ELECTRIC POWER UNIT COMMITMENT SCHEDULING USING A DYNAMICALLY EVOLVING MIXED INTEGER PROGRAM

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This study was done in association with the Electric Power Systems Engineering Laboratory and the Department of Civil Engineering (Ralph M. Parsons Laboratory for Water Resources and Hydrodynamics and the Civil Engineering Systems Laboratory).

ABSTRACT

A quasi-optimal technique ('quasi' in that the technique discards unreasonable optimums), realized by a dynamically evolving mixed integer program, is used to develop regional electric power unit commitment schedules for a one week time span. This sophisticated, yet computationally feasible, method is used to develop the hourly bulk dispatch schedules required to meet electric power demands at a given reliability level while controlling the associated dollar costs and environmental impacts.

The electric power system considered is a power exchange pool of closely coupled generation facilities supplying a region approximately the size of New England. Associated with a tradeoff between a given cost of production and the relevant ecological factors, an optimum generation schedule is formulated which considers fossil, nuclear, hydroelectric, gas turbine and pumped storage generation facilities; power demands, reliabilities, operating constraints, startup and shutdown factors, geographic considerations, as well as various contracts such as interregional power exchanges, interruptible loads, gas contracts and nuclear fuel optimum batch utilization.

A prerequisite of the model was that it be flexible enough for use in the evaluation of the optimum system performance associated with hypothesized expansion patterns. Another requirement was that the effects of changed scheduling factors could be predicted, and if necessary corrected with a minimal computational effort.

A discussion of other existing and potential solution techniques is included, with an example of the proposed solution technique used as a scheduler. Although the inputs are precisely defined, this paper does not deal with the explicit fabrication of inputs to the model, such as e.g. river flow prediction or load forecasting. Rather, it is meant as a method of incorporating those inputs into the optimum operation scheduling process.

Acknowledgements

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Table of Contents

page

1. Introduction 10
  1.1 Problem 11
  1.2 Historical Approaches 15
  1.3 Results 43
    1.3.1 Model Description 43
    1.3.2 Method of Solution 46
    1.3.3 Computational Feasibility 49
  1.4 Presuppositions 50

2. Model 54
  2.1 System Requirements 54
    2.1.1 Power Demands 55
    2.1.2 Reliability Requirements 56
  2.2 System Capabilities 57
    2.2.1 Capacity Levels 57
      2.2.1.1 Fossil Fueled Units 58
      2.2.1.2 Nuclear Energy Relegation 59
      2.2.1.3 Hydroelectric Capabilities 61
      2.2.1.4 Pumped Storage Constraints 63
  2.3 Startup Costs 63
  2.4 Inputs 65
    2.4.1 System Updates 65
    2.4.2 Power Demand Adjustments - Reserve Requirements 66
  2.5 Performance Index 70
    2.5.1 Operating Costs 70
    2.5.2 Transmission Costs 71
    2.5.3 Ecological Impact Units 73

3. Solution Technique 81
  3.1 Possible Optimization Approaches 81
  3.2 Adaptation of the Model 82

4. Application to a Sample Regional Scheduling Problem . 94
  4.1 Description of the Sample System 94
  4.2 Examples of Unit Commitment Schedules 99

5. Feasibility and Usefulness 125
  5.1 Cost Considerations 126
  5.2 Drawbacks 127

Glossary of Equation Nomenclature 129

Appendix A 134

Optional Appendix A A1-A19

page

Appendix B 137

Optional Appendix B B1-B3

References 138-148

List of Illustrations

page

Figure 1.1-1 Block diagram representation of the overall system operation procedure 12

Table 1.1-1 Input-output summary of the overall system operation procedure including program uses 13

Figure 1.2-1 Branch and bound search technique used for maintenance scheduling 21

Figure 1.2-2 Computational procedure for the solution of the unit commitment problem via maximum principle . . 23

Figure 1.2-3 First unit commitment 'shutdown' rule including turnoff specification 26

Figure 1.2-4 Graph showing the removal of units on a priority system 27

Figure 1.2-5 Definition of a unit commitment 'day' for use with multiple daily shutdowns 27

Figure 1.2-6 Heuristic approach to the scheduling problem by complete decomposition 28

Figure 1.2-7 Heuristic approach to the subdividing of the scheduling problem with provision for one minute and five minute spinning reserve requirements 29

Figure 1.2-8 Method of perturbing solutions from decomposed results so as to search for decreased costs 29

Figure 1.2-9 Comparison of fossil fuel and pumped hydro incremental costs 31

Figure 1.2-10 Incremental cost technique for monthly placement of hydro energy utilization, assuming this to be the cheapest form of power 31

Figure 1.2-11 Monthly placement of pumped storage energy utilization 32

Figure 1.2-12 Comparison of an actual operation schedule and hydro-thermal optimized schedule 33

Figure 1.2-13 Demonstration of the iterative method of plant removal using a security constraint 34

page

Figure 1.2-14 Comparison of strict unit priority method to the security constrained method 35

Figure 1.2-15 Method of optimum seeking using local linearizations 36

Figure 1.2-16 Incremental costs of power plants in integer mode formulation 37

Figure 1.2-17 Discrete breakdown of the probabilistic load forecasts for mixed integer program 38

Figure 1.2-18 Tons of NOx versus megawatt loading curve for a power plant 40

Figure 1.2-19 Hypothetical representation of pollution sources and points at which concentrations are to be predicted 41

Figure 1.3.2 Flow chart of the dynamic evolving mixed integer program used in the scheduling process 47

Figure 2.2.1.1-1 Piecewise linearization of a megawatt power versus cost loading curve 58

Figure 2.2.1.1-2 Dollar cost-gas usage curve representing a gas supply contract 59

Figure 2.2.1.2 Penalty function representation for considering discrepancies between fuel usage and quotas 60

Figure 2.3-1 Starting costs as a function of previous down time 64

Figure 2.3-2 Piecewise linear curve of startup cost versus previous plant down time 64

Figure 2.4.2-1 Loading curve of simplest type of plant showing spinning reserve capability 67

Figure 2.4.2-2 Increases in expected costs with changes in spinning reserve requirements 69

Figure 2.4.2-3 Expected energy not supplied for different spinning reserve requirements (in a 1.8 million megawatt hour schedule) 70

Figure 2.5.2 Piecewise linear representation of a quadratic function 72

page

Figure 2.5.3-1 Simplified general systematic representation of method for computing aquasphere impacts from electric power generation 74

Figure 2.5.3-2 Detail of biological model portion of the general schematic 75

Figure 2.5.3-3 Tradeoff curve representing all possible optimum consequences of dollar and water pollution strategies 77

Figure 2.5.3-4 Solid tradeoff curve representing all possible dollar, water pollution impact, air pollution impact, and reliability combinations 78

Figure 2.6 Power plant response rate characteristics . 80

Figure 3.1 Sequential decision process using a dynamically evolving series of static overviews 82

Figure 3.2-1 Loading curve for simple, single segment representation of a power plant 83

Figure 3.2-2 Loading curve for two segment, downward breaking loading curve of power plant 85

Figure 3.2-3 Piecewise linear curve of startup cost versus previous plant down time 87

Figure 3.2-4 Amounts of SO_2 impacting the environment scaled (nonlinearly) to reflect the escalating nature of its consequences 92

Figure 4.2-1 Representation of the closeness of the actual valid schedules to the optimal continuous degeneration of the scheduling problem 101

Figures 4.2-2a,b and c Row and column activity in the schedule for the first decision field of the unrevised problem 103, 104, 105

Figure 4.2-3 Alternative schedules for the second decision field, list of integer decision variables for these alternatives and their respective dollar costs 107

Figures 4.2-4a,b,c and d Partially completed and completed integer schedules for integer decision variables of the third decision field 109, 110, 111, 112

page

Figures 4.2-5a, and b Display of integer variables for the completed and partially completed schedules for the revised scheduling problem 113, 114

Figures 4.2-6a,b,c,d,e,f,g,h,i and j Row and column activity of the best schedule for the revised problem 115, 116, 117, 118, 119, 120, 121, 122, 123, 124

1. Introduction

A great problem to develop from this industrial era is the dilemma of the increasing demands for energy and the increasing demands that environmental qualities not be degraded. As the electric power industry assumes an ever increasing commitment to resolve the energy supply problem it is subjected to escalating societal pressures to:

(1) generate reliably a sufficient amount of electricity to meet any demands,

(2) retain or decrease its price rates, and

(3) minimize the impact of its generation efforts upon the ecosphere.

The solution to this problem will take a long and unremitting effort from all sectors of society. In the long-term (30 years) program of action must be included, among many other things, efforts to develop more efficient means of power generation and more efficient power utilization.[1] There can be no doubt that to reverse the trend of environmental deterioration a tremendous technological effort will be required.

There is, however, another aspect of the solution to the 'electric power-environment' dilemma which should be closely coordinated with (and is definitely not meant to be a replacement for) the technological advances, but is essentially a separate effort. This is the development of methods

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1. A detailed documentation of the course of action required from technological improvements is contained in a report by Philip Sporn, reference (1).

to assure the best possible operation of an imperfect power generation system. That is, until facilities which are perfectly compatible with the ecosystem are producing all of our power there must be a method for insuring that the imperfect plants are utilized in the least damaging manner. This effort breaks essentially into two segments. First, the plants must be sited to take the best advantage of the site options available.[2] Secondly, the operation of existing systems must be directed toward those objectives enumerated in the beginning of this section.

This optimum operation of existing systems is the overall project being undertaken in the author's Ph.D. thesis, of which this study is one portion.

1.1 Problem

For a more thorough description of the part this research effort will assume in the overall study of 'optimum operation of existing systems' the reader is directed to reference (4). However, a basic understanding of the interconnections involved can be gotten from figure 1.1-1and the descriptive outline in table 1.1-1.

Briefly, the problem undertaken in this study is the development of a scheduling and/or simulation tool which prepares, out to an indefinitely far horizon, hourly production

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2. This is a problem receiving a great deal of research effort, see for example reference (2). The author's particular project is also to be used as a simulation technique for the evaluation of specifically hypothesized expansion alternatives, as explained in reference (3).

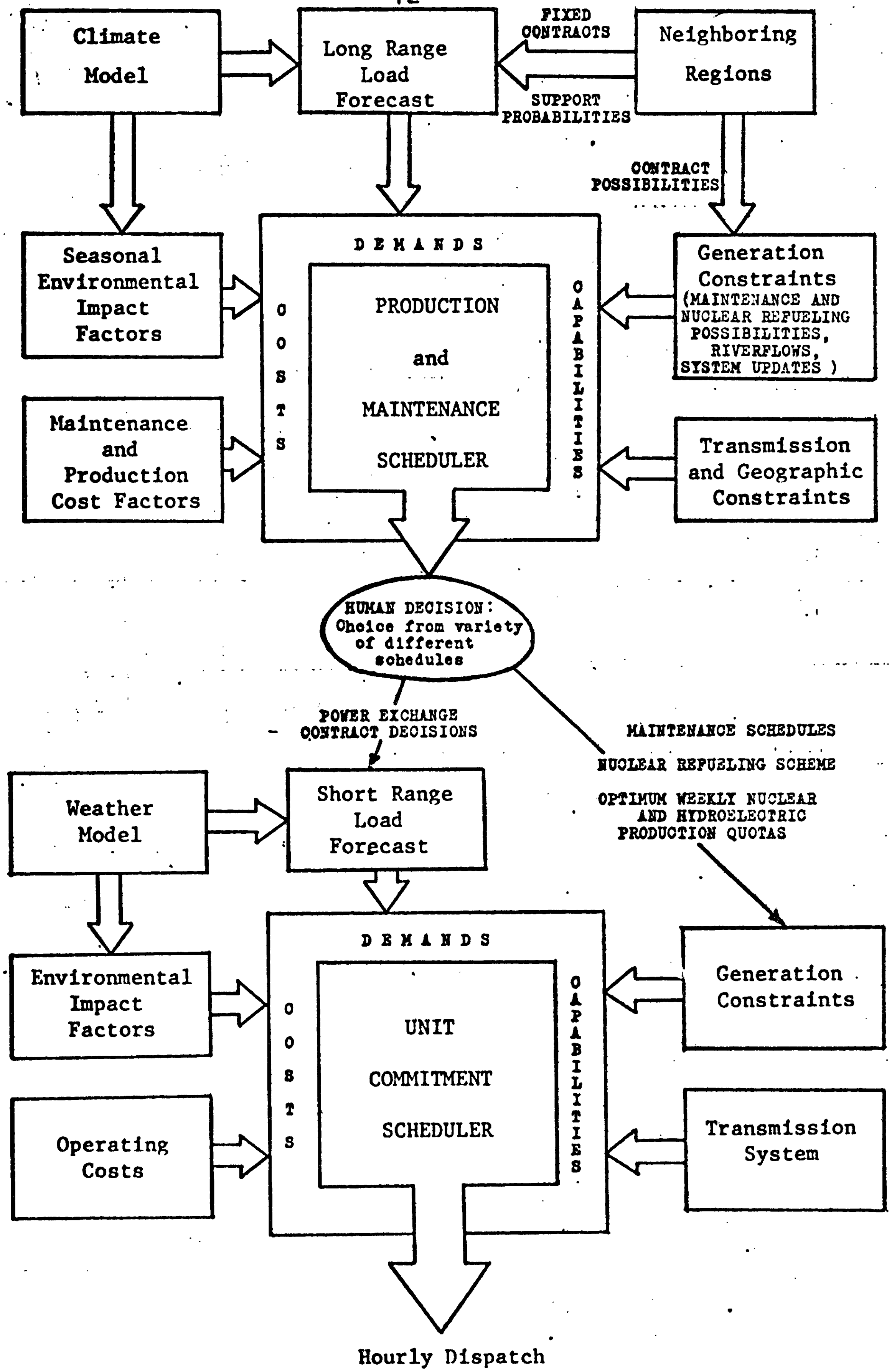

Figure 1.1-1 Block diagram representation of the overall system operation procedure

GIVEN:

1. Generation characteristics
   A. Capabilities and limitations
      i. Types of facilities
      ii. Output capacities
      iii. Maintenance and refueling possibilities
   B. Performance
      i. Dollar costs per megawatt
      ii. Costs of various maintenance and refueling schemes
      iii. Air and water emissions per megawatt

2. Transmission characteristics
   A. Capabilities and limitations
   B. Costs

3. Weather model (probabilistic)
   A. Air flow and temperature
   B. Water flow and temperature
   C. Upcoming weather patterns

4. Load model (probabilistic)
   A. Long range
   B. Short term forecasts

5. Interregional coordination
   A. Power exchange contract possibilities (probabilistic)
   B. Maintenance and production schedules

RESULTS:

1. Creates a variety of optimum maintenance and refueling schedules

2. Optimum unit commitment and hourly dispatch strategies

3. Performance in dollar costs, reliability and environmental impact

4. Shows system weaknesses, deficiencies and strengths

5. Makes power exchange contract decisions and coordinates system efforts with neighboring networks

USES OF PROGRAM:

1. Creates maintenance, production and hourly dispatch schedules

2. Simulates and evaluates performance of hypothesized system expansion configurations including generation and/or transmission additions

3. Evaluates tradeoffs available between dollar costs, reliability and environmental impact

4. Evaluates the possible dollar cost and environmental impact effects of proposed additions to the system such as pollution abatement equipment

5. In the licensing of new facilities (with commissions or in court problems):
   A. yields realistic pollution figures rather than worst case figures
   B. puts utility in position of defending its choice from among the alternatives, rather than defending its choice on its own grounds alone

6. Yields intangible benefits which result from being able to assure the public and the governmental agencies that the system could not be operating in a better manner

Table 1.1-1 Input-output summary of the overall system operation procedure, including program uses.

schedules for a regional electric power pool. These schedules are to be schemes which optimize the multiple-objective function including reliability, dollar and environmental considerations. "Optimize" is actually not a correct choice of words in that schedules which may perhaps be the exact optimum may in fact be very undesirable. For example, the mathematical optimum might depend for its slight edge over other schedules upon some very tenuous, unwaverable procedure over a long span of time. Thus, the need developes for the use of the term 'quasi-optimal,' that is, 'in-a-sense optimal,' for, what is really sought is a reasonable schedule (or simulation), respecting the vagaries of the future by offering a number of alternative schemes from each point.

One final consideration must be mentioned. Due to the number of ever changing factors which affect the generation schedule it would be very desirable to have a scheduling scheme which would be minimally disrupted by changes of the input factors. To achieve minimal disruption it would be necessary to decide <u>without computational efforts</u>:

(1) which future changing factors will be outside of the concern of the current schedule, and what point in the future they must be included,

(2) which factors will cause only slight schedule variations, and which scheduling decisions and parameters are most sensitive to these changes, and

(3) which future factors will require recomputation

of the schedule, and at what point in time must that recomputation start, and if possible stop,[3] to insure the total inclusion of the changing factor's sphere of influence.

This then is a short encapsulation of all the demands which are made upon an ideal generation schedule, and thus, represent the goal for this particular research effort.

1.2 Historical Approaches

With the operation and maintenance costs accounting for between 5 and 10% of the utility's expenditures,[4] the economic advantages of optimum production scheduling have long been recognized. Methods for the effective coordination of reserve requirements, forced outage probabilities and the millions of dollars worth of maintenance and fuel have been steadily increasing in complexity.

The problem of hour by hour scheduling out to a week horizon is greatly dependent upon the weekly production quotas and maintenance schedules which come from schedulers with longer time spans. Since the unit commitment problem and maintenance and production schedulers are so closely coupled, it is instructive to examine the different methods

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3. In generating a new schedule due to changing factors it would be desirable to be able to determine at what point in the future (if a point exists) the scheduling process has settled back to the pattern of the old schedule so computation can be stopped.

4. See, for example, reference (5).

of attacking this similar scheduling problem.

Despite the fact that large amounts of money are spent on maintenance, for example, a utility with 2000 megawatts of capacity spends in the vicinity of $6.6 million annually for maintenance,[5] there has been comparitively little effort put forth for the sophisticated optimization of the scheduling of this maintenance.

Very early scheduling efforts, when only a few power plants were considered, consisted of plotting the amount of capacity which could be spared to maintenance and then iteratively scheduling the largest facility in the largest space available. The technique worked well for small systems, using a minimum amount of clerical help, and had the advantage of more or less assuring that the largest facility would not be squeezed out of its slot by small changes in demand. But, there is no economic consideration in this technique, that is to say, leveling the oversupply is not necessarily consistent with any system performance measure except possibly maximum system reliability. And even at leveling the oversupply, this scheduling technique is not necessarily the optimum procedure.[6]

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5. See reference (6)

6. Consider, for a trivial example of the non-optimality of this procedure, the very simple system with plants of capacities 4, 3, and 2 to be fit into slots of 5 and 4. This algorithm would place the largest facility, 4, in the largest slot, 5, and would thus fail.

During the World War II hyperintensive energy using period new problems in the maintenance and production scheduling became evident, as explained in a 1942 Electrical World article[7] by Philip Sporn:

> "The object of any program of co-ordination of major unit outage is to maintain the maximum margin feasible between demand on a system and load capability of the various plants serving the system. For an individual system this means careful study and evaluation of the shapes of the annual load and capability curves. The latter involves taking into account not only seasonal variations in hydro capability but seasonal variations in steam-plant capability. However, in wartime, with rapidly growing loads, three other factors have to be taken into consideration. These are the rate of growth of new load, because such growth can overbalance the seasonal trend factor; the rate of bringing in new capacity on the system; and the broad integrated, regional-area picture."

Since World War II, little research has been done on the maintenance scheduling problem. Receiving much more attention has been the problem of simulating power system financial operations over the course of the year in a general probabilistic manner.[8] Some of the more sophisticated of these simulators recognize the need for having or creating a maintenance schedule to show the exact splicing together of the different generation facilities. One of these simulators uses a static linear program,[9] but unfortunately it is not directly adaptable to maintenance scheduling, being directed more toward system security

7. Excerpt from reference (7).
8. See references (8) through (15).
9. Contained in reference (16).

precautions. There is a production cost program[10] which describes a possible modification for use as a maintenance scheduler. The program uses a dynamic programming technique, and for large systems (gives a production cost example using six power plants) suggests incorporation of the method of successive approximations[11] to keep down the number of variables.

Of the maintenance programs developed as such there is none[12] which includes measures of dollar costs. In fact before 1972 there weren't <u>any</u> automatic scheduling mechanisms although the need for such a program had long been growing. Even among the few automated schedulers available today none is good enough to be popular and the problem has become so complex that what develops, as one regional exchange staff officer has told me, is a "horror show."

To demonstrate how little this field has progressed, consider what is done today by the regional power pool NEPEX, New England Power Exchange. They have been a pioneer in the use of sophisticated computation equipment for the purpose

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10. See reference (12)

11. See reference (17) or reference (18).

12. The author's own counterpart to this study, ref. (19), does include dollar costs, as well as environmental impacts.

13. Reference (20) in 1970 outlined the need for a good scheduling algorithm, using a static or dynamic technique, whichever would resolve the problem.

of system operation,[14] and they are responsible for, among other things, the coordination of the maintenance of 25 hydroelectric plants, and some 150 fossil and nuclear fueled generating stations. So, in this case, both the computational ability and the need exist for a viable scheduling technique. However, their maintenance schedule comes from staff members sitting in monthly, sometimes weekly, meetings studying forms on plant maintenance needs, which they have received from the superintendents of production in charge of the individual plants.

Within the last year, outside of the author's technique (reference 19), three automatic scheduling devises have appeared in the technical literature. These techniques utilize information on maximum and minimum times for maintenance, maintenance crew availability, relative importances of outages, 'must run' geographic considerations, forced outages,[15] and pool coordination of maintenance schedules, with no consideration for costs, environment, hydroelectric power, pumped hydro or nuclear plants, reservoir levels, or cycling capabilities of the configurations. Since none of these schedulers uses any dollar cost or environmental measures of desirability, they

14. See reference (21)

15. Basically included by the derating of the capacity of plants, at least this has been shown to perform as well as any other method, see reference (22)

search instead within desirable limits of system security. A comparison of these techniques is made in reference (22) and with the use of an example comes to the general conclusion that they are about equally good in levelizing risk although they use different security measures.

Reference (23) figures the effective capacities, after derating for forced outage, and proceeds to fit in the largest facility first, as previously described in the very early scheduling efforts.[16] Reference (24) goes about filling in the scheduling slots in a slightly different manner. First the crews are ranked with those serving the most capacity considered first. The units maintained by a single crew are then ranked from largest to smallest. Now with this priority list, a branch and bound search is made considering units in the order that those units are ranked, see figure 1.2-1 on the next page. The third of these recent maintenance schedulers, described in reference (22), uses a slightly more complicated priority listing, but uses about the same fill-in-the-valley method once it has the priority list. A search is made for the unit which, when scheduled out in its optimal position, leaves the highest risk factor for the system. Thus, this like the other techniques, is just another measure of

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16. A nearly identical technique uses the 'capacity times duration of outage' to figure the total shutdown energy as its measure of the'toughness of fit' for setting up the priority for filling plants into the schedule.

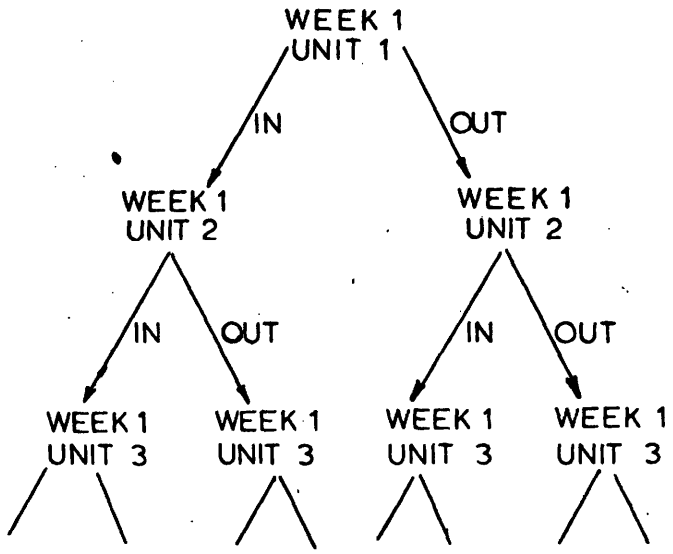

Figure 1.2-1 Branch and bound search technique used for maintenance scheduling in reference (24)[17]

'toughness of fitting' a unit into the schedule.

The scheduling mechanism offered in the author's previous paper, reference (19), does not require a priority list instead it considers all plants simultaneously with a sophisticated static technique which operates within a security constraint using a dollar cost and/or environmental impact measure of desirability. This method considers cycling and base loaded potentials and computes figures such as end-of-week reservoir storage quotas, hydroelectric production quotas, nuclear fuel consumption quotas, and buy and sell decisions on bulk power contracts. Because this technique yields these end-of-week quotas it fills needs usually relegated to special purpose

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17. From reference (24).

computer programs. For example, there is no need for a separate nuclear fuel relegation computation,[18] or for separate computations of the weekly reservoir levels at which to be aimed.[19] It must be considered that these separate special purpose programs cannot be perfectly spliced into a maintenance schedule, unless numerous iterations are performed between these separate procedures until they are in exact accord. Thus, a single program which incorporates these other problems must be considered to have an immediate advantage.

Especially since World War II, nearly every optimization technique available has been tried on the unit commitment problem, where every hundredth of a percent improvement in scheduling can mean literally thousands of dollars in savings. Nearly all of the successful unit commitment solution techniques have relied upon the extension of the incremental cost scheduling methods used in minute to minute economic dispatch.[20] Other dynamic solution approaches, such as dynamic programming, work well[21] until a large number of plants must be considered. Dynamic approaches with probabilistic load meeting requirements have also been

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18. Such as is in reference (25) or reference (26).
19. Such as is presented in (27), (28) or (29).
20. See references (7) and (30) through (33).
21. This opinion is contained in reference (34).

considered.[22] A limited amount of research in the use of the maximum principle is available in print, and, at least for the economic operation of hydroelectric plants seems to enjoy the advantage of greater accuracy than is available with dynamic programming.[23] However, outside of other weaknesses[24] that these techniques have, they may give rise (as do many dynamic techniques) to unstable or unrealizable solutions and may require tremendously complex solutions, such as two point boundary value problems or conjugate gradient searches for optimization of Hamiltonians, see figure 1.2-2.

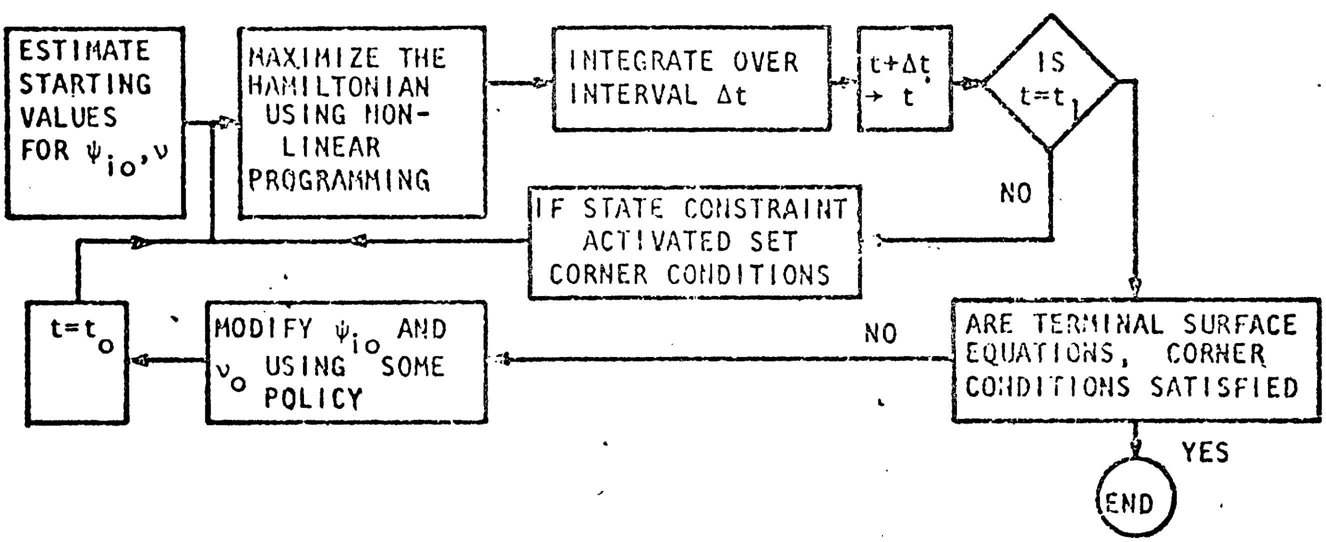

Figure 1.2-2 Computational procedure for the solution of the unit commitment problem via the Maximum Principle[25]

22. See refs. (35), (37), (38), or (28) with method in (36).

23. Refer to references (39) through (44).

24. See reference (45) or (46).

25. Excerpt from reference (47).

Static techniques also have been developed, with varying success, for solving the unit commitment problem. Over a daily interval, use of an interruptible gas supply has been considered.[26] Integer programming[27] and mixed integer programming[28] have been attempted for the solution to this problem, but because of the dynamic programming nature required to consider probabilistic demand curves and the more or less continuous nature of many of the variables, these techniques fall prey[29] to the same dimensionality and magnitude problems that plague the dynamic programming techniques. Other techniques that have been tried are gradient search [30] and minimum norm contraction mappings,[31] but neither approach appears to be promising for use over longer than daily time spans with large systems, that is, in a large week-long unit commitment problem.

However, to start at the beginning historically, the first realization that the unit commitment problem, with its particular startup and shutdown costs, should use a technique different from the usual incremental

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26. See reference (48).

27. This application was done in reference (49).

28. See reference (50).

29. See reference (51), page 321 for an authority for, and explanation of this opinion.

30. See references (52), (53) and (54).

31. See reference (100).

cost technique, was in 1959, reference (55).[32] Previously, using a straight incremental cost computation, when a plant dropped to 10% to 25% of its rated maximum capacity it was dropped entirely from the system, because this was considered to be the point at which the fixed operating costs were making it too expensive to operate this plant. The first unit commitment scheduler, as the load was decreasing, would determine the shutdown of generators based on the considerations:

(1) minimum down time
(2) startup cost
and (3) plant efficiencies.

According to these considerations the scheduler would build up a strict priority of shutdown "rule" for different "seasons," i.e. different daily load shapes, by considering whether or not it would be possible to restart the next most inefficient plant by the time the load again reached its present level, see figure 1.2-3 on the following page. Then it would compute whether or not the startup cost would wipe out this potential savings. This particular technique did not consider any possibility of spinning reserve requirements, hydroelectric or nuclear power, pumped hydro or gas turbines taking up slack, nonlinear loading curves, or a difference between startup and shutdown priorities, so other schemes followed.

32. Another that followed soon after was ref. (56), 1960.

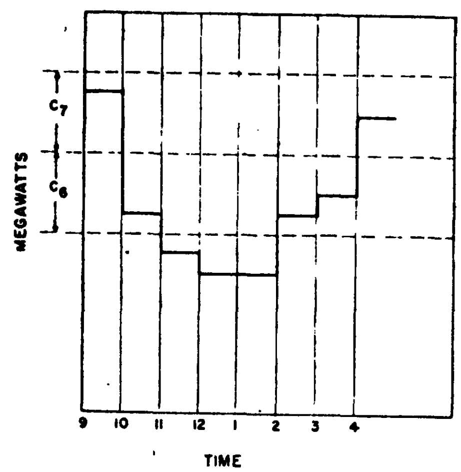

Figure 1.2-3 First unit commitment 'shutdown' rule involved turning off specified plants when certain demand levels were reached, reference (55).

Slightly more accuracy is obtained from a later work, reference (48) in 1965, in that spinning reserve, possible limitations of fuels (in particular gas), multiple daily shutdown possibilities (by defining unit commitment 'day' from peak to peak), and different startup and shutdown orders are possible. This method still, however, requires a priority of unit removal, and the removal of those units is just made so as to not violate the daily load forecast demands, see figures 1.2-4 and 1.2-5 on the following page.

