might have to follow the installation of a new facility, or might have to coincide with the contracted purchase of a large block of power, etc.

2.3.1.1 Planned Maintenance Outages

The most common maintenance constraint imposed by a power plant is that it must be serviced once, and only once within its window. This requirement reduces to:

$$u_j(k) = \begin{cases} 1 \\ 0 \end{cases} \quad \text{for all } k \in F_{ji} \qquad 2311\text{-}1$$

48. Reference (16) page 320 explains seasonal constraints on maintenance.

$$\sum_{k \in F_{ji}} u_j(k) = 1 \qquad 2311\text{-}2$$

If a facility has the option of either a single session or a multiple length session this can be treated as explained in equation 2211-1.

The increased forced outage rates in the system due to delayed maintenance has been discussed in the paragraph following equation 221-1. These adjustments need not be made prior to the maintenance window because the system is real and can not be anticipative. Shorter maintenance sessions, however, such as u^1 instead of u^2 could affect forced outages in the intervals following the session.

2.3.1.2 Maintenance Intersession Constraints

In reality, for a plant j, the position (and possibly size) of maintenance window $F_{j,i+1}$ will depend upon when maintenance occurred within $F_{j,i}$. To exactly (and simply) describe this interdependence of variables would require a nonlinearity, and so it becomes advantageous to find a linear approximation. One possibility is to slightly shrink the set of all possible intervals in $F_{j,i+1}{}^{*} = \bigcup_{\substack{u_j(k)=1 \\ \text{for all } k \in F_{j,i}}} F_{j,i+1}$. Then if an untenable alternative results from the optimum schedule, further restrictions could be used to constrain $F_{j,i+1}{}^{*}$.

An alternative approach requires one or more extra constraint equations. Again defining $F_{j,i+1}{}^{*}$ as the set of all possible maintenance intervals resulting from any of the possible

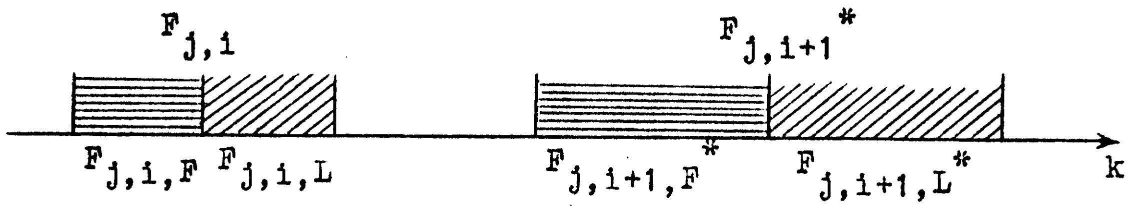

Figure 2.3.1.2 Intervals used for the linear approximation of the maintenance intersession constraints.

outages within $F_{j,i}$, the following equations constrain the potential maintenance combinations:

$$\sum_{k \in F_{j,i,L}} u_j(k) + \sum_{k \in F^*_{j,i+1,F}} u_j(k) = 1 \qquad 2312\text{-}1$$

or equivalently (adding both would add a redundancy in view of equation 2311-2),

$$\sum_{k \in F_{j,i,F}} u_j(k) + \sum_{k \in F^*_{j,i+1,L}} u_j(k) = 1 \qquad 2312\text{-}2$$

This extra equation insures against a maintenance outage planned at the end of one window and the beginning of the next (causing too short an intersession interval), or vice versa.

It may be observed that the windows can be subdivided into many parts and thus many constraint equations introduced to insure that maintenance will not fall on unattractive pairings. An exact intersession coordination <u>could</u> be built if the first window were partitioned into single intervals, but the benefit from this task must be weighed against the many added constraint equations.

As in the case of many of the approximations or relaxations of exactitude within this simulation, as the events to be decided come nearer to the present, the approximations tighten up and eventually become exact.

2.3.2 Maintenance Crews

The availability of labor[49] must be considered in the scheduling procedure. In the simplest case a single crew, say the i^{th} crew, is responsible for all plants j where j is a member of L_i, a set of plants. Then for each interval k

$$\sum_{j \in L_i} u_j(k) \leq 1 \qquad \text{for all } k \qquad 232\text{-}1$$

A more complicated set of constraint equations results if two or more crews may be able to work on the same plant, that is some of the sets L_i overlap one another. However, the number of equations required at each time interval is still held to the same number as the number of crews to be considered. Suppose there are m crews, then for each k

$$\sum_{j \in \left\{L_1 - \bigcup_{\text{all } i>1} L_i\right\}} u_j(k) \leq 1 \qquad 232\text{-}2$$

$$\sum_{j \in \left\{\bigcup_{i \leq p} L_i - \bigcup_{\text{all } i>p} L_i\right\}} u_j(k) \leq p \qquad 1 \leq p \leq m \qquad 232\text{-}3$$

$$\sum_{j \in \left\{\bigcup_{\text{all } i} L_i\right\}} u_j(k) \leq m \qquad 232\text{-}4$$

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49. Availability of equipment can be treated in an identical manner.

2.3.3 Geographic Constraints

In the interest of area security it may be necessary to assure that at least n of m plants within a geographic district D_i are operative at all k:

$$\sum_{j \in D_i} \left[1 - u_j(k)\right] \geq n \qquad 233\text{-}1$$

or equivalently

$$\sum_{j \in D_i} u_j(k) \leq m - n \qquad 233\text{-}2$$

This condition is readily adapted to a capacity limitation, using the addition of the $A_j(k)$ capacity factors.

For a power exchange pool of utilities, constraints may exist upon the maximum or minimum amount of planned outage capacity (or number of plants) allowed out of operating condition. This restriction translates mathematically into a condition identical to the geographic limitation type.

If particular transmission line limitations must be recognized (due to poor tie lines, line outages, etc.) they are introducible into the simulation but require the use of areally discretized load demands. In the simplest case, the plants $j \in D_i$ are in a district which is connected to the remainder of the system by the transmission line of limit $l_i(k)$.[50] If the local power consumption within D_i is forecasted as $P_{D_i}(k)$, then

$$\sum_{j \in D_i} A_j(k)\left[1 - u_j(k)\right] - P_{D_i}(k) \leq l_i(k) \qquad 233\text{-}3$$

50. Line limitations are time variable to account for the summer to winter temperature variations and scheduled restringing of lines.

The recognition of more complex configurations requires the inclusion of production and consumption of other districts so that the intersectoral flows can be predicted, as dictated by the network topology.

2.3.4 Scheduled Economic Shutdowns

It is necessary to make certain that the schedules not count on expensive units (i.e. inefficient performers) if it can at all be avoided. Thus, we would like to have the optimum schedule rearrange surplus generation capabilities so that maximum shutdown of the expensive units could be realized. That would mean that the inefficient components become the least needed and thus least used portions of the system.

Since the addition of new variables increases the time and cost involved in finding a solution, it is wise to limit the number of added variables and to maximize their usefulness if they are unavoidable.

At first glance it might seem as though shutting down facilities for an interval for economic reasons could be treated along with maintenance shutdowns. Unfortunately, economic shutdowns must be new variables in the system. They will enter into the cost functional in a vastly different form than the $u_j(k)$ terms. Also, economic shutdowns do not satisfy maintenance requirements, do not require maintenance crews and need not be limited to outage windows F_{ji}.

As a first step in the inclusion of this variable, notice that it is possible to estimate the total amount of economic shutdown that may be available to the system. Using the number of outage windows in the future of each plant we may estimate[51] the total loss of capacity from these planned outages over the simulation time horizon, M_n. Since we have a close measure of the total capacity available for outages, $\sum_{\text{all } k} M(k)$, then the difference between these two quantities will be the approximate total capacity available for economic shutdown.[52] To the extent that economic shutdown capacity is available, variables for that usage should be relegated starting with the poorest performing plants at the various intervals.

It will save computation time, and in fact will be more instructive, to allow the shutdown variable $v_j(k)$ the full range from 0 to 1. A nonintegral shutdown variable would have the physical interpretation of a planned outage over that fraction of the interval represented by its value.

A constraint such as used in equation 2214-6 is required to preclude the possibility of multiple outage accounting.

Now, in all the equations generated in this model where u appears, v must appear also, with the exception of the maintenance satisfaction equations (2311-2, 2211-1, 2212-8

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51. Only estimations are possible, due to the facts that some facilities will have multiple interval outage options and some window limits are sure to straddle the end of the planning span.

52. The assumption has been made that power purchases and sales will about cancel over the planning period. Any expected deviations from this presumption must be considered accordingly.

2212-9), the maintenance intersession constraints (2312-1 and 2312-2) and the maintenance crew assignments (section 2.3.2).

2.4 Inputs

The main thrust of this project is directed at the alignment of the input material and the optimal attack of the problem. So, for the most part, inputs to this simulation will be considered given. For a somewhat broader description of what the collection of input data will entail, or what the relevant influencing factors might be, consult reference (4). There is, however, a certain amount of input shaping which must be accomplished before this simulation can use that input. Because of this, input modifications will be presented to the extent that their shaping is peculiar to this analysis.

2.4.1 System Updates

All of the inputs to this model, i.e. power demand forecasts, river level forecasts, temperature predictions, environmental impacts, etc., will require updating as more accurate information becomes available.

The update data of the system's physical characteristics will include new generation, transmission line additions (where transmission constraints are used), retirement of facilities, forced outages, loss of transmission, and all other unpredictable changes, as well as planned changes and the times at which they will affect system operation.

2.4.2 Power Demand Adjustments

Once given the forecasted load level $P_d(t)$, a probability distribution at all points in time, computing the target level of power P(k) which should be aimed at during the interval k to meet reliability specifications is a crucial and complex issue.

Because of the stochastic nature of the power demand and the flexible interregional power support, a probabilistic approach to this problem might seem natural. But probability theory is cumbersome and confining when we wish to establish a measure of the 'goodness' of the supply covering various potential demand levels. A different measure of 'acceptibility' could be used which would still range between zero and one, but which could include a mixture of loss-of-load-probabilities LOLP, loss-of-energy-probabilities LOEP, and various 'good will' measures, concerning:

(1) preference of short outages at intervals to a single massive failure, although the LOEPs might be equal,

(2) disproportionately large avoidance of outages during times of greatest customer 'dependence', such as during severe winter nights when power loss would leave street lights out and homes without heat, etc.

A technique which seems particularly amenable to this type of problem treatment would require the measure and description of power demand in terms of fuzzy sets.[53] This development is, however, beyond the scope of this study.

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53. Footnote 53 is given on the bottom of the following page.

2.4.2.1 Time Adjustments

The definition of the percentage $C_{\%}(k)$ of the demand $P(k)$ which is of a cycling nature is a difficult task. Measuring this percentage as the peak-to-valley variation divided by the peak demand is for this model not an altogether acceptable method. For example, if the peak load within the week comes at 11:00 a.m. on Wednesday, and the lull is at 5:00 a.m. on Sunday, it would not be unreasonable to assume that even the strictest base loaded plant could be shut down within that span of time. Thus, every plant would be granted 100% cycling potential and the generation flexibility equation 22-7 would become meaningless. So it can be seen that the simulation will require coordination between the definitions of the plant cycling capabilities $c_{\%,j}(k)$ and the $C_{\%}(k)$ of the system. If for example $c_{\%,j}(k)$ reflects a plant's ability to change easily over three hour time spans, the $C_{\%}(k)$ must measure the demand variations over three hour spans. The time span chosen should be the most representative and effective measure of the load following problems experienced by the system.

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53. (from previous page) For a short description of fuzzy mathematics see reference (44). Reference (45) demonstrates this technique's usefulness in decision and control problems. The advantage of fuzzy control theory in its application to this problem is in its ability to handle vagaries in inputs and outputs which become more precise, i.e. finer, as time progresses. For example, maintenance or refueling windows, river level predictions, load and weather forecasts, flexible interregional exchange contracts, etc. would have associated with their possible occurance some measure, not necessarily a normalized probability distribution. Fuzzy sets are more amenable to dynamic techniques, and thus will be investigated in part II of this study. Some work on fuzzy control theory can be found in reference (46).

2.4.2.2 Power Contracts[54]

The reason for including power contracts in this simulation is not merely for the preservation of the model's validity, but also because this simulation can be a valuable tool in determining the advisability of renegotiating certain contracts, the need for various types of agreements, and the contract's possible impact upon the system from operational, economic and ecological standpoints.

2.4.2.2.1 Fixed Interregional Contracts

If a contract is absolutely binding, the specified bulk power exchanges between regions can be considered by appropriate adjustments to the power-level-to-be-met in the corresponding intervals.

In most cases, however, contracts are revokable, and if the advisability of such an action is to be considered this decision will require a new binary variable:

$$x_j = \begin{cases} 1 & \text{if the } j^{th} \text{ contract is to be honored} \\ 0 & \text{if the } j^{th} \text{ contract is to be revoked} \end{cases} \qquad 24221\text{-}1$$

Equations which must carry the x_j term will be all those capacity[55] equations 22-5 in the intervals spanned by the contract, as well as the total performance, viz. the cost portion of the performance function.

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54. A description of the philosophy behind, and types of, power interchange contracts see reference (47).

55. The generation flexibility equations 22-7 will in general not receive a contributing term from the variable x_j because bulk contracts usually involve the interchange of constant levels of power.

Slightly more flexibility is reflected in contracts to sell or purchase unfixed amounts of power, restricted only by some upper limit X_j.[56] This amount of power X_j will be either a positive or negative quantity depending upon its addition to of subtraction from the systems load meeting capabilities. The continuous variable, $x_{v,j}$ now represents the desired fraction of the limit X_j which will then be added to the capacity equation 22-5.

2.4.2.2.2 Flexible Interregional Agreements

Besides allowing planned bulk transfer of power, the interregional tie lines are also used to provide energy transfers in emergency situations. Of course, whether or not a j^{th} neighboring region will have any reserve power that can be tapped (or if this region in the simulation has emergency power which its neighbor can tap), and what amount of power is available defines a power availability probability distribution $P_{A,d,j}(k)$.[57] The derivative of this $P_{A,d,j}(k)$ with respect to varying power levels P is called the power availability density function $P_{A,f,j}(k)$

$$\frac{d\ P_{A,d,j}(k)}{d\ P} = P_{A,f,j}(k) \qquad 24222\text{-}1$$

Likewise,

$$\frac{d\ P_d(k)}{d\ P} = P_f(k) \qquad 24222\text{-}2$$

56. Note that X_j might be restricted by temperal transmission constraints on the tie lines.

57. This term represents the point within this simulation at which the maintenance schedules of neighboring regions will have their influence.

If we define as $\otimes$ the convolution [58] procedure with respect to the power level, then the total power density function which considers demand levels and flexible contracts is $P_{T,f}(k)$ where

$$P_{T,f}(k) = P_f(k) \otimes -P_{A,f,1}(k) \otimes \ldots \otimes -P_{A,f,j}(k) \quad 24222\text{-}3$$

and it is assumed the region under consideration has j neighbors. Now a total power distribution function $P_{T,d}(k)$ can be generated by integrating $P_{T,f}(k)$ up to each level of power and recording the probabilities.

Deciding upon the proper curves for $P_{A,d,j}(k)$ will be difficult, and in general will require a pessimistic underestimation. There are several reasons for the <u>under</u>estimation:

(1) even if power can be supplied by a neighboring region, they may have to withdraw their support if something should go wrong in their system

(2) emergency power is costly, and not something that should be planned on being used

(3) the convolution of probability curves for total probability presumes the independence of events, however, the effectors of the demand anomalies, such as unusually high temperatures, may be equally perturbing to neighboring regions.

In this probabilistic study of demand agreements it is

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58. $f(t) \otimes g(t) = h(x) = \int_0^x f(t)\, g(x-t)\, dt = \int_0^x g(t)\, f(x-t)\, dt$ is the convolution of f(t) and g(t). If the probabilities corresponding to the power demand levels are defined using discrete intervals Δz, i.e. $f(z_n)$ and $g(z_n)$ then $f(z_n) \otimes g(z_n) = h(z_n)$ where

$$h(z_n) = \sum_{p=-\infty}^{\infty} f(z_p)\, g(z_{n-p}).$$

not necessary to consider the probabilistic power level curve of emergency energy sent out of the district. This power is dispatched only when the system is not in distress and is immediately called back once the region requires its use.

2.4.2.2.3 <u>Interruptible Loads</u>

In some areas, industries are offered large amounts of power at lower than bulk rates with the provisal that this energy may not always be available. This interruptible load agreement usually requires only a five minute warning so these contracts will definitely affect the required cycling capacity.

To the extent that the load is interruptible the simulation could introduce a new generator, G(k), into its equations. There are a number of difficulties associated with this procedure. First, it is not likely that this interruptible load generator could be introduced into the interval capacity equation 22-5 unless it is intended that this load could be shut down for substantial periods of time. Secondly, the inclusion of the G(k) term within the generation flexibility equation 22-7 would depend to a large degree upon the time span[59] over which the cycling measurements are taken. Therefore, it remains in many cases for the interruptible loads to be accounted for within the probabilistic load demand calculations. That is, G(k) can be dropped from the very highest demand levels within the interval k. Because the peak demands take place

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59. See section 2.4.2.1 for an explanation of this time span.

almost exclusively during industrially loaded times, it will probably be valid to assume the interruptible load will always be available for dropping at peak demand times, thus avoiding further probabilistic computations.

2.4.2.3 Reliability Margins

Reliability margins have been one of the most thoroughly studied areas of the scheduling problem. There have been a number of different treatments used to develop risk level evaluations, some including forced outages.[60]

The most commonly used procedure involves developing the loss of load probabilities associated with different projected power demands, and to make the cutoff at the one-day-in-ten-years level. This is more a name for a unit of measure than a literally meaningful quantity, and any risk evaluation which more accurately accounts for the perturbances within the system should meet a more relaxed level.

A post-optimal analysis of the resulting schedule effects due to changes in the reliability level (and likewise the demand level) will be helpful in evaluation of the sensitivity of the scheduling process with respect to various reliability measures.

2.4.3 Weather Inputs

Another post-optimal study involves the variational

60. For the purposes of this simulation, any forced outage information given in a reliability level evaluation must be separated and included in forced outage derating terms, that is equation 221-1.

flexibility of the schedule with respect to deviations from the forecasted (or average) weather patterns. However, any introduced deviations, such as higher temperature levels, will have simultaneous effects, and thus must be introduced simultaneously, upon demand levels, capacity capabilities (i.e. warmer cooling waters), dollar cost changes (it is more expensive to produce power from warmer water), transmission limitations (i.e. line sagging), and environmental impacts. So in the preparation of the data for this simulation it will be necessary to accumulate variations which might be expected due to unpredicted weather changes.

Leaving those deviations aside for the moment, on the time scale used for this schedule it will be necessary to use historical averages[61] for the weather inputs to most of the model. Exactitude will be available as the interval in question gets closer to the current interval, and thus when exactitude is most needed. For the maintenance schedule time scale there is little that can be done except to use the averages available and respect the possible deviations in accordance with the magnitude of these historical variances.

