50] Equivalence Principle $\Rightarrow$ Gravitational Redshift

with $|a| = |g|$

Emit light at bottom of elevator, with wave-crests separated by time intervals δ (so $\omega = \frac{2\pi}{\delta}$)

To reach height h, takes $\frac{h}{c}$. By that time, detector has velocity $\frac{h}{c} g$, up. Thus the next peak must travel an extra interval $\frac{h}{c} g\delta$, which takes time $\frac{h}{c} g\delta / c$. Thus

$$\delta' = \delta + \frac{hg}{c^2}\delta = \delta\left(1 + \frac{hg}{c^2}\right)$$

$$\omega' = \frac{\omega}{1 + \frac{hg}{c^2}} \approx \omega\left(1 - \frac{hg}{c^2}\right).$$

This lessening of frequency as particle "climbs out" of gravitational field is the <u>gravitational</u> <u>redshift</u>.

Can be $\sim 10\%$ for neutron stars. ($\sim .1\%$ for white dwarfs)

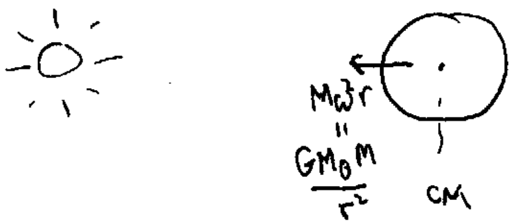
Photon interpretation. $E' = \hbar\omega' = \hbar\omega -$ work done against gravity

$$= \hbar\omega - \frac{\hbar\omega}{c^2} hg$$

↖ pseudo-mass!

Pound-Rebka exp't.

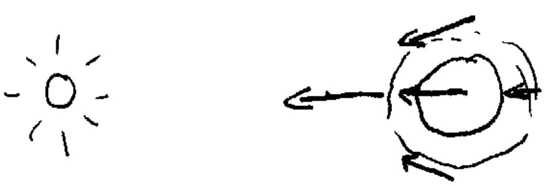
51) Tides



measure forces in frame accelerating with CM $\Rightarrow$ displacements relative to Earth - coordinates

$$\vec{A} = \overleftarrow{\omega^2 r}$$

$$-\vec{A} = \overrightarrow{\omega^2 r} = \frac{GM_\odot}{r^2}$$



gravity forces

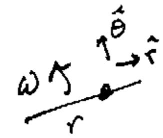
$\Rightarrow$ effective (real + fictitious) force

"quadrupole"

12-hr. period

52) Bead on Wire: Inertial vs. Rotating Descriptions

i)



old way (inertial)

$$m\ddot{\vec r} = m(\ddot r - \dot\theta^2 r)\hat r + m(r\ddot\theta + 2\dot r\dot\theta)\hat\theta$$

$$\parallel$$

$$\vec F = N\hat\theta$$

we write $\dot\theta = \omega$ and look at $\hat r, \hat\theta$ components:

$$0 = \ddot r - \omega^2 r$$

$$N = m(r\dot\omega + 2\dot r\omega)$$

new way (rotating frame)

$$m\ddot{\vec r} = \underset{(i)}{N\hat\theta} - \underset{(ii)}{m\frac{d\vec\omega}{dt}\times\vec r} - \underset{(iii)}{2m\vec\omega\times\vec v} - \underset{(iv)}{m\vec\omega\times(\vec\omega\times\vec r)}$$

Now $\vec r = r\hat r$, $\dot{\vec r} = \dot r\hat r$, $\ddot{\vec r} = \ddot r\hat r$!

and $\vec\omega = \omega\hat z$

use $\hat z\times\hat r = \hat\theta$

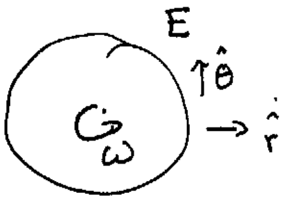
$\hat z\times\hat\theta = -\hat r$

$\hat r$ component: $m\ddot r = m\omega^2 r$ (from term (iv))

$\hat\theta$ component: $0 = \underset{(i)}{N} - \underset{(ii)}{m\dot\omega r} - \underset{(iv)}{2m\omega\dot r}$

Just the same!!