As more and more features were incorporated into the unit commitment problem, solution techniques were not capable of handling all of the complexity. Many techniques which then came into general usage were heuristic approaches which completely subdivided the problem into separate efforts for pumped hydro scheduling, hydro scheduling, etc.,

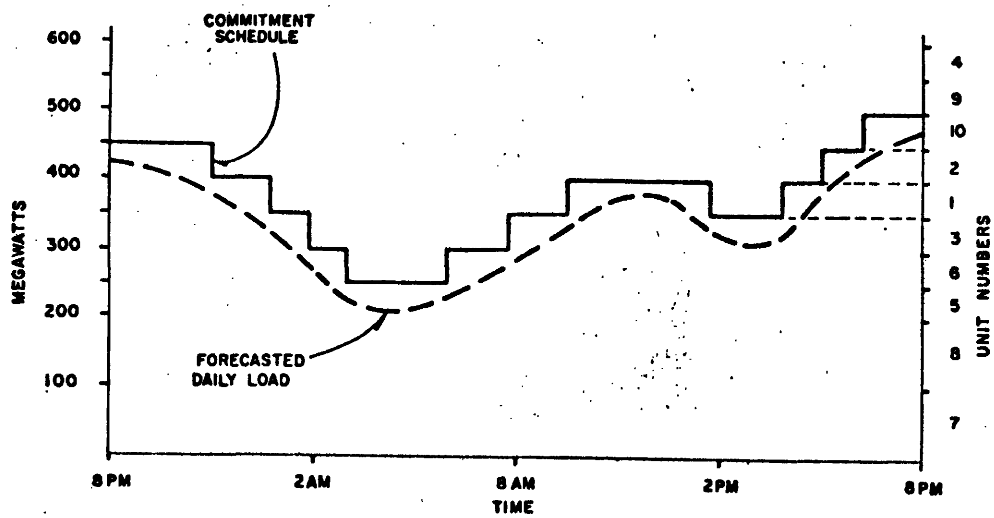

Figure 1.2-4 Graph showing the removal of units on a priority system so that the load can be followed[33]

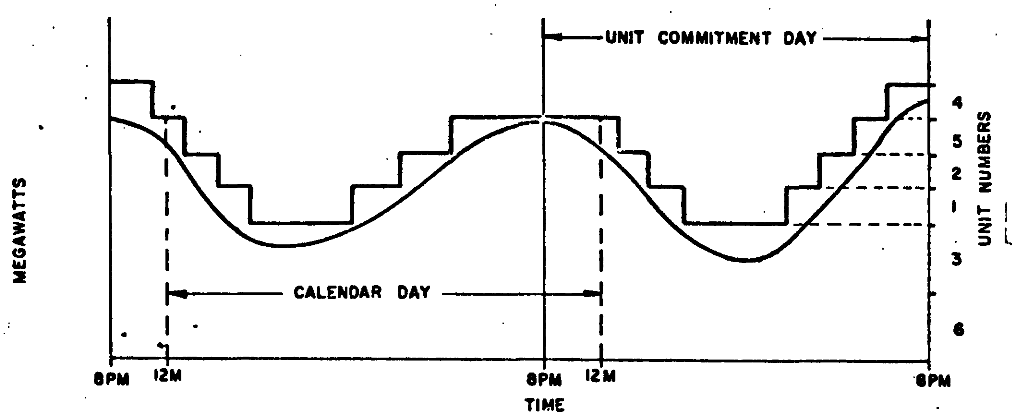

Figure 1.2-5 Definition of a unit commitment 'day' for use in the case of multiple daily shutdowns[33]

and after these productions had been deducted from the load-to-be-met, fossil fueled thermal power was added in quantities just sufficient to meet the system security constraints, see figures 1.2-6 and 1.2-7. Although these are relatively crude methods for the inclusion of hydro and pumped hydro, they were much better than not considering these aspects at all.

33. From reference (48), page 420.

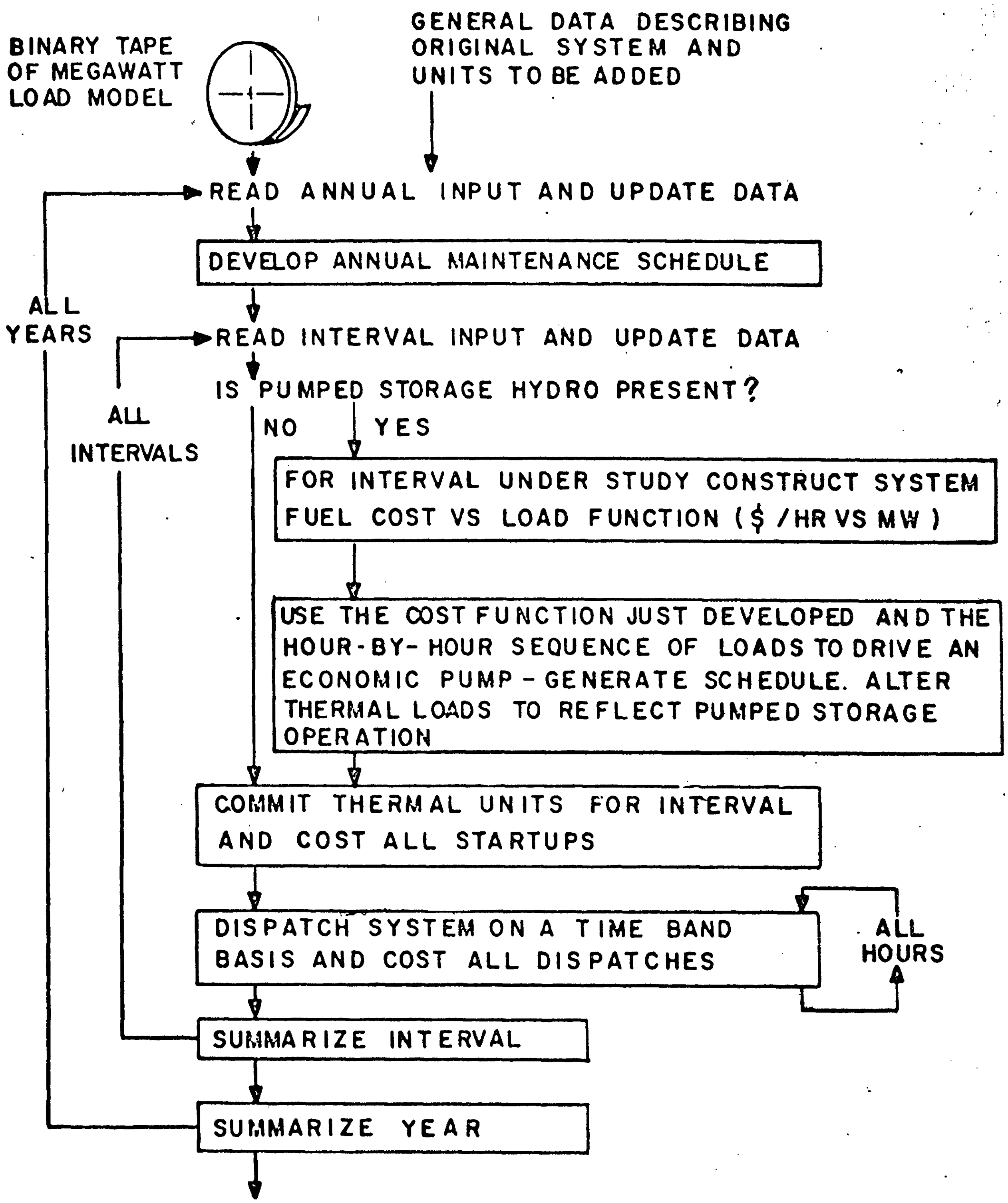

Figure 1.2-6 Heuristic approach to the scheduling problem completely decomposing system into its components, ref. (57).

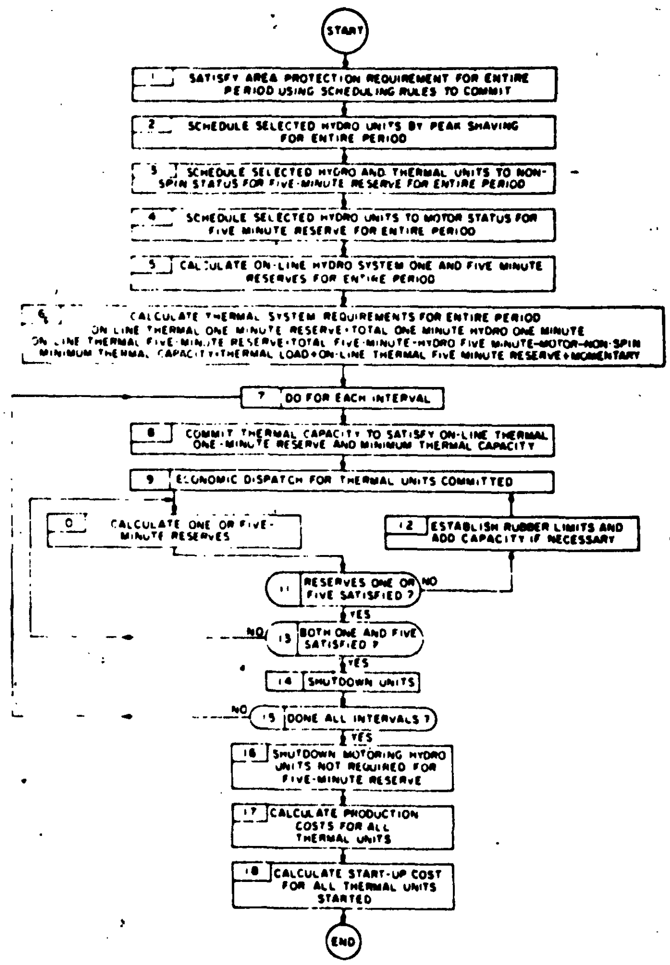

Figure 1.2-7. Heuristic approach to the subdividing of the scheduling problem with provision for one minute and five minute spinning reserve requirements[34]

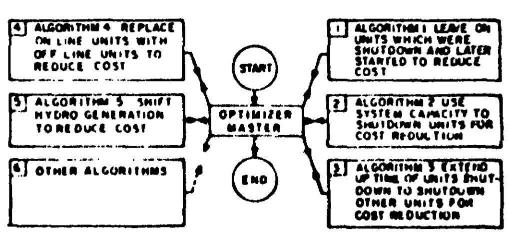

Figure 1.2-8 Method of perturbing solutions from the approach in figure 1.2-7 so as to search for decreased costs[34]

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34. From reference (58), page 1380.

The latest dynamic techniques, while they can deal with complex, nonlinear conditions, and probabilistic methods, nevertheless require discretization of the operating states, fake incremental costs for pumped hydro, hydro, and nuclear power,[35] and must search over a good portion of all possible ways of operating the system over a week, OR they must seek their optimum in a function space. For handling specific parts of the unit commitment problem these techniques can be workable. Thus, the method of attack they usually employ is to section out the hydro or pumped hydro aspect of the problem, either requiring a pseudo-incremental cost for water,[36] or computing such an incremental cost and iterating between the hydro or pumped hydro and thermal parts of the problem until the incremental costs match,[37] see figure 1.2-9 on the following page for a pumped hydro - fossil incremental cost comparison.

These hydro and pumped hydro incremental cost arguments have been extended to the monthly planning of water power usage so weekly quotas could be developed for

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35. Unless they meet quotas such as is presented in a production scheduler, like reference (19) has, and even then this would tremendously increase the number of discrete variables and thus astronomically increase the total number of possible operating combinations for the whole week.

36. As in reference (59).

37. See reference (30), or (60) and (61).

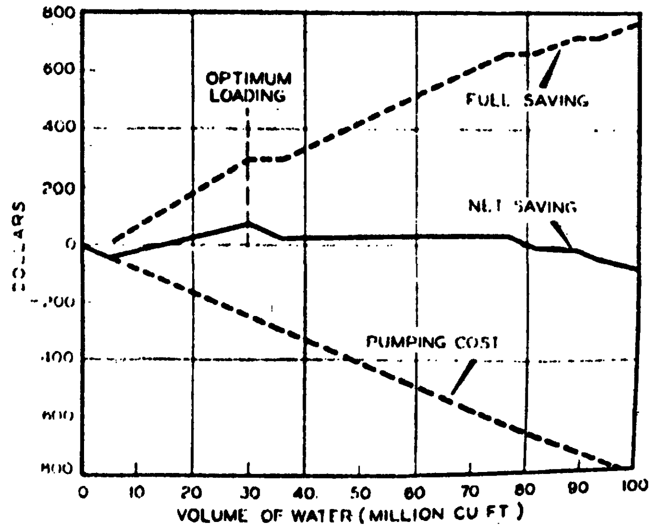

Figure 1.2-9 Comparison of fossil fuel and pumped hydro incremental costs[38]

unit commitment schedules. Here, typically,[39] the hydro power is planned to shave off the extreme peaks, and the pumped hydro is then used to levelize the remaining demand for power, see figures 1.2-10 and 1.2-11.

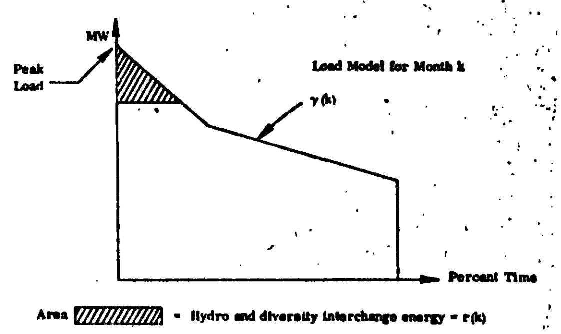

Figure 1.2-10 Incremental cost technique for monthly placement of hydro energy utilization, assuming this to be the cheapest form of power[40]

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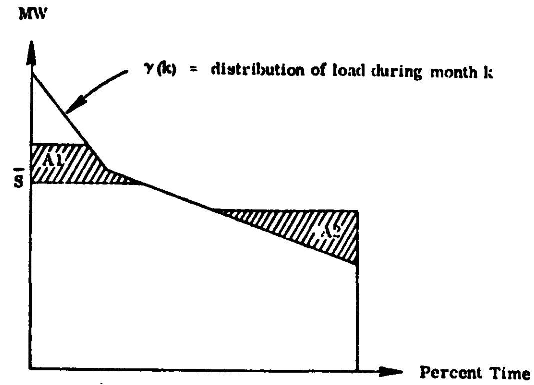
38. From reference (62), page 27, although this particular curve was meant to be a dispatching tool.

39. See, for example, reference (29).

40. From reference (29), page 28.



A1 area = pump storage generation energy
A2 area = pumping energy = A1/EFF

Figure 1.2-11 Monthly placement of pumped storage energy utilization after hydro has been removed from scheme[41]

The reason for the heavy concentration of effort on the optimization of hydro power is the large amounts of money which can be saved by proper treatment of this particular problem. Refer to figure 1.2-12 to see the tremendous difference in operating procedure that can result from a detailed optimization of hydroelectric power usage.

There are a number of dynamic solution techniques which avoid the problem of requiring pseudo-incremental water costs. Some of these techniques, such as the Maximum Principle in reference (64), can even treat the problem of delays of water from one reservoir to another on the same water system.[42] This hydraulic delayed coupling can be a significant factor at some

41. From reference (29), page 28.

42. Although (65) offers a less difficult solution technique than that proposed in reference (64).

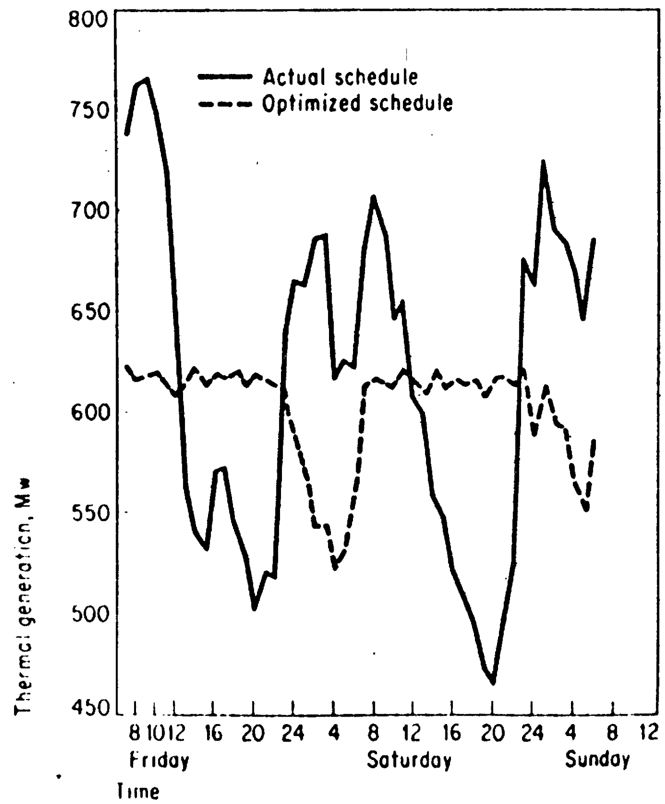


Figure 1.2-12 Comparison of an actual operation schedule and a hydro-thermal optimized schedule[43]

sites,[44] particularly where small streams are the water carrier, but apparently this is not frequently a large enough problem to warrant the use of the numerical complexity involved in functional analysis on a large system (especially considering that this problem can be modelled in a linear programming framework).

Another more recently developed dynamic technique using incremental costs sections off the system reliability problem, rather than the hydro aspect, as the angle from

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43. From reference (63), page 47.

44. See reference (66).

which to attack the unit commitment problem. Figures 1.2-13 and 1.2-14 show a method[45] which removes each plant, one at a time, for as long as it can be kept out of the system without violating the constraint on the security measure, and finds the one plant which realizes the most savings. It then removes this plant and starts again to find the next plant to take out. For a large system, the number of examples which must be

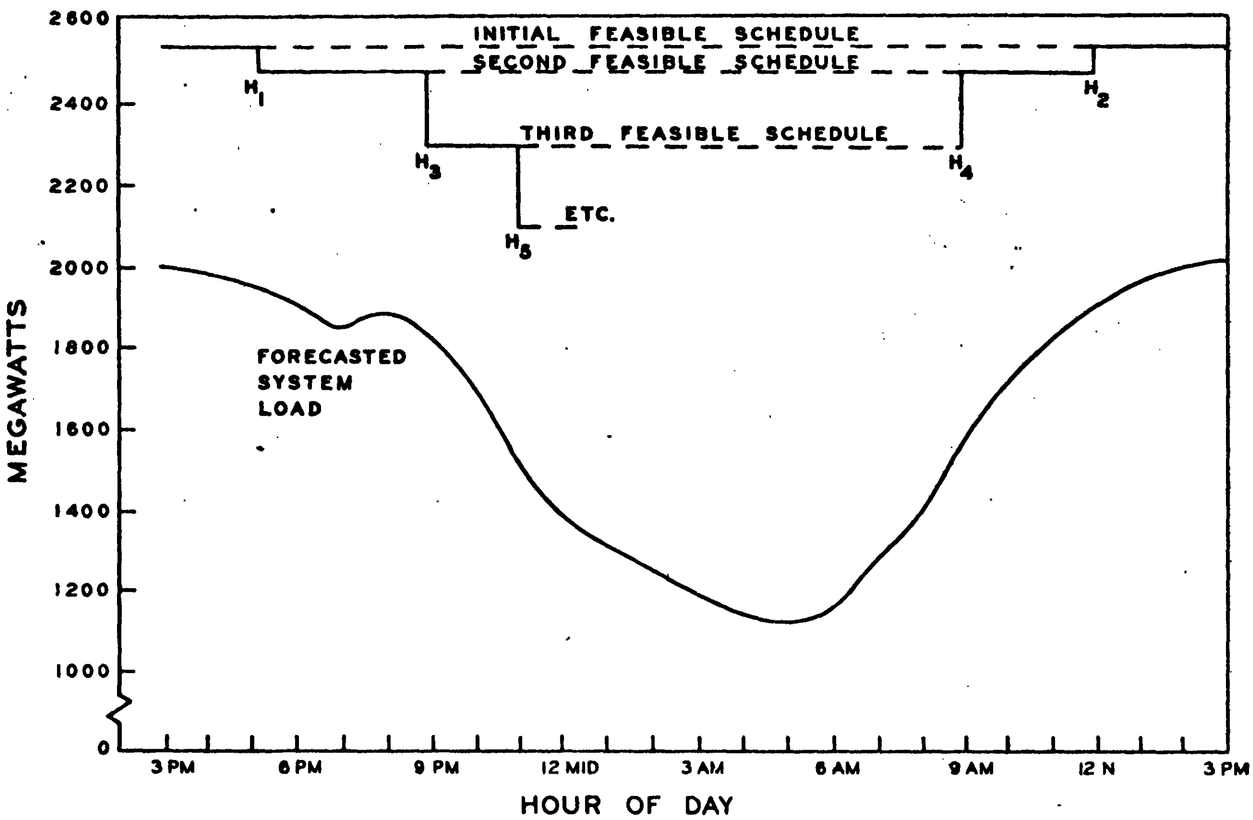

Figure 1.2-13 Demonstration of the iterative method of plant removal using a security constraint[46]

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45. See reference (67), also used in (68) and (69) with the technique described in (70).

46. From reference (67), page 1387.

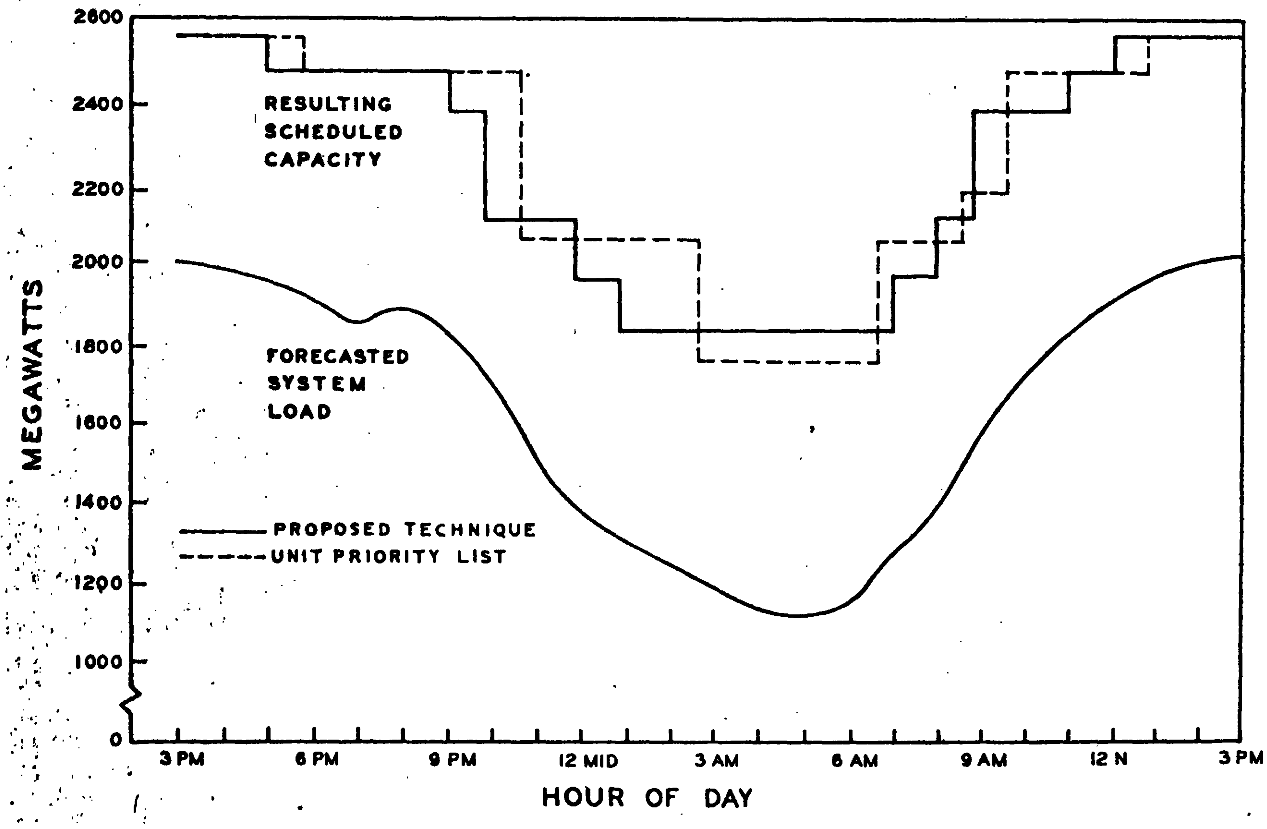

Figure 1.2-14 Comparison of strict unit priority method to the security function constrained method[47]

considered can be substantial, and nevertheless, none of these fill-in-the-valley one at a time programs can select the best schedule, or even an acceptable schedule, except by chance.[48]

A number of nonlinear solutions to the unit commitment problem have been proposed,[49] but these perform much better in on-line dispatch tasks, and involve

47. From reference (67), page 1387.

48. See footnote 6. on page 16 for a proof of non-optimality and non-viability of these techniques.

49. See references (33) and references (71) through (78).

too much computation for large (100 plant), week long, unit commitment problems. A nonlinear method[50] which goes so far as to include startup and shutdown rates, uses local linearizations to solve the nonlinear formulation, see figure 1.2-15

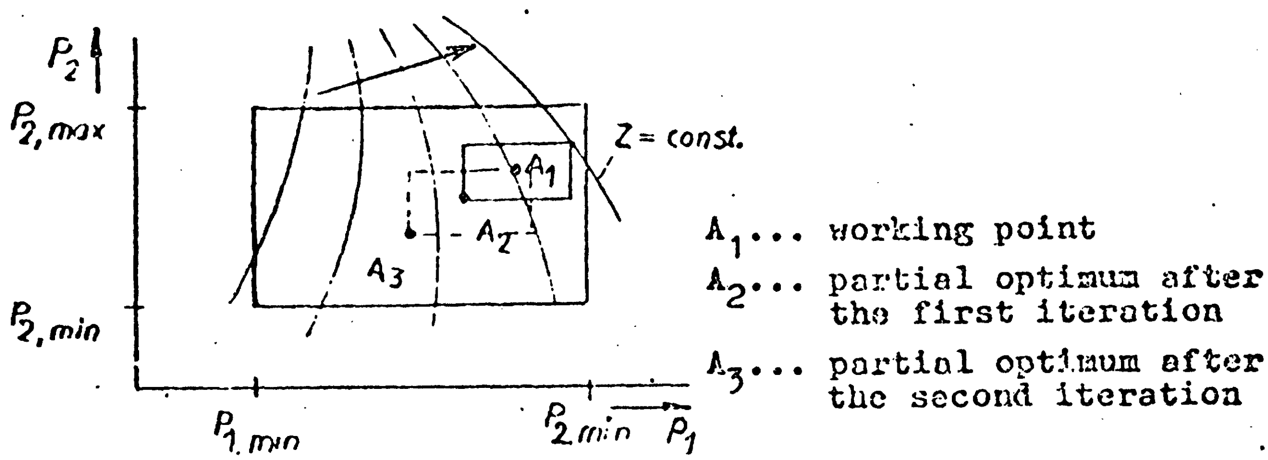

Figure 1.2-15 Method of optimum seeking using local linearizations of the nonlinear objective function[51]

Unfortunately, there is no proper provision for shutting down plants (this could be alleviated by the addition of integer variables) because this technique uses an unclear rule for shutting down plants, called "costly generation," which fall below minimum output requirements.

The static techniques, of which this study is one, appear to show the most promise for fast, accurate solutions to large unit commitment problems. Static studies previous to this current project were, unfortunately

50. See reference (79).

51. From reference (79), page 18.

forced into the use of pseudo-incremental costs or pseudo-limitations for the use of water (or nuclear) power. The first static technique, reference (49), severely restricted itself by using pure integer programming. Thus, there was no room for any continuous variables. The display of typical incremental costs for individual power generating units is given in figure 1.2-16.

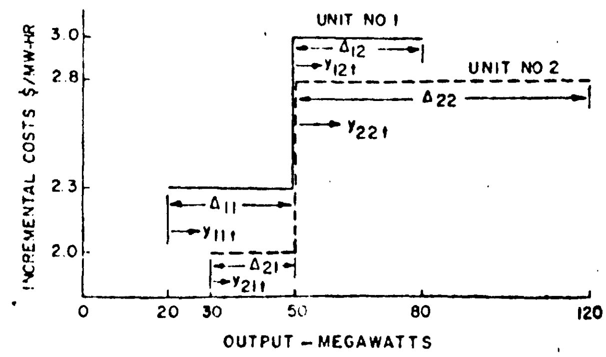

Figure 1.2-16 Incremental costs of power plants in integer mode formulation[52]

The integer solution technique, the tableau method, is very slow and cumbersome, involving rotations about each non-integer coefficient in the solution space.

A mixed integer formulation, in reference (50), does allow for continuous variables, and uses the much faster branch and bound solution method, but runs into dimensionality problems. There is no algorithm presented in that paper which facilitates the cutting up of large, week-long problems into reasonably sized chunks. Also, a discretization of the probability load curve, see

52. From reference (49), page 730.

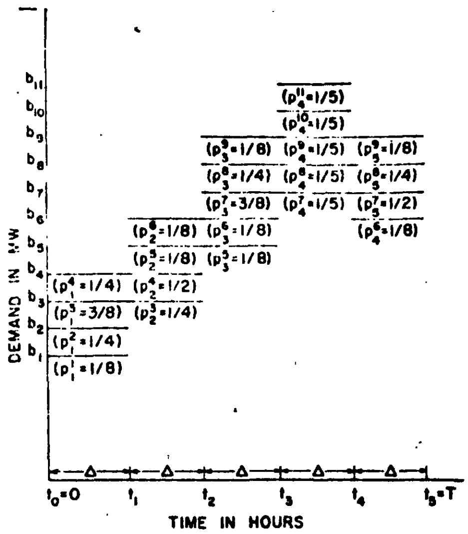

Figure 1.2-17 Discrete breakdown of the probabilistic load forecasts for use in a mixed integer program[53]

figure 1.2-17, forces the solution to be computed for <u>every</u> combination of load probabilities, an astronomical number, e.g. five discrete load probability levels for each of the 168 hours of a week would lead to 5^{168} (more than a googol) different demand curves which must be scheduled. A very good mixed integer formulation is contained in reference (80).[54] Unfortunately, since the time intervals that are considered are slightly more in the dispatch area (minute to minute) than in the unit commitment (hourly), transmission effects are included (10 nodes). The complexity added by this inclusion forces a breaking up of the problem into

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53. From reference (50), page 1969.

54. This technique is more fully described in reference (81), originally from (82), with a corresponding dispatch technique described in (83), and the splicing together of these different hierarchies described in reference (84).

separate thermal and hydroelectric studies with an eventual splicing. The mixed integer formulation is thus reduced to the task of computing incremental costs (using the dual variables) and thus is very similar to the early simple incremental cost techniques. There are a number of other weaknesses; pumped hydro cannot be considered, hydro is used only to "levelize" thermal outputs i.e. peak shave, no hydro network transmission is considered, each time interval is considered separately and then spliced to the others, there is no provision for bulk power purchases, and individual plant loading curves can only have one, linear, incremental cost segment.[55]

Moving now from the unit commitment problem to the dispatch problem, there are such a number[56] of these minute by minute dispatch techniques that if it is desirable to find a method which splices together well with the unit commitment technique, then it can be found. For example, there are several static programming dispatch methods.[57]

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55. It appears that this inaccurate linear loading curve requirement would introduce more error than could possibly be gained in the consideration of transmission losses.

56. Some include references (85) through (92).

57. Some are reference (54), references (93) through (99), although (99) is more of a fuel management transportation and consumption model.