2.5 Performance Index

Performance index, quality measure, objective function,

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61. As explained in reference (4), page 20, it is possible to obtain a limited amount of information on weather possibilities for the month ahead, and in some cases speculations to four month horizons. River level statistics are subject to a good deal of prediction, depending upon the precipitation, thickness of snow covers and temperatures.

cost functional, or whatever name is used, there can be no denying the fact that if a decision is to be made there must be some well-ordering measure which will rate the alternatives according to their desirability. Thus, the equating of a certain quantity of dollars with a certain quality of environment is unavoidable[62] and should not be deemed a shortcoming of this simulation. So in looking for areas of possible deficiencies, attention should be directed toward the collection and treatment of the data, but not in the mixing together of portions of the data.

Two conventions will be established at the outset. First, current dollar evaluation[63] will be used as the basic unit of measure for the performance index. This choice in no way is intended to bias the performance measure in favor of economic optimization and against ecological impact minimization. Choosing some 'neutral' measure and introducing a ratio of dollars to this neutral measure is no better a

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62. Some equating of environmental impacts to costs have been made, for example, it has been estimated , reference (48), that air pollution impact upon humans causes possibly $1990 million annually in costs of diseases.

63. For simulations running 2 to 5 years it is necessary to distinguish between future dollar prices and future dollar net worth in terms of present prices. As an example suppose inflation continues at 6% a year, it would be a misconceived decision to schedule maintenance or refueling 2 months earlier because it would 'cost' 1% less. If however a utility has reason to believe that the price of some portion of its operation will be unusually costly (or perhaps less expensive) in the future, the cost of operations could reflect this increase with an appropriate overestimation (or underestimation) of future costs in terms of dollars. Examples might include forecasted bond interest rates, or increased costs of some out-of-plant contracted wage laborers.

basic unit than dollars; and it is clearly undesirable to measure the entire system operation in terms of an ecological unit such as air pollution levels.

The second convention is that only net performance will be used as a measure, where net performance is the difference between the unavoidable fixed costs of the system and those costs contingent upon the decision variables. Examples of fixed costs would be the yearly salaries of workers, maintenance costs which must be borne at some time within the planning horizon, or the fixed ecological cost of unsightly transmission lines or hydroelectric dams.

A good test for determining the appropriateness of a cost to be included within the performance index is the ability to show that tradeoffs between this cost and other performance measures are reasonable.

Since the performance measure will be the operating cost in dollars this simulation, thus, naturally takes the form of a minimization problem.

The main detraction from this optimization procedure is the manner in which a linear cost function is made to fit a nonlinear curve. This linear approximation, however, becomes increasingly exact as it is determined more accurately exactly where on the nonlinear curve the linearization should be made.

2.5.1 Operating Expenses

In deciding the precedent for definitions of cost, first considered must be those contributors to the cost functional whose definitions are most inflexible. One of these pivotal, already determined standards is the fixed price of interregional exchanges of power, say $b_{c,j}$ for the j^{th} contract. This price is clearly and totally contingent upon the honoring, x_j , of the j^{th} contract. For moneys received from power sales, $b_{c,j}$ will be negative, and thus $b_{c,j}$ will carry a positive sign for purchases. The contribution from fixed power exchange contracts to the system's performance quality over the planning horizon will be:

$$q_d = \sum_{all\ j} b_{c,j} x_j \qquad 251\text{-}1$$

where q_d designates that this term is used in this form in the dollar performance quality equation.

There is money to be saved, $y_{v,j}(k)$, from the portion of interval k for which plant j is shut down. This quantity $y_{v,j}(k)$ must not include any prices fixed regardless of the shutdown decision. Included should be rewards for fuel savings, saving the auxiliary power requirements, and possibly also a savings in maintenance costs from fixtures not receiving wearing service.

$$q_d = - \sum_{j} \sum_{k} y_{v,j}(k)\ v_j(k) \qquad 251\text{-}2$$

An interesting and useful set of variables necessary in any event for the solution of the constraint inequalities are the slack variables. In static solutions of inequalities such as

$$f(\ x(k)\ ) \geq y(k) \qquad 251\text{-}3$$

the slack variables, or oversupply variables, say o(k) are introduced into the inequality to produce the equivalent conditions

$$f(\ x(k)\ ) + o(k) = y(k) \qquad 251\text{-}4$$

$$o(k) \geq 0 \qquad 251\text{-}5$$

These equations are now more easily handled in algorithmic form by solving for equation 251-4 in the half space defined by inequality 251-5. Thus, there is no need to justify the introduction of these 'new' variables, as they are already inherent in solution techniques. It happens, however, that the use of these oversupply variables in the cost equation would in any case justify their inclusion.

Take for example inequality 22-5. Define as $o_A(k)$ the oversupply in terms of capacity within the k^{th} interval, and as $o_c(k)$ the oversupply in terms of cycling capacity in the k^{th} interval from equation 22-7. These terms $o_A(k)$ and $o_c(k)$ are now the oversupply of the system <u>beyond</u> the projected reserve requirement. There is no cause to penalize these oversupplies, they are already penalized by not being absorbed by economic shutdowns. On the contrary, these terms and in particular $o_c(k)$ deserve a slight reward because their existence and amount preclude the expenses of interrupting the loads G(k) and of paying for emergency interregional power exchanges. Also some slight rewards must be given for the possibility of collecting payments from other region's emergency demands during times when oversupply is available. These rewards $y_{oc}(k)$ and $y_{Ao}(k)$ will be time variable and will be calculated from the prices and probabilities of need during the period k.

$$q_d = - \sum_k y_{oc}(k)\, o_c(k) - \sum_k y_{Ao}(k)\, o_A(k) \qquad 251\text{-}6$$

Another oversupply slack variable results from the failure to use all the contracted gas purchases in equation 2211-5. This variable o_{gc} will be costly to the extent of the penalty clause for not meeting the quota within the gas contract, say b_{gc} dollars per unused cubic foot of gas. Thus,

$$q_d = \sum_{\substack{\text{all gas} \\ \text{contracts}}} b_{gc} o_{gc} \qquad 251\text{-}7$$

If the penalty clause is severe, equation 2211-5 could be made an equality with no effect on the eventual schedule.

Two slack variables due to the non-use and overuse of nuclear fuel before refueling of the j^{th} reactor result from equation 2212-1. If these variables are defined as $o_{n+,j}$ and $o_{n-,j}$ for overuse and underuse, the associated costs $b_{n+,j}$ and $b_{n-,j}$ are not as severe as one might at first believe. Because of rotating refueling schemes, only a fraction of the core is replaced at any one refueling. Also the utility is monetarily rewarded to the extent that there exists recoverable fuel, such as U_{235}, in the removed fuel elements. There is, however, a definite cost [64] lost in not refueling when the

64. Costs associated with nuclear fuel usage can be found in reference (49).

optimum batch usage level is reached, and so

$$q_d = \sum_j b_{n-,j}\, o_{n-,j} + \sum_j b_{n+,j}\, o_{n+,j} \qquad 251\text{-}8$$

Although it certainly has the form and consequences of an oversupply variable, the spillage at a hydroelectric facility $s_j(k)$ is in equality 2213-1 and thus not a slack variable. An estimate of the dollar loss $b_{sj}(k)$ due to $s_j(k)$ must be made, and then

$$q_d = \sum_j \sum_k b_{sj}(k)\, s_j(k) \qquad 251\text{-}9$$

To the extent of the usage of the j^{th} pumped storage facility in interval k, i.e. $1-v_j(k)$, there is a cost in terms of power input and operation procedures. It is easier to include this in the performance measure as a reward for extent of nonusage, $y_{v,j}(k)$, so

$$q_d = -\sum_j \sum_k y_{v,j}(k)\, v_j(k) \qquad 251\text{-}10$$

and this equation is then similar to the economic shutdown equation 251-2.

Since shutdowns are considered 'rewards,' the system performance must be penalized the cost associated with the degree of overloading a plant, from equation 22-8

$$q_d = \sum_j \sum_k b_{x,j}(k)\, v_{x,j}(k) \qquad 251\text{-}11$$

where $b_{x,j}(k)$ represents the additional cost inherent in running plant j at the additional capacity $A_{x,j}(k)$ beyond its nominal maximum.

Certain plants do not have economic shutdown possibilities, and for these cases the expenses saved in fuel costs, etc. from not operating in interval k must[65] be rewarded to these plants during their maintenance shutdown period:

$$q_d = - \sum_j \sum_k y_{u,j}(k)\, u_j(k) \qquad 251\text{-}12$$

2.5.2 Maintenance Costs

A great percentage of the expense of maintaining or refueling a power plant is unavoidable and time invariant. These costs are of no significance to the optimization process.

One of the costs which must be considered is the maintenance or refueling cost which varies throughout the window, e.g. availability of overtime or regular-time labor, or penalty clauses in nuclear fuel contracts.

Another cost consideration involves the decision, e.g. of a two week or four week maintenance session. A third consideration could be the evaluation of forced outage costs inherent in different maintenance schemes. This would be an alternate, and less effective, way than that introduced at the end of section 2.3.1.1 for dealing with the increased forced outage penalties associated with delayed maintenance sessions. There are, however, strains to the equipment from these long sessions, and this is an extra cost. Section 2.6.2 yields another cost for this section. In any event, the cost $b_j^n(k)$ for performing a $u_j^n(k)$ session of maintenance is:

65. This reward is inherent in equation 251-2 for plants which have economic shutdown capabilities.

$$q_d = \sum_j \sum_k b_j^n(k)\ u_j^n(k) \qquad 252\text{-}1$$

2.5.3 Ecological Impact Units[66]

The quantification of the environmental impacts to the ecosphere due to electric generation is a topic which has prompted some research efforts.[67] The author is currently preparing a paper which will provide measures for these impacts.

Reaching a common denominator for all the environmental impacts is a difficult task. This proposed measure will here be called the ecological impact unit,[68] e.i.u. and can be envisioned as a difference, or derivative, of the already proposed environmental quality unit, e.q.u.[69]

In this simulation, stoppage or partial shutdown of the operations of a j^{th} generating facility during the k^{th} interval will be rewarded by

$$q_e = -\sum_j \sum_k z_j(k)\ v_j(k) \qquad 253\text{-}1$$

66. For a more detailed introduction to this section and a description of the types of considerations which must be made see reference (4) pages 23-30. Reference (4) pages 41-43 contains a list of references representative of the state-of-the-art of the work in this field.

67. Some efforts have already been made in the direction of reducing impacts upon the environment to single quantities, see references (50) and (51).

68. '<u>Ecological</u> impact units' rather than 'environmental' because it is not the impact to the environs, i.e. surroundings, but the impact to living organisms due to changes in the environment, i.e. ecology. It is in fact precisely this difference which will be taken advantage of in the optimum scheduler.

69. See reference (50).

where $z_j(k)$ represents the number of e.i.u. inherent in the full operation of the j^{th} plant in the k^{th} interval.[70]

It should be noted at this point that there are beneficial uses[71] for some forms of pollution, viz. thermal enrichment at some sites. Therefore, it is entirely possible that $z_j(k)$ could be a negative quantity.

For those plants which do not carry a shutdown indicator v, the associated reward for maintenance shutdowns is

$$q_e = - \sum_j \sum_k z_j(k)\, u_j(k) \qquad 253\text{-}2$$

These are the basic two equations (253-1,2) in the measurement of ecological impact. Refinements must be made in a few cases where generation levels might vary from interval to interval in other ways. Define as $Z_{g,j}(k)$, $Z_{n,j}(k)$ and $Z_{x,j}(k)$ the penalties associated with the usage, respectively, of $g_j(k)$ units of gas turbine operation, $n_j(k)$ units of nuclear operation and the additional burden to the environment due to production of $A_{x,j}(k)$ extra megawatts at plant j. Then,

$$q_e = + \sum_j \sum_k \left\{ Z_{g,j}(k) g_j(k) + Z_{n,j}(k) n_j(k) + Z_{x,j}(k) v_{x,j}(k) \right\} \qquad 253\text{-}3$$

The only additional contributions to the ecological impact involve the operation of the hydroelectric and pumped

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70. For an example of the reason for the time variability of $z_j(k)$ see reference (2) pages 28-29.

71. For a list of some beneficial uses of thermal effluents see reference (4) page 30.

facilities (assuming pondage accounting is necessary on this production schedule time scale). These two power sources are enough alike so their impacts can be considered in the same way. Recall that the variable $w_j(k)$ represents the quantity of water in the j^{th} reservoir above the minimum drawdown level. Therefore, there is an e.i.u. reward for each level of $w_j(k) > 0$, call this reward $z_{w,j}(k)$. Augmenting the minimum allowable river flow (cf. equation 2213-3) with the oversupply variable $o_{r,j}(k)$

$$h_j(k) + s_j(k) - o_{r,j}(k) = R_j(k) \qquad 253\text{-}4$$

$$o_{r,j}(k) \geq 0 \qquad 253\text{-}5$$

there is then a reward associated with the amount beyond the minimum streamflow requirement,[72] defined $z_{r,j}(k)$. Therefore,

$$q_e = -\sum_j \sum_k \left\{ z_{w,j}(k)\, w_j(k) + z_{r,j}(k)\, o_{r,j}(k) \right\} \qquad 253\text{-}6$$

The question now arises as to how these environmental performance measures q_e relate to the dollar operating performance measures of q_d. There must be an ecolo-economic index, $0 \leq \theta \leq \infty$, which relates the public preference for dollar to e.i.u. tradeoffs.[73]

$$Q = q_d + \theta q_e \qquad 253\text{-}7$$

72. Some references contend, e.g. reference (25) page 51, that there could be significant increases in the nitrogen levels of water that has spilled over dams. Since high nitrogen levels are a detriment to aquatic life, it might be that $s_j(k)$ could be penalized at those reservoirs where this is a problem.

73. A more complete description of the problems involved in calculation of this index can be found in reference (4) pages 10-12.

where Q is the total combined performance of the schedule.

It is obviously not an easy task to determine Θ, and for this reason, despite the extra computations required, it would be worthwhile to perform a sensitivity study of the changes in the schedule versus the parameterization of Θ. Even with just three of four values of Θ used, a transform curve of optimal q_d , q_e pairings could be approximated.

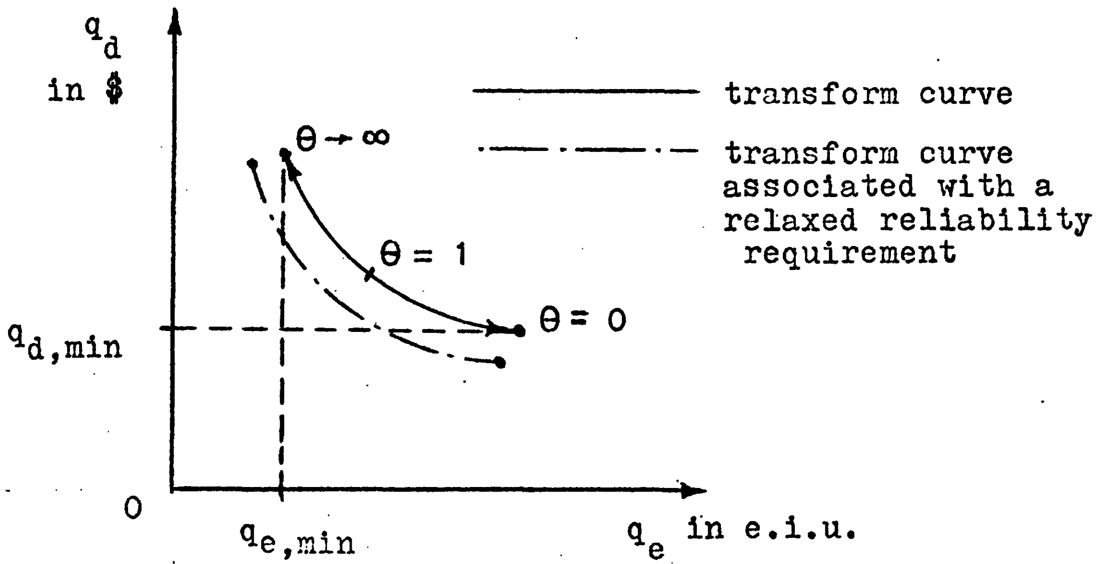

Figure 2.5.3 The tranform curve relating optimal q_d , q_e pairings associated with different Θ ecolo-economic weightings.[74]

The points $q_{d,min}$ and $q_{e,min}$ represent the absolute minimum attainable dollar cost schedule and environmental impact schedule, respectively.

The dot-dashed line in figure 2.5.3 represents the change

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74. Some preliminary studies, reference (52), on minimum versus economic dispatch of NO_x have yielded the concave toward the origin curvature represented here. The ramifications of this shape are obvious. This curve shape can be easily proved if a linear cost function such as used in this study is assumed. The key to the proof is in the convexity of the polytope.

of the transform curve which would result from a slackening of the reliability requirements.

2.5.4 Transmission Costs

For the sake of determining transmission costs it is reasonable to describe the load requirements as centered at several points around the region. This areal discretization and the resulting power flow equations required to generate an accurate transmission cost measure would unduly burden the production scheduling problem with additional variables. For this reason the approximation is usually made that the power demands from all sectors are summed and treated as a single demand, the transmission costs thus being ignored. This conglomeration presupposes an even distribution of generation facilities over the load area, even with a fraction of the plants not operating.

It is possible, and especially true in the case of proposed future off-shore nuclear generation facilities, that there can be a distinct transmission charge inherent in the use of a plant. In such cases, this cost can be considered as a 'nominal' operating expense, and as such, the network can be appropriately rewarded (or penalized) for the extent of non-use (or overextended usage) of that facility. Thus, the transmission cost is included in $y_{v,j}(k)$ of equation 251-2 or in $y_{u,j}(k)$ of equation 251-12.

2.6 Time Considerations

There are essentially two different problems associated with the choice of the time spans for this simulation. One involves determining how finely subdivided the intervals, or steps, through time should be - and what, if any, are the natural stepsizes. Secondly, the extent of the planning horizon must be determined. Despite the wide differences between these problems, they must be coordinated if the simulation is to be computationally feasible.

2.6.1 Basic Time Units

Deciding upon a basic time unit for the system is not a problem to be taken lightly. An obvious lower limit to the size of the interval is a one week time span. The pumped storage operating procedures cycle over a week, and more importantly, the load curves carry components which are distinctly cyclic over a week. Ideally, a simulation which could handle week intervals out to a two to five year horizon might be most desirable. However, this injects an enormous number of equations into the simulation, and thus presents a computational feasibility gap.

The upper size limit for the time unit falls in the four to eight week range (and possibly 12 weeks if the horizon is very far ahead). For intervals of larger size, most of the maintenance and refueling operations would fall within small fractions of that interval.

Although there has been no attempt presented in the literature a variable interval size seems most suited to this optimization. Within the first couple months when resolution is desired on the weekly time scale, a discrete time interval of one week is recommended. Then for several months, two week intervals could be used. Eventually three and four week time units are possible.

With variable time units the number of equations has been reduced considerably. We are also assured of ascribing equal time in the decision making process to equally massive blocks of information.

Some experience dealing with the variable time intervals has led to a feel for the tradeoffs involved and shows that the following general rule works well. It is quite reasonable to expand the basic time unit sizes in the future so as to always cover the maintenance windows with about three or four of these intervals. Thus, for maintenance close at hand and with only three or four weeks leeway an interval of one week is used; out at a year ahead with 16 week windows the interval would be four or five weeks.

It appears that the most desirable method of incorporating maintenance and other costs into larger intervals is not an average cost technique, but a summation of costs method.