53] Deflection of Dropped Ball



Looking down on equator from N pole

$\Rightarrow \hat{\theta}$ is "East"

$$m(\ddot{r} - \dot{\theta}^2 r)\hat{r} + m(2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{\theta}$$

$$= -mg\hat{r} - 2m\underbrace{\omega\hat{z}}_{\vec{\omega}} \times (\dot{r}\hat{r} + r\dot{\theta}\hat{\theta}) - m\omega^2\hat{z}\times(\hat{z}\times\vec{r})$$

with $\hat{z}\times\hat{r} = \hat{\theta}$, $\hat{z}\times\hat{\theta} = -\hat{r}$, we get:

$\hat{r}$ component: $\quad m\ddot{r} - \dot{\theta}^2 r = -mg + 2m\omega r\dot{\theta} + m\omega^2 r$

$\hat{\theta}$ component: $\quad m(2\dot{r}\dot{\theta} + r\ddot{\theta}) = -2m\omega\dot{r}$

In the 1st equation, drop $\dot{\theta} \ll \omega$ and use $r \approx R_e$

$\Longrightarrow \quad m\ddot{r} \approx -mg + m\omega^2 R_e \equiv -mg'$; $r = R_e + h - \frac{1}{2}g' t^2$

In the 2nd equation, drop $\dot{r}\dot{\theta}$ compared to $r\ddot{\theta}$, $r \approx R_e$

$\uparrow R_e!$

$$\Rightarrow \quad R_e\ddot{\theta} \approx -2\omega\dot{r} \approx 2\omega g' t$$

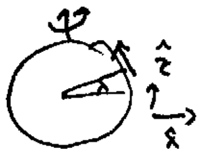
$$\theta \approx \frac{\omega g'}{3} t^3$$

For fall from height h, total $\theta \approx \frac{\omega g'}{3}\left(\frac{2h}{g}\right)^{3/2}$.

$(t = \sqrt{2h/g})$

54] Deflection of Rivers, and Weather Systems

↑ River flow N in N hemisphere



$$\vec{v} = (-v\sin\lambda, 0, v\cos\lambda)$$

$$\vec{\omega} = (0,0,\omega)$$

$$-2m\vec{\omega}\times\vec{v} = 2m\omega v\,(0, \sin\lambda, 0)$$

It is a deflection to the East.

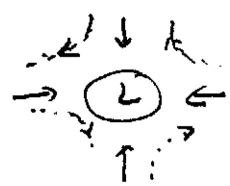
For flow S in N hemisphere, the flow is West.

There is a simple geometric interpretation.

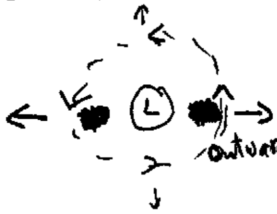
Vector with fixed direction w.r.t. Earth is getting tilted west w.r.t. inertial frame. Coriolis corrects for this.

[Similarly, for dropped ball at Equator!]

Air tries to rush into a low-pressure zone ...



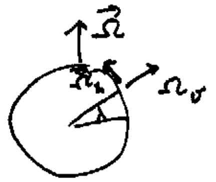
but Coriolis deflects, as shown with dotted paths. This makes the air-mass spin counterclockwise around the low.



In final configuration, Coriolis forces are outward, + support the air mass against pressure gradient.

[Actually, this is only the dominant effect at large distances, where centrifugal v^2/r gets small compared to Ωv. A "high" cyclone cannot be sustained, because both pressure and centrifugal forces blow out the core!]

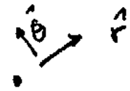
55] Foucault Pendulum



$$\vec{\Omega} = \vec{\Omega}_v + \vec{\Omega}_h$$

$|\vec{\Omega}_v| = \Omega \sin\lambda$ $\qquad$ $\vec{\Omega}_h$ only gives vertical forces from horizontal velocities

Looking down on $\vec{\Omega}_v$



with pendulum oscillating in $\hat{r}$ line,

$$r = A \sin \gamma t$$

$\frac{\text{Force}}{\text{mass}}$ in $\hat{\theta}$ direction:

$$\underbrace{r\ddot{\theta} + 2\dot{r}\dot{\theta}}_{= \frac{1}{r}(r^2\dot{\theta})^{\cdot}} = -2\dot{r}\,\Omega \sin\lambda$$

$$(r^2\dot{\theta} + r^2 \Omega \sin\lambda)^{\cdot} = 0$$

$$r^2\dot{\theta} + r^2\Omega\sin\lambda = \text{const.} \underbrace{=}_{\text{if } r \to 0} 0$$

$$\Rightarrow \dot{\theta} = -\Omega \sin\lambda$$