Only two of all of these dispatch methods (and no unit commitment or maintenance methods) include any consideration whatsoever for the environment. The first of these two to appear, reference (101) in December 1971, uses nothing more than an incremental cost dispatch, where instead of dollar costs it uses tons of nitrogen oxides which go up the stacks. So, replacing the dollar versus megawatt loading curve, is a 'tons of NO_x' versus 'megawatt' curve, see figure 1.2-18. Slightly more realistic than this is the study hypothesized in reference (102), July 1972. This technique uses wind directions and Gaussian dispersion models to predict the superimposed

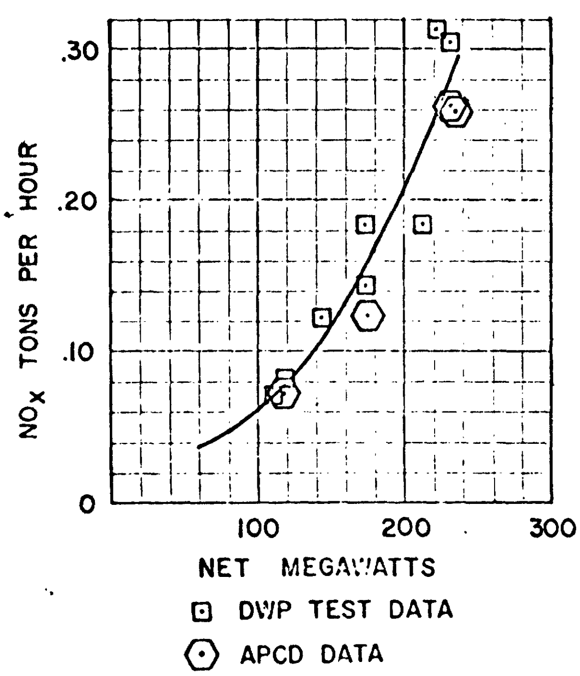

Figure 1.2-18 Tons of NO_x versus megawatt loading curve for a power plant, DWP is a Los Angeles county government test, APCD a U. S. government test[58]

58. From reference (101), page 2653.

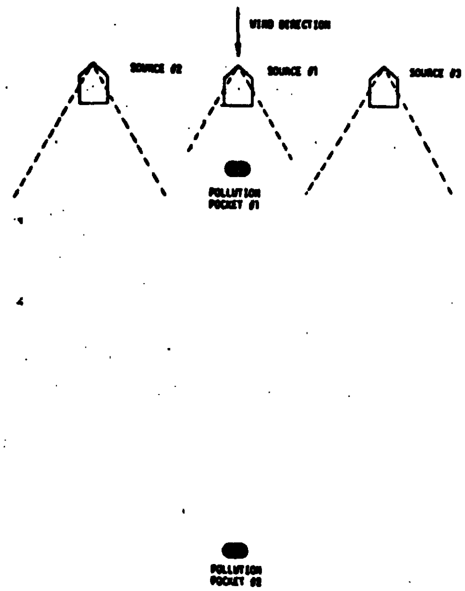

Figure 1.2-19 Hypothetical representation of pollution sources and points at which concentrations are to be predicted[59]

concentrations at one or two points from all power generation pollution sources, see figure 1.2-19. Otherwise, the solution technique is identical to existing dispatch mechanisms, using incremental pollution concentrations at selected points rather than incremental dollar costs.

So, in summary, there exists no unit commitment scheduling techniques which can handle week-long problems with optimal or near-optimal results. The dynamic techniques require crude discretization of individual plant output levels, and then still must search over enormous numbers of possible solutions, even for a single

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59. From reference (102), page 2.

day of scheduling. Static techniques also fall prey to the huge number of possibilities which exist over the course of the operating week, and if they do not use some integer variables, then they also require excessive simplification of such problems as minimum power outputs. Obviously, both techniques fail in that they cannot make firm decisions as they proceed through a week, or even a day. Heuristic techniques made specifically to cut the problem down into separate components, and usually smaller time horizons, can not approach optimality without tremendous numbers of adjustments back and forth between these separately considered - but obviously coupled - portions of the overall problem. So what is needed is a technique which can step along, making firm decisions as it proceeds, while keeping week-long problems in mind (e.g. weekly quotas or pumped hydro cycles), and which can consider all the intercoupled aspects of the problem simultaneously, e.g. thermal power outputs, hydro outputs, nuclear outputs, reservoir levels, pumped hydro usage, and overall system security requirements.

This unsolved problem is further complicated by the pressing environmental issues. A. H. Aymond, head of the Edison Electric Institute has pointed out that "the days are gone when a utilityman could sit confident that power is an undebatable blessing, accepted without argument or discussion by the people."[60] Thus, what is required now is a sophisticated technique which includes both economic and environmental performance measures.

1.3 Results

The results of this research project include:

(1) a modelling of all the components of the scheduling problem,

(2) a solution technique which reaches the desired quasi-optimal schedule and requires minimum readjustment for changed input factors, and

(3) a computer program realization of the solution technique, with a sample problem.[61]

1.3.1 Model Description

The model for the generation scheduling problem is set in a linear framework. Although this format is somewhat constricting upon some of the nonlinear scheduling factors, for the most part the nonlinearities approach linear functions before the scheduling decisions are made.

The forecasted demand to be met by the schedule is assumed known, and the necessary reserve requirements are included in the demand which must be met. Adjustments to the demand-

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60. Excerpt from reference (103), page 52.
61. For the comparison of the quasi-optimum technique to the optimum see reference (19).

to-be-met curve are made for fixed and flexible interregional power exchange contracts, probabilistic emergency support and interruptible loads. The solution technique makes decisions about which contracts to honor, and extent to which variable contracts should be subscribed, as well as indications of when oversupplies of power are available for bulk interregional sale possibilities. Contract possibilities are enumerated even at times when the region has no oversupply of power, with the final schedule yielding a list of all the intervals and the cost of producing more power in those intervals. Also, the cost of meeting extra unexpected demands is produced for each interval, pointing out the times when it might be prudent to overèstimate the reserve requirements.

The capabilities of the generating system in the model are time-varying to account for the weekly variations in output capabilities. Capacities of the plants are derated to the extent that they incur forced outages, or to the extent that they are debilitated during repair of support equipment. Each generating facility is fit with a piecewise linear loading curve, including provisions for minimum operating capacities. Rather than having a loading curve, the pumped hydro plants are operated under input pumping efficiency and output efficiency models with appropriate constraints on water usage, reservoir levels and output capacities. Quotas are obtained from the maintenance and production scheduler (reference (19)) for the weekly targets of nuclear fuel consumption, hydro reservoir usage, gas contract limited energies, and pumped hydro reservoir level targets for the end of the week. Penalties or rewards are available for deviations from these target levels.

A nonlinear startup cost is used to accurately predict restart charges based upon down times, and provisions are made for minimum down times, and startup rates. A single measure of spinning reserve is presented, although it is just as easy to introduce a second measure, e.g. one minute and five minute reserves (that is, spinning reserves available with that much advanced notice).

Geographic constraints, viz. 'must run' plants or minimum capacity requirements within a sector, as well as a certain amount of transmission limitation and losses, can also be modelled.

The time intervals vary in size over the span of time covered by the scheduler. As less information is known about the future, this changing size interval (from one hour long to eight hours long) insures that equal weightings are attached to equal amounts of information. This scheme is also used to reduce the number of variables which must be considered.

The quality measure of the simulation is measured in both dollar costs and ecological impact consequences, and the use of the presented solution techniques results in the determination of all possible optimum pairings of $ to impacts ranging from the minimum cost end to the minimum possible ecological impact for a given reliability level, (for more of the very specific scheduling and simulation studies performed with this scheduler refer to reference (104)).

1.3.2 Method of Solution

The method for the solution of the proposed model is a dynamically evolving decision process which uses mixed integer programming to make current decisions and linear programs to keep the future system within its restrictions (but not forcing decisions for the future system). This is then a quasi-optimal sequential process which requires operator participation at each iteration (about six hours covered per iteration).

A decision field is defined which includes all decisions within a time span (about six hours) as well as those outside the span which are directly or importantly coupled to the current decision-making process. Those firmly determined decisions within one field are fixed, and the process passes to the next field (which overlaps the previous field slightly in time).

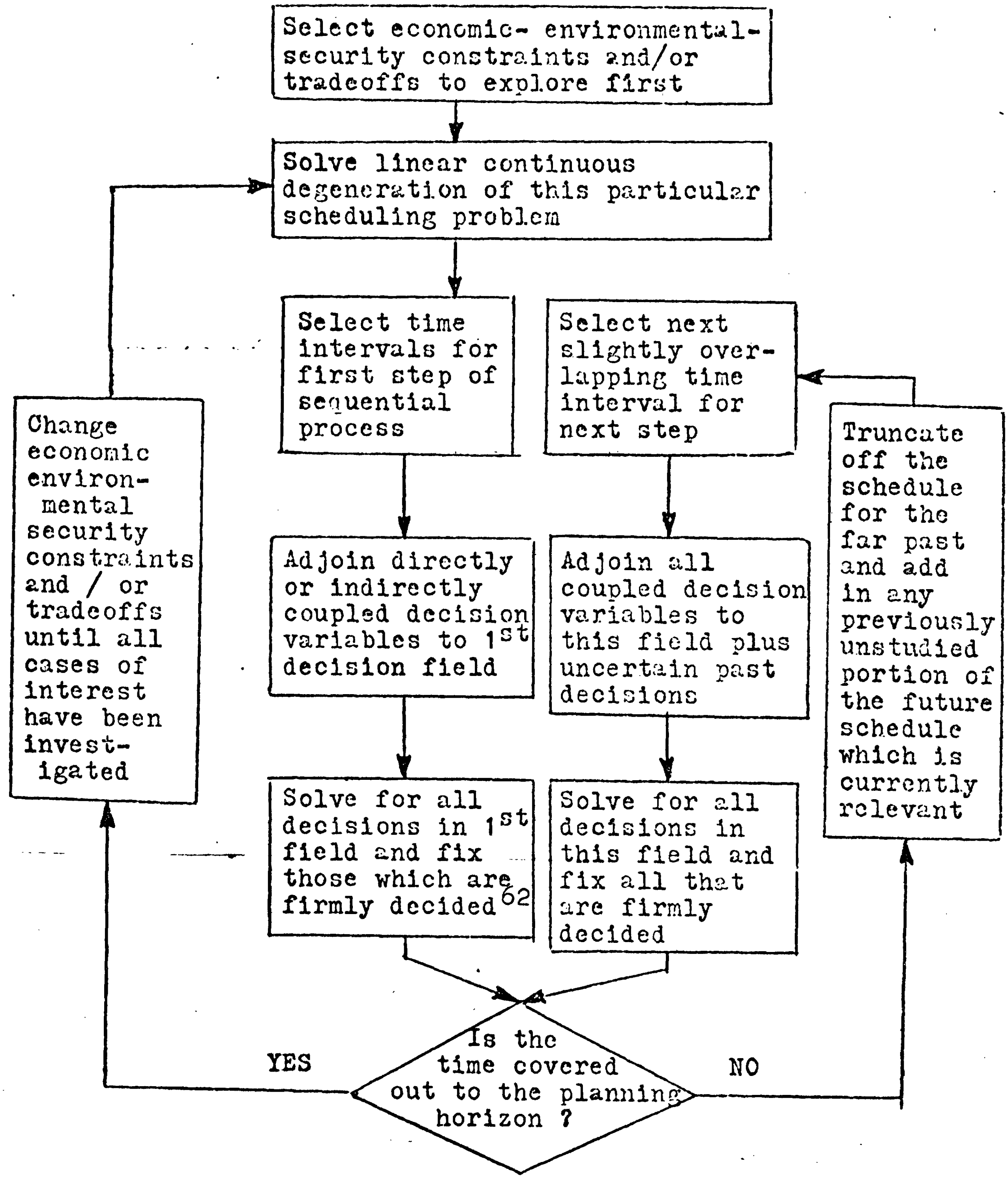

Figure 1.3.2 Flow chart of the dynamic evolving mixed integer program used in the scheduling process.

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62. Here terms such as indirectly coupled and firm or uncertain refer to closeness to the optimum supporting hyperplane, or the propensity to change, as measured by the solution to the dual problem.

When used as a scheduling tool it is only necessary to proceed far enough in the sequence to fix the current decisions, usually only two or three iterations. As a simulation tool, the model must be iterated over the entire time span in question, but has the advantage of computation time required being linearly (not exponentially) dependent upon the span of time considered.

Recomputation of a schedule due to changing factors requires a minimal computational effort. The dual solution to both the mixed integer and linear programs presents a sensitivity measure of the decisions to various changing input parameters (such as changes in forecasted demands, river levels, or new or bought capacities becoming unavailable). When it is determined that a recomputation is required, the solution to the decision fields previous to the disturbance can be salvaged intact, and if it happens that the perturbation has a short-lived effect, the old solution can be reclaimed for some of the future decision fields.

A solution to a small (eight power plants over one week) sample problem is presented. This demonstration system is meant only for giving an initial feel for the capabilities of the scheduler. A test of the validity of this quasi-optimal technique has already been performed in reference (19). The extensive use of this mechanism as a simulator and a scheduler on numerous sample problems is presented in reference (104). The tools required for the manufacture of the input data, such as load forecasters and river flow predictors, are available from other sources, with the quantification of the environmental impact to the air and water being presented in references (105) and (106). Thus, this paper is meant primarily as a detailed description of the modelling of the scheduling mechanism itself.

1.3.3 Computational Feasibility

Because this problem has been set up in a form for which the integer decisions are all bivalent, the computer time, and thus costs, are small. Besides the fact that with the pseudo-Boolean constraints all integer solutions are on the corners (the linear programming simplex method seeks out only corners) of the space of feasible solutions, the problem setup has a distinct mutual exclusivity, i.e. 'multiple choice,' characteristic which decreases to a small fraction the time required per integer decision.

Almost every computation facility has available the linear and mixed integer functions used in the solution technique presented in this project.[63] If, however, the facility to be used does not have sufficient capability there are a number of simplifications, in the form of

63. It would be possible to create a fairly good schedule without the mixed integer subroutine, i.e. with the linear and dual solutions alone, see reference (104) page 81.

approximations, which can be made, e.g. the decision fields could be cut in size.

1.4 Presuppositions

The most widespread assumption of this approach is the assumed linearity of the problem form, or to be more precise, the piecewise linearity and integer form. Fortunately, however, most of these approximations, if they prove to be too inaccurate, can just be modelled with further segments added to the piecewise linear model. Exceptions, such as the synergistic ecological effects of operating two plants in close proximity, can be dealt with to a certain extent by overestimating the costs of each plant operating alone, and preserving the linear pattern. In general, the solutions of nonlinear problems with the dimensionality considered here, are either not computationally feasible or are prohibitively time consuming procedures. One nonlinear possibility, however, for future considerations in this research area, would involve a linear problem setup with a nonlinear objective function.[64]

In the problem modelling process there have been many assumptions and approximations. For example, the reserve

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64. It is highly unlikely that attempts at problems which are either not quadratic or are inseparable would be fruitful. The most likely candidates for nonlinear objective functions would be those which were convex in nature, although even convex functions are fairly time consuming for linear programs to handle, let alone mixed integer programs.

requirement is assumed to be a function of the load and not of the plants in use at that particular time (which would have caused a nonlinearity). Similar linearity assumptions are explained throughout Chapter 2 as they are introduced into the model.

There is in this project no attempt to level the oversupply of power, that is, above and beyond the demand plus reserve requirements. If the reserve is not felt to be adequate it can be pushed up (until it is at a level where there is no feasible schedule in which case the ϵ-optimal solution is found), and in this way any particular desire for leveling the oversupply can be met. Any intervals for which there is particular concern can be granted extra added reserve allotments.

Forced outages have been averaged in as percentage plant capacity deratings,[65] instead of being treated probabilistically.

No attempt has been made to refine the time intervals down beyond one hour. Further refinements are possible, though, within the framework of the model.

Of course, the piecewise linearization of the plant's loading curves is an approximation to the actual nonlinear curve, but considering that most techniques can use only a single linear loading curve, this represents an improvement over many existing schemes. Piecewise linearization

65. There is some evidence which supports the contention that this adds negligible inaccuracies, see reference (22).

of the variable head effects on reservoir power productions is also an improvement over the linear schemes which have proved to be acceptable.[66] A transmission loss model is described, but has not been developed fully because of the negligible[67] addition in accuracy to a unit commitment scheduler that modelling of transmission incorporates, namely that the small improvement is lost compared to the load prediction inaccuracies at this time scale.

There are a number of future studies which could be carried out to refine this particular research project. Examples of some of these studies are the study of the possibilities for and effects of the inclusion of a more probabilistically oriented security assessment model, or the clarification and further definition of the precise role played by the dual space, so as to hopefully allow its inclusion in the rigid, mechanical algorithm, if this is deemed desirable. Of course, one obvious need for further work in this area involves the development of a minute by minute dispatch technique which includes environmental as well as economic assessments of operating consequences. Without such a dispatch scheduler tuned to the same

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66. See, for example, reference (27).

67. This contention is contained in reference (79) on page 4.

economic-environmental-security objectives as are aimed at in the unit commitment scheduler, much of the gain predicted by the unit commitment mechanism will be lost.

2. Model

In formulating the model for this scheduling problem it is not possible, and in fact not as instructive, to remain completely impartial to the theoretical and computational feasibilities of the various setup's solutions. The fact that abstract formulations do shed light upon the variety of possible solution techniques is granted, and for this reason is discussed in section 3.1. However, when aiming at a clear portrayal of the problem, it is best wherever possible to deal with physical or visualizable quantities. Inevitably implied in such a detailed problem formulation is a solution technique. And that this problem setup seems conducive to a dynamically evolving mixed integer program should not be viewed as a contrival intended to make this seem like the 'obvious' technique, but should be considered a foresight to the results of the survey of possible optimization methods.

2.1 System Requirements

A logical first step in the formulation of a system model is a detailed study of the requirements imposed upon that system from external sources. For this problem, these exogenous demands are in the form of minimum constraints upon the output, such as meeting all requests for energy with good quality (i.e. constant voltage), reliable electricity, and in the form of a minimization of the inputs, that is payments from customers and usage of the environment.

By incorporating within the system, endogenously, the predicted demand levels and the fixed reliability requirements, it is possible to measure the 'performance' of the system in terms of its decision making alternatives alone. Section 2.5 on performance levels deals with the collection and weighting of the various input terms, and the remainder of this section deals with the endogenous incorporation of the 'output' demands.

2.1.1 Power Demands

Power demands will be defined as encompassing any demands made on the power pool which are definitely obligatory. All non-binding contracts between regions and any interruptible loads will therefore not be included here. Refinements which are to be made of the 'power demanded' before it can be used directly in this model are outlined in section 2.4.2. Section 2.1.1 of reference (19) gives a detailed description of the 'power demand' components, and thus this will not be repeated here.

Although the means are available, the forecasting of the probabilistic power demand curves is not within the scope of this study, and thus the load forecast will be considered as an input. It is, however, important to have knowledge of the factors which contribute to the load forecast. For example, techniques are available which incorporate within the load forecasts the weather factors which might be of importance.[68] This weather information is necessarily included in the prediction of environmental factors as well, thus any parameterization of weather factors to gain insight into the weather sensitivity of any particular schedule must show simultaneous changes in the environmental impact factors as well as the power demand.

2.1.2 Reliability Requirements

The term 'reliability' is fully described in section 2.1.2 of reference (19). Briefly, it should here be noted that for this unit commitment problem the reliability measure will be satisfied by meeting a pre-forecasted demand-to-be-met level computed from the probabilistic demand curve. For example, the demand-to-be-met level could be the expected power level plus four standard deviations of the power demand level. If a then computed schedule does not meet a certain security standard, the demand-to-be-met can be increased - either in the intervals of the security problems or over the entire schedule.

Reliability levels are further affected by the amount of spinning reserve required of the system, these spinning reserve requirements are described in section 2.4.2.

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68. Such a forecaster is documented in reference (107).

2.2 System Capabilities

From section 2.4.2 can be obtained a number of megawatts P(k) which represents the power level in the k^{th} interval which must be supplied by the system in order to realize the prespecified reliability level (thus P(k) includes reserve requirements).

If $PA_i(k)$ represents the capacity of the i^{th} plant in the k^{th} interval (derated to average in the effects of its forced outage rate, if necessary), and if

$$UP_i(k) = \begin{cases} 0 & \text{if the plant i is not operating during interval k} \\ \text{otherwise, between 0 and 1, denoting the fractional portion of the plant in use} \end{cases} \qquad 22\text{-}1$$

then for the system capacity in the k^{th} interval to at least meet the demand level

$$\sum_{\text{all } i} \left[PA_i(k) \cdot UP_i(k) \right] \geq P(k) \qquad 22\text{-}2$$

2.2.1 Capacity Levels

Derating of capacity levels due to reserve requirements is explained in section 2.4.2. There will, however, be additional times when it will be necessary to derate the maximum capacity ratings for generating units, for example, derating may result from the scheduled maintenance of generator support equipment. For the most part, however, capacity levels are relatively unchanging and can be treated in the ways described in the following sections.

2.2.1.1 Fossil Fueled Units

Fossil fueled units can be described by their own particular capacity, or loading, curve.

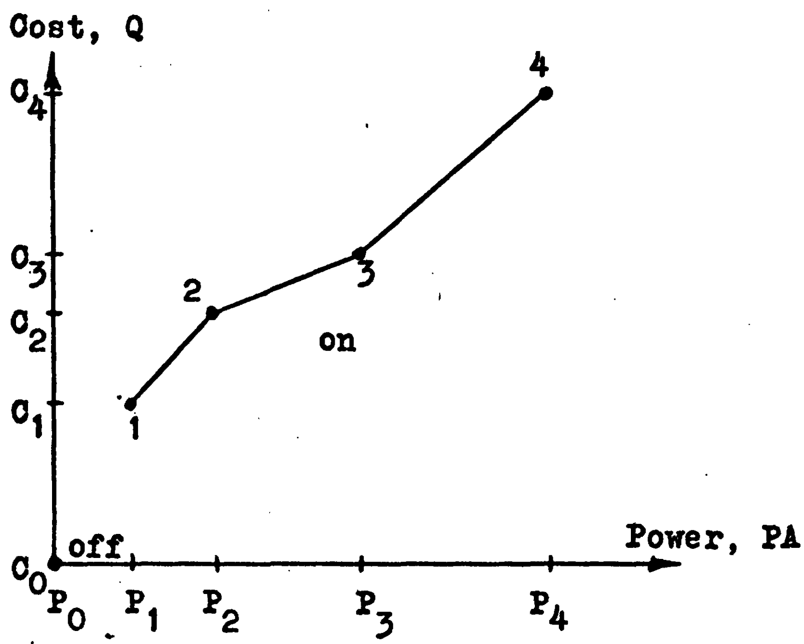

Figure 2.2.1.1-1 Piecewise linearization of a megawatt power versus cost loading curve for a fossil plant

First, it should be noted that the 'cost' in figure 2.2.1.1-1 may be either in dollar or some sort of environmental impact units. Secondly, there may be some power demand made by the facility even in the 'off' mode, thus P_0 may be negative. And, there is likely to be a cost associated with the plant being in the 'off' position, thus, C_0 may be greater than zero. These costs, however, may be assumed to be fixed, for they are not affected by the scheduling procedure. It is the quantities C_1-C_0 and P_1-P_0 which will be the important quantities in any decisions concerning plant operation.

For fossil fueled units using gas supplies there is the possibility of gas usage contracts either limiting the supply of gas and/or outlining a variety of fuel costs for various amounts of daily or weekly usage. An example of a dollar cost-gas usage curve over a time period (such as a week) is represented in figure 2.2.1.1-2.

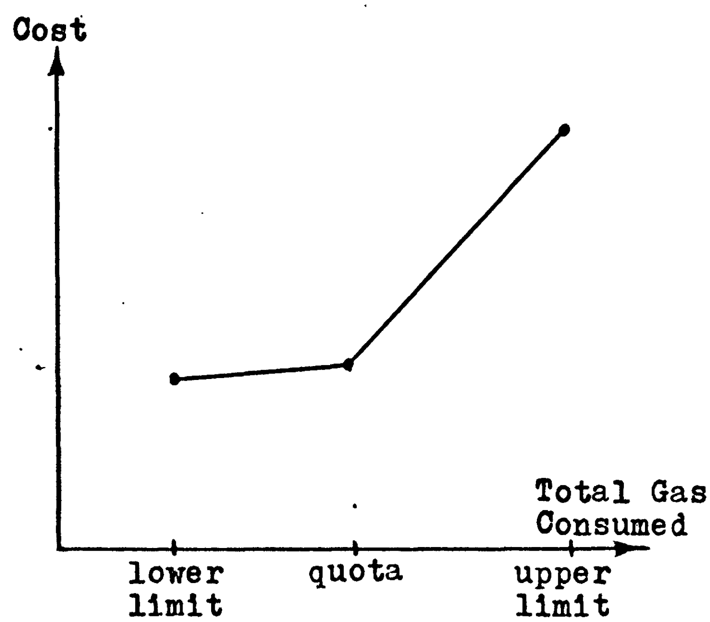

Figure 2.2.1.1-2 Dollar cost-gas usage curve which might be represented in a gas supply contract.

2.2.1.2 <u>Nuclear Energy Relegation</u>

Assuming that weekly nuclear energy usage quotas have been computed by a maintenance and production scheduler,[69] the unit commitment scheduler is responsible for determining the hour by hour usage strategy for this nuclear fuel so as meet these weekly quotas. However, for there to be a meaningful coupling between the unit commitment scheduler and the maintenance and production scheduler it is essential that the unit commitment scheduler not be totally constricted to a particular nuclear fuel weekly quota. Instead, within the unit commitment scheduler should be a mechanism which represents the appropriate penalties for not hitting the exact weekly quotas. Such a mechanism might be of the

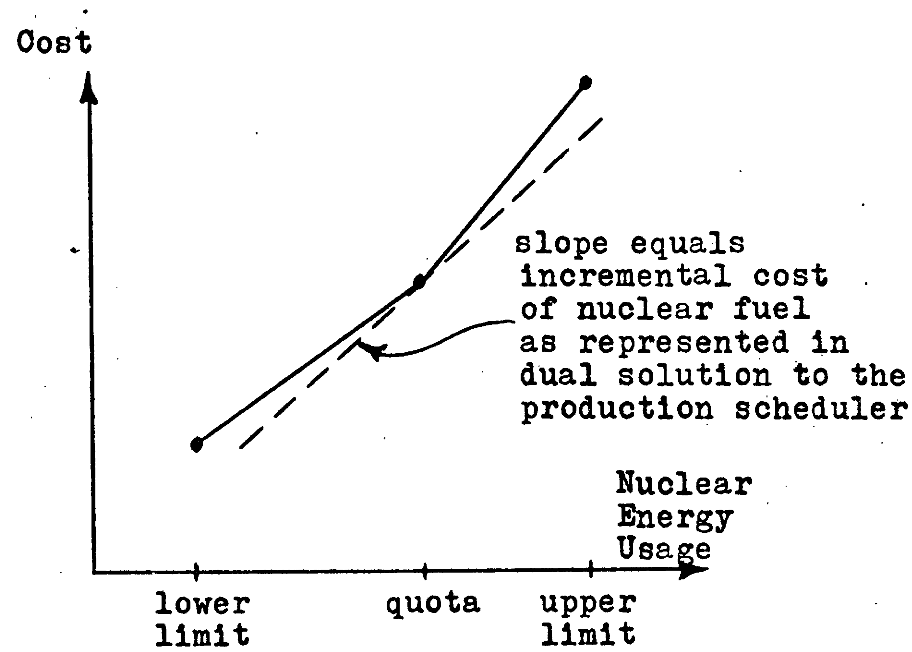

Figure 2.2.1.2 Penalty function representation for consideration of discrepancies between fuel usage and quotas.

69. For example, a scheduler such as is described in reference (19).

same form as the gas contract quota diagram, see figure 2.2.1.2.

Also to be considered in the scheduling of nuclear reactors are certain costs contingent only upon the on or off mode of reactor operation, or costs which may be dependent upon the entent of operation, but these costs are easily modelled in the linear - integer format.

It will be mentioned here, and not again in the hydroelectric section, that there may be consequential energy losses associated with the startup of facilities. In fossil fueled plants this can be considered as a pure dollar loss (assuming there is no inventory of fuel), but for facilities which must meet a weekly fuel quota these startup energy losses must also be included in the total weekly fuel usage.

2.2.1.3 Hydroelectric Capabilities

Because the maintenance and production scheduler yields hydroelectric quotas, in addition to the nuclear quotas, the same requirements apply here as are described in section 2.2.1.2, including the end-of-week disposition allowance penalties or rewards (like those displayed in figure 2.2.1.2.) The comments on operation costs are also applicable here.

Equations for the treatment of reservoir pondage accounting, including water inflows, spillage, and other reservoir requirements are given in section 2.2.1.3 of reference (19), and, thus, will not be repeated here. A problem inherent to hydroelectric unit commitment is the possibility of reservoir levels being close enough to upper or lower limits so as to require monitoring of the level during the scheduling process. This can be easily handled, however, by setting upper and lower bounds on the value of the reservoir level.

A more difficult problem peculiar to the hydroelectric situation is the effect of water pressure on the efficiency of power production. This effect, usually called the effect of variable head sizes, can be piecewise linearized if it is considered to be of significant importance. This can be accomplished by, in effect, defining different reservoirs associated with different sections of the head. The hydroelectric facility will then automatically deplete the higher, more efficient levels first. Care must be taken to preserve the proper loading order for the inflowing water. The only way this can be done, without the use of integer variables (in the same manner as the fossil fueled plant loading orders), is by assuming a knowledge of the approximate levels of the reservoir beforehand, and then inflowing into the proper stages.[70]

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70. This level approximation may not be a difficult task, especially in large reservoirs, because reservoir levels are known for the beginning and end of the week. Of course, if levels are known accurately then efficiencies can be changed.

2.2.1.4 Pumped Storage Constraints

The equations required for keeping track of a pumped storage facility are presented in section 2.2.1.4 of reference (19), so here they will only be quickly reviewed.

Assuming HL(t) is the water level for hour t, then the pondage accounting equations are

$$GH(t) - PA(t) + (\text{inflows}) - (\text{spillage}) + HL(t-1) = HL(t) \qquad 2214\text{-}1$$

where GH(t) is the amount of water pumped into the facility and PA(t) the amount drawn out for generating. Of course there are also physical limitations to each facility, such as

$$HL(t) \leq T \qquad 2214\text{-}2$$

where T is the total storage capacity of the unit.

The quantity PA(t) will then be put toward the total system production in interval t after it has been appropriately disproportioned for conversion losses. Likewise, GH(t) will be drawn out of the system's power production and must also be adjusted for conversion losses.

2.3 Startup Costs

In general, there is a cost associated with turning on a particular facility which will vary with the amount of time that that plant has been shut down. This cost is directly related to the cooling rate of the boilers, which is exponential in shape, see figure 2.3-1.

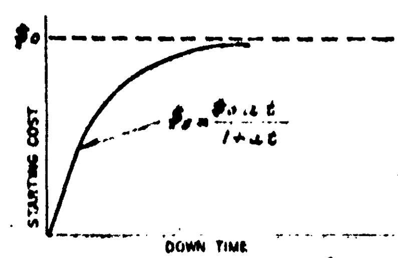

Figure 2.3-1 Starting costs as a function of previous down time[71]

Figure 2.3-2 represents a piecewise linear approximation to one of these startup cost curves (and since the smallest step size of the unit commitment scheduler is one hour, such a piecewise linearization is in effect an exact representation).

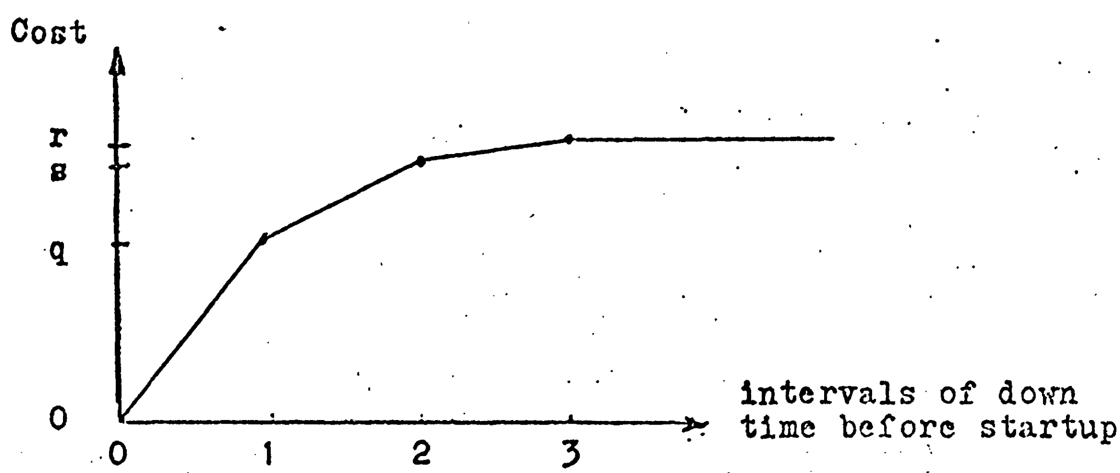

Figure 2.3-2 Piecewise linear curve of startup cost versus previous plant down time

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71. From reference (48), page 417.