The variable time size concept can be extended downward to include the unit commitment problem as well, viz. the first week of the schedule could be broken down into days and the first day into hours.[75] The reason for not attaching directly

75. One such gradual change of time intervals could start, for example, with intervals of 1 hour, 1 hour, 1 hour, 1 hour, 2 hours, 2 hours, 4 hours, 4 hours, 8 hours, 1 day, 1 day, 2 days, 3 days, 1 week, 1 week, etc.

the unit commitment problem to this one was that it had a number of problems best left uncoupled to the annual production schedule. For example, transmission costs, startup costs, minimum shutdown constraints, and so on are problems of great importance only to the hourly dispatch. Although these two problems are not spliced together in this study, it is still very advantageous to have a transition a smooth as this one, between these two problems.

2.6.2 Model Period Termination

Regardless of the extent to which the variable time units expand as years progress, it will obviously be necessary to terminate the simulation at some point. If this project is used to develop the schedule for the next two months, then the termination may come after only six or eight months. In any event, a final assessment is required measuring the desirability of the system at the termination of the model.

An example of the need for and evaluation of this system disposition at termination can be taken from the case of scheduling maintenance outages:

(1) An appropriate penalty must be assessed for schedules which push maintenance sessions past the end of the model period and thus out of the cost penalties of the performance measure.

(2) Schedules must be rewarded appropriately for leaving plants in good repair. This reward could be 'number of weeks from model termination to the center of the next projected window' times 'the average per week cost of the annual maintenance expense of the facility.'

(3) Penalties must be assessed for leaving the next projected maintenance window centered on a high load demand time in the future.

Some other examples of components of the terminal status inventory are the effects of the amount of nuclear fuel batch energy left, gas contracts partially fulfilled, and water reservoirs left at high or low levels.

In all cases, these termination disposition costs or rewards could be collected and totaled, or they could be incorporated within the interval model costs. For example, appropriate rewards can be granted maintenance sessions scheduled just before model termination and included in the $b_j^n(k)$ term of equation 252-1. If the costs are incorporated within costs in the model, as the horizon time is positioned farther in the future, these terminal disposition costs should clearly demarcate the terminal time at which their effect has now been included within the model period.

3. Optimization Techniques

At the outset it should be emphasized that this study encompasses only static solution techniques for arriving at the optimum production schedule. Only those dynamics which introduce themselves naturally into the implementation of the static technique will be included here. A second report is directed at dynamic techniques and a more probabilistic approach, and eventually, one chapter of the author's Ph.D. dissertation will reconcile the different approaches into a single technique.

Looking at the production scheduling problem from the most general viewpoint:

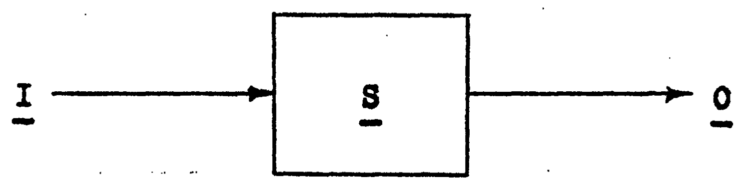

Figure 3.-1 Block diagram of most general system

Here O is the schedule generated to meet the demand for electricity at the acceptable reliability level and the optimum quality level of system performance. I is the set of constraints or the ranges of the operating procedures with which the system may function. S is the capability and description of the system. Now suppose figure 3.-1 is rearranged so that the acceptable reliability level and the demand constraints are endogenous, then the quality of performance Q can be the system output. Similarly, the constraints on the range of operating procedure can become state constraints of the system, and the input can represent the decisions $\underline{U}$ which are made about the schedule.

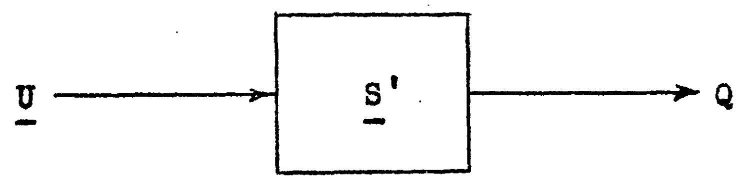

Figure 3.-2 System with decision inputs and quality level outputs

Perhaps the most difficult portion of the entire problem is the complexity of the set of all feasible inputs to the system. Some of these decision processes require the choice of elements from a set, some decisions are continuous, and all are limited to finite sets or ranges of activity.

To cut down the number of variables involved in the scheduling process it can be conceded that a certain amount of dynamic formulation will be necessary. One possibility for including dynamics is realized by breaking the entire problem into an evolving sequential decision process,[76] treating at each iteration the most important decisions left to be made.

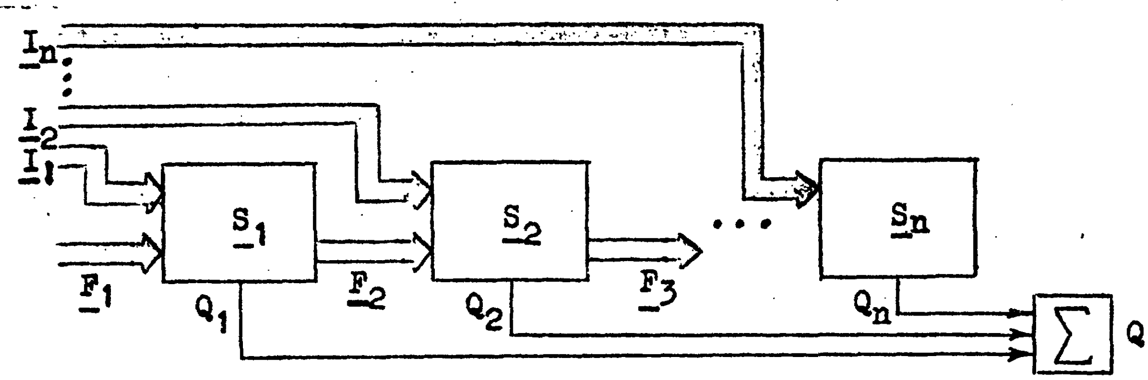

Figure 3.-3 Sequential decision process

76. See reference (53) for a description of the dynamics of sequential decision processes.

$\underline{S}_i$ now represents the system characteristics pertinent to the i^{th} group of decisions to be made (and may have a very limited horizon), i.e. the i^{th} decision field. $\underline{F}_i$ represents the set of all decisions fixed previous to the i^{th} step $\underline{S}_i$. $\underline{F}_1$, thus, signifies the system decisions already fixed at the beginning of the simulation.

It is possible to gain some feel for the sequential decision process by considering the $\underline{I}_i$ to be the decisions to be made about the schedule over some initial time span. After the first decision field, $\underline{I}_1$, has been fixed and its inherent quality measure Q_1 passed on to the total schedule performance, the second decision field $\underline{I}_2$ is considered. Using the immediacy of the decisions time as a measure of the decisions importance is a naîve concept, but valid to a certain extent. The crucial point in preparing an optimal, rather than a suboptimal, schedule requires, essentially, the breaking of the systems $\underline{S}_i$ into cleanly uncoupled portions.[77] The principle of optimality[78] assures us that optimizing the quality Q_i of each cleanly decoupled $\underline{S}_i$ will result in the total optimization of the entire problem. But a dimensionality problem arises, for to consider exactly all ramifications of decisions made in a subsection $\underline{S}_i$ would require essentially the use of all the system's variables. Thus, apparently no dimensionality gain arises unless a suboptimal solution is acceptable (or as it turns out, actually more advantageous).

77. An infinite horizon Markovian process results from the complete separability of the components of the chain.

78. See reference (54) page 313.

3.1 Possible Approaches to Optimization

Obviously, the problem setup has a great effect on the eventual optimization technique chosen. It is instructive to discuss the reasons for rejecting some approaches to defining the system states or different performance measures, but most were obviously not suitable, for example, defining as $x(k)$ the time of refueling of the k^{th} facility. There were, however, too many ideas rejected to go into them all.

A possibility for future work might be the use of a quadratic plus linear performance criteria

$$\min \left(q'u + u'C\,u \;\middle|\; A\,u \leq b \right) \qquad 31\text{-}1$$

This technique would be helpful for handling in a simple manner the maintenance intersession constraints, i.e. by heavily penalizing $u_j(k)\cdot u_j(i)$ terms where k and i do not leave an acceptable intersession span. This criteria was rejected due to the increased computational complexity and the relatively poor convergence rate[79] of quadratic programming techniques.

3.1.1 Different Methods

Dimensionality takes a heavy toll of techniques at the outset of this exploration of feasible solution techniques. Search techniques, including gradient searches, either do

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79. See reference (55) page 78. Reference (56), page 529, deals with a GRG program for a somewhat faster convergence, however, for reasonable results the number of variables and constraints are limited to about 50 apiece for results in less than 30 seconds. The search for integers would then multiply this 30 seconds to a substantial amount of time.

not handle constraints will, or become computationally infeasible due to the large number of dimensions inherent in this problem.[80] The complexity involved in nonlinear programming, geometric programming and quadratic programming[81] are not necessary because the problem has been kept in a linear form.

The large optimization area that remains is linear programming.[82] The format of the model is obviously that of the linear programming area, viz.

$$\begin{aligned} Q &= \underline{b}\,(\underline{u}+\underline{v}) \\ \underline{A}\,(\underline{u}+\underline{v}) &\leq \underline{m} \\ \underline{0} \leq \underline{u} &\leq \underline{1} \qquad u_i = \text{integer} \\ \underline{0} \leq \underline{v} &\leq \underline{1} \end{aligned} \qquad 311\text{-}1$$

In particular, since $\underline{u}$ is a vector of integers, this is a mixed integer programming problem.[83] More specifically, since the integers are all binary, this problem is called a mixed bivalent problem. The additional restrictions imposed by the maintenance coordination equations and the crew coordination equations, viz.

$$\sum_j u_j = 1 \qquad 311\text{-}2$$

make this, for the most part, what has become known as a mutual exclusivity, or multiple

80. See chapter 6 of reference (57).

81. See chapter 6, sections 2, 3 and 4 of reference (58).

82. For a basic introductory text on linear programming see reference (59).

83. Mixed integer programming was introduced in reference (60).

choice, problem.[84]

Tremendously efficient programs exist for solving multiple choice type problems, and this is the main reason the optimum production schedule was more or less forced to accept this configuration. Bivalent (or pseudo-Boolean) programming is itself so efficient that integers are often converted to binary numbers to take advantage of the efficiency of the bivalent techniques.[85]

So, the choice of optimization procedure was helped along partially by the faults of the rejected schemes, but was aided considerably by the positive attributes of the mixed bivalent technique (with its associated dual problem of appropriate measures of system tendencies as will be explained in section 312).

3.1.2 Mixed Integer Programming

One of the main reasons for choosing mixed integer linear programming as the optimization technique was because of the need for the quantities represented by the dual problem. The dual problem can not be fully explained in this paper, and in any event is described in almost every text on linear programming. Mixed integer programming techniques are also explained in many texts,[86] but since a certain amount of

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84. Multiple choice problems were first introduced into the literature in conjunction with mixed integer programming with reference (61) in 1964.

85. See reference (62) page 75.

86. See for example reference (63).

knowledge of solution techniques and dual spaces is crucial to an understanding of this paper, a very short introduction will be presented.

The general linear programming problem takes the form

$$\underline{a}_i \, \underline{u} \geq m_i$$
$$\underline{a}_j \, \underline{u} = m_j$$
$$u_k \geq 0 \qquad\qquad 312\text{-}1$$
$$\text{minimize} \quad Q = \sum_k b_k \, u_k$$

and the equivalent symmetric dual relationships are

$$\underline{x} \, \underline{a}_k \leq b_k$$
$$x_i \geq 0$$
$$x_j \text{ arbitrary} \qquad\qquad 312\text{-}2$$
$$\text{maximize} \quad w = \sum_i m_i \, x_i + \sum_j m_j \, x_j$$

At the outset the system is assumed to be nonredundant. The constraints and equalities imposed on $\underline{u}$ will now section off a portion of n-space within which $\underline{u}$ may vary and still satisfy those constraints. This sectioned off space is called the activity space, or space of all feasible solutions. It may be visualized as a portion of n-space cut off by hyperplanes, each defined by a constraint equation.

These hyperplanes support what is variously termed, besides activity space or space of feasible solutions, a convex hull, convex polyhedra, or polytope. This polytope can, and usually is, found by solving an appropriate set of equalities. This is possible once the inequalities are replaced by equalities

$$a_1 \underline{u} + u_{ei} = m_i$$
$$u_{ei} \geq 0 \qquad 312\text{-}3$$

where u_{ei} is called a slack variable.

Once the slack variables are introduced the system becomes an underspecified system of equations, and is constrained now only by the orthant conditions $u_i \geq 0$, $u_{ei} \geq 0$.

For a linear system the solutions to equation

$$Q = \sum_k b_k u_k \qquad 312\text{-}4$$

as Q is parameterized, represent a family of parallel hyperplanes in R^n, the n-space. The hyperplane in R^n associated with the optimal Q can be seen to be a supporting hyperplane of the system. For, if the Q_{opt} hyperplane truncated the polytope, activity normal to that hyperplane in the direction of better performance could be found within the polytope.

From the above argument it is possible to see that there always exists an optimal solution on some corner, i.e. extrema, of the activity space.[87] A great many of these corners of the activity space will, however, be at 0 or 1 decisions of bivalent variables. This fortuitous phenomon makes likely several integer decisions even before the integer requirements are imposed upon the solution!

It should be pointed out that in linear programming the

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87. Even if the optimal Q supporting hyperplane coincides with a face of the polytope, this face must necessarily also include corners.

insurance that an optimal solution will result comes only when problems are proved to provide convex activity spaces.

The simplex method results from following the edges of the activity space represented by the tight constraint equations 312-3. Because the simplex method never moves along an edge of the polytope which reduces performance, and because it avoids repeating any paths (cycling), it assures optimality. Typically, the simplex method converges after following approximately a number of edges about equal in order of magnitude to the number of nonorthant constraint equations.

After the optimal linear solution is gotten, the optimal integer solution is obtained by truncating out of the feasible space any fractional operating points.[88] There is a great variety of techniques available for making these cuts- ranging from Gomory's cuts to essentially branch and bound techniques. Different methods, obviously, have different types of problems for which they are especially suitable. The scheduling problem considered in this paper is best handled by the 'branch and bound' or 'separation and evaluation procedure' SEP[89] technique.

The symmetric dual to the primal for the problem attacks the optimization from the opposite end, as can be seen from equation 312-2, and actually creates lower limits to the optimal solution to go with the primal's upper bounds. Most computer programs for linear programming use the solutions to the dual system in their formulations, and so, fortunately

88. See for example reference (64)

89. See reference (56) page 419, or reference (65).

all this valuable dual information is readily available in the existing program 'packages.'

Specifically because the simulation is being torn into decision fields it is necessary to have a measure of the propensities to change, i.e. costliness and well or bad-fitting tendencies, of variables surrounding the decision field's time span. Those outside variables which are greatly sensitive to the current decision time range will be included in that decision field. Thus, decision fields will cover a span of time, but will also have a fuzzy edge, or fringe, caused partly by direct coupling, or linkage, of variables to those just outside the decision field's time block. In addition, partly adding to the fuzziness, those decisions just barely[90] connected to the fringe of the decision field will be included in that field if they represent still very 'jittery' not clearly resolved decisions from the immediate past, and very 'crucial' future considerations. Here, 'crucial' and 'jittery' are terms referring to the dual activity, or closeness to the optimum supporting hyperplane, as measured from a broad linear programming overview.

To see exactly what the dual activity means to the scheduling problem, consider a simple sample power plant scheduling problem.

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90. If the decisions are not at all connected to the fringe, that is, separable or non-interacting with the present decision field, then, in the past, they can be set at the decision value which minimizes Q, or in the future they can be dropped and considered in following decision fields with no effect (but not necessarily left out of the linear programming planning horizon, as will be explained in section 3.2.2).

Consider only the simplified problem where the intensity a_{ij} represents the megawatt capacity of the j^{th} plant in the i^{th} interval if that plant were to be shutdown. That is, a_{ij} represents a loss of capacity, a negative number; x_j corresponds to the extent of shutdown of plant j, d_i the amount (negative) of shutdown available, i.e. demand minus total system capacity, and c_j as the per unit cost of x_j. Then,

$$\sum_{j=1}^{n} x_j \underline{a}_j \geq d_i \quad i = 1,\ldots,m$$

$$x_j \geq 0 \quad j = 1,\ldots,n \qquad 312\text{-}5$$

$$\text{minimize} \sum_{j=1}^{n} c_j x_j = z$$

Now for the interpretation of the dual, the dual form is

$$\sum_{i=1}^{m} u_i a_{ij} \leq c_j \quad j = 1,\ldots,n$$

$$u_i \geq 0 \quad i = 1,\ldots,m \qquad 312\text{-}6$$

$$\text{maximize} \sum_{i=1}^{m} d_i u_i = w$$

The solution $\underline{u}$ of the dual problem may now be interpreted as the set of per unit shutdown prices associated with the intervals 1,...,m for the system.

If now the maintenance inequalities $u_j(k) \leq 1$ are introduced into the linear program, they too have physically interpretible dual activities. These activities show the per unit cost of an additional unit shutdown, if it were possible, of $u_j(k)$.

In other words, the dual cost is the difference between the cost of $u_j(k)$ and the cost level of the optimal supporting hyperplane of the polytope. Therefore, unless the cost level of $u_j(k)$ is changed by the amount represented by its dual activity, then the optimal basis, i.e. the maintenance schedule, will remain unchanged.

With the dual measuring the propensity to change of the different decisions in the maintenance schedule, it becomes the ideal tool for studying important future decisions and uneasy past decisions. Thus, the dual is an evaluation tool for determining which variables should be included within the decision field at any step in the quasi-optimal program.

3.2 Quasi-optimal Programming

This problem points out the need for a type of scheme for solving very large, more or less multiple choice type mixed bivalent problems. If it were not for the extreme efficiency that a well ordered multiple choice problem enjoys due to its special form, this form could be destroyed and the problem's matrix could be partitioned horizontally and vertically using the decomposition principle of Dantzig and Wolfe.[91] The large amount of special column linking, viz. equation 311-2, however, makes attempts at partially block angularizing the matrix extremely destructive to the multiple choice ordering.

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91. This is essentially a diakoptic technique. For more information on this decomposition principle from a simplified point of view see reference (63) page 212. Reference (66) contains general extentions of this decomposition technique.

The quasi-optimal technique postulated takes into account a certain amount of horizontal matrix decomposition into decision fields (corresponding more or less to decomposition by time spans), but also uses the dual activities of the decisions within linkage distance of the edge of the decision field as a mechanism to rate the concern due these 'fringe' decisions.

So, in summary, this quasi-optimal sequential technique makes decisions within each decision field based on the effect its decisions have on the total system. The program eventually passes over all the decision fields in the whole space, with decision fields overlapping to the extent that there is coupling between the fields.[92] Assignment of variables to any decision field is done primarily based on the time of the decision, and direct coupling to components within the decision time span, and secondarily, based on sensitivity studies.[93]

3.2.1 Adaptation of Model

It may not be altogether clear at this point how the model developed in chapter 2 is to be changed so as to make the quasi-optimal technique applicable.

First, recall that the economic shutdown variables are constrained by

$$v_j(k) \leq 1 \qquad \text{all } j, k \qquad 321\text{-}1$$

92. A measure of the amount of coupling between fields can be visualized as similar to the density measure of the matrix.

