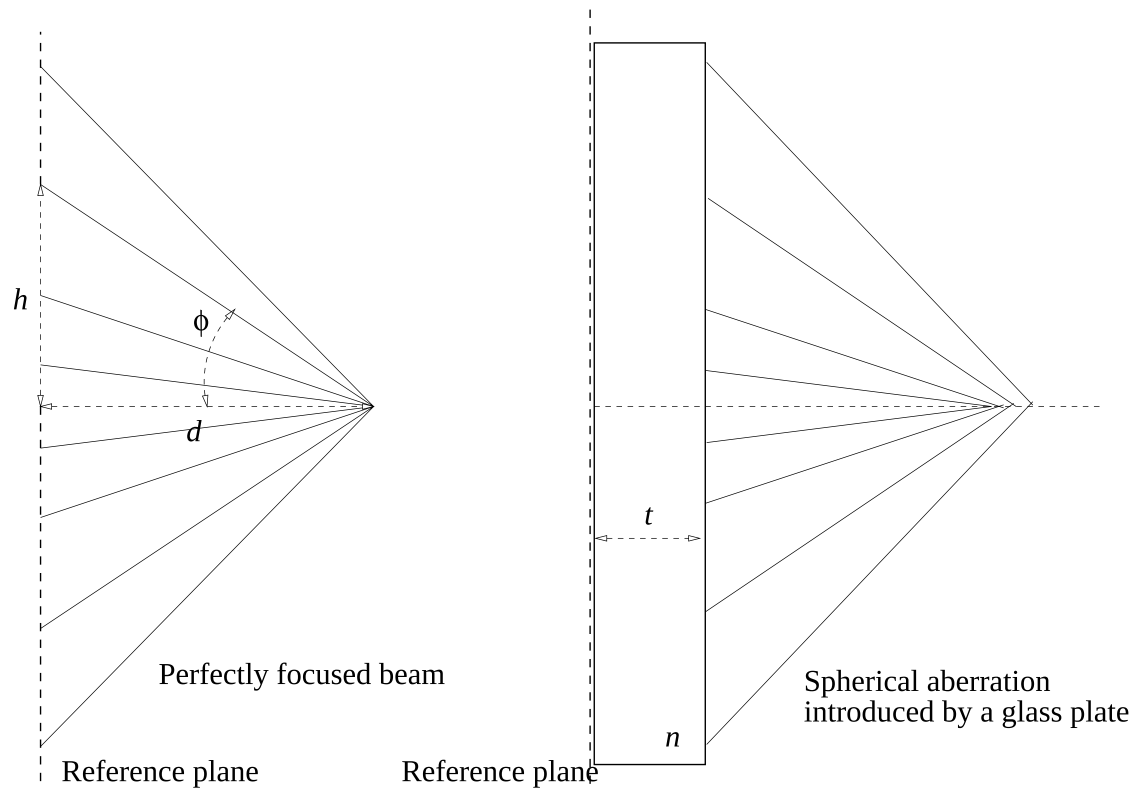
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

2.710 Optics **Fall '01**

Problem Set #5 Posted Oct. 18, 2001 — Due Wednesday Oct. 24, 2001

1. **Spherical aberration from a planar glass plate.** A beam is perfectly focused in free space so that all the rays converge towards the axis at distance d from a reference plane (see left figure below.) We insert a plate of thickness t and refractive index n in the path of the beam, just beyond the reference plane (see right figure below.) Compute the primary (Seidel) longitudinal spherical aberration introduced by the plate. Ignore reflections from the plate surfaces. [Hint: calculate the distance from the plate where the rays intersect the axis as function of the ray incidence angle ϕ and height h on the reference plane; expand to Taylor series in the small number (h/d), making sure that powers up to $(h/d)^2$ survive in the final result.]



2. Show that

$$\left|e^{i\phi_1} + e^{i\phi_2}\right|^2 = 4\cos^2\left(\frac{\phi_1 - \phi_2}{2}\right)$$

3. Plane waves and phasor representations Throughout this problem, by "complete expression" of a wave we mean the space-time representation, e.g. $f(x, y, z, t) = A\cos(kx - \omega t)$ is a plane wave of wave-vector magnitude k and frequency ω propagating in the direction of the $\hat{\boldsymbol{x}}$ coordinate axis. By "phasor expression" we mean the complex representation of the wave, e.g. Ae^{ikx} for the same wave.

3.a) Write the complete and phasor expressions for a plane wave $f_1(x, y, z, t)$ propagating at an angle $30°$ relative to the $\hat{\boldsymbol{z}}$ axis on the xz-plane (*i.e.*, the plane $y = 0$). The wavelength is $\lambda = 1\mu\text{m}$, and the wave speed is $c = 3 \times 10^8 \text{m} \cdot \text{sec}^{-1}$.

3.b) Write the complete and phasor expressions for a plane wave $f_2(x, y, z, t)$ of the same wavelength and wave speed as f_1 but propagating at angle $60°$ relative to the $\hat{\boldsymbol{z}}$ axis on the yz-plane.

3.c) Use the complete expression to plot $f_1(x, y, z = 0, t = 0)$ and $f_2(x, y, z = 0, t = 0)$ using MATLAB. (*Note:* you will probably need to use `surf` or an equivalent command.)

3.d) The plane $z = 0$ is illuminated by the superposition of the two waves f_1 and f_2. Plot the waveform received at points A, B, C, D, E with Cartesian coordinates, respectively,

$$(0,0,0), \left(\frac{1}{4}, -\frac{1}{4\sqrt{3}}, 0\right), \left(\frac{1}{2}, -\frac{1}{2\sqrt{3}}, 0\right), \left(\frac{3}{4}, -\frac{3}{4\sqrt{3}}, 0\right), \left(1, -\frac{1}{2\sqrt{3}}, 0\right)$$

(all units in microns) as function of time. What do you observe?