Special Problem 6: The Long Voyage Home

Consider a pendulum consisting of a bob of mass m connected to a massless rigid rod of length L pivoted at its endpoint, all in near-Earth gravity. There is an unstable equilibrium with the bob inverted, i.e. as high as it could be. Consider what happens if the bob is released very close to this equilibrium, say at angle ϵ from the top. Show that as ϵ gets small the time it takes for the bob to swing down and return gets very long, and find the functional form of the divergence as $\epsilon \rightarrow 0$.