93. Obviously, both these methods are sensitivity studies, because direct coupling of components will show them to be extremely sensitive to each other.

The maintenance variables are constrained by range, coordination of maintenance and crew coordination, and by district minimum constraints

$$u_j(k) \leq 1 \qquad \text{all } j, k \qquad 321\text{-}2$$

$$\sum_{k \in F} u_j(k) = 1 \qquad \text{all } j \qquad 321\text{-}3$$

$$\sum_{\substack{\text{all } j \text{ plants} \\ \text{assigned to crew } i}} u_j(k) \leq 1 \qquad \text{all } i, k \qquad 321\text{-}4$$

$$\sum_{j \in D_i} u_j(k) \leq m \qquad \text{all } k \qquad 321\text{-}5$$

also

$$u_j(k) - v_j(k) \leq 0 \qquad 321\text{-}6$$

Maintenance intersession constraints take the form of

$$\sum_{k \in F_1} u_j(k) + \sum_{k \in F_2} u_j(k) = 1 \quad \text{all } j \qquad 321\text{-}7$$

Different maintenance options, as explained in section 2.3.1.1 take the form of

$$\sum_{k \in F^1} u_j^1(k) + \sum_{k \in F^2} u_j^2(k) = 1 \quad \text{all } j \qquad 321\text{-}8$$

Other components of the v and u types are the extended capacities $v_x \leq 1$, contract agreements x_j which can be called $u_c \leq 1$, and $v_c \leq 1$.

So if $\underline{u}$ is the vector of all the u's and $\underline{v}$ the v's then the equations become (if the u_e slack variables are added):

$$\underline{\Gamma}_1 \begin{bmatrix} \underline{v} \\ \underline{u} \end{bmatrix} = \underline{\gamma}_1 \qquad 321\text{-}9$$

$$\underline{\Gamma}_2 \; \underline{u} = \underline{\gamma}_2 \qquad 321\text{-}10$$

u_i are all binary integers

where $\underline{\Gamma}_1$ and $\underline{\Gamma}_2$ are matrices of zeros, ones and minus ones, and $\underline{\gamma}_1$ and $\underline{\gamma}_2$ are given constraints.

Defining as $\underline{c}$ a large vector of continuous decision variables such as gas usage, nuclear usage, and hydro and pumped storage variables, then there exists a matrix $\underline{\Gamma}_3$ and vectors $\underline{\gamma}_3$ and oversupplies $\underline{o}_1$ and $\underline{o}_2$[94] such that

$$\underline{\Gamma}_3 \, \underline{c} + \underline{o}_1 - \underline{o}_2 = \underline{\gamma}_3 \qquad 321\text{-}11$$

Defining $\underline{A}$ as the matrix of appropriate capacity factors (along with many appropriate zeros), and $\underline{m}$ as the maintenance availabilities for base loaded and cycling levels at each interval, and $\underline{o}_3$ as the oversupply, then

$$\underline{A} \begin{bmatrix} \underline{u} \\ \underline{v} \\ \underline{c} \end{bmatrix} + \underline{o}_3 = \underline{m} \qquad 321\text{-}12$$

For appropriate $\underline{b}$, $\underline{y}$ and $\underline{z}$ vectors, the quality function becomes

$$Q = q_d + \theta q_e = \underline{b}' \begin{bmatrix} \underline{u} \\ \underline{v} \\ \underline{c} \\ \underline{o}_1 \\ \underline{o}_2 \\ \underline{o}_3 \end{bmatrix} - \underline{y}' \begin{bmatrix} \underline{u} \\ \underline{v} \\ \underline{o}_3 \end{bmatrix} + \underline{z}' \begin{bmatrix} \underline{u} \\ \underline{v} \\ \underline{c} \end{bmatrix} \qquad 321\text{-}13$$

This then defines the entire problem in vector form. It can be seen that indeed the system is linear[95] if the integer contraints on the elements of $\underline{u}$ are relaxed.

Crucial to the efficiency of the solution procedure is the ordering of the variables which must be integers. After the static optimization technique is completed, i.e.

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94. The nuclear equation, since it is unconstrained, requires two oversupply variables as previously explained, see equation 251-8.

95. Linearity obviously implies convexity of the polytope of feasible solutions, so the proof of convexity required to insure optimality is omitted.

the linear program, a searching technique is used to find the optimal solution with the integers constrained. For this searching technique to operate most effectively the strongest contingencies should be considered first. Therefore, it is best not to separate the decisions associated with any single maintenance window. The various plants coupled by crew, equipment or geographic constraints should be kept together as much as possible. Finally, the variables concerning the supply of power in the same time intervals should be kept approximately together.

3.2.2 Quasi-optimal Solution of the Scheduling Problem

This section deals with the procedure used to obtain a sequential quasi-optimal solution to the production scheduling problem.

The first procedure is the resolution of the time problems, i.e. the basic units and the horizon time. As previously explained, section 2.6.1, the interval size is chosen so that the maintenance windows use three to four intervals. The use of typical models has shown that horizon times for each decision field's linear considerations need not exceed 36 weeks. Once in operation it is relatively simple to observe whether or not the time to the planning horizon should be shortened or lengthened, by the amount of scheduling activity in the last interval.

Because of the type of matrix configuration the production schedule yields, the basic nucleus of each decision field could typically be set at 6 to 8 week time spans. The fringe of the decision field is built mainly around the column linkages, that is, the direct coupling effects of maintenance windows at a single plant.

The first step in the solution of the system is then to set as integers those maintenance decisions which fall within the first decision field. The other integers in the 36 week future (or possibly as short as 18 weeks) are allowed to slide around as continuous variables. The mixed integer program is then solved as the first evolving step in the decision process.

Several suboptimal (as well as the optimal if possible) solutions of this first mixed integer problem should be found. In this way it can be determined which decisions are on shaky grounds, and available for immediate use will be a list of several schedule options along with their associated performance levels. The solution to the dual of the original linear program will give an indication of which decisions are and are not obvious, and a measure of their propensities to change.[96] This dual solution should thus be used to decide which decisions should be incorporated in the adjacent decision fields, future and past, as well as in their own time span's field.

The second decision field may or may not overlap the first, but in any event should include past and future decisions which hold a high propensity to change.

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96. This is described in section 3.1.2.

The second evolving step[97] may or may not include an extension of the horizon time and/or a refinement of the basic time units. These adjustments will be dictated by the accuracy expected of, and reason for using, this scheduler.

In any event the second evolving step is solved by fixing the solid (i.e. not jittery) integer decisions from the first step and constraining to integers those which belong to the second decision field.

The rest of the iterations proceed in the same manner until the planning horizon has been covered.

3.3 Post-optimal Analysis

By post-optimal analysis is meant the variety of techniques used after the optimum is found, such as parameterization of variables and constraints, and analysis of the neighborhood of an optimum for sensitivities, or constrained stresses and tendencies of the solution.

Because of the linear setup of this problem, the post-optimal analysis possibilities are nearly limitless. Consider the parameterization of the 'dollar to environmental impact' tradeoff θ.

$$Q = q_d + \theta q_e \qquad 33\text{-}1$$

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97. It should be noted that the scheduling process could be terminated with the first evolving step, the second evolving step being performed when it is actually needed in real time. This method, although it would save on the computation effort, is not recommended especially if several of the decisions have large propensities to change.

It is possible to let θ slide from 0 to ∞ for the linear programming problem and watch the changes in the scheduling activity at each θ. There are critical ranges in θ associated with each of the scheduling variables, below and above which the decision is clearly determined (these ranges will include the theta equals zero or infinity endpoints for some of the variables).

Another possibility involves determining schedules, in whole but more likely in part, associated with several discrete mixes. For example, $\theta = 0$ would be the minimum dollar cost schedule, $\theta = 1/3$, $\theta = 1$ would be the equal mix schedule, $\theta = 3$, and $\theta \to \infty$ would be the minimum environmental impact schedule.

A certain amount of the information about changes in θ could be read directly from the dual activity, i.e. as the ecological impact costs are increased the dual activity will indicate which variables are likely to change i.e. those in the current basis, which will start coming into concern i.e. those whose cost is closing in on the optimum hyperplane, and which will become more firmly committed decisions i.e. those retreating from the current basis.

Another parameterization which is important is the variation of the reliability level, or, to be treated in the same manner, the possibility of variations from the predicted demand levels. Again either the original linear problem can be parameterized by these changes, or new schedules can be formulated. This parameterization is affected by varying the maintenance availability vector m in equation 321-12 by some scalar multiple of a new vector. Again the effects of variations in m and Q simultaneously can be predicted from the dual activity of the optimum continuous solution or they can be studied by creating several schedules. The need for this simultaneous parameterization can be demonstrated by considering the effects of temperature variations. An increase in the ecological costs for warmer water systems is the effect of temperature on Q, and the temperature effect on m is twofold, increasing the load and decreasing the generation capacity.

Although there are many more possibilities for post-optimal analysis, one in particular is of significance. This procedure, sometimes called 'ranging the optimal solution' determines the range over which each of the variables might travel without changing the optimality of the basic schedule, or alternatively, without breaking the feasibility[98] of the schedule.

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98. Feasibility implies that none of the system's constraints are violated.

4. Application to a Regional Scheduling Problem

There are enough peculiarities in this problem, e.g. its multiple choice characteristics[99] and the bivalent nature of the integer variables, that a prediction of the desirability of this technique, in particular the computer time required to reach a solution, would be very difficult without a sample simulation.

This section, therefore, deals with a simulation of this quasi-optimal solution technique. However, since at this stage of the rest of the work being done on the entire energy-environment dilemma much of the data is presently unobtainable, the simulation in this section will be carried out only insofar as it is necessary to evaluate the solution technique.

Evidently, a full scale mockup of this problem with data will be included in the author's Ph.D. dissertation.

4.1 Available Subroutines

Because linear programming is such a widely used and well defined problem most companies selling programs, and in fact many industries, have available linear programming packages.

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99. Another characteristic inherent in this problem is its close similarity to the zero-one knapsack problem, see reference (67). The zero-one knapsack problem involves making yes-no decisions about the inclusion or exclusion of certain sized items in a multidimensional container so that certain dimensions are not oversubscribed (analogous to maintenance availability) and so that the desirability of the decided collection is maximized. Since this type of problem reaches its optimum quickly one might speculate that the proposed scheduler would also be quickly run.

In 1965 such programs as 'Capline' at IBM, 'Bettina' at Shell, 'LP 90' at C.E.I.R.[100] and 'Ophelie' at CDC had reached the 1500 constraint capability level. Further improvements in efficiency, flexibility and size capabilities since then have made possible the solution of problems of astronomical size.[101] For an idea of solution speed, a problem with 1000 constraints can be solved in about 6.2 minutes on only a 300K byte storage system.[102]

It would be possible, but laborious, to solve the scheduling problem in its presented form using linear programming alone (using many of the indicators in the dual space). There are, fortunately, almost equally as many mixed integer programs as there are linear programming packages. Because the mixed integer programs have such widely differing solution techniques a word is necessary here about the suitability of the various methods with respect to the scheduling problem.

All solution methods are identical in their first step, which is the optimal solution of the linear programming problem with integer constraints ignored. Methods particularly suitable to widely ranging integer variables now introduce elaborate cuts which slice away from the polytope the fractional components of any integer variables which do not assume integers in the

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100. See reference (63) page 137.

101. The number of variables is virtually unlimited, the number of constrained equations being dependent upon the size of the memory core available. For an approximate size figure, a computer with 933K data bytes can handle a problem with 16,300 constraints.

102. See reference (68) page 25.

optimum continuous solution.

For closely constricted integer ranges a different procedure, the separation and evaluation procedure, SEP, is more suitable. In the SEP the range of any integer assuming non-integral values in the optimum linear solution will be divided into two new ranges, omitting the unit interval containing the fractional optimum solution. Thus, two subproblems result

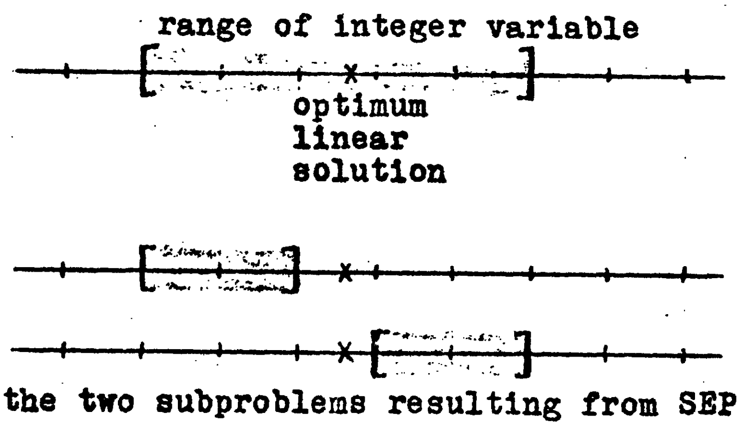

Figure 41-1 Range partitioning in the separation and evaluation procedure

and the first branch in a tree is formed. The two subproblems are then solved separately as linear programming problems, and if necessary further branching results. The point at which each branching is performed is called a node, and as is obvious, the performance index of nodes further down a tree, i.e. further restricted, cannot be better than the values of the previous nodes in its section of the tree.

Thus, in SEP, the optimum is proved once an integer solution is obtained, and all other branches of the tree have been chased down to nodes of qualities worse than the best integer solution's performance quality.

To say that this is the full extent of SEP is to be grossly misleading. In particular, there are schemes for estimating the further cost, called pseudo-cost, of travelling down a branch before hitting an integer solution. Thus, the most sophisticated forms of SEP can at any point in the procedure decide which branches show the best promise for high quality integer solutions. From this capability they can set up orderly candidate sets of waiting nodes and explore them systematically.

This type of a search technique is a particularly excellent choice for bivalent, or pseudo-Boolean, integer problems. Efficiency is further increased by the multiple choice characteristics of the scheduling problem. To illustrate this point consider the example of choosing one interval at which time to initiate maintenance out of a window of say 5 such intervals.

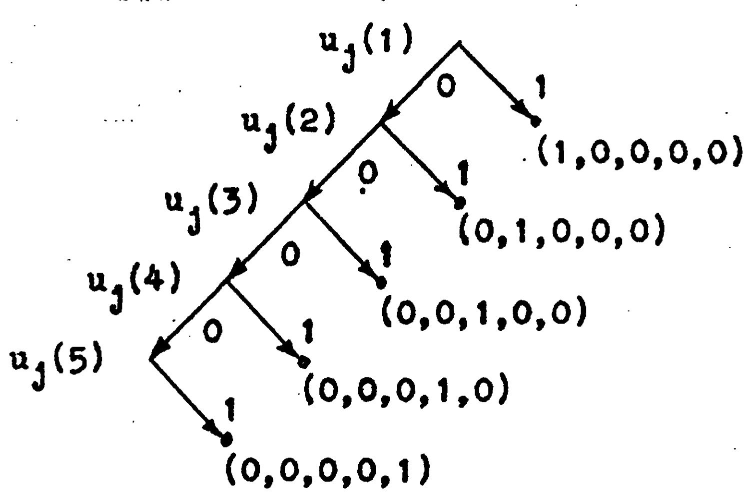

Figure 4.1-2 Separation and evaluation procedure for multiple choice problems

Thus a tree which even in a bivalent SEP problem might have created 32 nodes, uses only 5. Compound this gain by 'multiple choice'-type efficiency inherent in crew assignment, and it can be seen that the SEP method is ideally suited to the scheduling problem.[103]

Although others exist, the author is familiar with two SEP type mixed integer programs, the OPHELIE MIXTE[104] written for the CDC 6600 and MPSX-MIP[105], i.e. Mathematical Programming System Extended - Mixed Integer Programming written for IBM OS/360.[106]

4.2 Mathematical Programming System Extended - Mixed Integer Programming

IBM's MPSX-MIP has been available since February 1971. It has the capability of handling 4095 integer variables (although a reasonable limit is much less).[107] All of the post-optimal analysis techniques described in section 3.3 are available as options within the framework of MPSX-MIP, as well as routines for fixing integer solutions at any point in the solution procedure (MIXFIX) for sensitvity analysis. The activity of the dual space is also available for stress, sensitivity and tendency studies.

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103. The zero-one knapsack characteristics, as explained in footnote 99 add further efficiency to this technique.

104. See reference (56) page 419.

105. See reference (69).

106. The author is indebted to Nancy H. Bell of the State Street Bank in Boston for her help on the interpretation of the Job Control Language, JCL, used by the supervisor OS/360.

107. See reference (68) page 23.

MPSX-MIP will yield an ε-optimal solution if the optimum is not available.[108] Options are available for starting the search only after nodes of a certain minimum quality are reached, obtaining a fixed number of integer solutions and then terminating (possibly before determining the optimum), finding all solutions with a quality better than some pre-determined level, and a great many other possibilities.[109]

4.3 Sample Schedules and Examples

The thoroughness of this section will be something less than required to reproduce the examples given, but hopefully enough so that the techniques and programs covered can be understood.

It should be pointed out that the examples entered in this section do not represent real systems, but are meant only to be representative of the forms of the problems to be encountered in the scheduling process.

4.3.1 Quasi-optimal Performance Validity

For the sake of testing a quasi-optimal procedure a problem was devised requiring the scheduling of the maintenance of 12 power plants within a 39 week period. The size of the problem was kept small so that the solution to the entire pure optimum could be found for the sake of comparison.

The description of the problem in its entirety is not presented. A computer program and the data used are listed

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108. OPHELIE MIXTE also yields ε-optimal solutions, ref. (56) pg. 421

109. For other possibilities the reader is referred to ref. (69).

in Appendix B. Briefly: crew one is in charge of the maintenance of fossil plants: plant 1 of 225 megawatts, plant 2 of 125 megawatts, plant 3 of 150 megwatts and plant 4 of 350 megawatts. Crew two is in charge of the nuclear plants: plant 5 of 550 megawatts and plant 6 of 600 megawatts. Crew three maintains a very large pumped storage facility, plant 7, of 75 megawatts, and two 100 megawatt hydroelectric plants 8 and 9. Crew four maintains three plants of a mainly cycling nature: plants 10 and 12 both 85 megawatts and plant 11 of 100 megawatts. Cycling capabilities are defined for all the plants.

As an example of how the maintenance is handled consider plant 1. Plant 1 can begin its four week maintenance session at the first week of the simulation, unit one interval one: U0101 = 1, or at the second week U0102 = 1 or at the third week U0103 = 1. Plant 11, for example, has the option of starting maintenance anywhere from week 8 to 16: U1108=1, or U1110 = 1, or U1112 = 1, or U1114 = 1 or U1116 = 1. The rest of the plants also have similar windows, with the exception of unit 8 for which maintenance is optional, and unit 4 which has two windows in the next 39 weeks.

Some interregional power contracts are also included in this sample scheduling. UCB06 for example will equal 1 if the contract to buy power in the 6th week should be honored (200 megawatts of steady power, i.e. no cycling capabilities, at a cost of $20,130.) Power sales are also included, e.g. UCS10 represents a contract to sell 100 megawatts from the

10^{th} week to the 14^{th} week.

Variable contracts are also considered, and VCB04 for example represents the fractional amount of the 80 megawatt contract in the 4^{th} week that should be purchased where the price per megawatt used will be $8,230/80.