As mentioned in section 2.2.1.2 there may be a substantial energy cost in a plant startup procedure, and for facilities meeting weekly energy quotas this loss must be accounted.

2.4 Inputs

The main thrust of this project is directed at the alignment of the input material and the optimal attack of the problem. So, for the most part, inputs to this simulation will be considered given. For a somewhat broader description of what the collection of input data will entail, or what the relevant influencing factors might be, consult reference (4). There is, however, a certain amount of input shaping which must be accomplished before this simulation can use that input. Because of this, input modifications will be presented to the extent that their shaping is peculiar to this analysis.

2.4.1 System Updates

As described in reference (19), section 2.4.1, system updates must include all the changes that take place within the system, from the start of the scheduling procedure through to the end of the unit commitment horizon. Unpredictable changes, of course, must be included as soon as they are known, if the scheduler is to properly model the network.

2.4.2 Power Demand Adjustments - Reserve Requirements

The problems of properly handling fixed and flexible interregional contracts and interruptible loads are discussed in reference (19) section 2.4.2.2, and that material will not be repeated here.

Emergency support from neighboring power networks can be modelled as power plants within the system in question, but this will probably not be available in all intervals and undoubtedly it will be expensive enough to make its use infrequent. It may be necessary to define an additional pseudo-cost associated with this emergency support, if the unit commitment scheduler appears to be relying too heavily upon this support. This, however, is a question which must be handled after the measures of reliability and the costs of various schedules have been examined.

It may also be necessary to scale down the number of megawatts available from a facility, for example units representing more than 10% of total system capacity, for the system to realize the additional risk inherent in operating that plant (or alternatively, to make additional demands on the amount of spinning reserve which must be kept available when this plant is operating). Other than this derating (or linear spinning reserve addition), in order to preserve the linearity of the model, it is necessary not to use any nonlinear spinning reserve requirement formulas, such as making the spinning reserve requirement equal to 1½ times the largest unit which happens to be operating in any particular interval.

The reserve requirements of a system can be met by totaling, at each interval, the unused portions of those plants which are already on. Define

$$A_i(t) = \begin{cases} 1 \text{ if plant } i \text{ is on in interval } t \\ 0 \text{ if plant } i \text{ is off in interval } t \end{cases} \qquad 242\text{-}1$$

and

$$0 \leq J_i(t) \leq 1 \qquad 242\text{-}2$$

such that $J_i(t)$ represents the fractional usage of the plant's power over and above its minimum output in the 'on' mode. That is, considering the loading curve represented in figure 2.4.2-1,

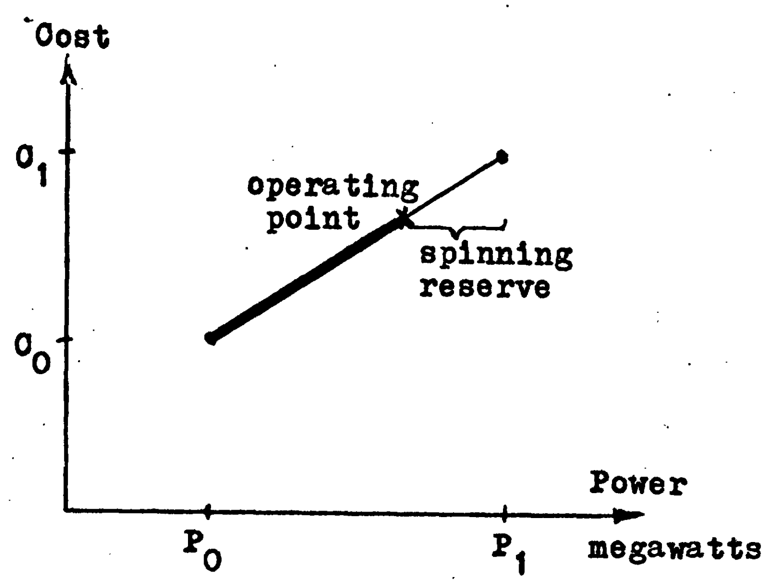

Figure 2.4.2-1 Loading curve of simplest type of plant showing spinning reserve capability

then the power output of this plant is

$$P_0 \cdot A_i(t) + (P_1 - P_0) \cdot J_i(t) \qquad 242\text{-}3$$

and this plant's contribution to the system's spinning reserve capability at time t will be

$$(P_1 - P_0)\,(J_i(t) - 1 + A_i(t)\,) \qquad 242\text{-}4$$

Of course, depending upon the type of generator being modelled, this spinning reserve capability from one facility may have to be upper bounded because of startup rate limitations (for example, no more than 15 megawatts can be added to the 3 minute reserve capability and 25 megawatts to the 5 minute reserve capability if the particular plant has a 5 megawatt per minute maximum rate for increasing capacity).

When considering the total spinning reserve available at time t (assuming no rate of change constraints), where P(t) is the total power demand at time t, the following formula can be used,

$$\left[\sum_{\text{all } i} P_{1i}(t) \cdot A_i(t)\right] - P(t) = SR(t) \qquad 242\text{-}5$$

where SR(t) is the spinning reserve at time t, and $P_{1i}(t)$ is the maximum power output of plant i. This equation is now true for systems with plants that have loading curves more complicated than that represented in figure 2.4.2-1, as long as

$A_i(t)$ is the on-off variable for plant i.

A post-optimal analysis of the resulting schedule effects due to changes in the reserve levels (and likewise the demand levels) will be helpful in the evaluation of the sensitivity of the schedules with respect to various reliability measures. Exactly what the spinning reserve requirements should be must be computed to suit the particular needs of a system. Reference (16) uses forced outage rates, tie load levels, and load duration curves to compute (for a typical 2700 megawatt system) expected cost values and loss of energies associated with changes in spinning reserve requirements. Obviously, there is a tradeoff involved between cost and reliability, see figures 2.4.2-2 and 2.4.2-3.

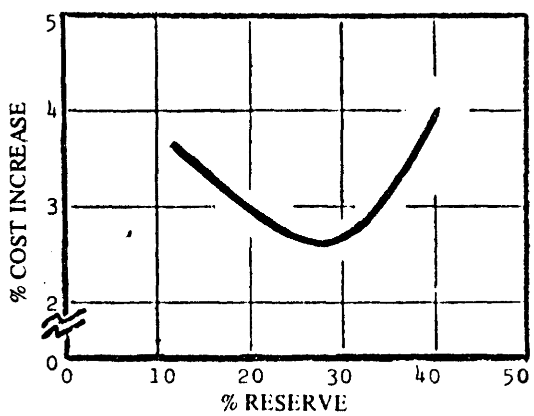

Figure 2.4.2-2 Increases in expected costs with changes in spinning reserve requirements[72]

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72. This figure and the computations upon which it was based are contained in reference (16), page 157.

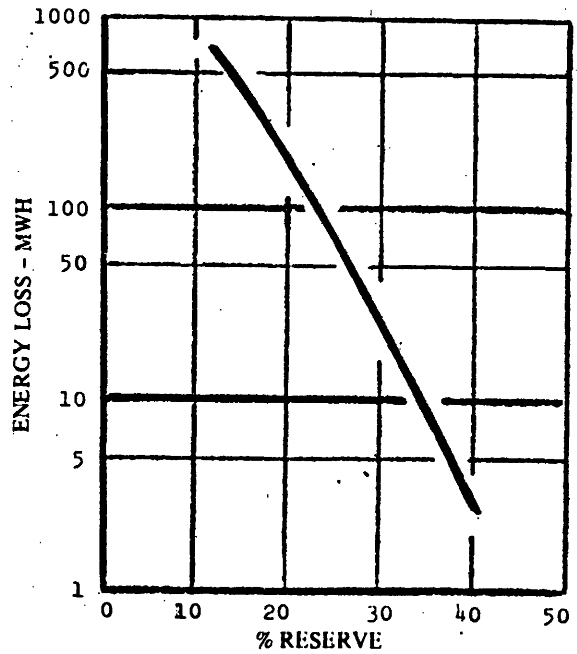

Figure 2.4.2-3 Expected energy not supplied for different spinning reserve requirements (in a 1.8 million megawatt-hour schedule)[73]

2.5 Performance Index

For the most part, section 2.5 of reference (19) contains this material, thus, it will not be included again here. Only costs which are new to this unit commitment scheduler will be discussed here.

2.5.1 Operating Costs

Unlike the convention used in the maintenance scheduler of reference (19), all[74] the contributions

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73. From reference (16), page 157.

74. Except in the case of possible rewards for non-use of hydroelectric or nuclear energies which can then be carried on into the next week to defray operating expenses at those times.

to the performance index will be in the form of penalties.[75] Thus, the costs here will include the dollar costs incurred in operation, such as those shown in the loading curve, figure 2.2.1.1-1, and in the startup cost curves, see figure 2.3-2.

Gas quota costs such as those in figure 2.2.1.1-2 are described in reference (19), section 2.5.1. For the most part, the fixed costs associated with hitting a quota will have no bearing on the scheduling mechanism, and may thus be omitted from the scheduler. Underusage and overusage penalty costs will play a definite role and should obviously be included.

2.5.2 Transmission Costs

As is usually done in the unit commitment problem the transmission costs will not be exactly represented. The reason these costs are usually left out of the unit commitment scheduler is that the inaccuracies in load forecasts for times this far into the future more than overshadow any small amounts of accuracy transmission considerations would add.[76]

In cases such as far removed facilities, such as

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75. That is, there will be no rewards for extent of non-use - as was appropriate for the scheduler which just chooses one interval in which it alleviates the environment of system operating consequences.

76. For this opinion see reference (79), page 4.

offshore nuclear reactors, the inevitable transmission costs, of course, should be included directly within the cost of producing that power. For systems with unusual network configurations, creating for example 'must run' situations, it may be worthwhile to areally discretize the power demands and groups of generators.[77]

The complexity involved in including transmission losses exactly in any formulation results from the quadratic form in which they must be represented. If it is deemed essential, there are at least two possible methods of including these transmission losses in this scheduling formulation

(1) the quadratic form can be approximated by a piecewise linearization of the quadratic loss shape, see figure 2.5.2

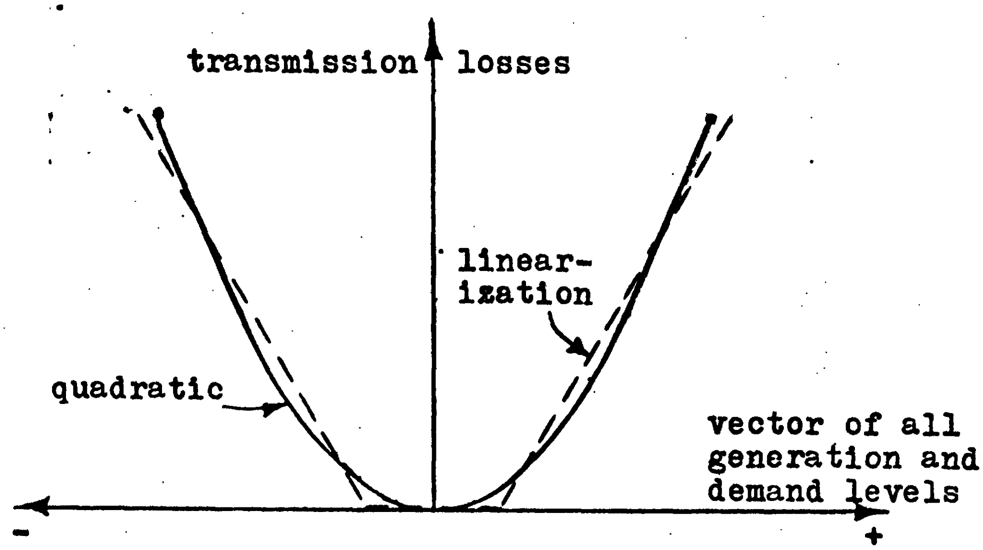

Figure 2.5.2 Piecewise linear representation of a quadratic function

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77. These methods are described in reference (19), sections 2.3.3 and 2.5.4; this method is used in reference (81).

(2) the transmission losses can be computed and compared for each of the otherwise attractive schedules, after those schedules have been computed.

Which method should be used, and in fact whether or not it is worthwhile even to consider transmission losses, is a question which must be answered by close examination and knowledge of the particular network under study.

2.5.3 Ecological Impact Units

The quantification of the environmental impacts to the ecosphere due to electric generation is a topic which has prompted several research efforts.[78]

Reaching a common denominator for all the environmental impacts is a task which might hopefully be avoided. Ideally the minimization of the various environmental ramifications can be kept as separate, i.e. multiple, objectives of a scheduler. It is, unfortunately, necessary to do some temporary collecting of different impacts into a single quantification for the purpose of decision making.

First, it is necessary to have a knowledge of the environmental impacts of the various possible schedules, in particular, the major ecological impacts. An outline

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78. Some efforts have already been made in the direction of reducing impacts upon the environment to single, or multiple vector, quantities, see references (105) and (106).

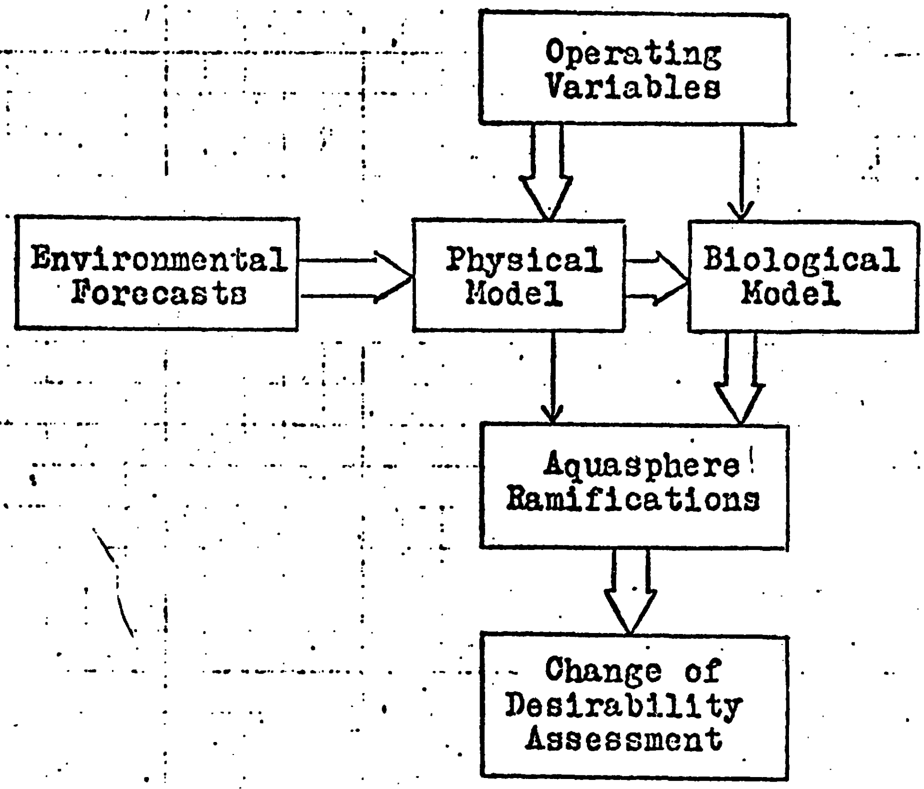

Figure 2.5.3-1 Simplified general systematic representation of method for computing aquasphere impacts from electric power generation[79]

of a plan of attack developed in reference (106) for such a study is presented in figure 2.5.3-1 with a more detailed display of the biological model in figure 2.5.3-2.

Once the aquatic and atmospheric environmental impacts have been calculated and quantified, they can be included as measures of desirability in the scheduler's decision making process by making the various environmental ramifications contingent upon the operating

79. From reference (106).

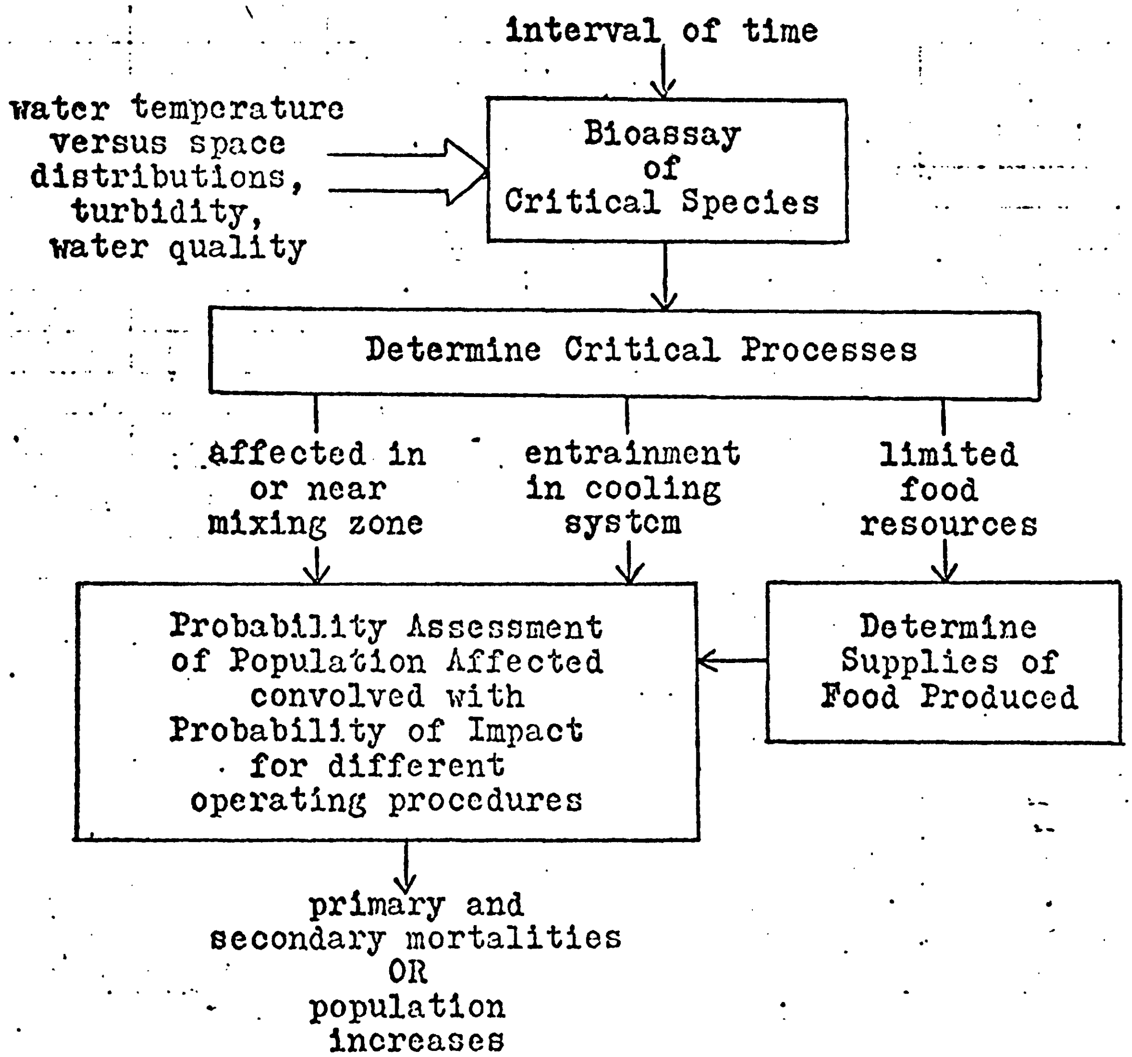

Figure 2.5.3-2 Detail of biological model portion of the general schematic for computing aquaspheric impacts of electric power generation[80]

variables which effect them. The question now arises as to how these various environmental performance measures, qe_i , relate to the dollar operating performance measure qd. In order to generate the spread of all possible optimum pairings of dollar and environmental impacts

80. From reference (106).

it will be necessary to explore all possible ecolo-economic indices, $0 \leq \theta_i \leq \infty$, which relate the relative weightings of dollars and environmental impacts in the desirability, or quality, measure used by the scheduler,

$$Q = qd + \sum_{i} \theta_i \cdot qe_i \qquad 253\text{-}1$$

where Q is the total combined desirability of the particular schedules.

It is obviously not intended that these θ_i should be fixed, or even operator regulated. Despite the additional computation required, it will be necessary to perform a number of studies corresponding to various values of θ_i so that an array can be shown of the possible operating consequences of various schedules. Consider, for example, the effect of this type of parameterization of θ in figure 2.5.3-3. Clearly, here three points, water impact only, water plus dollar costs equally weighted, and dollar costs only, with the corresponding slopes known for these points, slopes of ∞, 1 and 0 respectively, plus the knowledge of the inward curvature of the curve, yield a very good idea of its exact shape, and thus, all possible tradeoffs between these two measures of desirability. With the addition of other measures of desirability, for example air impacts or specific impact problems which can be singled out, the shape of this

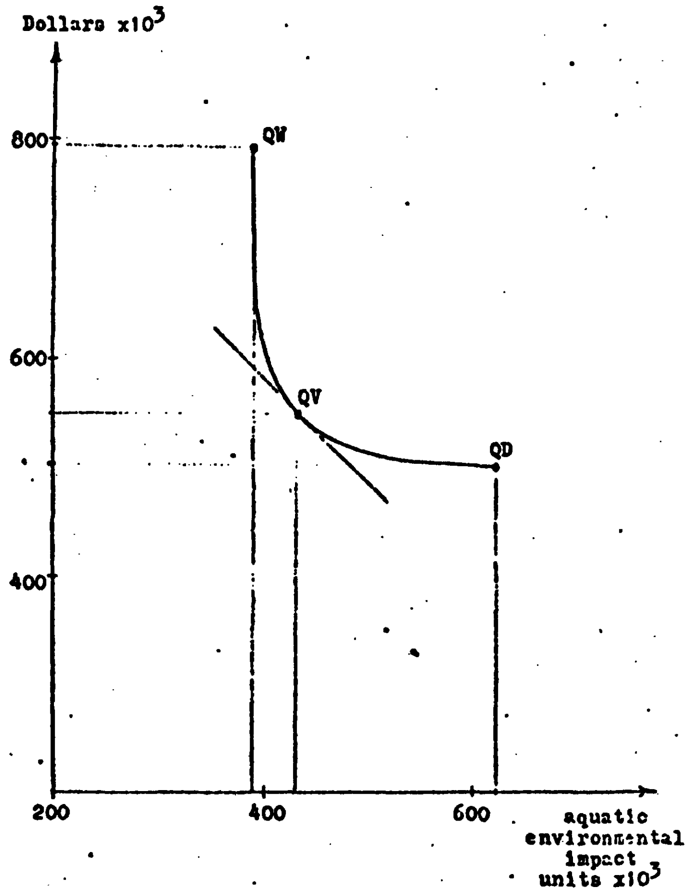

Figure 2.5.3-3 The tradeoff curve representing all possible optimum consequences of dollar and water pollution strategies, QW is the minimum water pollution strategy, QD the minimum cost consequences, and QV their equal weighting.[81]

surface of all possible tradeoffs will be extended to new dimensions (for some examples see reference (104)). And for changes in reliability levels these transform curves will make more or less concentric shiftings, see figure 2.5.3-4.

Many other possibilities for post-optimal studies are discussed in reference (19), section 3.3.

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81. From reference (104), page 30.

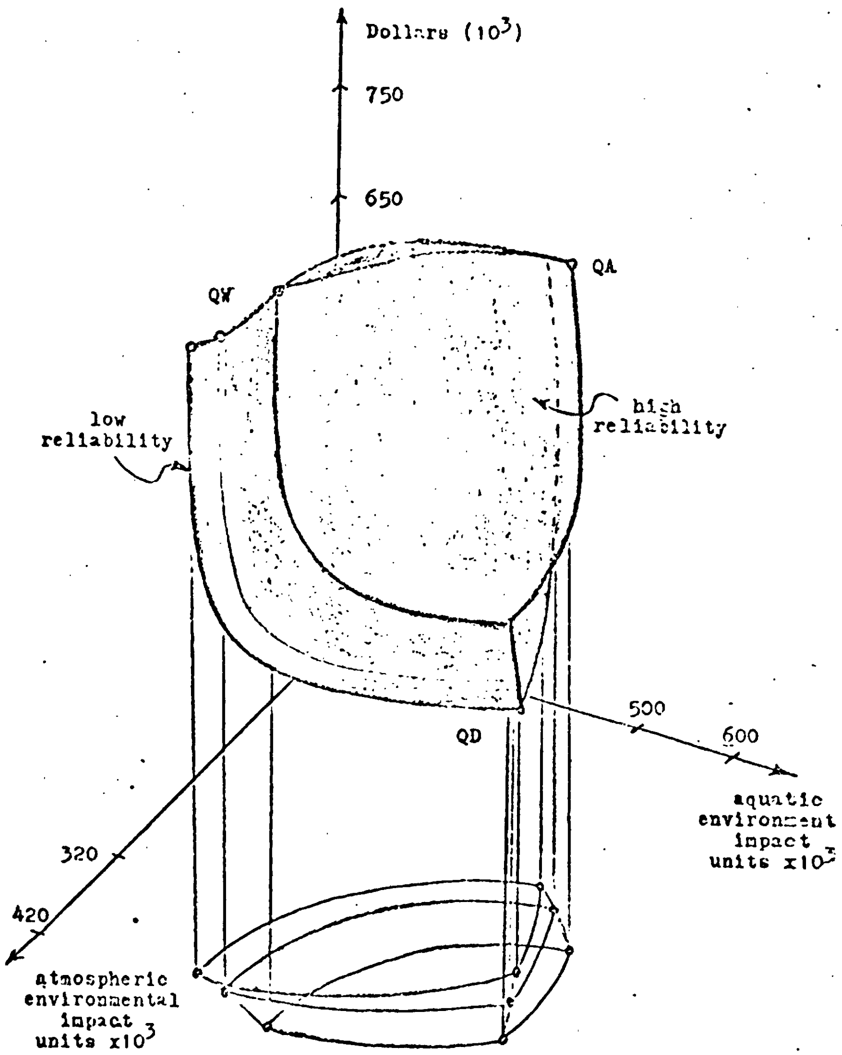

Figure 2.5.3-4 The solid tradeoff curve representing all possible dollar, water pollution impact, air pollution impact, and reliability combinations, QW is the minimum water pollution edge, QA the minimum air impact edge, and QD the minimum dollar costs associated with various levels of reliability.[82]

2.6 Time Considerations

In a manner like that described in reference (19), section 2.6, the time intervals of this scheduler were chosen to telescope from one hour units, for times close

82. From reference (104), page 49.

at hand, to eight hours long for intervals a week in the future. This variable interval size insures rapid computation, detailed information about close-at-hand times, and concentrates the computational effort on intervals when more detailed and more certain decision making information is available.

Rewards and penalties for the week-end disposition of the system, e.g. hydro reservoir levels at the termination of the model period, have been covered in the previous sections on system capabilities.

An additional time consideration is the recognition of the minimum down time requirements for certain facilities. These minimum down times are necessitated by slow startup rates and physical limitations, particularly on highly tuned, primarily base loaded, generating plants. To determine whether or not a particular facility has a load following speed which is slow enough to require inclusion in the unit commitment problem, a plot of its response characteristics can be made, such as is shown on the following page in figure 2.6. The modelling of this factor in terms of system equations is discussed in the section on the necessary adaptations of the system equations required to prepare this problem for the solution technique chosen, section 3.2.

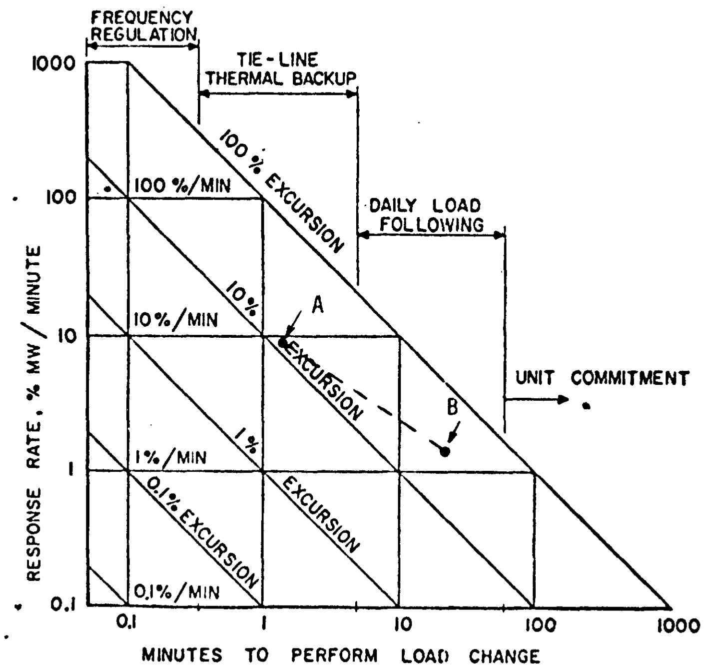

Figure 2.6 Power plant response rate characteristics plotted to demonstrate the scheduling hierarchies which must take this rate into consideration[83]

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83. From reference (108), page 455.

3. Solution Technique

The detailed account of the dynamically evolving mixed integer programming technique which is used for the solution of this scheduling problem is described in reference (19) chapter 3. Thus, the optimization technique will not be redescribed at this point. This chapter will deal primarily with the adaptation of the problem modelled in chapter 2 to the optimization technique proposed.

3.1 Possible Optimization Approaches

There are, of course, some advantages to a number of different possible approaches to the solution of this scheduling problem.

Dynamic solution techniques have the advantage of being able to deal more directly with probabilistic system problems, as well as being able to thrive upon complicated sequences of dependent procedures. However, for the problem presented by a 100 power plant system, and its inherent high dimensionality, the dynamic solution approaches require computation times which grow exponentially with the system sizes.

Linear static solution techniques must also be excluded from consideration due to the scope of this problem's size, and its nonlinear nature. And nonlinear static formulations are not solvable in reasonable amounts of computation time, for this type of broad, static overview of the entire scheduling space.

Thus, a combination of the dynamic and the static techniques is chosen for the solution method. Several static overviews of small, digestible portions of the schedule are coupled by the dynamic process represented in figure 3.1.

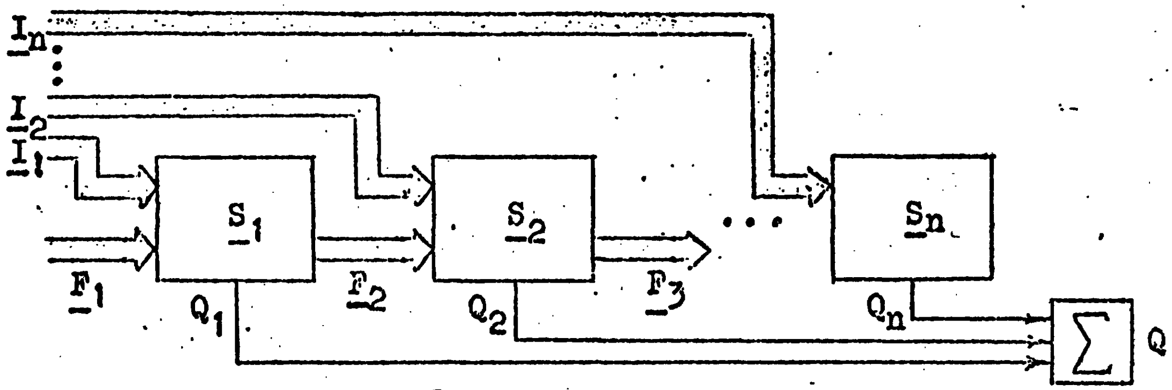

Figure 3.1 Sequential decision process using a dynamically evolving series of static overviews with input material I_n brought in at the appropriate stages, quality measures Q_n collected at each step, and coupling information fed forward F_n .

A more exact description of this technique can be found in reference (19), section 3.2.

3.2 Adaptation of the Model

The model developed in chapter 2 for this unit commitment problem will here be changed to fit into the format required by the chosen solution technique.