Economic shutdowns are also considered in this sample problem, for example plant 8 in week 6 = V0806. These shutdown variables are limited to only the most inefficient plants, and their shutdown rewards the schedules according to the amount of time these plants do not have to be operating to meet the load.

The dollar-environmental cost mix has already been decided for this sample schedule, and Q represents the cost (or reward if negative) associated with each unit of the system variables.

The maintenance availability is presented in megawatts for each interval, i.e. the difference between the system capacity in that interval and demand level which must be met to insure the chosen system will meet demand to the extent of the reliability level (see section 2.4.2). The availability for maintenance of the system's cycling capability is also introduced (the cycling constraints are ignored in the far future, i.e. from week 27 to week 39). A plot of the megawatt maintenance availability curve used in this sample problem is presented in figure 4.3.1-1, page 104.

To rigorously test the validity of the quasi-optimal sequential technique no overlapping of intervals was allowed, and the dual was not considered to include shaky decisions

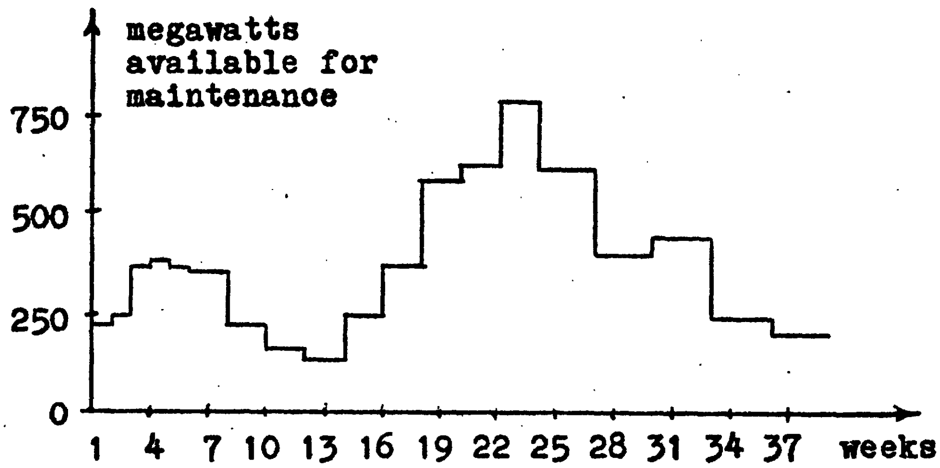

Figure 4.3.1-1 Maintenance availability curve used in the sample simulation

in the decision fields of neighboring time spans. So except for keeping the maintenance windows intact, the decision fields were built strictly from time spans.

The first decision field included the first twelve weeks which meant deciding on the maintenance sessions for plants 1, 2, 4, 7, 8(optional maintenance), the front slice of the long window of 11, and two power contracts UCS10 and UCB06. In Table 4.3.1-1 are two possible schedules over this decision period.[110] Node 5 was proved to be optimal, at $168,359.90 , more than $5,000 better than node 7.

The second decision field was formed after fixing the optimal values in field one, as well as fixing U1114 = 1 and U1116 = 0, two values that were identical to the two schedules. Decision field two covered approximately week 12 to week 24 and resulted in the scheduling of the decisions in table 4.3.1-2.

With only $115.30 difference between the optimal node 5 and node 4 this would have to be considered a 'jittery' decision.

110. Options are available for creating at least a minimum number of schedules, say 5, for each decision field.

NODE	7	5
FUNCTIONAL	173.4502	169.35[illegible]
114= U0101	.	.
115= U0102	1.0000	1.0000
116= U0103	.	.
117= U0206	.	.
118= U0208	1.0000	.
119= U0210	.	1.0000
120= U0401	.	.
121= U0406	1.0000	1.0000
122= U0706	.	.
123= U0708	.	1.0000
124= U0710	1.0000	.
125= U0804	.	.
126= U1108	.	.
127= U1110	.	.
128= U1112	.	.
129= UCS10	1.0000	.
130= UCB06	1.0000	1.0000

Table 4.3.1-1 Alternative schedules for the first decision field

NODE	4	5
FUNCTIONAL	205.5789	205.4636
121= UCB14	.	.
122= UCB16	1.0000	1.0000
123= U0516	1.0000	1.0000
124= U0518	.	.
125= U0520	.	.
126= U0620	1.0000	1.0000
127= U0622	.	.
128= U0624	.	.
129= U1022	.	1.0000
130= U1024	1.0000	.

Table 4.3.1-2 Alternative schedules for the second decision field

However, in this rigorous test it was instructive to fix the optimal values and proceed.

The third and last decision field[111] accounts for all the remaining decisions and proceeds to find six integer solutions before node 7 is definitely established as the optimum.

NODE	21	24	7
FUNCTIONAL	220.5332	222.5923	212.4042
ESTIMATION	INTEGER	INTEGER	INTEGER
114= U0924	1.0000	.	1.0000
115= U0927	.	1.0000	.
116= UCS24	1.0000	1.0000	1.0000
117= U0322	.	.	.
118= U0327	1.0000	.	.
119= U0330	.	.	1.0000
120= U0333	.	1.0000	.
121= UCS27	1.0000	1.0000	.
122= UCS30	.	.	1.0000
123= U0427	.	.	1.0000
124= U0430	1.0000	1.0000	.
125= U0433	.	.	.
126= U0436	.	.	.
127= U1227	1.0000	1.0000	.
128= U1230	.	.	1.0000
129= U1233	.	.	.
130= U1236	.	.	.

Table 4.3.1-3a Alternative schedules for the third decision field

111. The horizon time was not extended into the future, nor has more resolution been introduced into the later intervals of this problem because an optimum to the total span was to be calculated as a basis for comparing the pure optimum with the quasi-optimum.

NODE	15	18	20
FUNCTIONAL	214.3999	213.5933	220.5507
ESTIMATION	INTEGER	INTEGER	INTEGER
114= U0924	.	1.0000	.
115= U0927	1.0000	.	1.0000
116= UCS24	1.0000	1.0000	1.0000
117= U0322	.	.	.
118= U0327	1.0000	.	1.0000
119= U0330	.	.	.
120= U0333	.	1.0000	.
121= UCS27	1.0000	.	.
122= UCS30	.	1.0000	.
123= U0427	.	1.0000	.
124= U0430	1.0000	.	1.0000
125= U0433	.	.	.
126= U0436	.	.	.
127= U1227	.	.	1.0000
128= U1230	.	1.0000	.
129= U1233	.	.	.
130= U1236	1.0000	.	.

Table 4.3.1-3b Alternative schedules for the third decision field

Now to test the validity of the quasi-optimal schedule produced by the sequential decision technique, the pure optimum over the entire horizon time is run. The optimization procedure for this test developed 10 integer nodes and stopped with a description of all the nodes that had not yet been traced down fully. An indication is given of the estimated total cost required to trace down to the best integer solution in each branch of the tree that has not resulted so far in an integer solution, see tables 4.3.1-4a,b,c and d.

NODE	17	63	67	71	79	92
FUNCTIONAL	212.4062	219.3176	219.3351	219.5171	214.3999	213.5933
ESTIMATION	INTEGER	INTEGER	INTEGER	INTEGER	INTEGER	INTEGER
85= U0101	.	.	.	.	.	.
86= U0102	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
87= U0103	.	.	.	.	.	.
88= U0304	.	.	.	.	.	.
89= U0401	.	.	.	.	.	.
90= U0405	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
91= U0305	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
92= U0206	.	.	.	.	.	.
93= U0203	.	.	.	.	.	.
94= U0210	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
95= U0510	.	.	.	.	.	.
96= U0706	.	.	.	.	.	.
97= U0708	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
98= U0710	.	.	.	.	.	.
99= U1103	.	.	.	.	.	.
100= U1110	.	.	.	.	.	.
101= U1112	.	.	.	.	.	.
102= U1114	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
103= U1116	.	.	.	.	.	.
104= U0814	.	.	.	.	.	.
105= U0816	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
106= U0516	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
107= U0518	.	.	.	.	.	.
108= U0520	.	.	.	.	.	.
109= U0620	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
110= U0622	.	.	.	.	.	.
111= U0624	.	.	.	.	.	.
112= U1022	1.0000	.	.	.	1.0000	1.0000
113= U1024	.	1.0000	1.0000	1.0000	.	.
114= U0324	1.0000	1.0000	.	.	.	1.0000
115= U0927	.	.	1.0000	1.0000	1.0000	.
116= U0524	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
117= U0322	.	.	.	1.0000	.	.
118= U0327	.	1.0000	1.0000	.	1.0000	.
119= U0330	1.0000	.	.	.	.	.
120= U0333	.	.	.	.	.	1.0000
121= U0527	.	1.0000	.	1.0000	1.0000	.
122= U0530	1.0000	.	.	.	.	1.0000
123= U0427	1.0000	.	.	.	.	1.0000
124= U0430	.	1.0000	1.0000	1.0000	1.0000	.
125= U0433	.	.	.	.	.	.
126= U0436	.	.	.	.	.	.
127= U1227	.	.	.	.	.	.
128= U1230	1.0000	.	.	.	.	1.0000
129= U1233	.	.	.	.	.	.
130= U1236	.	1.0000	1.0000	1.0000	1.0000	.

Table 4.3.1-4a The best integer solutions to the optimization of the entire scheduling problem

NODE	85	86	37	51	30	32
FUNCTIONAL	222.5923	220.5507	224.6080	226.9226	221.4149	218.6519
ESTIMATION	INTEGER	INTEGER	INTEGER	INTEGER	225.903	221.327
85= U0101	.	.	.	.	.	.
86= U0102	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
87= U0103	.	.	.	.	.	.
88= U0204	.	.	.	.	.	.
89= U0401	.	.	.	.	.	.
90= U0406	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
91= U0805	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
92= U0206	.	.	.	.	.	.
93= U0208	.	.	1.0000	1.0000	1.0000	1.0000
94= U0210	1.0000	1.0000	.	.	.	.
95= U0510	.	.	1.0000	.	1.0000	1.0000
96= U0706	.	.	.	.	.	.
97= U0708	1.0000	1.0000	.	.	.	.
98= U0710	.	.	1.0000	1.0000	1.0000	1.0000
99= U1109	.	.	.	.	.	.
100= U1110	.	.	.	.	.	.
101= U1112	.	.	.	.	.	.
102= U1114	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
103= U1116	.	.	.	.	.	.
104= U0814	.	.	.	.	.	.
105= U0816	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
106= U0516	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
107= U0518	.	.	.	.	.	.
108= U0520	.	.	.	.	.	.
109= U0620	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
110= U0622	.	.	.	.	.	.
111= U0624	.	.	.	.	.	.
112= U1022	1.0000	1.0000	.	1.0000	.	.
113= U1024	.	.	1.0000	.	1.0000	1.0000
114= U0324	.	.	1.0000	1.0000	.	.
115= U0927	1.0000	1.0000	.	.	1.0000	1.0000
116= U0524	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
117= U0322	.	.	.	.	.	.
118= U0327	.	1.0000	1.0000	.	.	1.0000
119= U0330	.	.	.	1.0000	.1333	.
120= U0333	1.0000	.	.	.	.8667	.
121= U0527	1.0000	.	1.0000	.	1.0000	.
122= U0530	.	.	.	1.0000	1.0000	1.0000
123= U0427	.	.	.	1.0000	.	.
124= U0430	1.0000	1.0000	1.0000	.	.5857	.7429
125= U0433	.	.	.	.	.	.
126= U0436	.	.	.	.	.4143	.2571
127= U1227	1.0000	1.0000	.	.	.	.
128= U1230	.	.	.	1.0000	1.0000	.5882
129= U1233	.	.	.	.	.	.4118
130= U1236	.	.	1.0000	.	.	.

Table 4.3.1-4b The remaining completed schedules with their quality levels

NODE	34	40	44	47	49	55
FUNCTIONAL	226.4192	171.7577	219.9972	220.4854	231.9908	216.3246
ESTIMATION	226.419	233.614	230.440	233.073	236.781	220.719
85= U0101	.	.	.	.	.	.
86= U0102	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
87= U0103	.	.	.	.	.	.
88= U0504	.	.	.	.	.	.
89= U0601	.	.	.	.	.	.
90= U0406	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
91= U0806	1.0000	.	1.0000	1.0000	1.0000	1.0000
92= U0206	.	.	.	.	.	.
93= U0208	1.0000	1.0000	1.0000	1.0000	1.0000	.
94= U0210	.	.	.	.	.	1.0000
95= U0510	1.0000	.	.	.	.	.
96= U0706	.	.8267	.	.	.	.
97= U0709	.	.	.	.	.	1.0000
98= U0710	1.0000	.1733	1.0000	1.0000	1.0000	.
99= U1108	.	.	.	.	.	.
100= U1110	.	.	.	.	.	.
101= U1112	.	.	.	.	.	.
102= U1114	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
103= U1116	.	.	.	.	.	.
104= U0814	.	.	.	.	.	.
105= U0816	1.0000	.	1.0000	1.0000	1.0000	1.0000
106= U0516	1.0000	.4273	1.0000	1.0000	1.0000	1.0000
107= U0518	.	.5727	.	.	.	.
108= U0520	.	.	.	.	.	.
109= U0620	1.0000	.6273	1.0000	1.0000	1.0000	1.0000
110= U0622	.	.5727	.	.	.	.
111= U0624	.	.	.	.	.	.
112= U1022	.	1.0000	.	1.0000	1.0000	.
113= U1024	1.0000	.	1.0000	.	.	1.0000
114= U0924	1.0000	.	.	.	.	.
115= U0927	.	1.0000	1.0000	1.0000	1.0000	1.0000
116= U0524	1.0000	.8568	1.0000	1.0000	1.0000	1.0000
117= U0322	.	.	1.0000	.	.	.
118= U0327	1.0000	.	.	.1333	.	.
119= U0330	.	.8000	.	.	1.0000	.1333
120= U0333	.	.2000	.	.8667	.	.8667
121= U0527	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
122= U0530	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
123= U0427	.	.4571	.2143	.4000	.	.
124= U0430	.	.2857	.5286	.3428	.4571	.5857
125= U0433	.4571	.	.	.	.2857	.
126= U0436	.5428	.2571	.2571	.2571	.2571	.4143
127= U1227	.	.	.	.	1.0000	.
128= U1230	1.0000	1.0000	1.0000	1.0000	.	1.0000
129= U1233	.	.	.	.	.	.
130= U1236	.	.	.	.	.	.

Table 4.3.1-4c Some of the incompleted schedules which did not show a great deal of promise

NODE	69	87 P	60	64	25	27
FUNCTIONAL	221.5283	215.5107	221.3283	221.3463	210.5540	218.9338
ESTIMATION	221.528	220.438	221.329	221.366	220.505	221.509
85= U0101	.	.	.	.	.	.
86= U0102	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
87= U0103	.	.	.	.	.	.
88= U0304	.	.	.	.	.	.
89= U0401	.	.	.	.	.	.
90= U0406	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
91= U0806	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
92= U0206	.	.	.	.	.	.
93= U0208	.	.	.	.	1.0000	1.0000
94= U0210	1.0000	1.0000	1.0000	1.0000	.	.
95= U0510	.	.	.	.	1.0000	1.0000
96= U0706	.	.	.	.	.	.
97= U0708	1.0000	1.0000	1.0000	1.0000	.	.
98= U0710	.	.	.	.	1.0000	1.0000
99= U1108	.	.	.	.	.	.
100= U1110	.	.	.	.	.	.
101= U1112	.	.	.	.	.	.
102= U1114	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
103= U1115	.	.	.	.	.	.
104= U0814	.	.	.	.	.	.
105= U0816	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
106= U0516	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
107= U0518	.	.	.	.	.	.
108= U0520	.	.	.	.	.	.
109= U0620	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
110= U0622	.	.	.	.	.	.
111= U0624	.	.	.	.	.	.
112= U1022	.	1.0000	.	.	1.0000	.
113= U1024	1.0000	.	1.0000	1.0000	.	1.0000
114= U0924	.	.	1.0000	.	.	.
115= U0227	1.0000	1.0000	.	1.0000	1.0000	1.0000
116= U0524	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
117= U0322	1.0000	.	.	.	.1333	1.0000
118= U0327	.	1.0000	1.0000	1.0000	.	.
119= U0330	.	.	.	.	.6667	.
120= U0333	.	.	.	.	.2000	.
121= U0527	1.0000	.3750	1.0000	.	1.0000	1.0000
122= U0530	1.0000	.	1.0000	1.0000	1.0000	1.0000
123= U0627	.	.	.	.	.4571	.
124= U0630	.	1.0000	.	.	.2857	.7429
125= U0633	.4571	.	.4571	.4571	.	.
126= U0636	.5429	.	.5428	.5429	.2571	.2571
127= U1227	.	1.0000	.	.	.	.
128= U1230	1.0000	.	1.0000	1.0000	1.0000	.5882
129= U1233	.	.	.	.	.	.4118
130= U1236	.	.	.	.	.	.

Table 4.3.1-4d The remaining incomplete schedules to the total optimum schedule procedure

The result of this comparison shows that the best three solutions found to the complete problem (nodes 17, 79 and 82) are exactly the three best solutions from the quasi-optimal technique (decision field three, nodes 7, 15 and 18 from table 4.3.1-3a&b). The next three best integer nodes developed for the entire problem, as might almost have been expected, resulted from the other option in decision field two which had the price that was so close to the optimum.

The next two best solutions from the overall problem were again contained in the quasi-optimal's last decision field. Between these two in quality, however, was a schedule created by the quasi-optimal technique (node 21, table 4.3.1-3a) which had not been created by the total simulation (it is very close, both in schedule and price,[112] to the waiting node 87P, figures to be the integer solution estimated for node 87P to reach).

The next best integer solution to the total (node 37, table 4.3.1-4b), which was $12,003.20 more than the best solution resulted from the other option of the <u>first</u> decision field.

Further study of the schedules points to more reasons for accepting the quasi-optimal technique. For example, (1) there are definite patterns to the scheduling over different time spans and they seem more or less spliced together to come up with the variations to the best solution, and (2)

112. The estimated cost of 87P reaching an integer was $220,438 and node 21 from decision field three yielded a cost of $220,533.20 .

there is very little coupling between different time spans.