Define as D(t) the demand for power in interval t, where this demand has been chosen from the power demand probability distribution so as to cover power needs with a probability consistent with the security, i.e. reliability, measure imposed upon the system. Therefore, the summation of the power production from all of the units at time t must equal or exceed D(t).

To model the power production from each of the units, define An(t) as the on=1, off=0 mode of the n^{th} unit at time t. Let Jn(t) be the fractional portion of the first segment on the loading curve used by unit n in time t. Thus, for a most simply described facility with just an on-off indicator and a linear loading factor, the power generated in interval t would be represented by;

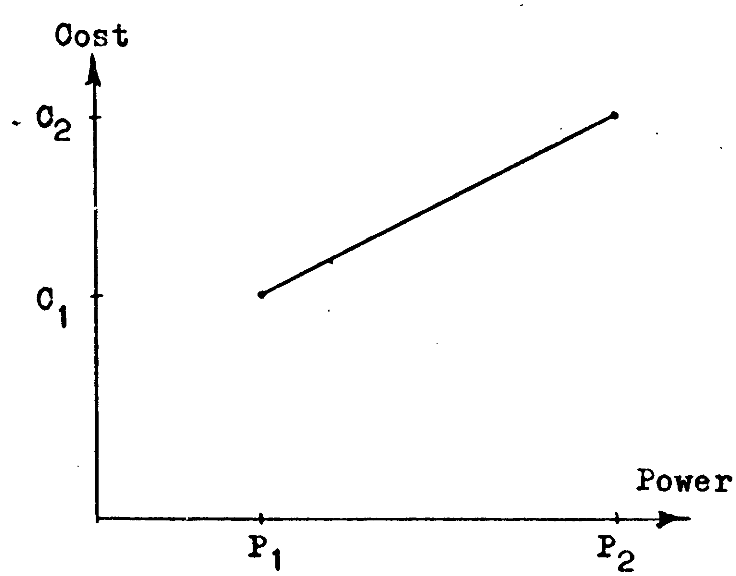

Figure 3.2-1 Loading curve for simple, single segment representation of a power plant

$$P_1 \cdot An(t) + (P_2 - P_1)\, Jn(t) \qquad 32\text{-}1$$

where

$$An(t) - Jn(t) \geq 0 \qquad 32\text{-}2$$

$$An(t) = 0\,,\,1 \qquad 32\text{-}3$$

$$0 \leq Jn(t) \leq 1 \qquad 32\text{-}4$$

and where P_1 is the minimum possible power output, P_2 being the maximum output level.

Suppose now that the loading curve of plant n has two segments, and breaks upward. Then the output power will be

$$P_1 \cdot An(t) + (P_2 - P_1)\, Jn(t) + (P_3 - P_2)\, Kn(t) \qquad 32\text{-}5$$

where Kn(t) is now the fractional portion of the second segment of the loading curve which is used, P_2 is now the power output at the breakpoint and P_3 is the maximum output. Here,

$$2\,An(t) - Jn(t) - Kn(t) \geq 0 \qquad 32\text{-}6$$

$$An(t) = 0\,,\,1 \qquad 32\text{-}7$$

$$0 \leq Jn(t) \leq 1 \qquad 32\text{-}8$$

$$0 \leq Kn(t) \leq 1 \qquad 32\text{-}9$$

If the loading curve has two segments, but happens to break downward, then the order of loading will not automatically be proper, because the scheduler will try to use the cheaper power first. Thus, another binary variable Bn(t) will be required to indicate that the system is operating on the second segment of the loading curve, and the equations which result are, for the power output:

$$P_1 \cdot An(t) + (P_2 - P_1)\, Jn(t) + (P_2 - P_1)\, Bn(t) + (P_3 - P_2)\, Kn(t) \qquad 32\text{-}10$$

where in equations given the name Mn(t)

$$An(t) - Jn(t) - Bn(t) \geq 0 \qquad 32\text{-}11$$

and in equations given the name Nn(t)

$$Bn(t) - Kn(t) \geq 0 \qquad 32\text{-}12$$

where

$$An(t) = 0 , 1 \qquad 32\text{-}13$$

$$Bn(t) = 0 , 1 \qquad 32\text{-}14$$

$$0 \leq Jn(t) \leq 1 \quad \text{and } 0 \leq Kn(t) \leq 1 \qquad 32\text{-}15$$

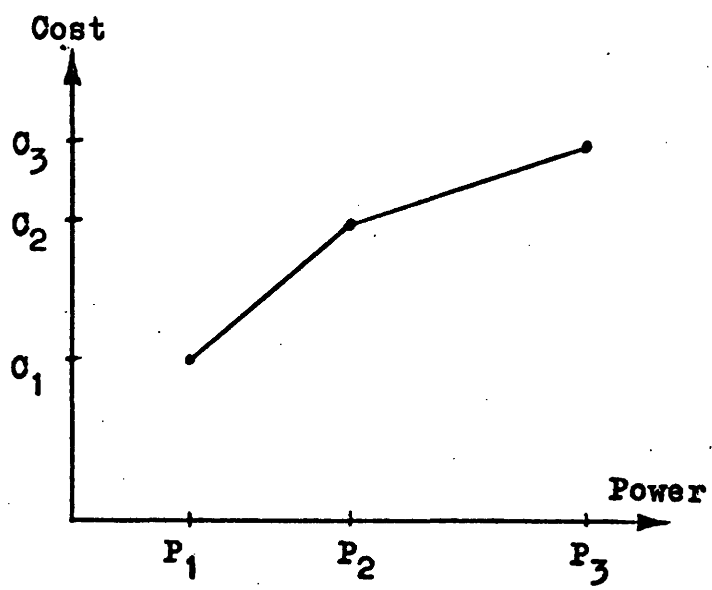

Figure 3.2-2 Loading curve for two segment, downward breaking loading curve of power plant

Additional segments to the loading curves are similarly constructed, each requiring new binary variables unless the additional segment is incrementally more expensive than the previous segments.

The consequences, dollar and environmental, of operating at any given point on the loading curve are then obtained by a collection of costs similar to the collection of output power levels. For example, in the last instance, with C_1 as the minimum operating cost, C_2 as the cost at the breakpoint, and C_3 as the cost of full production, the total cost of operation from plant n in time t is:

$$C_1 \cdot An(t) + (C_2-C_1)Jn(t) + (C_2-C_1)Bn(t) + (C_3-C_2)Kn(t) \qquad 32\text{-}16$$

For the computation of startup costs, it is advantageous to define a dummy variable Wn(t) in a logic equation which will be named Ln(t) as follows:

$$An(t) - An(t-1) - Wn(t) \leq 0 \qquad 32\text{-}17$$

$$0 \leq Wn(t) \leq 1 \qquad 32\text{-}18$$

There will then be a dollar and environmental consequence for the startup of a facility, and thus W will take on the value 1 only when it absolutely has to, that is, when both An(t) = 1 and An(t-1) = 0, i.e. when the unit has just been turned on.

To model the time-varying startup cost represented in figure 2.3-2, and again here:

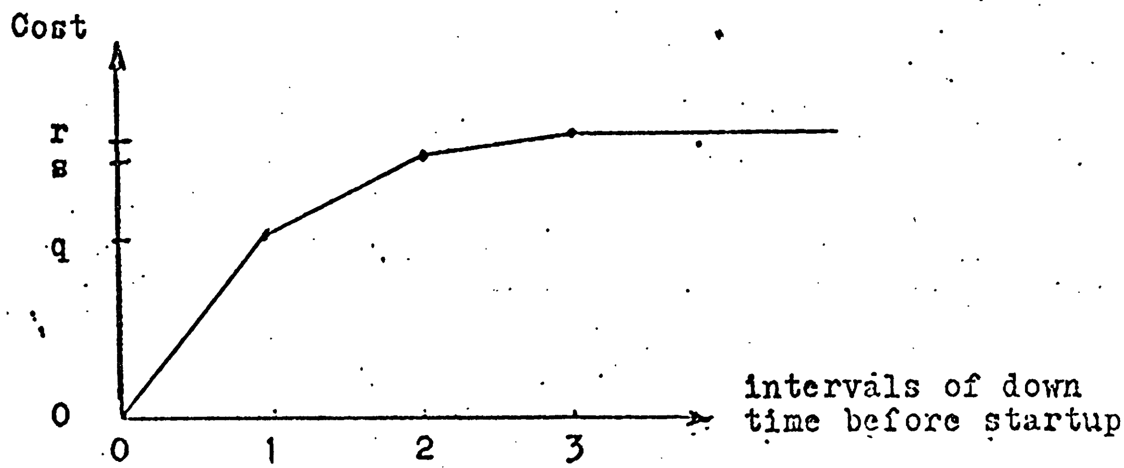

Figure 3.2-3 Piecewise linear curve of startup cost versus previous plant down time

using the dummy variable Wn(t), the startup cost at time t from unit n is:

$$r \cdot Wn(t) - (s-q)An(t-2) - (r-s)An(t-3) + (r-s)Wn(t-2) \qquad 32\text{-}19$$

and this equation will hold as long as (r-s) is less than q,[84] but this is obviously a reasonable assumption.[85]

For those plants which have a slow startup rate, there are a number of modelling alternatives. One possibility is to make after-the-fact feasibility checks of minimum down time requirements. Another modelling

84. Otherwise the Wn variable will not be a valid indicator of system startup, as it will have more to gain from a false indication.

85. If this were not the case then it would mean that the plant would cool down more during the third hour after it had been turned off, than in the first hour.

method involves the definition of a dummy variable, say En(t), where, for example, if the minimum down time is four intervals long:

$$\frac{1}{4} En(t) - An(t) \geq 0 \qquad 32\text{-}20$$

$$0 \leq En(t) \leq 4 \qquad 32\text{-}21$$

$$En(t) - En(t-1) - Wn(t) \leq 0 \qquad 32\text{-}22$$

and then En(t) would reflect the cost of maintaining a plant in the partially operative mode. The expense associated with En(t) would force this variable to zero in non-operative intervals, and then the upper limit of 'one' on the Wn's would pace the plant startup to take the allotted time span. This type of modelling preserves the on-off binary variable An(t), which is an important consideration, because An(t) serves a number of other purposes in the modelling.

One of these additional uses for An(t) is in the spinning reserve requirement equations SR(t). In the simplest formulation, as described in section 2.4.2, the spinning reserve potential of a system is the summation over all n of the An(t) times the maximum power output of each n minus the power demanded at time t. Since it is also possible to purchase external emergency support power, ES(t), at some times, this would then have to be added directly into the SR(t) equation, and be reflected in the dollar cost of the schedules.[86]

The collection of energy usages from limited sources, and the penalties or rewards for over or underusage of allotted quotas uses the following terminology. The summation of the amount of energy used from any of these sources, minus the overuse beyond the quota, plus the underuse of the quota, must then equal the quota. For nuclear, hydro and pumped hydro, the overuses are designated OSN, OSH, OSPH respectively, and the underuses are termed USN, USH, USPH respectively in the quota equations NUTOT, HYTOT, and PHTOT respectively. These quantities of underusages and overusages are then available for penalizing or rewarding in the cost functional.

The terminology of the pumped hydro accounting equations X(t), as described in section 2.2.1.4, is

$$Gn(t) - An(t) + HL(t-1) - HL(t) = 0 \qquad 32\text{-}23$$

where Gn(t) is the pumped input into the reservoir, An(t) is the outtake and HL(t) is the hydro level at time t (assuming no inflow or spillage). Gn(t) depletes the power available from the system at time t and in the power demand equation D(t) Gn(t) is scaled up to reflect the input inefficiency of the facility.

86. The use of emergency support probably should not be added to the environmental impact consequences, since no ecological consequences take place in the scheduler's region.

An(t) enhances D(t) and is scaled down to reflect hydro generating inefficiencies of the facility.

The consequences of various operating variables contribute to the equations which measure the desirability of the schedules; equations Q, QA and QW, that is, the dollar costs, air impact consequences, and water impact consequences (plus whatever other measures are wanted). For quick access, rows representing some mixtures of these various consequences were also defined, QB as the equally weighted combination of Q and QA, QV as Q plus QW, QE as QA plus QW, thus QE is the total environmental impact combined measure. In addition, a QT was defined as Q plus QA plus QW, i.e. the equal weighting of all three variables.

Such a combination of variables in separate rows is not really necessary. For example, define a row named Q as zero equals all dollar consequences of the schedule minus a new variable QX. Now QX will be forced to be equal to the dollar cost of the schedule, and can be manipulated as any other variable. For example, if QAX and QWX are likewise computed for air and water impact totals, then a new objective function can be formed of

$$a \cdot QX + b \cdot QAX + c \cdot QWX \qquad 32\text{-}24$$

where a, b, and c are the

weightings to be associated with the various consequences, and their parameterizations will yield a display of all possible measures of desirability.

It is likewise also possible to section out particular power generation consequences for constraining and/or penalizing. For example, the SO_2 concentration at a given time (or SO_2 total production over some specific time span) that affects a particular city could be collected separately and forced, as above, to equal some amount QSO2, which now can be manipulated. For example, with the predicted external concentration (or production) from background sources, say SO2X, could then be to compute

$$QSO2 + SO2X \qquad 32\text{-}25$$

which would then be the predicted total SO_2 level, and that could now be constrained to be less than a certain dangerous amount, or it could be penalized in a manner appropriate to its impact.

As an example of a penalty function which might be appropriate, consider the example given where QSO2 is the SO_2 impacting[87] upon an area over the course of a day as a result of a particular power plant's production schedule. Let SO2X be the total impact from external, background sources. Suppose that figure 3.2-4 represents

87. See reference (105) for a more exact description of how such an impact measure can be defined.

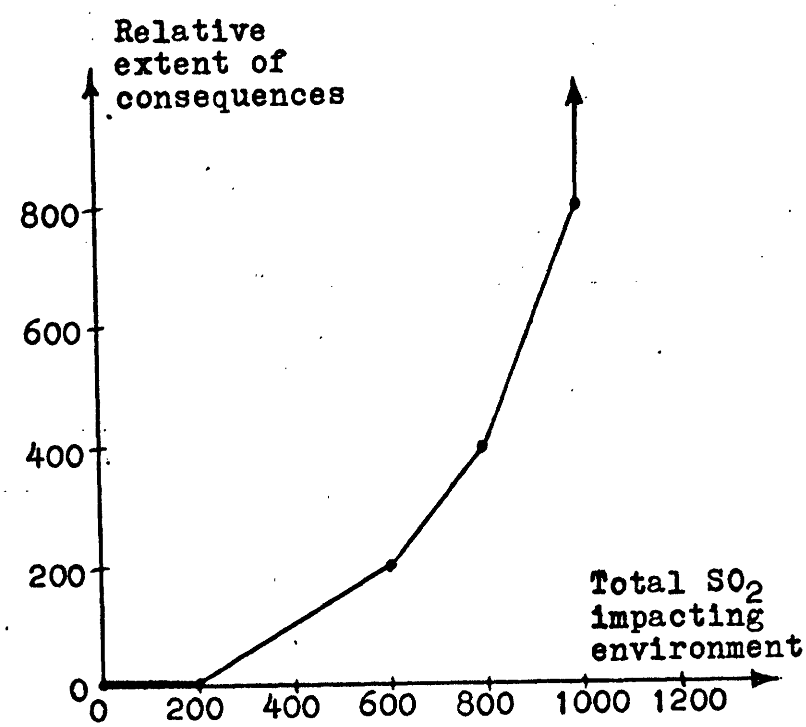

Figure 3.2-4 Amounts of SO_2 impacting the environment scaled (nonlinearly) to reflect the escalating nature of its consequences[88]

the relative impacts to the environment of the various levels of SO_2. Now let QS02X be the relative extent of the consequences from the external, background sources alone, as computed from S02X and the graph of relative consequences in figure 3.2-4. Then define S02A, S02B, and S02C by the following bounds and equation:

$$0 \leq S02A \leq 400 \qquad 32\text{-}26$$

$$0 \leq S02B \leq 200 \qquad 32\text{-}27$$

$$0 \leq S02C \leq 200 \qquad 32\text{-}28$$

$$S02X + QS02 - 200 - S02A - S02B - S02C \leq 0 \qquad 32\text{-}29$$

88. This curve represents an absolute constraint of the SO_2 level at the level 1000.

So now the change in consequences from SO_2 production caused by power generation in the schedule being investigated is

$$\Delta QSO2 = \tfrac{1}{2}SO2A + SO2B + 2SO2C - QSO2X \qquad 32\text{-}30$$

Thus, from this simple example, it can be seen that the environmental consequences of various schedules can be examined for any type and/or combination of pollutants, at any point in time and/or collected over time spans, and at any particular place in the region and/or over areas of the region. These consequences can be viewed in terms of total amounts of pollutant, levels of polluant, and/or in terms of some impact measure.

4. Application to a Sample Regional Scheduling Problem

Contained in reference (19), chapter 4, is a description of the type of computer program used, a discussion of the techniques and subroutines available for solving this problem, arguments concerning the validity of the quasi-optimal programming technique, various dual space aids to the scheduling operator, and a survey of post-optimal study opportunities. Thus, these will not be repeated here, with only a description of the sample system and some of the scheduling results being presented in this chapter.

4.1 Description of the Sample System

Although it would have been no more difficult to have introduced any amount of time variability into this sample system, for this first trial pass the system was kept more or less time invariant, with only demand for power changing significantly with time. Modifications both in the performance of the system and in the time variation of system characteristics for this unit commitment scheduling technique can be found in reference (104). The few system changes and time variations which are tested in this simple sample scheduling example will be described as they were made in the course of running the various examples.

Obviously, there is no unique method for formulating any particular feature of the scheduling mechanism. The adaptations presented here are merely suggestions, and any modelling of a real system should be done to accurately describe, and perhaps take advantage of, any particular peculiarities of that system. In adapting any special feature into the model one must be certain to recognize the needs for

(1) accuracy of representation

(2) resultant speed of computation

and (3) ease of inputting information into the new format.

There are eight power plants which are currently operating (some plants are obviously out for maintenance in this system, but these do not enter into the formulation) in this system over the week of concern. A detailed description of the environmental consequences of different plant operations is given here in this system description, although in this simple example only the dollar consequences will take part in the determination of the desirability of the various schedules. The environmental consequences do, however, play a big role in the further extensive testing of this system which can be found in reference (104).

Plant 1 of this system is a relatively expensive to operate unit, about $6. per megawatt hour, fossil fueled plant of 160 megawatts capacity, with a moderately heavy air pollution factor, about 6 air pollution environmental impact units per megawatt hour. Because of a cooling tower the water impact is cut to .75 units per megawatt hour. The exact loading curve has a 70 megawatt on-off variable with 90 additional megawatts available at costs of $282 and $455 respectively for full use of these variables. The startup of the facility takes one hour and costs $330.

Facility 2 has 70 megawatts of maximum capability, uses low sulfur content fuel and has sulfur precipitators, thus has on the average about 3.3 water units and 3.1 air units per megawatt hour. The actual loading curve includes 30 megawatts of on-off capability plus 40 megawatts of variable loading at costs of $157 and $221, respectively. The startup cost is $112.

Plant 3 is 120 megawatts and has air and water impacts of 7.2 and 4.7 units, respectively. Its loading curve has two segments, 30 megawatts on-off for $85, 20 megawatts of variable loading for $80 if used fully, then 70 megawatts of variable loading for $390 if used fully. Startup costs $185.

Plant 4 is an 80 megawatt total combination of a group of gas turbines with 2.7 water units and 2.2 air units per megawatt hour. Its loading curve has 20 megawatts on-off at $100, 30 megawatts at $178 per hour, then 30 megawatts at $190 per hour. Its startup cost is $150.

Plant 5 is a 240 megawatt base loaded type of highly tuned fossil fueled plant with 120 megawatts on-off at $210 per hour, then 80 megawatts for $390 per hour, and 40 megawatts for $161 per hour. Plant 5 costs $402 to start up, and has water and air indices of 5.9 and 6.6 per megawatt hour, respectively.

The nuclear plant, number 6, has 560 megawatts of capacity divided into 60 megawatts on-off and 500 megawatts of additional variable power. Over the course of the week the goal set by the maintenance and production scheduler is to use 84,000 megawatt hours of nuclear power, and penalties are set at $8.6 per megawatt hour for overuse, $2.0 for underuse. The air and water indices are about 1.3 and 7.9 units per megawatt hour respectively. The startup cost is $1019.

A hydroelectric facility is plant number 7, with 100 megawatts potential, 5 on-off and 95 variable, and a goal for the week of 16,800 megawatt hours. The penalty for overuse of water power energy is $7.6 per megawatt hour and there is a $1.1 per megawatt hour reward for any water energy used less than the quota which can then be saved for later weeks. The startup cost is $184 and the air and water indices are .1 and 1.1 respectively, per megawatt hour.

The pumped storage facility, number 8, has 80 megawatt hours of storage potential and of water usage potential, with storing capacity of 1000 megawatt hours. The input efficiency is 83.4% and the output efficiency 80.0%. Plant 8's startup cost is $119.

The initial conditions of the system include the pumped hydro facility with 205 megawatt hours worth of stored water, and all plants are on except plants 4 and 8.

The time intervals in the schedule progress by one hour increments until hour four, then in increments of two hours, two hours, four hours, four hours, and finally, eight hour intervals for the rest of the week. The first interval is 8:00 pm on a Tuesday.

The megawatt hours of power required of the system (average values are given for those intervals longer than one hour) are: 1230, 1205, 1100, 990, 780, 740, 1200, 1310, 1250, 760, 1400, 1260, 800, 1310, 1100, 550, 1265, 1020, 680, 1000, 1200, 425, 950, 1000, 890, 1285, and 1120 megawatts.

There is an opportunity to buy 200 megawatts of power starting at hour 24, for eight hours, at a cost of $1150 per hour.

There are limits on the amount of underextending and overextending that can be done to the weekly energy quotas. The nuclear quota cannot be overused by more than 5000 megawatt hours or underused by more than 6200 megawatt hours. The hydro over and under usage limits are 2000 and 1200 megawatt hours respectively. The ideal quota for the pumped hydro reservoir is to leave 160 megawatt hours of storage at the end of the week, with $5.5 penalty for each megawatt hour less than this amount stored, and $5.2 reward for every megawatt hour more than this which is stored.

4.2 Examples of Unit Commitment Schedules

The schedules created for this section were directed toward the examination of the quasi-optimal programming technique and its effectiveness in producing unit commitment schedules. A greater variety of scheduling strategies, as well as a demonstration of the use of this mechanism as a simulator, including in both cases some air and water environmental objectives, is presented in reference (104).

The first scheduling attempted was that of the system described in the previous section. The first evolving decision field was confined to the first two time periods of the week, with the entire remainder of the week being carried in the linear scheduling mode. The most important result to come of this sample run was the demonstration of the fact that the scheduler tends toward integer values for the integer variables, even when it is in the linear mode. In fact, for this example, through hour 64, all integer variables assumed integer values. That is, even in the linear, continuous program optimal solution, the first 112 integer variables in the schedule assumed integer values.

This closeness of the linear, continuous solution to the integer solution is more than a coincidence, it is a characteristic of this particular scheduling mechanism. Figure 4.2-1 shows further evidence of the closeness of the integer and the optimal continuous solutions, closeness plotted on the graph of the resulting consequences of the schedules (using the same example as is given in figure 2.5.3-3 of this paper). There are several reasons for this fortunate behavior of the scheduling mechanism:

(1) The startup costs associated with the changes in the plant on-off variable tend to make it desirable for a plant to either turn full on and stay on, or turn off, because moving this variable around costs money. Thus, this integer on-off variable tends to remain either full on or full off even in the linear program degeneration of the actual scheduling process.

(2) The loading logic equation 32-2 forces the plant to turn full on for full use of the incremental

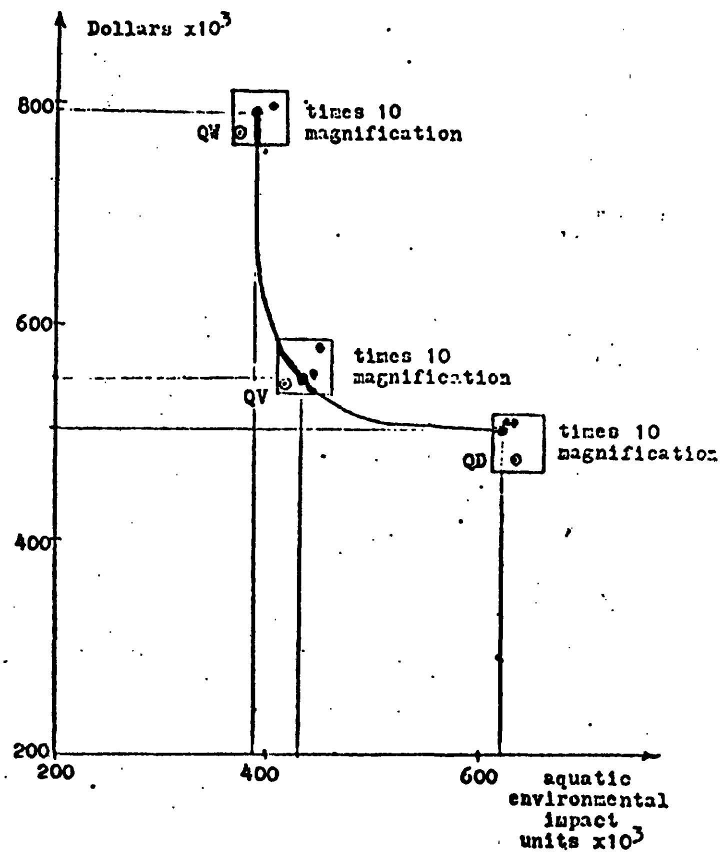

Figure 4.2-1 The representation of the closeness of the actual valid schedules (represented by •) to the optimal continuous degeneration of the scheduling problem (represented by ⊙) as seen the their respective measures of scheduling consequences (see figure 2.5.3-3 or see Appendix E of reference (104) for the original source)

loading variable Jn(t).

(3) At any one particular time, those plants which are operating are largely chosen from a consideration of which are cheapest to operate. Thus, generally, even in the linear scheduling mode there is only one plant which may be in an indecisive partially loaded, i.e. partially on, position. This more or less assures the full on operation of cheaper plants and full off operation of plants with greater

consequences. Thus, this is an additional motive for the scheduling mechanism to force many on-off integers to 0 or 1 positions.

(4) The spinning reserve requirements, when used, measure the power producing potential of the system in terms of the on-off variables. This then provides an additional impetus for the scheduling program to move a fractional on-off status up to a full on position so as to realize the additional credit this gives to the spinning reserve available to the system.

(5) From a mathematical point of view, since the integers are binary, there are no integer solutions which are hidden within the polytope of all feasible solutions. That is, all possible integer solutions are on corners of the polytope, and are thus likely to represent the optimal value of the objective function (since corners are sought out by the supporting hyperplane which represents the maximum value of the objective function).

As an example of the kind of results that are contained in this first schedule, the nuclear facility, plant 6, was scheduled 'on' over the entire first 56 hours. It was not, however, fully loaded over that span. The values of J6(1) through J6(54), the fraction of the loading curve used, were: 1, 1, 1, .84, .42, .34, 1, 1, 1, .38, 1, 1, and .46. An abbreviated sample of some of the rest of the results of this schedule is given in figure 4.2-2 (the notation used is the same as that used in the equations in section 3.2 and in the rest of chapters 2 and 3. A summary of the notation can be found in the glossary of symbols).

...ROW..	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
C	BS	335940.31440	335940.31440-	NONE	NONE	1.00000
D001	LL	1230.00000	.	1230.00000	NONE	5.57143-
D002	LL	1205.00000	.	1205.00000	NONE	5.57143-
D003	LL	1100.00000	.	1100.00000	NONE	5.05556-
D004	LL	1980.00000	.	1980.00000	NONE	4.94444-
D006	LL	1560.00000	.	1560.00000	NONE	4.94444-
D008	LL	2960.00000	.	2960.00000	NONE	4.94444-
D012	LL	4800.00000	.	4800.00000	NONE	5.57143-
D016	LL	10480.00000	.	10480.00000	NONE	6.33333-
D024	LL	10000.00000	.	10000.00000	NONE	5.57143-
D032	LL	6080.00000	.	6080.00000	NONE	4.94444-
D040	LL	11200.00000	.	11200.00000	NONE	6.33333-
D048	LL	10080.00000	.	10080.00000	NONE	5.57143-
D056	LL	6400.00000	.	6400.00000	NONE	4.94444-
D064	LL	10480.00000	.	10480.00000	NONE	6.33333-
D072	LL	8800.00000	.	8800.00000	NONE	5.05556-
D080	LL	4400.00000	.	4400.00000	NONE	4.94444-
D088	LL	10080.00000	.	10080.00000	NONE	5.60000-
D096	LL	8160.00000	.	8160.00000	NONE	5.00174-
D104	LL	5440.00000	.	5440.00000	NONE	4.94444-
D112	LL	8000.00000	.	8000.00000	NONE	4.94444-
D120	LL	9600.00000	.	9600.00000	NONE	5.60000-
D128	LL	3400.00000	.	3400.00000	NONE	4.59167-
D136	LL	7600.00000	.	7600.00000	NONE	5.05556-
D144	LL	8000.00000	.	8000.00000	NONE	5.05556-
D152	LL	7120.00000	.	7120.00000	NONE	4.94444-
D160	LL	10280.00000	.	10280.00000	NONE	5.93333-
D168	LL	8960.00000	.	8960.00000	NONE	5.05556-
NLTOT	EQ	80000.00000	.	80000.00000	80000.00000	4.94444
HYTOT	EQ	14000.00000	.	14000.00000	14000.00000	4.89021
PHTOT	EQ	160.00000	.	160.00000	160.00000	4.04444-
M1001	BS	.	.	.	NONE	.
M1002	BS	.	.	.	NONE	.
M1003	BS	.06667	.06667-	.	NONE	.
M1004	BS	1.00000	1.00000-	.	NONE	.
M1006	BS	1.00000	1.00000-	.	NONE	.
M1008	BS	1.00000	1.00000-	.	NONE	.
M1012	LL	.	.	.	NONE	185.71429-
M1016	BS	.	.	.	NONE	.
M1024	BS	.	.	.	NONE	.
M1032	BS	1.00000	1.00000-	.	NONE	.
M1040	BS	.	.	.	NONE	.
M1048	BS	.	.	.	NONE	.
M1056	BS	1.00000	1.00000-	.	NONE	.
M1064	BS	.	.	.	NONE	.
M1072	BS	.39026	.39026-	.	NONE	.
M1080	BS	1.00000	1.00000-	.	NONE	.
M1088	BS	.	.	.	NONE	.
M1096	BS	1.00000	1.00000-	.	NONE	.

Figure 4.2-2a Row activity in the schedule for the first decision field, quality of schedule in $ Q, demands over intervals in total megawatt hours D(t) with dual activity representing the incremental cost of power in those intervals, and activity of Mn(t) equations (see equation 32-11).