There is of course no reason to believe that scheduling efforts for larger systems should be this 'decoupled.' So it is wise to study the information available from the dual

NUMBER	...ROW..	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
1	Q	BS	167.27234	167.27234-	NONE	NONE	1.00000
2	DR01	BS	100.00000	125.00000	NONE	225.00000	.
3	DC01	UL	30.00000	.	NONE	30.00000	.40667
4	DR02	UL	250.00000	.	NONE	250.00000	.11920
5	DC02	BS	61.25000	3.75000	NONE	65.00000	.
6	DR03	BS	350.00000	15.00000	NONE	365.00000	.
7	DC03	UL	65.00000	.	NONE	65.00000	.39296
8	DR04	BS	225.00000	150.00000	NONE	375.00000	.
9	DC04	BS	33.75000	36.25000	NONE	70.00000	.
10	DR05	BS	225.00000	140.00000	NONE	365.00000	.
11	DC05	BS	33.75000	51.25000	NONE	85.00000	.
12	DR06	UL	360.00000	.	NONE	360.00000	.10045
13	DC06	UL	100.00000	.	NONE	100.00000	.02108
14	DR08	UL	225.00000	.	NONE	225.00000	.10035
15	DC08	UL	85.00000	.	NONE	85.00000	.00991
16	DR10	UL	155.00000	.	NONE	155.00000	.10999
17	DC10	UL	80.00000	.	NONE	80.00000	.00201
18	DR12	UL	125.00000	.	NONE	125.00000	.11392
19	DC12	BS	31.25000	53.75000	NONE	85.00000	.
20	DR14	BS	225.00000	5.00000	NONE	230.00000	.
21	DC14	BS	87.50000	7.50000	NONE	95.00000	.
22	DR16	UL	360.00000	.	NONE	360.00000	.09571
23	DC16	BS	49.25000	15.75000	NONE	65.00000	.
24	DR18	UL	575.00000	.	NONE	575.00000	.08813
25	DC18	BS	27.50000	17.50000	NONE	45.00000	.
26	DR20	BS	571.36364	38.63636	NONE	610.00000	.
27	DC20	BS	28.56818	21.43182	NONE	50.00000	.
28	DR22	UL	780.00000	.	NONE	780.00000	.09288
29	DC22	BS	115.00000	5.00000	NONE	120.00000	.
30	DR24	UL	600.00000	.	NONE	600.00000	.09605
31	DC24	BS	102.18182	67.81818	NONE	170.00000	.
32	DR27	UL	380.00000	.	NONE	380.00000	.10721
33	DR30	UL	430.00000	.	NONE	430.00000	.08210
34	DR33	UL	230.00000	.	NONE	230.00000	.09517
35	DR36	UL	190.00000	.	NONE	190.00000	.10361
36	CM01	EQ	1.00000	.	1.00000	1.00000	72.49510-
37	CM02	EQ	1.00000	.	1.00000	1.00000	29.13625-
38	CM03	EQ	1.00000	.	1.00000	1.00000	28.55500-
39	CM0401	EQ	1.00000	.	1.00000	1.00000	70.26530-
40	CM0402	EQ	1.00000	.	1.00000	1.00000	60.06500-
41	CM05	EQ	1.00000	.	1.00000	1.00000	207.58333-
42	CM06	EQ	1.00000	.	1.00000	1.00000	233.56800-
43	CM07	EQ	1.00000	.	1.00000	1.00000	9.79100-
44	CM09	EQ	1.00000	.	1.00000	1.00000	19.35500-
45	CM10	EQ	1.00000	.	1.00000	1.00000	23.29905-
46	CM11	EQ	1.00000	.	1.00000	1.00000	18.39091-
47	CM12	EQ	1.00000	.	1.00000	1.00000	17.29917-
48	CC0101	BS	.	1.00000	NONE	1.00000	.
49	CC0102	UL	1.00000	.	NONE	1.00000	.08770
50	CC0104	BS	1.00000	.	NONE	1.00000	.
51	CC0105	UL	1.00000	.	NONE	1.00000	13.00500
52	CC0106	BS	1.00000	.	NONE	1.00000	.
53	CC0127	BS	.	1.00000	NONE	1.00000	.
54	CC0130	BS	.80000	.20000	NONE	1.00000	.
55	CC0133	BS	.20000	.80000	NONE	1.00000	.
56	ND0206	BS	1.00000	1.00000-	.	NONE	.
57	CC0220	UL	1.00000	.	NONE	1.00000	54.38000
58	CC0222	BS	1.00000	.	NONE	1.00000	.
59	ND0227	LL	.	.	.	NONE	9.98500-
60	CC0427	BS	.	1.00000	NONE	1.00000	.

Table 4.3.1-5 Row activity in the optimum linear schedule

NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
61	V0201	BS	.80000	15.25000-	.	1.00000	.
62	V0202	BS	.20000	14.90000-	.	1.00000	.
63	V0206	UL	1.00000	14.11000-	.	1.00000	.73825-
64	V0214	UL	1.00000	16.10000-	.	1.00000	16.10000-
65	V0216	UL	1.00000	15.09000-	.	1.00000	3.12636-
66	V0927	UL	1.00000	17.39000-	.	1.00000	16.65357-
67	V0930	BS	.05000	8.21000-	.	1.00000	.
68	V0933	UL	1.00000	9.79000-	.	1.00000	.27333-
69	V0830	LL	.	10.21000-	.	1.00000	.05250
70	V0833	LL	.	11.34000-	.	1.00000	.55583
71	V0836	LL	.	11.66000-	.	1.00000	1.51179
72	V0803	BS	1.00000	12.28000-	.	1.00000	.
73	V0806	BS	.56000	13.26000-	.	1.00000	.
74	V0808	UL	1.00000	14.00000-	.	1.00000	1.18083-
75	V0810	UL	1.00000	13.92000-	.	1.00000	.10833-
76	V0812	BS	1.00000	14.26000-	.	1.00000	.
77	VC508	BS	.02500	20.07000-	.	1.00000	.
78	VC519	BS	.16667	13.22000-	.	1.00000	.
79	VC522	BS	.38000	23.22000-	.	1.00000	.
80	VC533	UL	1.00000	14.50000-	.	1.00000	4.99333-
81	VC536	UL	1.00000	13.71000-	.	1.00000	3.34857-
82	VC805	LL	.	5.57000	.	1.00000	5.57000
83	VC806	LL	.	8.23000	.	1.00000	8.23000
84	VC820	LL	.	4.31000	.	1.00000	4.31000
85	U0101	BS	.	18.59000	.	1.00000	.
86	U0102	BS	1.00000	19.31000	.	1.00000	.
87	U0103	BS	.	22.86000	.	1.00000	.
88	U0806	LL	.	11.68000	.	1.00000	11.68000
89	U0401	LL	.	29.71000	.	1.00000	15.69573
90	U0406	BS	1.00000	34.30000	.	1.00000	.
91	UC806	BS	.92500	20.13000	.	1.00000	.
92	U0206	LL	.	29.22000	.	1.00000	.08575
93	U0208	BS	.76000	16.26000	.	1.00000	.
94	U0210	BS	.24000	15.31000	.	1.00000	.
95	UC510	LL	.	20.72000-	.	1.00000	1.67100
96	U0706	BS	.13333	8.21000	.	1.00000	.
97	U0708	BS	.33667	9.13000	.	1.00000	.
98	U0710	BS	.53000	9.64000	.	1.00000	.
99	U1109	LL	.	8.78000	.	1.00000	.86676
100	U1110	LL	.	9.62000	.	1.00000	2.32876
101	U1112	LL	.	8.90000	.	1.00000	1.90109
102	U1114	UL	1.00000	9.21000	.	1.00000	9.18091-
103	U1116	BS	.	8.82000	.	1.00000	.
104	UC814	LL	.	12.10000	.	1.00000	12.10000
105	UC816	LL	.	23.21000	.	1.00000	4.06819
106	U0516	BS	.42727	106.47000	.	1.00000	.
107	U0518	BS	.57273	104.73000	.	1.00000	.
108	U0520	LL	.	108.57000	.	1.00000	6.45067
109	U0620	BS	.42727	123.44000	.	1.00000	.
110	U0622	BS	.57273	120.19000	.	1.00000	.
111	U0626	LL	.	131.36000	.	1.00000	19.77057
112	U1022	BS	1.00000	7.23000	.	1.00000	.
113	U1024	LL	.	6.62000	.	1.00000	.60841
114	U0925	BS	.	9.75000	.	1.00000	.
115	U0927	BS	1.00000	9.37000	.	1.00000	.
116	UC526	BS	.85682	19.21000-	.	1.00000	.
117	U0322	LL	.	13.90000	.	1.00000	13.68450
118	U0327	LL	.	14.21000	.	1.00000	1.73714
119	U0330	BS	.80000	16.24000	.	1.00000	.
120	U0333	BS	.20000	14.28000	.	1.00000	.
121	UC527	UL	1.00000	13.44000-	.	1.00000	.57429-
122	UC530	UL	1.00000	16.50000-	.	1.00000	4.64800-
123	U0427	BS	.45714	22.52000	.	1.00000	.
124	U0430	BS	.28571	31.31000	.	1.00000	.
125	U0433	LL	.	33.02000	.	1.00000	6.28333
126	U0436	BS	.25714	23.79000	.	1.00000	.
127	U1227	LL	.	9.53000	.	1.00000	1.39605
128	U1230	UL	1.00000	9.73000	.	1.00000	.59067-
129	U1233	BS	.	9.21000	.	1.00000	.
130	U1236	LL	.	9.03000	.	1.00000	.59805

Table 4.3.1-6 Column primal and dual activity in the optimum linear schedule

to the continuous linear program solution for the system.

It will be most instructive to begin the examination of the dual with the study of a scheduling decision that is already 'locked in' at the optimal continuous solution stage. Such a decision is the maintenance of plant 11 which has settled into the 14[th] week. There is almost no diference in the costs of doing maintenance in the range from week 8 to week 16, ranging from $8,780 to $9,620, and in fact the 14[th] week had one of the higher costs $9,210. The dual activity, i.e. "reduced cost," however shows that because of peculiarities in the schedule the cost of U1114 would have to nearly <u>double</u> before the decision would require reexamination. Graphing the dual activity associated with the maintenance window of plant 11, figure 4.3.1-2, shows indeed why there was never any question in any of the schedules as to when plant 11 should have its maintenance session.

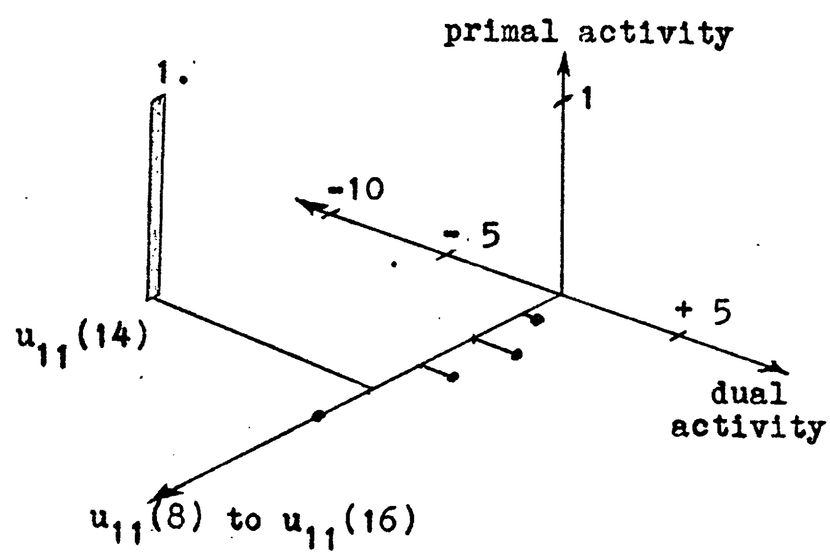

Figure 4.3.1-2 Dual activity associated with the maintenance window of plant 11

Of course it is not always this easy to settle decisions, or there would be no need for going beyond this examination of the dual. System requirements, as well as other parallel processes must be considered before such decisions can be made with certainty. Because the system cost will go up more than $45,000 from the continuous solution to the optimal integer solution and since none of the dual activities exceed this figure, it can be seen that <u>any</u> of the variables might (but are not likely to) change before the scheduling is finished. As an example consider the variable U0322 which shows a cost reduction of $13,684.50 necessary before it would become a factor. But as the system cost goes up this 'reduction' is apparent to U0322 which actually does participate in the sixth best schedule produced.

The cost increase, or pseudo-cost, associated with any single decision field, however, is much smaller than $45,000. Thus, many of the dual activities are excellent indicators.

More important than making decisions ahead of the decision field is the predicting of the jittery decisions in the future. An example of one variable over which much doubt existed in the scheduling process was the indecision between U0208 and U0210.

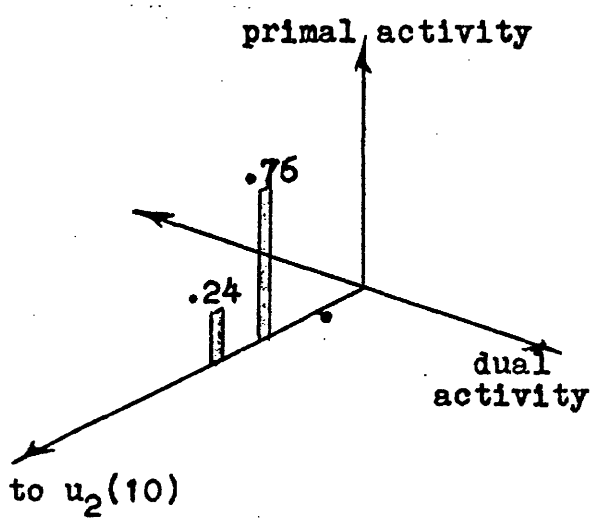

Figure 4.3.1-3 Dual activity associated with the indecisiveness within maintenance window for plant 2

The placing of the second plant's maintenance session is an example of a variable which should be carried along and reconsidered in the second decision field. This action will insure a better overall schedule and will also yield scheduling alternatives for both paths of this 'fork' in the scheduling process.

Looking at the dual activity in the 'columns' alone is not nearly as instructive as looking at the dual 'matrix.' For example,

U0208 P .0 D .76	U0210 P .0 D .24
U0708 P .0 D .34	U0710 P .0 D .53

Table 4.3.1-7 Primal-dual activity in matrix form

this portion of the dual matrix shows that interchange between the U0208-U0710 pairing and the U0210-U0708 pairing is very likely in final schedules.

Even these crude studies of the dual matrix, although very instructive, are not necessary, as the test of the quasi-optimal technique without their use has shown.

Further gains are possible through the study of dual quantities in the finished schedule. Consider the dual activity of the best schedule in tables 4.3.1-8 and -9. The negative costs in the dual activity, i.e. 'reduced cost,' associated with the economic shutdowns(in table 4.3.1-9) V0206, V0214 and V0933 show the need for the definition of more shutdown capabilities in intervals 6, 14 and 33. Asking any more cycling capability in the first week DC01 will be expensive.

NUMBER	...ROW..	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
1	O	BS	212.40422	212.40422-	NONE	NONE	1.00000
2	CR01	BS	100.00000	125.00000	NONE	225.00000	.
3	CC01	UL	30.00000	.	NONE	30.00000	.40667
4	CR02	UL	250.00000	.	NONE	250.00000	.11920
5	CC02	BS	41.25000	3.75000	NONE	45.00000	.
6	CR03	BS	350.00000	15.00000	NONE	365.00000	.
7	CC03	UL	65.00000	.	NONE	65.00000	.39296
8	CR04	BS	225.00000	150.00000	NONE	375.00000	.
9	CC04	BS	33.75000	36.25000	NONE	70.00000	.
10	CR05	BS	225.00000	140.00000	NONE	365.00000	.
11	CC05	BS	33.75000	51.25000	NONE	85.00000	.
12	CR06	UL	360.00000	.	NONE	360.00000	.10592
13	CC06	BS	93.75000	6.25000	NONE	100.00000	.
14	CR08	UL	225.00000	.	NONE	225.00000	.10035
15	CC08	UL	85.00000	.	NONE	85.00000	.04660
16	CR10	UL	155.00000	.	NONE	155.00000	.11136
17	CC10	BS	45.00000	35.00000	NONE	80.00000	.
18	CR12	UL	125.00000	.	NONE	125.00000	.11392
19	CC12	BS	31.25000	53.75000	NONE	85.00000	.
20	CR14	BS	225.00000	5.00000	NONE	230.00000	.
21	CC14	BS	87.50000	7.50000	NONE	95.00000	.
22	CR16	UL	360.00000	.	NONE	360.00000	.12072
23	CC16	BS	30.50000	34.50000	NONE	65.00000	.
24	CR18	UL	575.00000	.	NONE	575.00000	.08813
25	CC18	BS	27.50000	17.50000	NONE	45.00000	.
26	CR20	BS	600.00000	10.00000	NONE	610.00000	.
27	CC20	BS	30.00000	20.00000	NONE	50.00000	.
28	CR22	UL	780.00000	.	NONE	780.00000	.09288
29	CC22	BS	115.00000	5.00000	NONE	120.00000	.
30	CR24	BS	385.00000	215.00000	NONE	600.00000	.
31	CC24	BS	115.00000	55.00000	NONE	170.00000	.
32	CR27	UL	380.00000	.	NONE	380.00000	.17390
33	CR30	UL	430.00000	.	NONE	430.00000	.08210
34	CR33	UL	230.00000	.	NONE	230.00000	.09072
35	CR36	UL	190.00000	.	NONE	190.00000	.09152
36	CM01	EQ	1.00000	.	1.00000	1.00000	72.96960-
37	CM02	EQ	1.00000	.	1.00000	1.00000	29.23000-
38	CM03	EQ	1.00000	.	1.00000	1.00000	27.88800-
39	CM0401	EQ	1.00000	.	1.00000	1.00000	71.39200-
40	CM0402	EQ	1.00000	.	1.00000	1.00000	55.81200-
41	CM05	EQ	1.00000	.	1.00000	1.00000	153.20333-
42	CM06	EQ	1.00000	.	1.00000	1.00000	175.91800-
43	CM07	EQ	1.00000	.	1.00000	1.00000	8.21000-
44	CM09	EQ	1.00000	.	1.00000	1.00000	9.37000-
45	CM10	EQ	1.00000	.	1.00000	1.00000	21.40150-
46	CM11	EQ	1.00000	.	1.00000	1.00000	20.29200-
47	CM12	EQ	1.00000	.	1.00000	1.00000	16.86920-
48	CC0101	BS	.	1.00000	NONE	1.00000	.
49	CC0102	UL	1.00000	.	NONE	1.00000	.57200
50	CC0104	BS	1.00000	.	NONE	1.00000	.
51	CC0105	UL	1.00000	.	NONE	1.00000	13.00500
52	CC0106	UL	1.00000	.	NONE	1.00000	.01000
53	CC0127	BS	.	1.00000	NONE	1.00000	.
54	CC0130	BS	1.00000	.	NONE	1.00000	.
55	CC0133	BS	.	1.00000	NONE	1.00000	.
56	ND0206	BS	1.00000	1.00000-	.	NONE	.
57	CC0220	BS	1.00000	.	NONE	1.00000	.
58	CC0222	BS	1.00000	.	NONE	1.00000	.
59	ND0327	BS	.30000	.30000-	.	NONE	.
60	CC0427	BS	.	1.00000	NONE	1.00000	.

Table 4.3.1-8 Row primal and dual activity in the optimum completed schedule

NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
61	V0201	BS	.80000	15.25000-	.	1.00000	.
62	V0202	BS	.20000	14.90000-	.	1.00000	.
63	V0206	UL	1.00000	14.11000-	.	1.00000	.87000-
64	V0214	UL	1.00000	16.10000-	.	1.00000	16.10000-
65	V0216	BS	.08000	15.02000-	.	1.00000	.
66	V0227	BS	.30000	17.32000-	.	1.00000	.
67	V0230	BS	.75000	8.21000-	.	1.00000	.
68	V0233	UL	1.00000	9.79000-	.	1.00000	.71800-
69	V0239	LL	.	10.21000-	.	1.00000	.05250
70	V0333	BS	.24000	11.34000-	.	1.00000	.
71	V0336	BS	.72000	11.64000-	.	1.00000	.
72	V0503	BS	1.00000	12.23000-	.	1.00000	.
73	V0806	BS	.68000	13.24000-	.	1.00000	.
74	V0809	BS	.32000	14.00000-	.	1.00000	.
75	V0910	BS	.24000	13.92000-	.	1.00000	.
76	V0912	BS	1.00000	14.24000-	.	1.00000	.
77	V0509	BS	.92500	20.07000-	.	1.00000	.
78	V0519	BS	.16667	13.22000-	.	1.00000	.
79	V0522	BS	.39000	23.22000-	.	1.00000	.
80	V0533	UL	1.00000	14.50000-	.	1.00000	5.62800-
81	V0536	UL	1.00000	13.71000-	.	1.00000	4.55900-
82	V0805	LL	.	5.57000	.	1.00000	5.57000
83	V0304	LL	.	8.23000	.	1.00000	8.23000
84	V0820	LL	.	4.31000	.	1.00000	4.31000
85	U0101	IV	.	18.59000	.	1.00000	.
86	U0102	IV	1.00000	19.31000	.	1.00000	.
87	U0103	IV	.	22.86000	.	1.00000	.
88	U0804	IV	.	11.68000	.	1.00000	11.68000
89	U0501	IV	.	29.71000	.	1.00000	14.85333
90	U0406	IV	1.00000	34.30000	.	1.00000	.
91	U0806	IV	1.00000	20.13000	.	1.00000	1.05600-
92	U0206	IV	.	29.22000	.	1.00000	.
93	U0209	IV	.	16.26000	.	1.00000	1.32125
94	U0210	IV	1.00000	15.31000	.	1.00000	.
95	U0510	IV	.	20.72000-	.	1.00000	1.80800
96	U0706	IV	.	8.21000	.	1.00000	.
97	U0708	IV	1.00000	9.13000	.	1.00000	4.41500
98	U0710	IV	.	9.66000	.	1.00000	1.43000
99	U1108	IV	.	8.78000	.	1.00000	.85300
100	U1110	IV	.	9.62000	.	1.00000	.66400
101	U1112	IV	.	8.90000	.	1.00000	.
102.	U1114	IV	1.00000	9.21000	.	1.00000	11.08200-
103	U1116	IV	.	8.92000	.	1.00000	.60000
104	U0814	IV	.	12.10000	.	1.00000	12.10000
105	U0816	IV	1.00000	23.21000	.	1.00000	.93600-
106	U0516	IV	1.00000	106.67000	.	1.00000	68.13600
107	U0518	IV	.	104.73000	.	1.00000	.
108	U0520	IV	.	108.57000	.	1.00000	6.65067
109	U0620	IV	1.00000	123.44000	.	1.00000	3.25000
110	U0622	IV	.	120.19000	.	1.00000	.
111	U0624	IV	.	131.36000	.	1.00000	59.78200
112	U1022	IV	1.00000	7.23000	.	1.00000	6.27670-
113	U1024	IV	.	6.62000	.	1.00000	.
114	U0924	IV	1.00000	9.75000	.	1.00000	.38000
115	U0927	IV	.	9.37000	.	1.00000	.
116	U0524	IV	1.00000	19.21000-	.	1.00000	19.21000-
117	U0322	IV	.	13.90000	.	1.00000	.05600-
118	U0327	IV	.	14.21000	.	1.00000	12.40700
119	U0330	IV	1.00000	16.26000	.	1.00000	.66700
120	U0333	IV	.	14.28000	.	1.00000	.
121	U0527	IV	.	13.66000-	.	1.00000	7.62800
122	U0530	IV	1.00000	14.50000-	.	1.00000	4.66800-
123	U0627	IV	1.00000	22.52000	.	1.00000	27.57300
124	U0630	IV	.	31.31000	.	1.00000	4.23300
125	U0633	IV	.	33.02000	.	1.00000	8.96000
126	U0636	IV	.	23.78000	.	1.00000	.
127	U1227	IV	.	9.58000	.	1.00000	7.49230
128	U1230	IV	1.00000	9.73000	.	1.00000	.16070-
129	U1233	IV	.	9.21000	.	1.00000	.05200
130	U1236	IV	.	9.09000	.	1.00000	.

Table 4.3.1-9 Column primal and dual activity in the optimum completed schedule

A great number of additional results are obvious from further examination of these results. For example, the slack activity of 150 and 140 megawatts in weeks 4 and 5 indicate that here exists the possibility for an interregional bulk power sale, which perhaps had been overlooked.

4.3.2 Leveling Reserves

The practice of leveling the oversupply of power at the various times within a schedule has been a widely used technique. Thus it deserves looking into, and can possibly be useful, to see how this practice could be included in the linear programming format, either (1) as an addition to the presented performance measure, or (2) as the only component of a new performance measure. Consider case (2), for then case (1) is just an obvious extension.

Suppose the elements of a vector $\Delta \underline{m}$ represent the desired level of oversupply in the intervals of the schedule. Therefore, $\underline{m} - \Delta \underline{m}$ is the new desirable level of equation 22-5. So,

$$\begin{aligned} \underline{A}\,\underline{u} - \underline{o}_- + \underline{o}_+ &= \underline{m} - \Delta\underline{m} \\ \underline{0} \leq \underline{o}_- &\leq \Delta\underline{m} \qquad\qquad 432\text{-}1 \\ \underline{0} &\leq \underline{o}_+ \end{aligned}$$

where $\underline{o}_+$ is the oversupply of power beyond the desired level, and $\underline{o}_-$ is the oversupply less than the most desirable oversupply level (but never allowing the schedule to be infeasible, i.e. $\underline{o}_- = \Delta\underline{m}$).

The performance measure of this system would then be

$$Q = \underline{c}_-' \, \underline{o}_- + \underline{c}_+' \, \underline{o}_+ \qquad 432\text{-}2$$

where it is likely that the penalties

of undersupplying the desired level $\underline{c}'_-$ will be much stiffer than the penalties for going beyond the desired level $\underline{c}'_+$ (which may in fact be $\underline{Q}$).

4.3.3 Post-optimal Analysis

A very simple two interval four plant system was set up, see Appendix C, to demonstrate the form of the program which parameterized the dollar-environmental mixes of the objective function. The results of this sample system, figure 4.3.3, show the form the solution to a parameterization like $Q = q_d + \theta q_e$ would take.

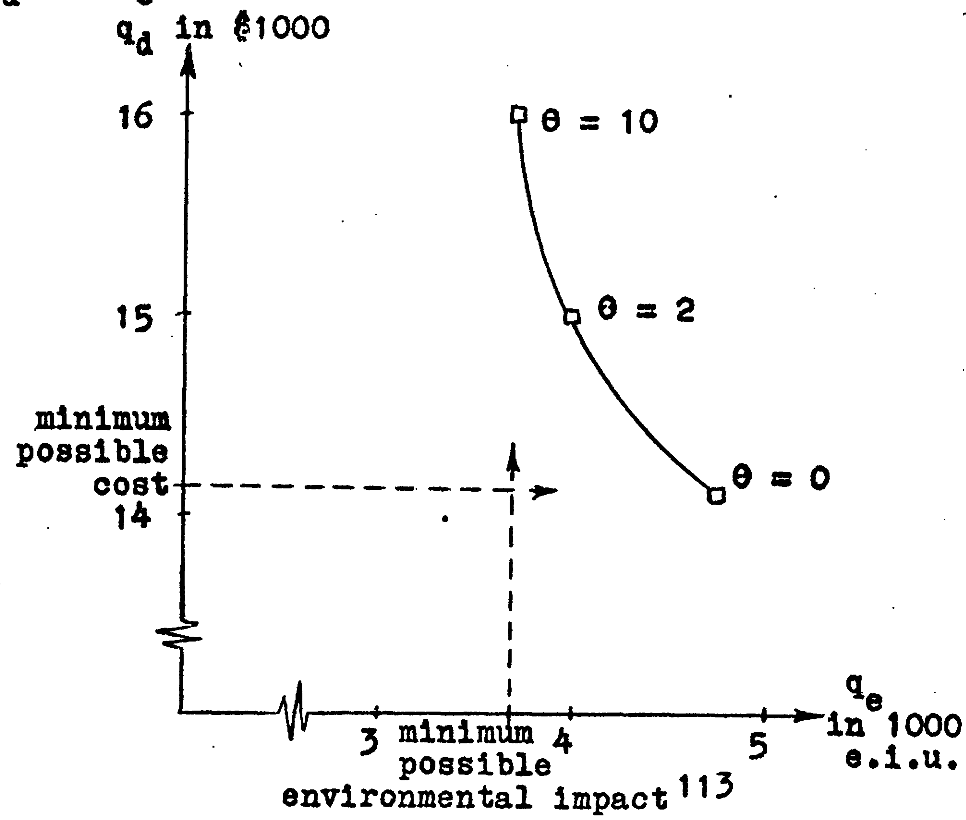

Figure 4.3.3 Range of all dollar-environmental impact pairings for optimum schedules

113. In this problem weighting the ecological impact units ten times the dollar costs resulted in the absolute minimum impact schedule. That is, in this problem for $\theta = 10$ and all larger θ the mix of variables which could be used to the best advantage of the environment did not change.

5. Feasibility and Usefulness

This study was undertaken as an attempt to included environmental costs in the production scheduling process. Because it accomplishes this goal, the procedure developed should prove useful. The scheduling technique presented also offers a technique for including major production scheduling variables which were previously not manageable, such as interregional contract decisions, nuclear and hydroelectric production quotas, and a number of other variable cost and capability considerations.

This technique is also usable as a simulation tool with computation efforts increasing only linearly with expanded time horizons. That is, it is more than a short term maintenance decision mechanism, but also a long range system performance evaluation tool.

5.1 Cost Considerations and Comparison to Dynamic Techniques

There should be no concern over the cost and time involved in running this scheduling program. If it is assumed that the decision field of concern, say a two month period, has maintenance windows which average about 2½ intervals in size, then 18 windows can be considered using 45 integer variables. Eighteen windows, or 18 plants, in two months becomes 108 plants with annual maintenance requirements. A scheduling problem with 46 integer variables was introduced at the MIT Information Processing Center and the CPU time for execution was 37 seconds. The total program cost from card reading down to handling was $11.63 .

The major concern, in the cost area, will probably be the cost of the mixed integer program product itself. It is possible that at some time in the future mixed integer options on systems will be free, as are linear programs now. Presently, however, MPSX-MIP costs $225 per month. If this cost is a consideration there are three options available. (1) The schedule can be formed from the linear program alone (see page 97). (2) It might be worthwhile to develop the integer programs starting with available linear programming subroutines. (3) Time might be rented at a user center where the program is available.

The results of the dynamic technique counterpart to this project are not yet available, but they do not appear to be headed in a promising direction. Dimensionality appears to be the main stumbling block, because as the system progresses a tremendous collection of discrete possibilities must be handled by dynamic techniques.

5.2 Drawbacks

Outside of any computational cost drawbacks (which don't appear to be a problem) there are few disadvantages to this scheduling procedure. Perhaps one objection could be the complete difference of this technique from those now existing, thus requiring time consuming initial problem setups. However, the significant and lasting gains to be made seem to justify the initial time investment.

Another problem is that the input data is not readily available. For example, reserve requirements in megawatts, maintenance costs, environmental costs, etc. will require a real collection and computation effort. This data collection, for example the ecological impact figures, is something which sooner or later must be reckoned with if the system is to operate most effectively. That is, this data requirement is not a fabrication of this particular scheduling scheme.

The quasi-optimal, i.e. 'in a sense optimal,' solutions which are of a suboptimal nature can not, it appears, be considered a drawback. Not only does this technique minimize the recomputational effort required due to changes in input factors, but consider which pure optimal solutions would be lost by this suboptimal process. An optimum would be lost, for example, which was tenuously relying upon an otherwise unexpected scheduling move made more than an entire decision field time span in the future(or the past). This characteristic of the solution technique could be considered an attractive factor in the scheduling procedure for it introduces a healthy respect for the uncertainties in the far future - a respect which any complex real-world system deserves.

Thus, this technique is more 'sensible' from the scheduling point of view, and this 'sensibility' also makes it more realistic from the simulation viewpoint.

Nomenclature for Equations

$A_{d,i,m}(k)$	the estimated additional derating of capacity due to forced outages in interval k, because maintenance on plant i has been delayed to interval m
$A_i(k)$	capacity of plant i in interval k, derated to account for average outage effects
$A_{in,j}(k)$	maximum input consumption in megawatts at a pumped storage station j
$A_{max,i}(k)$	forecasted maximum capacity of unit i in interval k
$A_{out,j}(k)$	the maximum contribution to the cycling equation 22-7 from the j^{th} pumped storage facility in the k^{th} interval
$A_{x,j}(k)$	the maximum additional (derated for forced outages) capacity which can be gotten from plant j beyond its nominal capacity factor
$b_{c,j}$	the dollar amount of money received from the j^{th} interregional power exchange contract, or negative the amount paid
b_{gc}	the penalty cost in the gas contract, dollars per unused cubic foot of gas
$b_j^n(k)$	the cost associated with the maintaining or refueling of the j^{th} plant for n intervals starting in the k^{th} interval
$b_{n+,j}$	the penalty cost in dollars resultant from the use of more of the nuclear energy batch than has been optimally determined
$b_{n-,j}$	the penalty cost in dollars resultant from the nonuse of some of the nuclear energy batch before refueling, as determined by the optimal burndown level
$b_{sj}(k)$	the estimated cost in dollars lost because of spillage over the j^{th} hydroelectric reservoir in the k^{th} interval
$b_{x,j}(k)$	the cost associated with the maximum possible overextension beyond the nominal capacity of the j^{th} generator in the k^{th} interval
$c_{\%,i}(k)$	percentage cycling capabilities of plant i in interval k
$C_{\%}(k)$	percent of demand P(k) of a cycling nature

D_i	the set of plants within geographic district i
$d_{\%,i}(k)$	percentage derating of maximum capacity of unit i in interval k to reflect forced outage rates
$e_j(k)$	electrically pumped water input to pumped storage reservoir j
$E_j(k)$	fuel to electricity conversion efficiency of plant j in interval k
$E_{p,j}(k)$	input pumping efficiency of electric power to water at the j^{th} pumped storage facility
e.i.u.	ecological impact units
F_{ji}	the window of possible refueling or maintenance intervals for the j^{th} plant in the i^{th} refueling or maintenance session
$G(k)$	the total amount of capacity contracted under interruptible load agreements
$g_j(k)$	gas usage of plant j in interval k
$h_j(k)$	the volume of water consumed within interval k by the j^{th} hydroelectric facility
$l_i(k)$	the transmission capability limitation of transmission line i during interval k
L_i	the set of plants for which the i^{th} maintenance crews is responsible
$M(k)$	the total capacity available for maintenance or economic shutdowns in interval k
M_n	the estimate of the total capacity which will be lost to maintenance and refueling over the planning time span
$n_{e,j,i}(k)$	the estimated nuclear allotment to the k^{th} interval from the j^{th} reactor if refueling takes place in the i^{th} interval
$n_j(k)$	the allotment of nuclear energy to the k^{th} interval for the j^{th} reactor
N_{ji}	the total batch of energy left of the i^{th} batch in the j^{th} reactor
$o_A(k)$	the oversupply beyond reserve requirements of power in the interval k

$o_c(k)$	the oversupply, beyond reserve requirements, of the cycling power in the k^{th} interval
o_{gc}	the slack variable representing the amount of gas left at the end of the contract period
$o_{n+,j}$	the amount of nuclear energy used beyond the optimal batch allotment
$o_{n-,j}$	the amount of nuclear energy left unused before refueling, as determined by comparison to the optimum batch burndown level
$o_{r,j}(k)$	the slack variable representing the oversupply of the streamflow beyond the minimum requirement
$P(k)$	the demand for power in the interval k which must be met to insure the prespecified level of reliability
$P_{A,d,j}(k)$	the probability distribution of power levels available from neighboring region j in interval k
$P_{A,f,j}(k)$	the probability density function associated with $P_{A,d,j}(k)$
$P_d(t),P_d(k)$	the forecasted set of power demands for time t, or interval k, each level with an associated probability of being greater than the actual load
$P_{T,d}(k)$	the probability distribution of total power levels demanded by the system in interval k after adjusting for the support which might be received from neighboring regions
$P_{T,f}(k)$	the probability density function associated with $P_{T,d}(k)$
Q	the total combined performance index of the system
q_d	the dollar performance index, or quality measure, of a maintenance and production schedule
q_e	the environmental performance index, or quality measure, of the maintenance and production schedule, measured in ecological impact units, e.i.u.
$r_j(k)$	the inflow into the j^{th} reservoir within the k^{th} interval
$R_j(k)$	minimum tailwater flow requirement for the k^{th} interval downstream from the j^{th} hydro facility

$S(k)$ the total system capacity updated to interval k and derated to account for all system outage probabilities

$s_j(k)$ the spillage, inadvertent or intentional, at the j^{th} reservoir during the k^{th} interval

T_j the maximum limit of water available to the j^{th} reservoir facility

θ the ecolo-economic tradeoff measured in dollars per environmental impact, that is $/e.i.u.