...ROW..	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
M6032	BS	.62000	.62000-	.	NONE	.
M6040	BS	.	.	.	NONE	.
M6048	BS	.	.	.	NONE	.
M6056	BS	.54000	.54000-	.	NONE	.
M6064	LL	.	.	.	NONE	366.55555-
M6072	LL	.	.	.	NONE	444.44445-
M6080	BS	.99025	.99025-	.	NONE	.
M6088	BS	.	.	.	NONE	.
M6096	LL	.	.	.	NONE	229.18033-
M6104	BS	.68000	.68000-	.	NONE	.
M6112	BS	.04000	.04000-	.	NONE	.
M6120	LL	.	.	.	NONE	649.98470-
M6128	BS	1.00000	1.00000-	.	NONE	.
M6136	LL	.	.	.	NONE	444.44445-
M6144	BS	.	.	.	NONE	.
M6152	BS	.11475	.11475-	.	NONE	.
M6160	LL	.	.	.	NONE	181.77778-
M6168	LL	.	.	.	NONE	444.44445-
M7001	LL	.	.	.	NONE	11.59390-
M7002	BS	.	.	.	NONE	.
M7003	BS	.	.	.	NONE	.
M7004	LL	.	.	.	NONE	10.30464-
M7006	BS	.	.	.	NONE	.
M7008	LL	.	.	.	NONE	20.60929-
M7012	BS	.	.	.	NONE	.
L1168	UL	.	.	NONE	.	330.00000
L2001	UL	.	.	NONE	.	22.14286
L2002	UL	.	.	NONE	.	12.00000
L2003	BS	.	.	NONE	.	.
L2004	UL	.	.	NONE	.	5.33333
L2006	UL	.	.	NONE	.	22.66667
L2008	UL	.	.	NONE	.	40.00000
L2012	UL	.	.	NONE	.	74.66667
L2016	UL	.	.	NONE	.	34.09524
L2024	UL	.	.	NONE	.	81.14286
L2032	BS	.	.	NONE	.	.
L2040	UL	.	.	NONE	.	69.33333
L2048	UL	.	.	NONE	.	112.00000
L2056	UL	.	.	NONE	.	30.85714
L2064	UL	.	.	NONE	.	112.00000
L2072	BS	.82923-	.82923	NONE	.	.
L2080	UL	.	.	NONE	.	42.66667
L2088	UL	.	.	NONE	.	112.00000
L2096	BS	.85714-	.85714	NONE	.	.
L2104	UL	.	.	NONE	.	55.58251
L2112	UL	.	.	NONE	.	112.00000
L2120	UL	.	.	NONE	.	112.00000
L2128	BS	.	.	NONE	.	.
L2136	UL	.	.	NONE	.	112.00000
L2144	UL	.	.	NONE	.	112.00000
L2152	UL	.	.	NONE	.	112.00000
L2160	UL	.	.	NONE	.	168.00000
L2168	BS	1.00000-	1.00000	NONE	.	.
L3001	BS	.	.	NONE	.	.
L3002	UL	.	.	NONE	.	185.00000
L3003	UL	.	.	NONE	.	185.00000
L3004	UL	.	.	NONE	.	185.00000
L3006	UL	.	.	NONE	.	126.66667
L3008	BS	.	.	NONE	.	.
L3012	UL	.	.	NONE	.	185.00000
L3016	BS	.	.	NONE	.	.
L3024	UL	.	.	NONE	.	185.00000
L3032	UL	.	.	NONE	.	185.00000
L3040	UL	.	.	NONE	.	185.00000
L3048	UL	.	.	NONE	.	185.00000
L3056	UL	.	.	NONE	.	185.00000
L3064	UL	.	.	NONE	.	185.00000
L3072	UL	.	.	NONE	.	185.00000
L3080	UL	.	.	NONE	.	185.00000

Figure 4.2-2b More row activity from schedule for first decision field, including activities of some Mn(t) equations and from some Ln(t) equations, the startup logic equations, see equation 32-17.

.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
A1000	EQ	1.00000	.	1.00000	1.00000	330.00000-
A2000	EQ	1.00000	.	1.00000	1.00000	22.14286-
A3000	EQ	1.00000	.	1.00000	1.00000	.
A5000	EQ	1.00000	.	1.00000	1.00000	402.00000-
A6000	EQ	1.00000	.	1.00000	1.00000	1019.00000-
A7000	EQ	1.00000	.	1.00000	1.00000	.
HL000	EQ	205.00000	.	205.00000	205.00000	4.88C73-
J1001	UL	1.00000	455.00000	.	1.00000	46.42857-
W1001	BS	.	330.00000	.	1.00000	.
J2001	UL	1.00000	221.00000	.	1.00000	1.85714-
W2001	LL	.	112.00000	.	1.00000	89.85714
J3001	UL	1.00000	80.00000	.	1.00000	31.42857-
K3001	BS	.42857	390.00000	.	1.00000	.
W3001	LL	.	185.00000	.	1.00000	185.00000
J4001	LL	.	178.00000	.	1.00000	10.85714
K4001	LL	.	190.00000	.	1.00000	22.85714
W4001	BS	1.00000	150.00000	.	1.00000	.
J5001	BS	.	390.00000	.	1.00000	.
K5001	BS	1.00000	161.00000	.	1.00000	.
W5001	BS	.	402.00000	.	1.00000	.
J6001	BS	1.00000	.	.	1.00000	.
W6001	BS	.	1019.00000	.	1.00000	.
J7001	UL	1.00000	.	.	1.00000	53.12191-
W7001	LL	.	184.00000	.	1.00000	184.00000
A8001	LL	.	.	.	1.00000	33.88690
G8001	LL	.	.	.	1.00000	144.39881
HL001	BS	205.00000	.	.	1000.00000	.
W8001	LL	.	119.00000	.	1.00000	119.00000
J1002	UL	1.00000	455.00000	.	1.00000	46.42857-
W1002	BS	.	330.00000	.	1.00000	.
J2002	BS	1.00000	221.00000	.	1.00000	.
W2002	LL	.	112.00000	.	1.00000	100.00000
J3002	UL	1.00000	80.00000	.	1.00000	31.42857-
K3002	BS	.07143	390.00000	.	1.00000	.
W3002	BS	.	185.00000	.	1.00000	.
J4002	LL	.	173.00000	.	1.00000	10.85714
K4002	LL	.	190.00000	.	1.00000	22.85714
W4002	LL	.	150.00000	.	1.00000	11.42857
J5002	BS	.	390.00000	.	1.00000	.
K5002	BS	1.00000	161.00000	.	1.00000	.
W5002	LL	.	402.00000	.	1.00000	402.00000
J6002	BS	1.00000	.	.	1.00000	.
W6002	LL	.	1019.00000	.	1.00000	319.11111
J7002	UL	1.00000	.	.	1.00000	64.71581-
W7002	LL	.	184.00000	.	1.00000	184.00000
A8002	LL	.	.	.	1.00000	33.88690
G8002	LL	.	.	.	1.00000	144.39881
HL002	BS	205.00000	.	.	1000.00000	.
W8002	LL	.	119.00000	.	1.00000	119.00000

Figure 4.2-2c Column activity from schedule for first decision field, including at various times, 000, 001 and 002, the on-off variables An(ttt), extents of incremental loadings Jn(ttt) and Kn(ttt), hydro reservoir levels (pumped) HL(ttt), extent of pumping hydro storage Gn(ttt), and startup variables Wn(ttt) for various plants n.

In the second evolving step, the values of the decision variables chosen for the first 56 hours of the schedule were fixed. A few new initial conditions were set to reflect the new position the system had been left in, e.g. the pumped hydro level at 100 megawatt hours, and some of the weekend quotas were adjusted to reflect the usages in the previous portion of the schedule, e.g. 28,580 megawatt hours of the nuclear total and 6,300 megawatt hours of the hydro total were used in the first 56 hours of the week, so they were deducted from the weekly quotas to establish new target figures.

After the quotas were adjusted and new initial conditions established, the next three time intervals were set up as the second evolving step decision field, again with the rest of the week carried in the linear mode. Three valid integer schedules resulted from the computational search before the optimum was established, and the integer portions of these schedules are given in figure 4.2-3.

From these few sample schedules it can be seen that the size of the decision field should not be dictated by how <u>large</u> a number of decisions can be handled by the program, but instead by how <u>small</u> a block can be sectioned out and still preserve the integrity of the process. For, by concentrating on small blocks, the greater number of alternative schedules produced yields a greater amount of information about near-optimal interchangeability of decision variables for use in the event of unexpected outages, and this is gotten for a small amount of computation time. For example, if only three schedules are computed for one whole day

FUNCTIONAL	196149.0984	196191.7650	196175.5972
ESTIMATION	INTEGER	INTEGER	INTEGER
653= A1064	1.0000	1.0000	1.0000
654= A2064	1.0000	1.0000	1.0000
655= A3064	1.0000	1.0000	1.0000
656= A4064	1.0000	1.0000	1.0000
657= A5064	1.0000	1.0000	1.0000
658= B5064	1.0000	1.0000	1.0000
659= A6064	1.0000	1.0000	1.0000
660= A7064	1.0000	1.0000	1.0000
661= A1072	1.0000	1.0000	1.0000
662= A2072	.	1.0000	1.0000
663= A3072	1.0000	1.0000	1.0000
664= A4072	1.0000	1.0000	1.0000
665= A5072	1.0000	1.0000	1.0000
666= B5072	1.0000	1.0000	1.0000
667= A6072	1.0000	1.0000	1.0000
668= A7072	1.0000	1.0000	1.0000
669= A1080	1.0000	1.0000	1.0000
670= A2080	.	.	1.0000
671= A3080	1.0000	1.0000	1.0000
672= A4080	1.0000	1.0000	1.0000
673= A5080	1.0000	1.0000	1.0000
674= B5080	1.0000	1.0000	1.0000
675= A6080	1.0000	1.0000	1.0000
676= A7080	1.0000	1.0000	1.0000

Figure 4.2-3 Alternative schedules for the second decision field, list of integer decision variables for these alternatives and their respective dollar costs

this would not demonstrate alternative schedules as well as if three schedules where generated for the first eight hours of the day, three schedules for the second eight hours, and three for the last eight hours. Splicing these schedules together would yield, in effect, 27 alternative daily scheduling possibilities

To demonstrate the expense that would be involved in computing schedules for large systems, a 112 integer decision field was entered for scheduling. This problem was simply the entire completion of the scheduling problem described in section 4.1, that is, from time 64 to time 168. The resultant schedules, and some of the partially completed schedules,[89] are given in figure 4.2-4. The completed all-integer, valid schedules cost about $3 apiece to generate for this example.

At this point in the testing of the scheduling mechanism, a revision of the system was enacted. The resulting program is summarized in Appendix A, and listed exactly in Optional Appendix A. (The cards which were changed from the previously described program are summarized in Appendix B, and listed in Optional Appendix B if there is any desire to reconstruct either program exactly). This change was made to be certain that the low costs of the integer on-off variables, An(t), were not a primary factor in the closeness of the linear program schedules to the integer schedules, nor a reason for the quickness of the computation procedure (as it

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89. The system was told to stop after computing six complete, valid, integer schedules, and was then told to print all pending partial schedules.

NODE	36	44	73	80	92	119
FUNCTIONAL	196317.2128	196393.6572	196311.2128	196428.8795	196296.6572	196320.1017
ESTIMATION	INTEGER	INTEGER	INTEGER	INTEGER	INTEGER	INTEGER
565= A1064	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
566= A2064	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
567= A3064	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
568= A4064	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
569= A5064	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
570= B5064	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
571= A6064	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
572= A7064	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
573= A1072	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
575= A3072	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
576= A4072	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
577= A5072	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
578= B5072	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
579= A6072	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
580= A7072	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
581= A1080	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
583= A3080	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
584= A4080	1.0000	.	1.0000	.	1.0000	1.0000
585= A5080	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
586= B5080	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
587= A6080	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
588= A7080	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
589= A1088	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
590= A2088	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
591= A3088	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
592= A4088	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
593= A5088	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
594= B5088	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
595= A6088	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
596= A7088	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
597= A1096	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
598= A2096	.	.	.	.	.	.
599= A3096	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
600= A4096	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
601= A5096	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
602= B5096	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
603= A6096	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
604= A7096	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
605= A1104	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
606= A2104	.	.	.	.	.	.
607= A3104	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
608= A4104	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
609= A5104	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
610= B5104	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
611= A6104	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
612= A7104	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
613= A1112	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
614= A2112	.	.	.	.	.	.
615= A3112	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Figure 4.2-4a First portion of the completed integer valued schedules for the integer decisions of the third decision field

NODE continued	36	44	73	80	92	119
616= A4112	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
617= A5112	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
618= B5112	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
619= A6112	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
620= A7112	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
621= A1120	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
622= A2120	.	.	.	.	.	.
623= A3120	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
624= A4120	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
625= A5120	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
626= B5120	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
627= A6120	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
628= A7120	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
629= A1128	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
630= A2128	.	.	.	.	.	.
631= A3128	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
632= A4128	1.0000	1.0000	.	.	.	.
633= A5128	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
634= B5128	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
635= A6128	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
636= A7128	.	.	.	.	.	.
637= A1136	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
638= A2136	.	.	.	.	.	.
639= A3136	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
640= A4136	1.0000	1.0000	1.0000	1.0000	.	.
641= A5136	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
642= B5136	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
643= A6136	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
644= A7136	.	.	.	.	.	.
645= A1144	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
646= A2144	.	.	.	.	.	.
647= A3144	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
648= A4144	1.0000	1.0000	1.0000	1.0000	.	1.0000
649= A5144	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
650= B5144	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
651= A6144	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
652= A7144	.	.	.	.	.	.
653= A1152	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
654= A2152	.	.	.	.	.	.
655= A3152	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
656= A4152	1.0000	1.0000	1.0000	1.0000	.	1.0000
657= A5152	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
658= B5152	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
659= A6152	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
660= A7152	.	.	.	.	.	.
661= A1160	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
662= A2160	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
663= A3160	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
664= A4160	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
665= A5160	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
666= B5160	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
667= A6160	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
668= A7160	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
669= A1168	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
670= A2168	.	.	.	.	.	.
671= A3168	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
672= A4168	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
673= A5168	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
674= B5168	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
675= A6168	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
676= A7168	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Figure 4.2-4b Continuation of integer values of the integer decision variables for the third decision field, these are the completed valid schedules

NODE	6	8	13 P	15 P	17 P	18
FUNCTIONAL	196274.6228	196426.0795	196277.3239	196293.9350	196297.6255	196380.0684
ESTIMATION	195410.	196433.	196312.	196338.	196336.	196439.
565= A1064	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
566= A2064	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
567= A3064	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
568= A4064	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
569= A5064	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
570= B5064	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
571= A6064	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
572= A7064	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
573= A1072	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
575= A3072	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
576= A4072	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
577= A5072	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
578= B5072	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
579= A6072	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
580= A7072	1.0000	.9500	1.0000	1.0000	1.0000	.8625
581= A1080	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
583= A3080	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
584= A4080	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
585= A5080	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
586= B5080	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
587= A6080	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
588= A7080	1.0000	.9500	1.0000	1.0000	1.0000	.8625
589= A1088	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
590= A2088	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
591= A3088	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
592= A4088	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
593= A5088	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
594= B5088	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
595= A6088	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
596= A7088	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
597= A1096	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
598= A2096	.	.	.	.	.	.
599= A3096	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
600= A4096	1.0000	1.0000	1.0000	1.0000	1.0000	.
601= A5096	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
602= B5096	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
603= A6096	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
604= A7096	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
605= A1104	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
606= A2104	.	.	.	.	.	.
607= A3104	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
608= A4104	1.0000	1.0000	1.0000	1.0000	1.0000	.6562
609= A5104	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
610= B5104	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
611= A6104	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
612= A7104	.	1.0000	1.0000	1.0000	1.0000	1.0000
613= A1112	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
614= A2112	.	.	.	.	.	.
615= A3112	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
616= A4112	1.0000	1.0000	1.0000	1.0000	1.0000	.6562
617= A5112	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
618= B5112	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
619= A6112	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
620= A7112	.5000	1.0000	1.0000	1.0000	1.0000	1.0000

Figure 4.2-4c Some of the partially completed schedules, integer variables in the third decision field

NODE continued	6	8	13 P	15 P	17 P	18
621= A1120	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
622= A2120	.	.	.	.	.	.
623= A3120	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
624= A4120	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
625= A5120	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
626= P5120	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
627= A6120	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
628= A7120	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
629= A1128	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
630= A2128	.	.	.	.	.	.
631= A3128	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
632= A4128	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
633= A5128	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
634= B5128	.6814	.	1.0000	1.0000	1.0000	1.0000
635= A6128	.9592	1.0000	1.0000	1.0000	1.0000	1.0000
636= A7128	.3361	.	.0488	.	.	.
637= A1136	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
638= A2136	.	.	.	.	.	.
639= A3136	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
640= A4136	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
641= A5136	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
642= B5136	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
643= A6136	.9592	1.0000	1.0000	1.0000	1.0000	1.0000
644= A7136	.3361	.	.0488	.0500	.	.
645= A1144	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
646= A2144	.	.	.	.	.	.
647= A3144	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
648= A4144	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
649= A5144	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
650= B5144	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
651= A6144	.9592	1.0000	1.0000	1.0000	1.0000	1.0000
652= A7144	.3361	.	.0488	.0500	.1000	.
653= A1152	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
654= A2152	.	.	.	.	.	.
655= A3152	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
656= A4152	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
657= A5152	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
658= B5152	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
659= A6152	.9592	1.0000	1.0000	1.0000	1.0000	1.0000
660= A7152	.3361	.	.0488	.0500	.1000	.
661= A1160	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
662= A2160	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
663= A3160	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
664= A4160	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
665= A5160	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
666= B5160	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
667= A6160	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
668= A7160	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
669= A1168	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
670= A2168	.	.	.	.	.	.
671= A3168	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
672= A4168	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
673= A5168	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
674= B5168	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
675= A6168	.9521	1.0000	1.0000	1.0000	1.0000	1.0000
676= A7136	1.0000	.9000	1.0000	1.0000	1.0000	.9000

Figure 4.2-4d Continuation of the display of the integer variables for some of the partially completed schedules in the third decision field

turned out it was actually not a factor). Thus, in this example the on-off prices were set much higher (as can be seen in the comparison of Appendices A and B).

Here the decision field was chosen to go from the time of 64 hours to 96 hours. The resultant schedules are given in figure 4.2-5. The price of computation

NODE	11	14	16	18	20	23
FUNCTIONAL	257260.6354	257285.3854	258681.2729	257909.9211	257272.3354	257297.5854
ESTIMATION	INTEGER	INTEGER	INTEGER	INTEGER	INTEGER	INTEGER
637= A1064	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
638= A2064	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
639= A3064	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
640= A4064	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
641= A5064	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
642= B5064	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
643= A6064	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
644= A7064	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
645= A1072	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
647= A3072	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
648= A4072	.	.	.	.	.	.
649= A5072	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
650= B5072	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
651= A6072	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
652= A7072	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
653= A1080	.	.	.	.	.	.
655= A3080	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
656= A4080	.	.	.	.	.	.
657= A5080	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
658= B5080	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
659= A6080	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
660= A7080	.	.	.	.	1.0000	1.0000
661= A1088	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
662= A2088	.	1.0000	1.0000	1.0000	.	1.0000
663= A3088	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
664= A4088	1.0000	.	1.0000	1.0000	1.0000	.
665= A5088	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
666= B5088	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
667= A6088	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
668= A7088	1.0000	1.0000	.	1.0000	1.0000	1.0000
669= A1096	.	.	.	.	.	.
670= A2096	.	.	.	.	.	.
671= A3096	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
672= A4096	.	.	.	.	.	.
673= A5096	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
674= B5096	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
675= A6096	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
676= A7096	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Figure 4.2-5a Display of the integer variables for the completed schedules of the revised problem with their respective costs, partially completed schedules are displayed on the following page

NODE	19 P	22	6	8
FUNCTIONAL	258059.1533	257486.7104	257315.4604	257280.0063
ESTIMATION	258215.	257904.	257466.	257431.
637= A1064	1.0000	1.0000	1.0000	1.0000
638= A2064	1.0000	1.0000	1.0000	1.0000
639= A3064	1.0000	1.0000	1.0000	1.0000
640= A4064	1.0000	1.0000	1.0000	1.0000
641= A5064	1.0000	1.0000	1.0000	1.0000
642= B5064	1.0000	1.0000	1.0000	1.0000
643= A6064	1.0000	1.0000	1.0000	1.0000
644= A7064	1.0000	1.0000	1.0000	1.0000
645= A1072	1.0000	1.0000	.	1.0000
647= A3072	1.0000	1.0000	1.0000	1.0000
648= A4072	.	.	1.0000	.2500
649= A5072	1.0000	1.0000	1.0000	1.0000
650= B5072	1.0000	1.0000	1.0000	1.0000
651= A6072	.9821	1.0000	1.0000	1.0000
652= A7072	1.0000	1.0000	1.0000	.
653= A1080	.	.	.	.
655= A3080	1.0000	1.0000	1.0000	1.0000
656= A4080	.	.	.	.
657= A5080	1.0000	1.0000	1.0000	1.0000
658= B5080	1.0000	1.0000	1.0000	1.0000
659= A6080	.9821	1.0000	1.0000	1.0000
660= A7080	.	1.0000	1.0000	.
661= A1088	1.0000	.5625	1.0000	1.0000
662= A2088	1.0000	1.0000	.5714	.5714
663= A3088	1.0000	1.0000	1.0000	1.0000
664= A4088	1.0000	1.0000	.5000	.5000
665= A5088	1.0000	1.0000	1.0000	1.0000
666= B5088	1.0000	1.0000	1.0000	1.0000
667= A6088	.9821	1.0000	1.0000	1.0000
668= A7088	1.0000	1.0000	1.0000	1.0000
669= A1096	.	.	.	.
670= A2096	.	.	.	.
671= A3096	1.0000	1.0000	1.0000	1.0000
672= A4096	.	.	.	.
673= A5096	1.0000	1.0000	1.0000	1.0000
674= B5096	1.0000	1.0000	1.0000	1.0000
675= A6096	1.0000	1.0000	1.0000	1.0000
676= A7096	1.0000	1.0000	1.0000	1.0000

Figure 4.2-5b Values for the integer variables of partially completed schedules for the revised scheduling problem

did not change, nor did the magnitude of the variance of the linear schedule from either of the integer formulations (the first and the revised) increase significantly.

Figure 4.2-6 displays the best computed schedule with the primal and dual activities of its rows and variables. The names used for these rows and variables are the same as those described in section 3.2 and listed in the glossary of symbols. The interpretation of the dual variables is described in detail in reference (19), the maintenance and production scheduling counterpart to this document.

Additional schedules, simulations and interpretations of results, as well as examples of the use of spinning reserve requirements and effects of changes in reliability measures, can be found in reference (104).

...ROW..	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
Q	BS	257260.63542	257260.63542-	NONE	NONE	1.00000
D064	LL	10480.00000	.	10480.00000	NONE	6.33333-
D072	LL	8800.00000	.	8800.00000	NONE	5.57143-
D080	LL	4400.00000	.	4400.00000	NONE	5.57143-
D088	LL	10080.00000	.	10080.00000	NONE	7.34531-
D096	LL	8160.00000	.	8160.00000	NONE	5.73393-
D104	LL	5440.00000	.	5440.00000	NONE	5.57143-
D112	LL	8000.00000	.	8000.00000	NONE	5.73643-
D120	LL	9600.00000	.	9600.00000	NONE	7.35781-
D128	LL	3400.00000	.	3400.00000	NONE	5.57143-
D136	LL	7600.00000	.	7600.00000	NONE	5.78393-
D144	LL	8000.00000	.	8000.00000	NONE	5.98768-
D152	LL	7120.00000	.	7120.00000	NONE	5.57143-
D160	LL	10280.00000	.	10280.00000	NONE	7.39063-
D168	LL	8960.00000	.	8960.00000	NONE	6.04688-
NUTOT	EQ	51420.00000	.	51420.00000	51420.00000	5.57143
HYTOT	EQ	7700.00000	.	7700.00000	7700.00000	5.48643
PHTOT	EQ	160.00000	.	160.00000	160.00000	5.72656-
M1064	LL	.	.	.	NONE	920.00000-
M1072	LL	.	.	.	NONE	371.42857-
M1080	LL	.	.	.	NONE	371.42857-
M1088	LL	.	.	.	NONE	1648.62500-
M1096	LL	.	.	.	NONE	488.42857-
M1104	LL	.	.	.	NONE	886.00000-
M1112	LL	.	.	.	NONE	490.22857-
M1120	LL	.	.	.	NONE	1306.99643-
M1128	LL	.	.	.	NONE	905.00000-
M1136	LL	.	.	.	NONE	524.42857-
M1144	LL	.	.	.	NONE	1194.90000-
M1152	BS	.	.	.	NONE	.
M1160	LL	.	.	.	NONE	1681.25000-
M1168	LL	.	.	.	NONE	713.75000-
M2064	LL	.	.	.	NONE	258.66667-
M2072	LL	.	.	.	NONE	14.85714-
M2080	LL	.	.	.	NONE	14.85714-
M2088	LL	.	.	.	NONE	582.50000-
M2096	LL	.	.	.	NONE	957.85714-
M2104	LL	.	.	.	NONE	816.85714-
M2112	LL	.	.	.	NONE	67.65714-
M2120	LL	.	.	.	NONE	645.12500-
M2128	LL	.	.	.	NONE	1246.85714-
M2136	LL	.	.	.	NONE	82.85714-
M2144	LL	.	.	.	NONE	750.95714-
M2152	LL	.	.	.	NONE	14.85714-
M2160	LL	.	.	.	NONE	475.25000-

Figure 4.2-6a Row activity of best schedule for the revised problem, Q row is quality of schedule, D(ttt) the demand at time ttt, NUTOT, HYTOT and PHTOT the nuclear, hydro and pumped hydro quotas, Mn(ttt) the logic equation (see equation 32-11)

...ROW..	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
M2168	LL	.	.	.	NONE	167.00000-
M3064	LL	.	.	.	NONE	373.33333-
M3072	LL	.	.	.	NONE	.
M3080	BS	.	.	.	NONE	.
M3088	LL	.	.	.	NONE	535.25000-
M3096	LL	.	.	.	NONE	91.00000-
M3104	BS	.	.	.	NONE	.
M3112	LL	.	.	.	NONE	92.40000-
M3120	LL	.	.	.	NONE	537.25000-
M3128	BS	.50000	.50000-	.	NONE	.
M3136	LL	.	.	.	NONE	119.00000-
M3144	BS	.	.	.	NONE	.
M3152	BS	.	.	.	NONE	.
M3160	BS	.	.	.	NONE	.
M3168	LL	.	.	.	NONE	18.87500-
M4064	BS	.66667	.66667-	.	NONE	.
M4072	BS	.	.	.	NONE	.
M4080	BS	.	.	.	NONE	.
M4088	LL	.	.	.	NONE	242.87500-
M4096	BS	.	.	.	NONE	.
M4104	BS	.	.	.	NONE	.
M4112	BS	.	.	.	NONE	.
M4120	LL	.	.	.	NONE	341.87500-
M4128	BS	.	.	.	NONE	.
M4136	LL	.	.	.	NONE	103.28571-
M4144	LL	.	.	.	NONE	312.48571-
M4152	BS	.	.	.	NONE	.
M4160	LL	.	.	.	NONE	349.75000-
M4168	LL	.	.	.	NONE	27.25000-
N5064	LL	.	.	.	NONE	738.66667-
N5072	LL	.	.	.	NONE	494.85714-
N5080	LL	.	.	.	NONE	494.85714-
N5088	LL	.	.	.	NONE	1062.50000-
N5096	LL	.	.	.	NONE	546.85714-
N5104	LL	.	.	.	NONE	494.85714-
N5112	BS	.	.	.	NONE	.
N5120	LL	.	.	.	NONE	1066.50000-
N5128	LL	.	.	.	NONE	494.85714-
N5136	LL	.	.	.	NONE	562.85714-
N5144	LL	.	.	.	NONE	.
N5152	LL	.	.	.	NONE	494.85714-
N5160	LL	.	.	.	NONE	1077.00000-
N5168	LL	.	.	.	NONE	.
M5064	LL	.	.	.	NONE	1672.00000-
M5072	LL	.	.	.	NONE	940.57143-
M5080	LL	.	.	.	NONE	940.57143-
M5088	LL	.	.	.	NONE	2643.50000-
M5096	LL	.	.	.	NONE	1096.57143-
M5104	LL	.	.	.	NONE	940.57143-
M5112	LL	.	.	.	NONE	551.31429-
M5120	LL	.	.	.	NONE	2655.50000-
M5128	LL	.	.	.	NONE	940.57143-
M5136	LL	.	.	.	NONE	581.71429-
M5144	LL	.	.	.	NONE	712.11429-
M5152	LL	.	.	.	NONE	940.57143-
M5160	LL	.	.	.	NONE	2687.00000-
M5168	LL	.	.	.	NONE	750.00000-
M6064	LL	.	.	.	NONE	1001.28571-
M6072	BS	.16000	.16000-	.	NONE	.
M6080	BS	.74000	.74000-	.	NONE	.
M6088	LL	.	.	.	NONE	261.07143-
M6096	LL	.	.	.	NONE	650.00000-
M6104	BS	.48000	.48000-	.	NONE	.
M6112	LL	.	.	.	NONE	660.00000-
M6120	BS	.	.	.	NONE	.
M6128	BS	.92000	.92000-	.	NONE	.
M6136	LL	.	.	.	NONE	850.00000-
M6144	BS	.	.	.	NONE	.
M6152	BS	.52500	.52500-	.	NONE	.

Figure 4.2-6b Continuation of row activity of best schedule for the revised problem, Mn(ttt) is the logic equation for insuring proper loading order, as is Nn(ttt), and Ln(ttt) is the startup logic equation

...ROW..	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
M6160	BS	.	.	.	NONE	.
M6168	LL	.	.	.	NONE	873.57143-
M7064	LL	.	.	.	NONE	418.12381-
M7072	LL	.	.	.	NONE	64.60000-
M7080	LL	.	.	.	NONE	64.60000-
M7088	LL	.	.	.	NONE	353.64464-
M7096	LL	.	.	.	NONE	188.10000-
M7104	LL	.	.	.	NONE	136.60000-
M7112	LL	.	.	.	NONE	190.00000-
M7120	LL	.	.	.	NONE	320.14464-
M7128	LL	.	.	.	NONE	78.60000-
M7136	LL	.	.	.	NONE	226.10000-
M7144	LL	.	.	.	NONE	380.95000-
M7152	LL	.	.	.	NONE	64.60000-
M7160	LL	.	.	.	NONE	227.83214-
M7168	LL	.	.	.	NONE	408.58214-
L1064	BS	.	.	NONE	.	.
L1072	BS	.	.	NONE	.	.
L1080	BS	1.00000-	1.00000	NONE	.	.
L1088	UL	.	.	NONE	.	330.00000
L1096	BS	1.00000-	1.00000	NONE	.	.
L1104	UL	.	.	NONE	.	330.00000
L1112	UL	.	.	NONE	.	330.00000
L1120	UL	.	.	NONE	.	1027.37143
L1128	BS	1.00000-	1.00000	NONE	.	.
L1136	UL	.	.	NONE	.	330.00000
L1144	UL	.	.	NONE	.	330.00000
L1152	UL	.	.	NONE	.	330.00000
L1160	UL	.	.	NONE	.	330.00000
L1168	BS	.37500-	.37500	NONE	.	.
L2064	BS	.	.	NONE	.	.
L2072	BS	1.00000-	1.00000	NONE	.	.
L2080	UL	.	.	NONE	.	112.00000
L2088	UL	.	.	NONE	.	112.00000
L2096	BS	.	.	NONE	.	.
L2104	UL	.	.	NONE	.	112.00000
L2112	UL	.	.	NONE	.	112.00000
L2120	UL	.	.	NONE	.	112.00000
L2128	UL	.	.	NONE	.	112.00000
L2136	UL	.	.	NONE	.	112.00000
L2144	UL	.	.	NONE	.	112.00000
L2152	UL	.	.	NONE	.	112.00000
L2160	UL	.	.	NONE	.	112.00000
L2168	BS	1.00000-	1.00000	NONE	.	.
L3064	BS	.	.	NONE	.	.
L3072	UL	.	.	NONE	.	185.00000
L3080	UL	.	.	NONE	.	136.14286
L3088	BS	.	.	NONE	.	.
L3096	UL	.	.	NONE	.	185.00000
L3104	UL	.	.	NONE	.	185.00000
L3112	UL	.	.	NONE	.	58.85714
L3120	BS	.	.	NONE	.	.
L3128	UL	.	.	NONE	.	185.00000
L3136	UL	.	.	NONE	.	154.85714
L3144	UL	.	.	NONE	.	185.00000
L3152	UL	.	.	NONE	.	185.00000
L3160	BS	.	.	NONE	.	.
L3168	UL	.	.	NONE	.	185.00000
L4064	BS	.	.	NONE	.	.
L4072	BS	1.00000-	1.00000	NONE	.	.
L4080	UL	.	.	NONE	.	150.00000
L4088	UL	.	.	NONE	.	150.00000
L4096	BS	1.00000-	1.00000	NONE	.	.
L4104	UL	.	.	NONE	.	150.00000
L4112	UL	.	.	NONE	.	150.00000
L4120	UL	.	.	NONE	.	150.00000
L4128	BS	.25000-	.25000	NONE	.	.
L4136	UL	.	.	NONE	.	150.00000
L4144	UL	.	.	NONE	.	150.00000
L4152	BS	.	.	NONE	.	.
L4160	UL	.	.	NONE	.	150.00000

Figure 4.2-6c Continuation of the row activity of the best schedule for the revised problem, Lm(ttt) is the startup logic equation

...ROW..	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
L4168	BS	.31250-	.31250	NONE	.	.
L5064	BS	.	.	NONE	.	.
L5072	BS	.	.	NONE	.	.
L5080	UL	.	.	NONE	.	402.00000
L5088	UL	.	.	NONE	.	402.00000
L5096	BS	.	.	NONE	.	.
L5104	UL	.	.	NONE	.	402.00000
L5112	BS	.	.	NONE	.	.
L5120	UL	.	.	NONE	.	402.00000
L5128	UL	.	.	NONE	.	402.00000
L5136	BS	.	.	NONE	.	.
L5144	UL	.	.	NONE	.	402.00000
L5152	UL	.	.	NONE	.	402.00000
L5160	BS	.	.	NONE	.	.
L5168	UL	.	.	NONE	.	402.00000
L6064	UL	.	.	NONE	.	1019.00000
L6072	UL	.	.	NONE	.	159.00000
L6080	UL	.	.	NONE	.	630.00000
L6088	UL	.	.	NONE	.	1019.00000
L6096	UL	.	.	NONE	.	453.46429
L6104	UL	.	.	NONE	.	269.46429
L6112	UL	.	.	NONE	.	781.46429
L6120	UL	.	.	NONE	.	490.26429
L6128	UL	.	.	NONE	.	122.80000
L6136	UL	.	.	NONE	.	710.80000
L6144	UL	.	.	NONE	.	188.80000
L6152	UL	.	.	NONE	.	533.00000
L6160	UL	.	.	NONE	.	1019.00000
L6168	UL	.	.	NONE	.	652.78571
L7064	BS	.	.	NONE	.	.
L7072	BS	.	.	NONE	.	.
L7080	BS	1.00000-	1.00000	NONE	.	.
L7088	UL	.	.	NONE	.	184.00000
L7096	BS	.	.	NONE	.	.
L7104	BS	1.00000-	1.00000	NONE	.	.
L7112	UL	.	.	NONE	.	184.00000
L7120	UL	.	.	NONE	.	184.00000
L7128	BS	1.00000-	1.00000	NONE	.	.
L7136	UL	.	.	NONE	.	184.00000
L7144	UL	.	.	NONE	.	184.00000
L7152	BS	.07500-	.07500	NONE	.	.
L7160	UL	.	.	NONE	.	184.00000
L7168	UL	.	.	NONE	.	184.00000
L8064	UL	.	.	NONE	.	119.00000
L8072	UL	.	.	NONE	.	119.00000
L8080	UL	.	.	NONE	.	119.00000
L8088	UL	.	.	NONE	.	119.00000
L8096	UL	.	.	NONE	.	23.20000
L8104	UL	.	.	NONE	.	119.00000
L8112	UL	.	.	NONE	.	119.00000
L8120	UL	.	.	NONE	.	102.20000
L8128	BS	.	.	NONE	.	.
L8136	UL	.	.	NONE	.	119.00000
L8144	UL	.	.	NONE	.	119.00000
L8152	UL	.	.	NONE	.	119.00000
L8160	UL	.	.	NONE	.	119.00000
L8168	BS	.15625-	.15625	NONE	.	.
X064	EQ	.	.	.	.	5.72656-
X072	EQ	.	.	.	.	5.72656-
X080	EQ	.	.	.	.	5.72656-
X088	EQ	.	.	.	.	5.72656-
X096	EQ	.	.	.	.	5.72656-
X104	EQ	.	.	.	.	5.72656-
X112	EQ	.	.	.	.	5.72656-
X120	EQ	.	.	.	.	5.72656-
X128	EQ	.	.	.	.	5.72656-
X136	EQ	.	.	.	.	5.72656-
X144	EQ	.	.	.	.	5.72656-
X152	EQ	.	.	.	.	5.72656-
X160	EQ	.	.	.	.	5.72656-
X168	EQ	.	.	.	.	5.72656-

Figure 4.2-6d Remainder of the row activity of the best schedule for the revised problem, Ln(ttt) is the startup logic equation and X(ttt) is the hydro reservoir accounting equation

.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
A1056	EQ	1.00000	.	1.00000	1.00000	.
A2056	EQ	1.00000	.	1.00000	1.00000	.
A3056	EQ	1.00000	.	1.00000	1.00000	.
A4056	EQ	1.00000	.	1.00000	1.00000	.
A5056	EQ	1.00000	.	1.00000	1.00000	.
A6056	EQ	1.00000	.	1.00000	1.00000	1019.00000-
A7056	EQ	1.00000	.	1.00000	1.00000	.
HL056	EQ	100.00000	.	100.00000	100.00000	5.72656-
J1064	BS	1.00000	3640.00000	.	1.00000	.
W1064	LL	.	330.00000	.	1.00000	330.00000
J2064	BS	1.00000	1768.00000	.	1.00000	.
W2064	LL	.	112.00000	.	1.00000	112.00000
J3064	BS	1.00000	640.00000	.	1.00000	.
K3064	UL	1.00000	3120.00000	.	1.00000	53.33333-
W3064	LL	.	185.00000	.	1.00000	185.00000
J4064	UL	1.00000	1424.00000	.	1.00000	96.00000-
K4064	BS	.33333	1520.00000	.	1.00000	.
W4064	LL	.	150.00000	.	1.00000	150.00000
J5064	LL	.	3120.00000	.	1.00000	738.66667
K5064	BS	1.00000	1288.00000	.	1.00000	.
W5064	LL	.	402.00000	.	1.00000	402.00000
J6064	UL	1.00000	.	.	1.00000	2046.33333-
W6064	BS	.	1019.00000	.	1.00000	.
J7064	UL	1.00000	.	.	1.00000	225.52381-
W7064	LL	.	184.00000	.	1.00000	184.00000
A8064	LL	.	.	.	1.00000	422.33333
G8064	LL	.	.	.	1.00000	1199.00000
HL064	BS	100.00000	.	.	1000.00000	.
W8064	BS	.	119.00000	.	1.00000	.
J1072	BS	1.00000	3640.00000	.	1.00000	.
W1072	LL	.	330.00000	.	1.00000	330.00000
J2072	BS	.	1768.00000	.	1.00000	.
W2072	LL	.	112.00000	.	1.00000	112.00000
J3072	UL	1.00000	640.00000	.	1.00000	251.42857-
K3072	BS	1.00000	3120.00000	.	1.00000	.
W3072	BS	.	185.00000	.	1.00000	.
J4072	LL	.	1424.00000	.	1.00000	86.85714
K4072	LL	.	1520.00000	.	1.00000	182.85714
W4072	LL	.	150.00000	.	1.00000	150.00000
J5072	LL	.	3120.00000	.	1.00000	494.85714
K5072	BS	1.00000	1288.00000	.	1.00000	.
W5072	LL	.	402.00000	.	1.00000	402.00000
J6072	BS	.84000	.	.	1.00000	.
W6072	LL	.	1019.00000	.	1.00000	860.00000
J7072	BS	1.00000	.	.	1.00000	.
W7072	LL	.	184.00000	.	1.00000	184.00000
A8072	LL	.	.	.	1.00000	812.42857
G8072	LL	.	.	.	1.00000	613.85714
HL072	BS	100.00000	.	.	1000.00000	.
W8072	BS	.	119.00000	.	1.00000	.
J1080	BS	.	3640.00000	.	1.00000	.
W1080	LL	.	330.00000	.	1.00000	330.00000
J2080	BS	.	1768.00000	.	1.00000	.
W2080	BS	.	112.00000	.	1.00000	.
J3080	UL	1.00000	640.00000	.	1.00000	251.42857-
K3080	UL	1.00000	3120.00000	.	1.00000	.
W3080	LL	.	185.00000	.	1.00000	48.85714
J4080	LL	.	1424.00000	.	1.00000	86.85714
K4080	LL	.	1520.00000	.	1.00000	182.85714
W4080	BS	.	150.00000	.	1.00000	.
J5080	LL	.	3120.00000	.	1.00000	494.85714
K5080	BS	1.00000	1288.00000	.	1.00000	.
W5080	BS	.	402.00000	.	1.00000	.
J6080	BS	.26000	.	.	1.00000	.
W6080	LL	.	1019.00000	.	1.00000	389.00000

Figure 4.2-6e Column activity of the best schedule for the revised problem, with An(ttt) the on-off variables, Bn(ttt) on or off the second segment of the loading curve, extents of incremental loadings Jn(ttt) and Kn(ttt), extent of pumping hydro storage Gn(ttt), pumped hydro reservoir levels HL(ttt), and startup variables Wn(ttt) for plants n at times ttt.

.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
J7080	BS	.	.	.	1.00000	.
W7080	LL	.	184.00000	.	1.00000	184.00000
A8080	LL	.	.	.	1.00000	812.42857
G8080	LL	.	.	.	1.00000	613.85714
HL080	BS	100.00000	.	.	1000.00000	.
W8080	BS	.	119.00000	.	1.00000	.
J1088	BS	1.00000	3640.00000	.	1.00000	.
W1088	BS	1.00000	330.00000	.	1.00000	.
J2088	BS	.	1768.00000	.	1.00000	.
W2088	BS	.	112.00000	.	1.00000	.
J3088	BS	1.00000	640.00000	.	1.00000	.
K3088	UL	1.00000	3120.00000	.	1.00000	458.12500-
W3088	LL	.	185.00000	.	1.00000	185.00000
J4088	UL	1.00000	1424.00000	.	1.00000	96.00000-
K4088	BS	1.00000	1520.00000	.	1.00000	.
W4088	BS	1.00000	150.00000	.	1.00000	.
J5088	LL	.	3120.00000	.	1.00000	1062.50000
K5088	BS	1.00000	1288.00000	.	1.00000	.
W5088	BS	.	402.00000	.	1.00000	.
J6088	UL	1.00000	.	.	1.00000	6834.46429-
W6088	BS	.	1019.00000	.	1.00000	.
J7088	UL	1.00000	.	.	1.00000	1059.10714-
W7088	BS	1.00000	184.00000	.	1.00000	.
A8088	BS	.	.	.	1.00000	.
G8088	LL	.	.	.	1.00000	1976.20000
HL088	BS	100.00000	.	.	1000.00000	.
W8088	BS	.	119.00000	.	1.00000	.
J1096	BS	.	3640.00000	.	1.00000	.
W1096	LL	.	330.00000	.	1.00000	330.00000
J2096	LL	.	1768.00000	.	1.00000	891.00000
W2096	LL	.	112.00000	.	1.00000	112.00000
J3096	UL	1.00000	640.00000	.	1.00000	186.42857-
K3096	BS	1.00000	3120.00000	.	1.00000	.
W3096	BS	.	185.00000	.	1.00000	.
J4096	LL	.	1424.00000	.	1.00000	47.85714
K4096	LL	.	1520.00000	.	1.00000	143.85714
W4096	LL	.	150.00000	.	1.00000	150.00000
J5096	LL	.	3120.00000	.	1.00000	546.85714
K5096	BS	1.00000	1288.00000	.	1.00000	.
W5096	LL	.	402.00000	.	1.00000	402.00000
J6096	BS	1.00000	.	.	1.00000	.
W6096	LL	.	1019.00000	.	1.00000	565.53571
J7096	BS	1.00000	.	.	1.00000	.
W7096	LL	.	184.00000	.	1.00000	184.00000
A8096	LL	.	.	.	1.00000	633.42857
G8096	LL	.	.	.	1.00000	738.65714
HL096	BS	100.00000	.	.	1000.00000	.
W8096	LL	.	119.00000	.	1.00000	95.80000
A1104	BS	.	4006.00000	.	1.00000	.
J1104	LL	.	3640.00000	.	1.00000	514.57143
W1104	BS	.	330.00000	.	1.00000	.
A2104	BS	.	2154.00000	.	1.00000	.
J2104	LL	.	1768.00000	.	1.00000	802.00000
W2104	BS	.	112.00000	.	1.00000	.
A3104	BS	1.00000	1211.00000	.	1.00000	.
J3104	UL	1.00000	640.00000	.	1.00000	251.42857-
K3104	UL	1.00000	3120.00000	.	1.00000	.
W3104	BS	.	185.00000	.	1.00000	.
A4104	LL	.	1514.00000	.	1.00000	622.57143
J4104	LL	.	1424.00000	.	1.00000	86.85714
K4104	LL	.	1520.00000	.	1.00000	182.85714
W4104	BS	.	150.00000	.	1.00000	.
A5104	UL	1.00000	3008.00000	.	1.00000	2879.14286-
J5104	LL	.	3120.00000	.	1.00000	494.85714
B5104	BS	1.00000	3120.00000	.	1.00000	.
K5104	BS	1.00000	1288.00000	.	1.00000	.
W5104	BS	.	402.00000	.	1.00000	.
A6104	BS	1.00000	512.00000	.	1.00000	.
J6104	BS	.52000	.	.	1.00000	.
W6104	LL	.	1019.00000	.	1.00000	749.53571
A7104	BS	.	324.00000	.	1.00000	.

Figure 4.2-6f Continuation of column activity of the best schedule for the revised problem

.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
J7104	LL	.	.	.	1.00000	72.00000
W7104	LL	.	184.00000	.	1.00000	184.00000
A8104	LL	.	.	.	1.00000	812.42857
G8104	LL	.	.	.	1.00000	613.85714
HL104	BS	100.00000	.	.	1000.00000	.
W8104	BS	.	119.00000	.	1.00000	.
A1112	BS	.	4400.00000	.	1.00000	.
J1112	BS	.	3640.00000	.	1.00000	.
W1112	BS	.	330.00000	.	1.00000	.
A2112	LL	.	2000.00000	.	1.00000	555.60000
J2112	BS	.	1768.00000	.	1.00000	.
W2112	BS	.	112.00000	.	1.00000	.
A3112	UL	1.00000	1373.00000	.	1.00000	129.68571-
J3112	UL	1.00000	640.00000	.	1.00000	185.42857-
K3112	BS	1.00000	3120.00000	.	1.00000	.
W3112	LL	.	185.00000	.	1.00000	126.14286
A4112	LL	.	1622.00000	.	1.00000	704.17143
J4112	LL	.	1424.00000	.	1.00000	47.25714
K4112	LL	.	1520.00000	.	1.00000	143.25714
W4112	BS	.	150.00000	.	1.00000	.
A5112	UL	1.00000	3002.00000	.	1.00000	3458.28571-
J5112	BS	.	3120.00000	.	1.00000	.
B5112	UL	1.00000	3120.00000	.	1.00000	.
K5112	UL	1.00000	1288.00000	.	1.00000	547.65714-
W5112	LL	.	402.00000	.	1.00000	402.00000
A6112	BS	1.00000	448.00000	.	1.00000	.
J6112	BS	1.00000	.	.	1.00000	.
W6112	LL	.	1019.00000	.	1.00000	237.53571
A7112	BS	.80000	200.00000	.	1.00000	.
J7112	BS	.80000	.	.	1.00000	.
W7112	BS	.80000	184.00000	.	1.00000	.
A8112	LL	.	.	.	1.00000	744.74857
G8112	LL	.	.	.	1.00000	740.57714
HL112	BS	100.00000	.	.	1000.00000	.
W8112	BS	.	119.00000	.	1.00000	.
A1120	BS	1.00000	4400.00000	.	1.00000	.
J1120	UL	1.00000	3640.00000	.	1.00000	350.62857-
W1120	UL	1.00000	330.00000	.	1.00000	697.37143-
A2120	BS	.	2411.00000	.	1.00000	.
J2120	LL	.	1768.00000	.	1.00000	58.62500
W2120	BS	.	112.00000	.	1.00000	.
A3120	UL	1.00000	1121.00000	.	1.00000	1904.37500-
J3120	BS	1.00000	640.00000	.	1.00000	.
K3120	UL	1.00000	3120.00000	.	1.00000	463.12500-
W3120	LL	.	185.00000	.	1.00000	185.00000
A4120	BS	.25000	1711.00000	.	1.00000	.
J4120	BS	.50000	1424.00000	.	1.00000	.
K4120	LL	.	1520.00000	.	1.00000	96.00000
W4120	BS	.25000	150.00000	.	1.00000	.
A5120	UL	1.00000	3100.00000	.	1.00000	6619.00000-
J5120	LL	.	3120.00000	.	1.00000	1066.50000
B5120	BS	1.00000	3120.00000	.	1.00000	.
K5120	BS	1.00000	1288.00000	.	1.00000	.
W5120	BS	.	402.00000	.	1.00000	.
A6120	BS	1.00000	490.00000	.	1.00000	.
J6120	UL	1.00000	.	.	1.00000	7145.53571-
W6120	LL	.	1019.00000	.	1.00000	528.73571
A7120	BS	1.00000	211.00000	.	1.00000	.
J7120	UL	1.00000	.	.	1.00000	1102.10714-
W7120	BS	.20000	184.00000	.	1.00000	.
A8120	BS	.	.	.	1.00000	.
G8120	LL	.	.	.	1.00000	1985.80000
HL120	BS	100.00000	.	.	1000.00000	.
W8120	LL	.	119.00000	.	1.00000	16.80000
A1128	BS	.	4355.00000	.	1.00000	.
J1128	LL	.	3640.00000	.	1.00000	533.57143
W1128	LL	.	330.00000	.	1.00000	330.00000
A2128	BS	.	2584.00000	.	1.00000	.
J2128	LL	.	1768.00000	.	1.00000	1232.00000
W2128	BS	.	112.00000	.	1.00000	.
A3128	BS	1.00000	1307.00000	.	1.00000	.

Figure 4.2-6g Continuation of column activity of the best schedule for the revised problem

.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
J3128	UL	1.00000	640.00000	.	1.00000	251.42857-
K3128	BS	.50000	3120.00000	.	1.00000	.
W3128	BS	.	185.00000	.	1.00000	.
A4128	LL	.	1659.00000	.	1.00000	617.57143
J4128	LL	.	1424.00000	.	1.00000	86.85714
K4128	LL	.	1520.00000	.	1.00000	182.85714
W4128	LL	.	150.00000	.	1.00000	150.00000
A5128	UL	1.00000	2583.00000	.	1.00000	2904.14286-
J5128	LL	.	3120.00000	.	1.00000	494.85714
B5128	BS	1.00000	3120.00000	.	1.00000	.
K5128	BS	1.00000	1288.00000	.	1.00000	.
W5128	BS	.	402.00000	.	1.00000	.
A6128	BS	1.00000	588.00000	.	1.00000	.
J6128	BS	.08000	.	.	1.00000	.
W6128	LL	.	1019.00000	.	1.00000	896.20000
A7128	BS	.	266.00000	.	1.00000	.
J7128	LL	.	.	.	1.00000	14.00000
W7128	LL	.	184.00000	.	1.00000	184.00000
A8128	LL	.	.	.	1.00000	693.42857
G8128	LL	.	.	.	1.00000	613.85714
HL128	BS	100.00000	.	.	1000.00000	.
W8128	LL	.	119.00000	.	1.00000	119.00000
A1136	LL	.	4312.00000	.	1.00000	548.57143
J1136	BS	.	3640.00000	.	1.00000	.
W1136	BS	.	330.00000	.	1.00000	.
A2136	LL	.	1632.00000	.	1.00000	161.00000
J2136	BS	.	1768.00000	.	1.00000	.
W2136	BS	.	112.00000	.	1.00000	.
A3136	UL	1.00000	1120.00000	.	1.00000	536.28571-
J3136	UL	1.00000	640.00000	.	1.00000	166.42857-
K3136	BS	1.00000	3120.00000	.	1.00000	.
W3136	LL	.	185.00000	.	1.00000	30.14286
A4136	BS	.	1132.00000	.	1.00000	.
J4136	LL	.	1424.00000	.	1.00000	139.14286
K4136	LL	.	1520.00000	.	1.00000	235.14286
W4136	BS	.	150.00000	.	1.00000	.
A5136	UL	1.00000	5220.00000	.	1.00000	1316.28571-
J5136	BS	.	3120.00000	.	1.00000	.
B5136	UL	1.00000	3120.00000	.	1.00000	562.85714-
K5136	BS	1.00000	1288.00000	.	1.00000	.
W5136	LL	.	402.00000	.	1.00000	402.00000
A6136	BS	1.00000	430.00000	.	1.00000	.
J6136	BS	1.00000	.	.	1.00000	.
W6136	LL	.	1019.00000	.	1.00000	308.20000
A7136	BS	.30000	238.00000	.	1.00000	.
J7136	BS	.30000	.	.	1.00000	.
W7136	BS	.30000	184.00000	.	1.00000	.
A8136	LL	.	.	.	1.00000	703.62857
G8136	LL	.	.	.	1.00000	777.05714
HL136	BS	100.00000	.	.	1000.00000	.
W8136	BS	.	119.00000	.	1.00000	.
A1144	BS	.	4548.00000	.	1.00000	.
J1144	LL	.	3640.00000	.	1.00000	523.77143
W1144	BS	.	330.00000	.	1.00000	.
A2144	BS	.	2188.00000	.	1.00000	.
J2144	LL	.	1768.00000	.	1.00000	602.90000
W2144	BS	.	112.00000	.	1.00000	.
A3144	UL	1.00000	1127.00000	.	1.00000	310.04286-
J3144	UL	1.00000	640.00000	.	1.00000	318.02857-
K3144	UL	1.00000	3120.00000	.	1.00000	233.10000-
W3144	BS	.	185.00000	.	1.00000	.
A4144	BS	.	1433.00000	.	1.00000	.
J4144	LL	.	1424.00000	.	1.00000	299.44286
K4144	LL	.	1520.00000	.	1.00000	395.44286
W4144	BS	.	150.00000	.	1.00000	.
A5144	UL	1.00000	4886.00000	.	1.00000	1574.28571-
J5144	BS	.	3120.00000	.	1.00000	.
B5144	BS	1.00000	3120.00000	.	1.00000	.
K5144	UL	1.00000	1288.00000	.	1.00000	628.05714-
W5144	BS	.	402.00000	.	1.00000	.
A6144	BS	1.00000	544.00000	.	1.00000	.

Figure 4.2-6h Continuation of column activity of the best schedule for the revised problem

.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
J6144	UL	1.00000	.	.	1.00000	1665.00000-
W6144	LL	.	1019.00000	.	1.00000	830.20000
A7144	BS	.80000	217.00000	.	1.00000	.
J7144	BS	.80000	.	.	1.00000	.
W7144	BS	.50000	184.00000	.	1.00000	.
A8144	LL	.	.	.	1.00000	599.30857
G8144	LL	.	.	.	1.00000	933.53714
HL144	BS	100.00000	.	.	1000.00000	.
W8144	BS	.	119.00000	.	1.00000	.
A1152	UL	1.00000	2256.00000	.	1.00000	864.00000-
J1152	UL	1.00000	3640.00000	.	1.00000	371.42857-
W1152	BS	1.00000	330.00000	.	1.00000	.
A2152	LL	.	2354.00000	.	1.00000	1002.00000
J2152	BS	.	1768.00000	.	1.00000	.
W2152	BS	.	112.00000	.	1.00000	.
A3152	UL	1.00000	1124.00000	.	1.00000	28.14286-
J3152	UL	1.00000	640.00000	.	1.00000	251.42857-
K3152	UL	1.00000	3120.00000	.	1.00000	.
W3152	BS	.	185.00000	.	1.00000	.
A4152	LL	.	1289.00000	.	1.00000	247.57143
J4152	LL	.	1424.00000	.	1.00000	86.85714
K4152	LL	.	1520.00000	.	1.00000	182.85714
W4152	LL	.	150.00000	.	1.00000	150.00000
A5152	UL	1.00000	3109.00000	.	1.00000	2778.14286-
J5152	LL	.	3120.00000	.	1.00000	494.85714
B5152	BS	1.00000	3120.00000	.	1.00000	.
K5152	BS	1.00000	1288.00000	.	1.00000	.
W5152	BS	.	402.00000	.	1.00000	.
A6152	BS	1.00000	486.00000	.	1.00000	.
J6152	BS	.47500	.	.	1.00000	.
W6152	LL	.	1019.00000	.	1.00000	486.00000
A7152	BS	.72500	252.00000	.	1.00000	.
J7152	BS	.72500	.	.	1.00000	.
W7152	LL	.	184.00000	.	1.00000	184.00000
A8152	LL	.	.	.	1.00000	812.42857
G8152	LL	.	.	.	1.00000	613.85714
HL152	BS	100.00000	.	.	1000.00000	.
W8152	BS	.	119.00000	.	1.00000	.
A1160	UL	1.00000	4237.00000	.	1.00000	1253.00000-
J1160	BS	1.00000	3640.00000	.	1.00000	.
W1160	BS	.	330.00000	.	1.00000	.
A2160	BS	1.00000	2137.00000	.	1.00000	.
J2160	UL	1.00000	1768.00000	.	1.00000	121.75000-
W2160	BS	1.00000	112.00000	.	1.00000	.
A3160	UL	1.00000	1237.00000	.	1.00000	721.75000-
J3160	UL	1.00000	640.00000	.	1.00000	542.50000-
K3160	UL	1.00000	3120.00000	.	1.00000	1018.75000-
W3160	LL	.	185.00000	.	1.00000	185.00000
A4160	BS	.31250	1732.00000	.	1.00000	.
J4160	BS	.62500	1424.00000	.	1.00000	.
K4160	LL	.	1520.00000	.	1.00000	96.00000
W4160	BS	.31250	150.00000	.	1.00000	.
A5160	UL	1.00000	3217.00000	.	1.00000	6967.00000-
J5160	LL	.	3120.00000	.	1.00000	1077.00000
B5160	BS	1.00000	3120.00000	.	1.00000	.
K5160	BS	1.00000	1288.00000	.	1.00000	.
W5160	LL	.	402.00000	.	1.00000	402.00000
A6160	BS	1.00000	507.00000	.	1.00000	.
J6160	UL	1.00000	.	.	1.00000	7276.78571-
W6160	BS	.	1019.00000	.	1.00000	.
A7160	BS	1.00000	304.00000	.	1.00000	.
J7160	UL	1.00000	.	.	1.00000	1219.35714-
W7160	BS	.27500	184.00000	.	1.00000	.
A8160	BS	.15625	.	.	1.00000	.
G8160	LL	.	.	.	1.00000	2011.00000
HL160	BS	.	.	.	1000.00000	.
W8160	BS	.15625	119.00000	.	1.00000	.
A1168	BS	.62500	4100.00000	.	1.00000	.
J1168	BS	.62500	3640.00000	.	1.00000	.
W1168	LL	.	330.00000	.	1.00000	330.00000
A2168	LL	.	2040.00000	.	1.00000	421.75000

Figure 4.2-6i Continuation of column activity of the best schedule for the revised problem

.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
J2168	BS	.	1768.00000	.	1.00000	.
W2168	LL	.	112.00000	.	1.00000	112.00000
A3168	BS	1.00000	1304.00000	.	1.00000	.
J3168	UL	1.00000	640.00000	.	1.00000	308.62500-
K3168	UL	1.00000	3120.00000	.	1.00000	247.37500-
W3168	BS	.	185.00000	.	1.00000	.
A4168	LL	.	1601.00000	.	1.00000	579.00000
J4168	BS	.	1424.00000	.	1.00000	.
K4168	LL	.	1520.00000	.	1.00000	96.00000
W4168	LL	.	150.00000	.	1.00000	150.00000
A5168	UL	1.00000	3117.00000	.	1.00000	3036.00000-
J5168	BS	.	3120.00000	.	1.00000	.
B5168	BS	1.00000	3120.00000	.	1.00000	.
K5168	UL	1.00000	1288.00000	.	1.00000	647.00000-
W5168	BS	.	402.00000	.	1.00000	.
A6168	BS	1.00000	449.00000	.	1.00000	.
J6168	UL	1.00000	.	.	1.00000	1028.21429-
W6168	LL	.	1019.00000	.	1.00000	366.21429
A7168	BS	1.00000	247.00000	.	1.00000	.
J7168	UL	1.00000	.	.	1.00000	17.35714-
W7168	BS	.	184.00000	.	1.00000	.
A8168	LL	.	.	.	1.00000	569.00000
G8168	LL	.	.	.	1.00000	979.00000
HL168	BS	.	.	.	1000.00000	.
W8168	LL	.	119.00000	.	1.00000	119.00000
OSN	LL	.	8.60000	.	12000.00000	3.02857
USN	LL	.	2.00000	.	18000.00000	7.57143
OSH	LL	.	7.60000	.	2000.00000	2.11357
USH	LL	.	1.10000-	.	3400.00000	4.38643
CSPH	LL	.	5.00000	.	1000.00000	10.72656
USPH	UL	160.00000	5.50000-	.	160.00000	11.22656-
A1064	IV	1.00000	4512.00000	.	1.00000	45.33333
A2064	IV	1.00000	2512.00000	.	1.00000	733.33333
A3064	IV	1.00000	1360.00000	.	1.00000	1091.66667-
A4064	IV	1.00000	1600.00000	.	1.00000	586.66667
A5064	IV	1.00000	3360.00000	.	1.00000	4392.00000-
B5064	IV	1.00000	3120.00000	.	1.00000	.
A6064	IV	1.00000	507.00000	.	1.00000	.
A7064	IV	1.00000	452.00000	.	1.00000	.
A1072	IV	1.00000	4388.00000	.	1.00000	896.57143
A2072	EQ	.	2507.00000	.	.	1043.00000
A3072	IV	1.00000	1199.00000	.	1.00000	89.28571-
A4072	IV	.	1532.00000	.	1.00000	490.57143
A5072	IV	1.00000	2963.00000	.	1.00000	3728.14286-
B5072	IV	1.00000	3120.00000	.	1.00000	.
A6072	IV	1.00000	471.00000	.	1.00000	.
A7072	IV	1.00000	258.00000	.	1.00000	190.00000
A1080	IV	.	4033.00000	.	1.00000	211.57143
A2080	EQ	.	2047.00000	.	.	695.00000
A3080	IV	1.00000	1201.00000	.	1.00000	.
A4080	IV	.	1581.00000	.	1.00000	689.57143
A5080	IV	1.00000	3008.00000	.	1.00000	3281.14286-
B5080	IV	1.00000	3120.00000	.	1.00000	.
A6080	IV	1.00000	389.00000	.	1.00000	.
A7080	IV	.	245.00000	.	1.00000	7.00000-
A1088	IV	1.00000	4772.00000	.	1.00000	660.00000-
A2088	IV	.	2159.00000	.	1.00000	74.37500-
A3088	IV	1.00000	1092.00000	.	1.00000	1926.37500-
A4088	IV	1.00000	1511.00000	.	1.00000	.
A5088	IV	1.00000	2994.00000	.	1.00000	6299.00000-
B5088	IV	1.00000	3120.00000	.	1.00000	.
A6088	IV	1.00000	547.00000	.	1.00000	.
A7088	IV	1.00000	244.00000	.	1.00000	.
A1096	IV	.	4660.00000	.	1.00000	630.57143
A2096	IV	.	2446.00000	.	1.00000	.
A3096	IV	1.00000	1166.00000	.	1.00000	392.14286-
A4096	IV	.	1617.00000	.	1.00000	549.57143
A5096	IV	1.00000	3121.00000	.	1.00000	3882.14286-
B5096	IV	1.00000	3120.00000	.	1.00000	.
A6096	IV	1.00000	544.00000	.	1.00000	.
A7096	IV	1.00000	198.00000	.	1.00000	.

Figure 4.2-6j Remainder of column activity for the best schedule for the revised problem, OSN, OSH and OSPH are overshoots of quota targets for nuclear, hydro and pumped hydro energy levels at end of week, USN, USH and USPH are under achievements of target levels in the same catagories

5. Feasibility and Usefulness

This study was undertaken as an attempt to include environmental costs in the unit commitment scheduling process, as well as to build such a scheduler which is compatible with the maintenance and production program of reference (19). Because it accomplishes this goal, the procedure developed should prove useful. The scheduling technique presented also offers a technique for including major production scheduling variables, such as nuclear, hydroelectric, and pumped storage production levels, without the need of pseudo-incremental costs, and does not require such artificial information as initial feasible schedules, priorities of unit startups and removals, nor does it require iterations to attempt to couple portions of the problem that are usually treated separately, because here they are treated simultaneously.