$u_i^n(k)$ a bivalent variable, one if plant i is to initiate n consecutive intervals of maintenance in interval k, otherwise zero

$v_j(k)$ a variable between 0 and 1 which designates the fractional extent of the interval k that the j^{th} plant should be shut down for economic reasons

$v_{x,j}(k)$ the fractional extent usage of the possible additional capacity beyond the nominal that the j^{th} plant is capable of producing in the interval k

$w_j(k)$ the j^{th} reservoir's head water level after the interval k

$W_j(k)$ total wattage output of plant j in interval k

$W_{in,j}(k)$ electric power consumption of the j^{th} pumped storage facility

x_j a binary variable, one if the j^{th} contract is to be honored, zero otherwise

X_j the maximum limit on a power exchange contract, positive or negative depending upon whether or not it adds or subtracts capacity to the system

$x_{v,j}$ a continuous variable, representing the fractional extent to which a contract will be honored

$y_{oA}(k)$ the reward in dollars for the oversupply $o_A(k)$

$y_{oc}(k)$ the reward in dollars for the oversupply $o_c(k)$

$y_{u,j}(k)$ the reward in dollars for the non-use of the j^{th} plant in the k^{th} interval

$y_{v,j}(k)$ the reward in dollars for the $v_j(k)$ fractional shutdown of the j^{th} plant in the interval k, that is, the price for a total shutdown

$z_j(k)$ — the environmental reward associated with full shutdown of the j^{th} plant in interval k

$Z_{g,j}(k)$ — the ecological penalty in e.i.u. for the impact on the environment associated with the use of one gas energy unit in a gas turbine

$Z_{n,j}(k)$ — the e.i.u. penalty for operating the j^{th} reactor in the k^{th} interval for one units worth of nuclear fuel consumption

$z_{r,j}(k)$ — the environmental reward associated with the amount beyond the minimum streamflow requirement

$z_{w,j}(k)$ — the ecological reward for the quantity in reservoir j above the minimum drawdown level

$Z_{x,j}(k)$ — the additional environmental burden associated with the use of extra capacity $A_{x,j}(k)$

<u>Symbols for Equations</u>

$\bar{x}$ — the negation of x, for a binary variable $1 - x = \bar{x}$

$\cup$ — the union, or collection, of all elements within the sets considered

$\cap$ — the intersection, or collection of only those elements which are common to all the sets considered

$\in$ — is a member of the set

A - B — in set operations this means 'subtracting any elements from set A which also exist as elements in set B'

<u>Nomenclature for Computer Programs</u>

BD — the vector of bounds on a variable vector

CCnnmm — the row which coordinates the use of the nn^{th} crew in the mm^{th} interval

CMnnmm — the row coordinating the maintenance, that is, which insures there will be exactly one maintenance in the mm^{th} window for the nn^{th} plant

CONVERT — used to convert the input data into an internal format on the problem file

DBnn — the row which insures that there will be no more maintenance assigned than the system can allow and still meet the load in the nn^{th} interval

DOnn	the row which insures the system will have sufficient cycling capability in the nnth interval
DEBE	designates the start of the integer variables in the data set
E	signals the row is an equality
FINE	designates the end of the integer variables in the data set
G	signals a greater than or equal to row
INIMIX	sets MIP parameters to standard values and establishes standard processing procedure
INITIALZ	system macro of MPSX which sets up the strategy for solving the linear program
L	signals a less than or equal to row
MA	the vector of maintenance availability megawatts
MIXFLOW	searches for integer solutions
MIXSAVE	saves the current status of the tree of nodes
MIXSTART	initiates the search for integers
MIXSTATS	prints status of the nodes
N	signals a nonconstraint row
NDnnmm	the row which forces shutdown of the nnth plant in the mmth interval when there is maintenance scheduled there
OPTIMIZE	optimizes the continuous problem
OS/360	IBM Operating System/360 is the supervisor
PARAOBJ	the parametric variation of the objective function
Q	the dollar costs of the schedule created
QE	the ecological costs associated with the schedule
SETUP	used to initiate the problem on the machine
SOLUTION	prints the solution obtained
UCBmm	a yes=1 or no=0 decision on an interregional buying contract initiated in the mmth interval

UCSmm	a binary decision about the bulk interregional power exchange contract in the mmth interval
Unnmm	binary variable equalling one if maintenance is to be initiated in the mmth interval at the nnth plant
UP	signals an upper bound
Vnnmm	a continuous variable representing the fractional portion of interval mm that plant nn should be shut down
VCBmm	a variable form of contract, like UCBmm
VCSmm	a variable contract of the form of UCSmm
XOHROW	the row used to make the parametric change on the objective function
XMXDROP	the minimum acceptable value of the schedules sought
XPARAM	the initial value of the variable parameter
XPARDELT	the incremental increase in the parameter
XPARMAX	the maximum value of the parameter which should be used

NEPEX FORM D

APPENDIX A

NEW ENGLAND POWER EXCHANGE

APPLICATION FOR OUTAGE OF GENERATION EQUIPMENT

Time of request ______________

Day of request ______________

Application No. ______________

Unit requested and nature of work ____

____

____

To be out of service

from ________________________ to ________________________
(hour) (date) (hour) (date)

Must start dropping load at ________________________________
(hour) (date)

Name of person requesting outage ________________ Satellite ____________

Can this work be postponed ______________ If no, why ______________

____

____

NEPEX Forecaster or Pool Coordinator
receiving request ____

Outage granted ______________________ If no, why ______________________

____

Actual work accomplished ______________________ Completed ____________
(Date)

Unit returned to service ______________________________
(hour) (date)

NEPEX Forecaster or Pool
Coordinator signature ____

______________ ______________
(hour) (date)

Appendix B

```
//JOBLIB  DD  DSNAME=SYS2.MPSX.LOAD,DISP=(SHR,PASS)
//OPPROS01  EXEC MPSX
//MPSCOMP.SYSIN  DD  *,DCB=(RECFM=FB,LRECL=80,BLKSIZE=2000)
         PROGRAM
* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
*                                                                   *
*  THIS PROGRAM IS DESIGNED TO                                      *
*       1- SET UP THE MIXED INTEGER PROGRAM ASSOCIATED WITH THE     *
*          COMPLETE OPTIMUM PRODUCTION SCHEDULE - OPPROS.           *
*        2- SOLVE FOR THE OPTIMUM SCHEDULE IGNORING THE INTEGER     *
*          CONSTRAINT SETS                                          *
*        3- THEN OBTAIN UP TO 10 INTEGER SOLUTIONS , IF THEY EXIST, *
*          WITH DOLLAR PLUS ENVIRONMENTAL QUALITY MEASURES OF NOT   *
*          MORE THAN THE QUALITY OF A HAND COMPUTED SCHEDULE        *
*          USING SCHEDULING TECHNIQUES CURRENTLY IN COMMON USAGE    *
*          (A QUALITY MEASURE EQUALLING 272K DOLLARS).              *
*                                                                   *
* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
*
         INITIALZ
         MOVE(XDATA,'MODEL')
         MOVE(XPBNAME,'PB1')
         CONVERT
         SETUP('BOUND','BD')
         MOVE(XOBJ,'Q')
         MOVE(XRHS,'MA')
         OPTIMIZE
         SOLUTION
         SAVE('NAME','OPTC')
         INIMIX
         MIXSTART('MATRIX')
         XMXDROP=272.
         CT=0
         MVADR(XDOPRINT,INT)
         MIXFLOW
STOP     MIXSAVE('NAME','TREE1')
         MIXSTATS('NODES')
         EXIT
INT      SOLUTION
         XMXDROP=272.
         CT =CT+1
         IF(CT.EQ.10,STOP)
         CONTINUE
*
CT       DC(0)
         PEND
/*
//MPSEXEC.MATRIX2  DD  UNIT=SYSDA,SPACE=(CYL,(5))
//MPSEXEC.MIXWORK  DD  UNIT=SYSDA,SPACE=(CYL,(5))
//MPSEXEC.SYSIN  DD  *,DCB=(RECFM=FB,LRECL=80,BLKSIZE=2000)
```

```
NAME          MODEL
ROWS
 N  Q
 L  DB01
 L  DC01
 L  DB02
 L  DC02
 L  DB03
 L  DC03
 L  DB04
 L  DC04
 L  DB05
 L  DC05
 L  DB06
 L  DC06
 L  DB08
 L  DC08
 L  DB10
 L  DC10
 L  DB12
 L  DC12
 L  DB14
 L  DC14
 L  DB16
 L  DC16
 L  DB18
 L  DC18
 L  DB20
 L  DC20
 L  DB22
 L  DC22
 L  DB24
 L  DC24
 L  DB27
 L  DB30
 L  DB33
 L  DB36
 E  CM01
 E  CM02
 E  CM03
 E  CM0401
 E  CM0402
 E  CM05
 E  CM06
 E  CM07
 E  CM09
 E  CM10
 E  CM11
 E  CM12
 L  CC0101
 L  CC0102
 L  CC0104
 L  CC0105
 L  CC0106
 L  CC0127
 L  CC0130
 L  CC0133
 G  ND0206
 L  CC0220
 L  CC0222
 G  ND0927
 L  CC0427
COLUMNS
    V0201     DB01        125.000     DC01          37.500
    V0201     Q           -15.250
    V0202     DB02        125.000     DC02          37.500
    V0202     Q           -14.900
    V0206     DB06        125.000     DC06          37.500
    V0206     ND0206        1.000     Q            -14.110
    V0214     DB14        125.000     DC14          37.500
    V0214     Q           -16.100
    V0216     DB16        125.000     DC16          37.500
    V0216     Q           -15.090
```

V0927	DB27	100.000	ND0927	1.000
V0927	Q	-17.390		
V0930	DB30	100.000	Q	-8.210
V0933	DB33	100.000	Q	-9.790
V0830	DB30	125.000	Q	-10.210
V0833	DB33	'25.000	Q	-11.340
V0836	DB36	125.000	Q	-11.44
V0803	DB03	125.000	DC03	31.250
V0803	Q	-12.280		
V0806	DB06	125.000	DC06	31.250
V0806	Q	-13.240		
V0808	DB08	125.000	DC08	31.250
V0808	Q	-14.000		
V0810	DB10	125.000	DC10	31.250
V0810	Q	-13.920		
V0812	DB12	125.000	DC12	31.250
V0812	Q	-14.240		
VCS08	DB08	200.000	Q	-20.070
VCS18	DB18	150.000	Q	-13.220
VCS22	DB22	250.000	Q	-23.220
VCS33	DB33	100.000	Q	-14.500
VCS36	DB36	100.000	Q	-13.710
VCB05	DB05	-50.000	Q	5.570
VCB04	DB04	-80.000	Q	8.230
VCB20	DB20	-50.000	Q	4.310
DEBE	'MARKER'		'INTORG'	
U0101	DB01	225.000	DC01	33.750
U0101	DB02	225.000	DC02	33.750
U0101	DB03	225.000	DC03	33.750
U0101	DB04	225.000	DC04	33.750
U0101	CM01	1.000	CC0104	1.000
U0101	CC0101	1.000	CC0102	1.000
U0101	Q	18.590		
U0102	DB02	225.000	DC02	33.750
U0102	DB03	225.000	DC03	33.750
U0102	DB04	225.000	DC04	33.750
U0102	DB05	225.000	DC05	33.750
U0102	CM01	1.000	CC0104	1.000
U0102	CC0105	1.000	CC0102	1.000
U0102	Q	19.310		
U0103	DB03	225.000	DC03	33.750
U0103	DB04	225.000	DC04	33.750
U0103	DB05	225.000	DC05	33.750
U0103	DB06	225.000	DC06	33.750
U0103	CM01	1.000	CC0106	1.000
U0103	CC0104	1.000	CC0105	1.000
U0103	Q	22.860		
U0804	DB04	125.000	DC04	31.250
U0804	DB05	125.000	DC05	31.250
U0804	Q	11.680		
U0401	DB01	350.000	DC01	35.000
U0401	DB02	350.000	DC02	35.000
U0401	CM0401	1.000	CC0101	1.000

U0401	CC0102	1.000	Q	29.710
U0406	DB06	350.000	DC06	35.000
U0406	CM0401	1.000	CC0106	1.000
U0406	Q	34.300		
UCB06	DB06	-200.000	Q	20.130
U0206	CM02	1.000	CC0106	1.000
U0206	ND0206	-1.000	Q	29.220
U0208	DB08	125.000	DC08	37.500
U0208	CM02	1.000	Q	16.260
U0210	DB10	125.000	DC10	37.500
U0210	CM02	1.000	Q	15.310
UCS10	DB10	100.000	DB12	100.000
UCS10	Q	-20.720		
U0706	DC06	75.000	CM07	1.000
U0706	Q	8.210		
U0708	DC08	75.000	CM07	1.000
U0708	Q	9.130		
U0710	DC10	75.000	CM07	1.000
U0710	Q	9.640		
U1108	DB08	100.000	DC08	50.000
U1108	CM11	1.000	Q	8.780
U1110	DB10	100.000	DC10	50.000
U1110	CM11	1.000	Q	9.620
U1112	DB12	100.000	DC12	50.000
U1112	CM11	1.000	Q	8.900
U1114	DB14	100.000	DC14	50.000
U1114	CM11	1.000	Q	9.210
U1116	DB16	100.000	DC16	50.000
U1116	CM11	1.000	Q	8.820
UCB14	DB14	-100.000	Q	12.100
UCB16	DB16	-200.000	Q	23.210
U0516	DB16	550.000	DC16	27.500
U0516	DB18	550.000	DC18	27.500
U0516	CM05	1.000	Q	106.470
U0518	DB18	550.000	DC18	27.500
U0518	DB20	550.000	DC20	27.500
U0518	CM05	1.000	CC0220	1.000
U0518	Q	104.730		
U0520	DB20	550.000	DC20	27.500
U0520	DB22	550.000	DC22	27.500
U0520	CM05	1.000	CC0220	1.000
U0520	CC0222	1.000	Q	108.570
U0620	DB20	600.000	DC20	30.000
U0620	DB22	600.000	DC22	30.000
U0620	CM06	1.000	CC0220	1.000
U0620	CC0222	1.000	Q	123.440
U0622	DB22	600.000	DC22	30.000
U0622	DB24	600.000	DC24	30.000
U0622	CM06	1.000	CC0222	1.000
U0622	Q	120.190		
U0624	DB24	600.000	DC24	30.000
U0624	DB27	600.000	CM06	1.000
U0624	Q	131.360		

	U1022	DB22	85.000	DC22	85.000
	U1022	DB24	85.000	DC24	85.000
	U1022	CM10	1.000	Q	7.230
	U1024	DB24	85.000	DC24	85.000
	U1024	CM10	1.000	Q	6.620
	U1024	DB27	85.000	CC0427	1.000
	U0924	DB24	100.000	DC24	30.000
	U0924	CM09	1.000	Q	9.750
	U0927	CM09	1.000	ND0927	-1.000
	U0927	Q	9.370		
	UCS24	DB24	200.000	Q	-19.210
	U0322	DB22	150.000	DC22	37.500
	U0322	DB24	150.000	DC24	37.500
	U0322	CM03	1.000	Q	13.900
	U0327	DB27	150.000	CC0127	1.000
	U0327	CM03	1.000	Q	14.210
	U0330	DB30	150.000	CC0130	1.000
	U0330	CM03	1.000	Q	16.240
	U0333	DB33	150.000	CC0133	1.000
	U0333	CM03	1.000	Q	14.280
	UCS27	DB27	120.000	Q	-13.440
	UCS30	DB30	120.000	Q	-14.500
	U0427	DB27	350.000	CM0402	1.000
	U0427	Q	22.520		
	U0430	DB30	350.000	CM0402	1.000
	U0430	Q	31.310		
	U0433	DB33	350.000	CM0402	1.000
	U0433	Q	33.020		
	U0436	DB36	350.000	CM0402	1.000
	U0436	Q	23.780		
	U1227	DB27	85.000	CM12	1.000
	U1227	Q	9.580	CC0427	1.000
	U1230	DB30	85.000	CM12	1.000
	U1230	Q	9.730		
	U1233	DB33	85.000	CM12	1.000
	U1233	Q	9.210		
	U1236	DB36	85.000	CM12	1.000
	U1236	Q	9.090		
	FINE	'MARKER'		'INTEND'	
RHS					
	MA	DB01	225.000	DC01	30.000
	MA	DB02	250.000	DC02	45.000
	MA	DB03	365.000	DC03	65.000
	MA	DB04	375.000	DC04	70.000
	MA	DB05	365.000	DC05	85.000
	MA	DB06	360.000	DC06	100.000
	MA	DB08	225.000	DC08	85.000
	MA	DB10	155.000	DC10	80.000
	MA	DB12	125.000	DC12	85.000
	MA	DB14	230.000	DC14	95.000
	MA	DB16	360.000	DC16	65.000
	MA	DB18	575.000	DC18	45.000
	MA	DB20	610.000	DC20	50.000

```
     MA        DB22        780.000     DC22        120.000
     MA        DB24        600.000     DC24        170.000
     MA        DB27        380.000     DB30        430.000
     MA        DB33        230.000     DB36        190.000
     MA        CM01          1.000     CM02          1.000
     MA        CM03          1.000     CM0401        1.000
     MA        CM0402        1.000     CM05          1.000
     MA        CM06          1.000     CM07          1.000
     MA        CM09          1.000     CM10          1.000
     MA        CM11          1.000     CM12          1.000
     MA        ND0206        0.000     ND0927        0.000
     MA        CC0101        1.000     CC0102        1.000
     MA        CC0104        1.000     CC0105        1.000
     MA        CC0106        1.000     CC0127        1.000
     MA        CC0130        1.000     CC0133        1.000
     MA        CC0222        1.000     CC0220        1.000
     MA        CC0427        1.000
BOUNDS
 UP BD         U0101         1.000
 UP BD         U0102         1.000
 UP BD         U0103         1.000
 UP BD         U0206         1.000
 UP BD         U0208         1.000
 UP BD         U0210         1.000
 UP BD         U0322         1.000
 UP BD         U0327         1.000
 UP BD         U0330         1.000
 UP BD         U0333         1.000
 UP BD         U0401         1.000
 UP BD         U0406         1.000
 UP BD         U0427         1.000
 UP BD         U0430         1.000
 UP BD         U0433         1.000
 UP BD         U0436         1.000
 UP BD         V0201         1.000
 UP BD         V0202         1.000
 UP BD         V0206         1.000
 UP BD         V0214         1.000
 UP BD         V0216         1.000
 UP BD         U0516         1.000
 UP BD         U0518         1.000
 UP BD         U0520         1.000
 UP BD         U0620         1.000
 UP BD         U0622         1.000
 UP BD         U0624         1.000
 UP BD         U0706         1.000
 UP BD         U0708         1.000
 UP BD         U0710         1.000
 UP BD         U0804         1.000
 UP BD         U0924         1.000
 UP BD         U0927         1.000
 UP BD         V0927         1.000
 UP BD         V0930         1.000
```

```
UP  BD      V0933      1.000
UP  BD      V0830      1.000
UP  BD      V0833      1.000
UP  BD      V0836      1.000
UP  BD      V0803      1.000
UP  BD      V0806      1.000
UP  BD      V0808      1.000
UP  BD      V0810      1.000
UP  BD      V0812      1.000
UP  BD      U1108      1.000
UP  BD      U1110      1.000
UP  BD      U1112      1.000
UP  BD      U1114      1.000
UP  BD      U1116      1.000
UP  BD      U1024      1.000
UP  BD      U1022      1.000
UP  BD      U1227      1.000
UP  BD      U1230      1.000
UP  BD      U1233      1.000
UP  BD      U1236      1.000
UP  BD      UCS10      1.000
UP  BD      UCB06      1.000
UP  BD      VCS08      1.000
UP  BD      VCS18      1.000
UP  BD      VCS22      1.000
UP  BD      UCS24      1.000
UP  BD      UCS27      1.000
UP  BD      UCS30      1.000
UP  BD      VCS33      1.000
UP  BD      VCS36      1.000
UP  BD      VCB05      1.000
UP  BD      VCB04      1.000
UP  BD      UCB14      1.000
UP  BD      UCB16      1.000
UP  BD      VCB20      1.000
ENDATA
/*
```

Appendix C

This is the program that parameterizes the objective function to include dollar-ecological impact mixes. The data presented is that used in the sample problem.

```
                    PROGRAM
                    INITIALZ
                    MOVE(XDATA,'MODEL')
                    MOVE(XPBNAME,'PB1')
                    CONVERT
                    SETUP('BOUND','BD')
                    MOVE(XOBJ,'QD')
                    MOVE(XRHS,'MA')
                    OPTIMIZE
                    SOLUTION
                    SAVE('NAME','OPTC')
          TESTOBJ   TITLE('THETA')
                    RESTORE('NAME','OPTC')
                    MOVE(XCHROW,'QE')
                    XPARAM = 0.
                    XPARDELT = 2.
                    XPARMAX = 10.
                    PARAOBJ
                    SOLUTION
                    EXIT
                    PEND
NAME                 MODEL
ROWS
 N  QD
 N  QE
 G  BD1
 G  BD2
COLUMNS
    U1               BD1            1.000     BD2             2.000
    U1               QD             3.000     QE              1.000
    U2               BD1            5.000     BD2             2.000
    U2               QD            12.000     QE              3.000
    U3               BD1            3.000     BD2             4.000
    U3               QD            10.000     QE              2.000
    U4               BD1            2.000     BD2             1.000
    U4               QD             4.000     QE              2.000
RHS
    MA               BD1            6.000     BD2             4.000
BOUNDS
 UP BD               U1             1.000
 UP BD               U2             1.000
 UP BD               U3             1.000
 UP BD               U4             1.000
ENDATA
/*
```

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