This mechanism is also usable as a simulation tool with computation efforts increasing only linearly with expanded time horizons. An example of a simulation on this unit commitment level is given in reference (104), where in a hypothetical example it is shown that an 11% error in cost could have been made by present simulation techniques which consider only peak load and total energy demands on a system, and which do not delve into the problem of a system's capabilities for handling projected load shapes, as this project's simulator does.

5.1 Cost Considerations

There should be no concern over the cost and time involved in running this scheduling program. The largest problem handled for this project used 112 decision variables which were simultaneously constrained to be integers and the resulting completed system schedules were generated at a cost of about $3 per schedule.

The major concern, in the cost area, will probably be the cost of the mixed integer program itself. It is possible that at some time in the future mixed integer program products will come with other portions of system libraries at no additional cost, as is the case with linear programs presently. The product used, MPSX-MIP, however, currently costs $225 per month. If this cost is a consideration there are then three options available. (1) The schedule can be formed from the linear program alone, see figure 4.2-1, and the error associated with this approximation could be very small, especially considering that a valid schedule made up by taking the linear optimal solution and moving the appropriate variables to the nearest integer values would bring the linear solution points in figure 4.2-1 much closer to the optimum integer solution. (2) It might be worthwhile to develop the integer programs starting with available linear programming subroutines. (3) Time might be rented at a user center where a mixed integer program is available.

5.2 Drawbacks

Outside of any computational cost drawbacks (which don't appear to be a problem) there are few disadvantages to this scheduling procedure. Perhaps one objection could be the difference of this technique from those now existing, thus requiring time consuming initial problem setups. However, the significant and lasting gains to be made seem to more than justify the initial time investment.

Another problem is that the input data is not all readily available. For example, reserve requirements in megawatts may be difficult to obtain, and certainly the atmospheric and aquatic environmental consequences of power generation will require a real collection and computation effort (see references (105) and (106)). This data collection in the case of the ecological impact figures is, however, something which sooner or later must be performed if the system is to operate in a manner consistent with energy-environmental priorities. That is, this data requirement is not a fabrication of this particular scheduling scheme, but is a necessity for effecting a proper balance between the environment and environmentally imperfect means of power generation.

The quasi-optimal, i.e. in a sense optimal, solutions which are of a suboptimal nature cannot, it appears, be considered a drawback. Not only does this technique minimize the recompuational effort required due to changes in input factors, but it should be considered which of the pure optimal solutions would be lost by this suboptimal process. An optimum would be lost, for example, if for its small gain over other near-optimal schedules it was relying tenuously upon an otherwise unexpected scheduling move to be made more than an entire decision field time span in the future (or the past). This characteristic of the solution technique of bypassing narrow, unwaverable optimum paths could be considered an attractive factor in the scheduling process, for it introduces a healthy respect for the uncertainties of the future - a respect any complex, real-world system deserves.

Thus, this technique is more 'sensible' from the scheduling point of view, and this 'sensibility' also makes it more realistic from the simulation viewpoint.

Glossary of Equation Nomenclature

$A_i(t)$	binary, on=1 off=0, variable for plant i in interval t
Anttt	computer symbol for $A_n(ttt)$
$B_n(t)$	binary variable indicator of whether system is on=1 or off=0 the second segment of the loading curve for plant n at time t
Bnttt	computer symbol for $B_n(ttt)$
C_i	costs of various plants loading curves at particular points i on those curves
D(t)	demand for power in interval t, including reserve requirement
Dttt	computer symbol for D(ttt)
$E_n(t)$	dummy variable which paces the startup of plant n so it conforms to its startup rate requirements
ES(t)	external emergency support power purchased at time t
ESttt	computer symbol for ES(ttt)
GH(t)	amount of water pumped into a pumped storage facility's reservoir
$G_n(t)$	fractional extent of use of pumped storage facility number n 's input capabilities at time t
Gnttt	computer symbol for $G_n(ttt)$
HL(t)	water storage amount contained in reservoir at time t
HLttt	computer symbol for HL(ttt) at the pumped hydro storage facility number 8
HYTOT	computer symbol representing the hydroelectric energy usage quota for the remainder of the week
$J_i(t)$	fractional extent of use of the first segment of the loading curve that plant i has at time t

Jnttt	computer symbol for $J_n(ttt)$
$K_n(t)$	fractional extent of use of the second segment of the loading curve for plant n at time t
Knttt	computer symbol representing $K_n(ttt)$
$L_n(t)$	name of the row or equation which keeps track of the startup logic for plant n at time t
Lnttt	computer symbol for row $L_n(ttt)$
$M_n(t)$	name of row or equation which preserves the proper loading of plant n at time t in that it requires plant turnon before the first segment of the loading curve can be used
Mnttt	computer symbol for row $M_n(ttt)$
$N_n(t)$	name of row or equation which preserves the proper loading order of plant n at time t in that it requires the plant to use the first segment of the loading curve before taking advantage of cheaper incremental power in the second segment
Nnttt	computer symbol for row $N_n(ttt)$
NUTOT	computer symbol representing the nuclear energy usage quota for the remainder of the week
OSH	computer symbol representing the over use of the hydro energy beyond the allotted quota for the week
OSN	computer symbol representing the over use of the nuclear energy beyond the allotted quota for the week
OSPH	over supply of water storage at pumped hydro reservoir beyond the quota set for the end of the week
P_i	power levels at various points i on the loading curve of a particular facility
P(k)	power demanded by the system at interval k including the appropriate reserve requirement for the system at that time

$PA_i(k)$	capacity of the i^{th} plant in the k^{th} interval, derated to average in the effects of the forced outage rate for that particular plant
PA(t)	amount of water drawn out of a reservoir for use in power production at time t
PHTOT	computer symbol representing the pumped hydro reservoir storage quota at the end of the week
Q	the dollar quality of a particular schedule
QA	the amount which represents the total air pollution environmental impact of a particular schedule
QAX	variable which is forced to take on the value of the air pollution quality of a particular schedule
QB	the equal weighting of the dollar quality measure and the atmospheric quality measure of a particular schedule
qd	total dollar quality of a particular schedule
QD	the point which represents the minimum dollar quality schedule
QE	the point which represents the best quality where quality is measure by equally weighted aquatic and atmospheric environmental impact measures
qe_i	quality of a schedule as measured by a specifically monitored, i^{th}, environmental quality measure
QS02	the quality of a schedule as measured by the SO_2 level at a certain time
QS02X	relative extent of the consequences of SO_2 levels that have resulted from levels predicted to be caused by external, background sources
QT	the point representing the best quality where quality is measured by equal weightings of the three measures: total dollar quality of a schedule, and the atmospheric and aquaspheric quality measures of the schedule

QV	the point representing the best quality of system operation where quality is measured by the equally weighted sum of the total dollar quality of a schedule and the aquatic impact quality
QW	the point which represents the best aquatic environmental impact consequences of all possible
QWX	the variable which is forced to take on the value of the water pollution quality of a particular schedule
QX	the variable which is forced to take on the value of the dollar cost quality of a particular schedule
SO2X	input variable representing the predicted SO_2 level from background sources
SR(t)	total amount of spinning reserve available on the power system at time t
SRttt	computer symbol for row which collects the spinning reserve capabilities of each of the machines at each time ttt
T	total storage capacity of pumped hydro reservoir
$UP_i(k)$	fractional extent of operation of plant i in the k^{th} interval
USH	computer symbol representing the under use of the hydro energy below the allotted quota for the week
USN	computer symbol representing the under use of the nuclear energy below the allotted quota for the week
USPH	under supply of water stored in the pumped hydro reservoir below the quota set for the end of the week
$W_n(t)$	binary variable indicating whether the n^{th} plant has been started up at time t, variable equals 1, or has not been started up at time t, equals 0
Wnttt	computer symbol representing $W_n(ttt)$

X(t)	name of equations or rows which keep track of the pumped hydro reservoir water accounting at t
Xttt	computer symbol for row X(ttt), that is at time ttt
ΔQSO2	change in the consequences from SO_2 caused solely by power plant operation
θ_i	parameter representing the weighting given to the specific environmental quality problem qe_i in the total quality measure used for scheduling

Appendix A

The following is the program which was used for the unit commitment example shown in chapter 4.

```
/*MAIN  TIME=20.LINES=8
//JOBLIB  DD  DSNAME=SYS2.MPSX.LOAD.DISP=(SHR,PASS)
//OPTUCS01  EXEC MPSX
//MPSCOMP.SYSIN  DD  *,DCB=(RECFM=FB,LRECL=80,BLKSIZE=2000)
         PROGRAM
* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
*                                                                     *
*  THIS PROGRAM IS DESIGNED TO                                        *
*        1- SET UP THE MIXED INTEGER PROGRAM ASSOCIATED WITH THE      *
*          THIRD EVOLVING STEP IN THE SOLUTION OF THE OPTIMUM UNIT    *
*          COMMITMENT SCHEDULER - OPTUCS.                             *
*        2- SOLVE FOR THE OPTIMUM SCHEDULE IGNORING THE INTEGER       *
*          CONSTRAINT SETS                                            *
*        3- THEN OBTAIN UP TO  6 INTEGER SOLUTIONS, IF THEY EXIST,    *
*          WITH DOLLAR PLUS ENVIRONMENTAL QUALITY MEASURES OF NOT     *
*          MORE THAN THE COST OF A HAND COMPUTED SCHEDULE             *
*                                                                     *
* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
*
          INITIALZ
          MOVE(XDATA,'MODEL')
          MOVE(XPBNAME,'PB1')
          CONVERT
          SETUP('BOUND','BD')
          MOVE(XOBJ,'Q')
          MOVE(XRHS,'MA')
          OPTIMIZE
          SOLUTION
          SAVE('NAME','OPTC')
          INIMIX
          MIXSTART('MATRIX')
          XMXDROP=450000.
          CT=0
          MVADR(XDOPRINT,INT)
          MIXFLOW
STOP      MIXSAVE('NAME','TREE1')
          MIXSTATS('NODES')
          EXIT
INT       SOLUTION
          XMXDROP=450000.
```

```
         CT =CT+1
         IF(CT.EQ.6,STOP)
         CONTINUE
*
CT       DC(0)
         PEND
/*
//MPSEXEC.MATRIX2  DD  UNIT=SYSDA,SPACE=(CYL,(5))
//MPSEXEC.MIXWORK  DD  UNIT=SYSDA,SPACE=(CYL,(5))
//MPSEXEC.SYSIN DD  *,DCB=(RECFM=FB,LRECL=80,BLKSIZE=2000)
```

A brief summary of the data used to describe the system in the above program is contained below.

Minimum turn-on requirements and costs

Plant	Megawatt Minimum output	Average dollar cost, $	Average aquasphere cost	Average atmosphere cost	Turn-on cost, $
1	70	501	45	450	330
2	30	283	100	100	112
3	30	151	150	230	185
4	20	189	50	45	150
5	120	378	250	1250	402

First segment of loading curves

Plant	Megawatt output of segment	Average dollar cost, $	Average aquasphere cost	Average atmosphere cost
1	90	455	65	500
2	40	224	125	100
3	20	80	100	150
4	30	178	75	65
5	80	390	500	125

Second segment of loading curves

Plant	Megawatt output of segment	Average dollar cost, $	Average aquasphere cost	Average atmosphere cost
3	70	390	330	500
4	30	190	75	65
5	40	161	750	180

Nuclear and Hydro requirements and costs

Plant	Minimum megawatt output	Additional $ cost above quota $	Extent of mW additional loading	Startup cost
6	60	64	500	1019
7	5	41	95	184

Pumped Hydro Statistics

Plant	Pumping power used, max.	Input to storage per hour	Output from storage per hour, max.	Max. input to system	Startup cost $
8	96	80	80	64	119

Penalties for missing quotas

	Dollars	Water	Air
Overuse of nuclear energy	8.6	7.9	1.3
Underuse of nuclear energy	2.0	-7.9	-1.3
Overuse of hydro energy	7.6	1.1	0.1
Underuse of hydro energy	-1.1	-1.1	-0.1
Overstorage in pumped hydro res.	5.0	-1.1	-0.1
Understorage in pumped hydro res.	-5.5	1.1	0.1

Nuclear energy usage target quota = 51,420 megawatt hours
Hydro energy usage target quota = 7,700 megawatt hours
Pumped hydro reservoir target level = 160 megawatt hours
Total storage capacity of reservoir = 1,000 megawatt hours
Initially all plants on except plant 8
Initially 100 megawatt hours in reservoir

There are 19 pages of additional data available for this particular example. This data is in the form of the exact computer listing of the program used. This listing, with pages numbered from A1 to A19 is available upon request.

Appendix B

Contained in this appendix is a summary of the data which appeared in the original example of chapter 4 but which was replaced in the revision which resulted in the program shown in Appendix A, i.e. the revised problem.

Minimum turn-on requirements and costs

Plant	Minimum Megawatt output	Average dollar cost, $
1	70	282
2	30	157
3	30	85
4	20	100
5	120	210

Nuclear and Hydro requirements and costs

Plant	Minimum Megawatt output	Additional $ cost above quota $
6	60	32
7	5	15

There are 3 pages of additional data available for this revision. This is in the form of the exact listing of the changed data cards. This listing is available upon request and consists of pages B1 through B3 of the Optional Appendix B.

References

(1) Sporn, P., "Our environment - options on the way into the future," IEEE Spectrum, volume 8, number 5, pages 49-58, May 1971

(2) White, D. C. (principal investigator), "Dynamics of energy systems, supplement-working papers," program of research by M.I.T. School of Engineering, et al, December 1971

(3) National Academy of Engineering, Committee on Power Plant Siting, "Engineering for the resolution of the energy-environment dilemma," N.A.E., Washington, D. C., 1972

(4) Gruhl, J., "Minimizing cost and environmental impact of electric power system operation," (unpublished) Ph.D. thesis preliminary working paper, M.I.T. Department of Electrical Engineering, 45 pages, March 1972

(5) Sporn, P., Vistas in Electric Power, volume 2, Pergamon Press, New York, 1968

(6) Federal Power Commission, "Steam-electric plant construction cost and annual production expenses, 21st annual supplement - 1968," published by the Federal Power Commission, Table 8, November 1969

(7) Sporn, P., "Co-ordination of major unit overhaul," Electric World, October 31, 1942

(8) Groff, G. L. and Robinson, J. L., "Generation planning parameters by systems simulation," PICA Conference Proceedings, pages 646-652, 1967

(9) Hano, I., Tamura, Y., Narita, S., Oh, Y. and Sing, K., "Application of the maximum principle to power systems analysis - its computational aspects," 2nd PSCC Proceedings, part 2, 1966

(10) Booth, R. R., "Optimum generating planning considering uncertainty," PICA Proceedings, pages 84-90, 1971

(11) Day, J. T., "Forecasting minimum production costs with linear programming," IEEE Transactions, volume PAS-90, number 2, pages 814-832, March 1971

(12) Booth, R. R., "Power system simulation model based on probability analysis," IEEE Transactions, volume PAS-91, number 1, pages 62-69, January 1972

(13) Booth, R. R., "Optimal generation planning considering uncertainty," (same as (10)), IEEE Transactions, volume PAS-91, number 1, pages 70-77, January 1972

(14) Evans, R. A., "A quadratic programming algorithm for optimal generation planning," IEEE Winter Conference paper number C73 112-0, January 1973

(15) Rees, F. J., and Goodrich, J. M., "Stochastic production costing, security assessment and maintenance scheduling of a large system of hydro-thermal-nuclear plants," J. Automatic Control Conference, paper number 112-A4, pages 12-13, August 12, 1971

(16) Sager, M. A., and Wood, A. J., "Power system production cost calculations - sample studies recognizing forced outages," IEEE Transactions, volume PAS-92, number 1, pages 154-158, January 1973

(17) Larson, R. E., "A dynamic programming successive approximation technique," Proceedings Joint Automatic Control Conference, University of Michigan, June 1968

(18) Larson, R. E., "A survey of dynamic programming computational procedures," IEEE Transactions on on Automatic Control, pages 767-774, December 1967

(19) Gruhl, J., "Electric generation production scheduling using a quasi-optimal sequential technique," Report MIT-EL 73-003, Energy Laboratory, M.I.T., Cambridge, Massachusetts, 154 pages, April 1972

(20) Anderson, S. W., Jenkins, R. T., Joy, D., Merrill, E. T., and Wolsko, T. D., "Methods of planning generation unit operation," American Power Conference, April 21, 1970

(21) N.E.P.E.X., "New England Power Exchange: NEPEX," descriptive pamphlet distributed by NEPEX, 12 pages, West Springfield, Massachusetts, 1971

(22) Patton, A. D., and Ali, J., "Comparison of methods for generator maintenance scheduling," IEEE Conference Paper C72 452-1, Power Apparatus and Systems Power Meeting, July 1972

(23) Garver, L. L., "Adjusting maintenance schedules to levelize risk," IEEE Transactions, volume PAS-91, number 5, pages 2057-2063, October 1972

(24) Christiaanse, W. R., and Palmer, A. H., "A technique for the automated scheduling of the maintenance of generating facilities," PICA Proceedings, pages 319-327, 1971

(25) Wall, I. and Fenech, H., "The application of dynamic programming to fuel management optimization," Nuclear Science and Engineering, volume 22, pages 285-297, 1965

(26) Rees, F. J., and Goodrich, J. M., "Power system approach to optimal nuclear fuel management," final report, Systems Control, Inc., SCI Project T279, June 1970

(27) Miller, R. H., and Thompson, R. P., "Long-range scheduling of power production," IEEE PAS Winter Conference paper number C72 160-5, 1972

(28) Larson, R. E., and Keckler, W. G., "Applications of dynamic programming to the control of water resource systems," Automatica, volume 5, pages 15-26, 1969

(29) Rees, F. J., and Larson, R. E., "Application of dynamic programming to the optimal dispatching of electric power from multiple types of generation," IEEE Transactions, volume PAS-90, number 2, pages 891-899, March 1971

(30) Davidson, P. M., et al, "Unit commitment start-stop scheduling in the Pennsylvania-New Jersey- Maryland interconnection," PICA Proceedings, pages 127-132, 1967

(31) Kirchmayer, L. F., Economic Operation of Power Systems, John Wiley & Sons, Publishers, New York, 1958

(32) Moskalev, A. G., "Principles of the most economic distribution of the active and reactive loads in automatic control power systems," Elektrichestvo, volume 12, pages 24-33, 1963

(33) Dopazo, J. G., Klitin, O. A., Stagg, G. W., and Watson, M., "An optimization technique for real and reactive power allocation," Proceedings IEEE, volume 54, pages 1877-1885, 1967

(34) Lowery, P. G., "Generating unit commitment by dynamic programming," IEEE Transactions, volume PAS-85, number 5, pages 422-426, May 1966

(35) Hara, K., Kimura, M., and Honda, N., "A method for planning economic unit commitment and maintenance of thermal systems," IEEE Transactions, volume PAS-85, number 5, pages 427-436, May 1966

(36) Larson, R. E., and Korsak, A. J., "A dynamic programming successive approximations technique with convergence proofs," Automatica, volume 6, pages 245-252, 1970

(37) Kickler, W. G., and Larson, R. E., "Dynamic programming application to water reservoir system operation and planning," Journal Mathematical Analysis Applications, volume 24, pages 80-109, 1968

(38) Petersen, H., "An example of the use of dynamic programming for economic operation of a hydro-thermal system," PSCC, Sweden, Report 6.4, 1966

(39) Hano, I., Tamura, Y., and Narita, S., "An application of the maximum principle to the most economic operation of power systems," IEEE Transactions, volume PAS-85, number 5, pages 486-494, May 1966

(40) Narita, S., Oh, Y., Hano, I., and Tamura, Y., "Optimal system operation by the discrete maximum principle," PICA Proceedings, pages 189-208, 1967

(41) Belaev, L. S., "Application of Pontryagin's Maximum Principle for the optimization of the operation of complex electric power systems," Isvestia Academy, Nauk, U.S.S.R. Energetica and Transport number 5, pages 13-22, 1965

(42) Dahlin, E. B., and Shen, D. W. C., "Optimal solution to the hydro-steam dispatch problem for certain practical systems," IEEE Transactions, volume PAS-85, pages 437-458, 1966

(43) Narita, S., "An application of the maximum principle to the most economic operation of power systems," Electrical Engineering in Japan, volume 85, pages 23-33, 1964

(44) Oh, Y. N., "An application of the discrete maximum principle to the most economic power system operation," Electrical Engineering in Japan, volume 87, pages 17-28, 1967

(45) Dillon, T., and Morsztyn, K., "Optimal control of integrated power systems," Monash University Electrical Engineering Department Report MEE 69-3, 1969

(46) Dillon, T. and Morsztyn, K., "Mathematical solution of the problem of optimal control of integrated power systems with generalized maximum principle," International Journal of Control, volume 13, pages 833-851, 1971

(47) Dillon, T. S., and Morsztyn, K., "New developments in the optimal control of integrated (hydro-thermal) power systems including a comparison of different computational procedures," 4th PSCC Proceedings, paper 2.1/10, 1972

(48) Kerr, R. H., Scheidt, J. L., Fontana, A. J., Wiley, J. K., "Unit commitment," IEEE Transactions, volume PAS-85, number 5, pages 417-421, May 1966

(49) Garver, L. L., "Power generation scheduling by integer programming," AIEE Transactions, paper number 62-212, pages 730-735, February 1963

(50) Muckstadt, J. A., and Wilson, R. C., "An application of mixed-integer programming duality to scheduling thermal generating systems," IEEE Transactions, volume PAS-87, number 12, pages 1968-1978, December 1968

(51) Christiaanse, W. R., and Palmer, A. H., "A technique for the automated scheduling of the maintenance of generating facilities," (or see reference (24)), IEEE Transactions, volume PAS-91, number 1, pages 137-145, January 1972

(52) Sokkappa, B. G., "Optimum scheduling of hydrothermal systems - a generalized approach," IEEE Transaction, volume PAS-82, pages 97-104, April 1963

(53) Krumm, L. A., "Generalization of the gradient method of optimization of mode of operation of integrated power systems," Isvestia Academy Nauk, U.S.S.R. Energetica and Transport, number 3, pages 3-16, 1965

(54) Tyren, L., "Short-range optimisation of a hydro-thermal system by a gradient method combined with linear programming," paper OS3 in _Proceedings of the 3rd Power Systems Computation Conference_, Rome, 1969

(55) Baldwin, C. J., Dale, K. M., Dittrich, R. F., "A study of the economic shutdown of generation units in daily dispatch," _AIEE Transactions_, volume 78, part III, pages 1272-1284, 1959

(56) Osterle, W. H., Geiser, J., Dale, K. M., DeSalvo, C. A., "Computer selection of generating units to be operated - Part II. By dynamic programming on a large computer," _Proceedings of AIEE Power Industry Computer Applications Conference_, St. Louis, 1960

(57) System Planning and Control Section, "Generation transmission planning and economic evaluation," Electric Utility Engineering Operation, General Electric Company, Schenectady, N. Y., June 1969

(58) Happ, H. H., Johnson, R. C., and Wright, W. J., "Large scale hydro-thermal unit commitment - method and results," _IEEE Transactions_, volume PAS-90, pages 1373-1384, 1971

(59) Bonaert, A. P., El-Abiad, A. H., Koivo, A. J., "Effects of hydro dynamics on optimum scheduling of thermo-hydro power systems," _IEEE Transactions_, volume PAS-91, number 4, August 1972

(60) Bonaert, A. P., El-Abiad, A. H., Koivo, A. J., "Optimal scheduling of hydro-thermal power systems," _IEEE Transactions_, volume PAS-91, number 1, pages 263-270, February 1972

(61) Hoffman, A. G., Guberman, R. P., Coulter, J. C., et al, "Hydro-thermal optimization of the PG&E system," presented at the IEEE summer conference, paper number c72 491-9, 1972

(62) Bernard, P. J., Dopazo, J. F., and Stagg, G. W., "A method for economic scheduling of a pumped hydro and steam generating system," _IEEE Transactions_, paper 63-959, pages 23-30, January 1964

(63) Rees, F. J., and Fate, M. E., "Programs optimize hydro-thermal mix," _Electrical World_, pages 47-48, May 11, 1970

(64) same reference as (42)

(65) El-Hawary, M. E., Christensen, G. S., "Functional optimization of common-flow hydro-thermal systems," IEEE Transactions, volume PAS-91, number 5, pages 1833-1839, October 1972

(66) Aresmunander, A., "Discussion of Optimum operation of a hydro-thermal system," AIEE Transactions, volume 80, part III, pages 242-250, August 1962

(67) Guy, J. D., "Security constrained unit commitment," IEEE Transactions, volume PAS-90, pages 1385-1389, 1971

(68) Ayoub, A. K., Patton, A. D., "Optimal thermal generation unit commitment," IEEE Transactions, volume PAS-90, number 4, pages 1752-1756, 1971

(69) Jain, A. V., and Billinton, R., "Unit commitment reliability in a hydro-thermal system," presented at IEEE winter conference meeting, paper C73 096-5, January 1973

(70) Jain, A. V., and Billinton, R., "Spinning reserve allocation in a complex power system," presented at IEEE winter conference meeting, paper C73 097-3, January 1973

(71) Dillon, T., and Morsztyn, K., "Application of the sequential unconstrained minimization technique for nonlinear programming to the optimization of a thermal power system," Proceedings Third Hawaii International Conference on System Sciences, pages 949-952, 1970

(72) Dillon, T., and Morsztyn, K., "Active and reactive load scheduling in a thermal power system in the presence of autotransformers using nonlinear programming," to appear in International Engineering of Aust.

(73) Dommel, H. W. and Tinney, W. F., "Optimal power flow solutions," IEEE Transactions, volume PAS-87, pages 1866-1874, 1968

(74) El-Abiad, A. H., and Jaimes, F. J., "A method for optimal scheduling of power and voltage magnitude," IEEE Transactions, volume PAS-88, pages 413-422, 1969

(75) Peschon, J., Piercy, D. S., Tinney, W. F., Tviet, O. J., and Cuenard, M, "Optimal control of reactive power flow," IEEE Transactions, volume PAS-87, pages 40-48, 1968

(76) Sasson, A. M., "Nonlinear programming solutions for load-flow, minimum-loss and economic dispatch problems," IEEE Transactions, volume PAS-88, pages 399-409, 1969

(77) Dauphin, G., Feingold, D., and Spohn, G., "Method of optimizing the production of generating stations of a power network," PICA Conference Record, pages 133-140, 1967

(78) Shen, C. M., and Laughton, M. A., "Determination of optimum power system operating conditions under constraints," Proceedings IEE (England), volume 116, pages 225-233, 1969

(79) Heuck, K., "Optimal scheduling of thermal power stations," paper presented at PSCC conference, paper number 2.1/2, Grenoble, France, September 11-16, 1972

(80) Dodu, J. C., Martin, P., Merlin, A., Pouget, J., "An optimal formulation and solution of short-range operating problems for a power system with flow constraints," Proceedings IEEE, volume 60, number 1, pages 54-63, January 1972

(81) Martin, P., Merlin, A., "Méthode combinatoire de répartition à court terme d'un ensemble de production thermique hydraulique," Revue Gen. Elect., volume 79, number 9, pages 741-754, October 1970

(82) Merlin, A., Martin, P., "Méthode de répartition journaliére d'un ensemble de moyers de production thermique et hydraulique," Electricité de France, Bull. Etudes Recherch., Ser. B., 1969

(83) Dodu, J. C., and Merlin, A., "Méthode de résolution du dispatching économique dans l'approximation du courant continu (Modéle MAYA)," Electricité de France, Bull. Etudes Recherch., Ser. B., no. 3, 1970

(84) Boullé, D., Vincent, M., "Les dispatchings d'électricité de France - Description générale de l'ensemble des programmes de calcul utilisés par le dispatching

central d'Électricité de France," Revue Gen. Elect., volume 80, number 5, pages 383-392, May 1971

(85) Stone, D. G., "Economic scheduling by a network flow method," Control Systems Research Report, Centre for Computing and Automation, Imperial College, London, 1967

(86) Brewer, C., Charles, G. I., Parish, C. C. M., Prewett, J. N., Salthouse, D. C., and Terry, B. J., "Performance of a predictive automatic load dispatching system," Proceedings IEE (England), volume 115, pages 1577-1586, 1968

(87) same as reference (78)

(88) Benthall, T. P., "Automatic load scheduling in a multiarea power system," Proceedings IEE (England), volume 115, number 4, pages 592-596, 1968

(89) Wells, D. W., "Method for economic secure loading of a power system," Proceedings IEE (England), volume 115, number 8, pages 1190-1194, 1968

(90) Thanikachalam, A., and Tudor, J. R., "Optimal rescheduling of power for system reliability," Transactions IEEE, volume PAS-90, number 5, pages 2186-2192, 1971

(91) Aldrich, J. F., Happ, H. H., Leuer, J. F., "Multi-Area Dispatch," Transactions IEEE, volume PAS-90, number 6, pages 2661-2670, December 1971

(92) Happ, H. H., "Multi-level tearing and applications," IEEE Transactions, volume PAS-92, number 2, pages 725-733, April 1973

(93) Ferrara, E., and Galiero, R., "Application of linear programming and special load flow algorithms to optimum scheduling computations," Proceedings 3rd PSCC, Rome, paper number OM7, 1969

(94) Taylor, D. G., "A linear programming model suitable for power flow problems," paper presented at 3rd PSCC, paper number OM2, Rome, 1969

(95) Fanshel, S., and Lynes, E. S., "Economic power generation using linear programming," IEEE Transactions, volume PAS-83, pages 347-356, 1964

(96) Diperna, A., and Ferrara, E., "Optimum scheduling of a hydroelectric system by linear programming," Proceedings 2nd PSCC, Sweden, paper 6.14, 1966

(97) Fukao, T., and Yamazaki, T., "A computational method of economic operation of hydro-thermal power systems including flow interconnected hydro power plants," Electro-Tech. J. Japan, volume 6, pages 22-26, 1960

(98) Shen, C. M., and Laughton, M. A., "Power-system load scheduling with security constraints using dual linear programming," Proceedings IEE (England), volume 117, number 11, page 2120-2127, November 1970

(99) Hayward, A. P., Taylor, C. E., Kerr, R. H., and Kirchmayer, L. K., "Minimisation of fuel costs by the technique of linear programming," Transactions AIEE, volume 76, pages 1288-1295, 1957

(100) El-Hawary, M. E., and Christensen, G. S., "Application of functional analysis to optimization of electric power systems," International Journal of Control (Great Britain), volume 16, number 6, pages 1063-1072, December 1972

(101) Gent, M. R., and Lamont, J. W., "Minimum emission dispatch," IEEE Transactions, volume PAS-90, number 6, pages 2650-2660, December 1971

(102) Sullivan, R. L., "Minimum pollution dispatching," paper presented at IEEE Power Engineering Society summer meeting, paper C72 468-7, 4 pages, 1972

(103) Congressional Quarterly section on Electric Power, "Man's control of the environment," Congressional Quarterly, Washington, D. C., pages 50-53, August 1970

(104) Gruhl, J., "Economic-environmental-security transform curves of electric power system production schedules and simulations," Report MIT-EL 73-006, Energy Laboratory, M.I.T., Cambridge, Massachusetts, 90 pages, November 1972

(105) Gruhl, J., "Quantification of atmospheric environmental impact of electric power generation," Report MIT-EL 73-005, Energy Laboratory, M.I.T., Cambridge, Massachusetts, March 1973

(106) Gruhl, J., "Quantification of aquatic environmental impact of electric power generation," Report MIT-EL 73-004, Energy Laboratory, M.I.T., Cambridge, Massachusetts, 176 pages, April 1973.

(107) Galiana, F. D., "An application of system identification and state prediction to electric load modelling and forecasting," Ph.D. Thesis M.I.T. Dept. of Electrical Engineering, Electric Power Systems Engineering Laboratory Report 28, Cambridge, Massachusetts, March 1971

(108) Working Group on Power Plant Response to Load Changes, "Megawatt response of fossil fueled steam units," <u>IEEE Transactions</u>, volume PAS-92, number 2, pages 455-463, April 1973