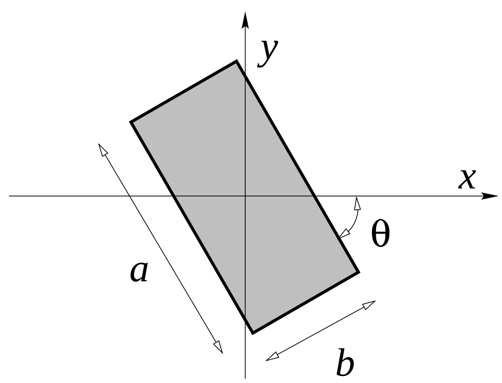
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

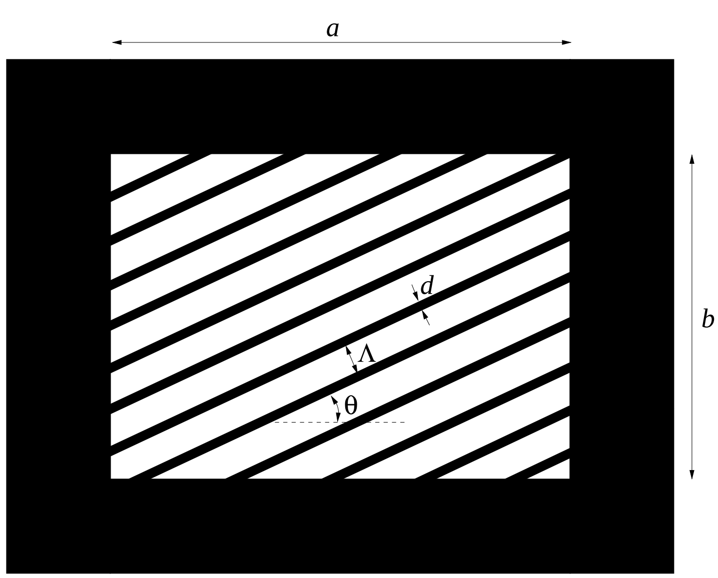
2.71/2.710 Optics **Fall '01**

Problem Set #7 Posted Oct. 31, 2001 — Due Friday Nov. 9, 2001

1. **Tilted aperture.** Caclulate analytically and sketch the Fourier transform of the tilted aperture shown below (the aperture has value one inside the tilted rectangle and zero outside). The edge lengths are $a = 10\mu$m, $b = 5$mu$m$ and the tilt is $\theta = 60°$. *Hint:* First calculate the Fourier transform of the same aperture oriented upright; then rotate the (x, y) coordinates.



2. **Tilted binary grating.** Calculate analytically and sketch the Fourier transform of the limited-aperture grating shown below (the aperture has value one at the locations shown as white and zero everywhere else). Assume spatial period $\Lambda = 10\mu$m, stripe size $d = 2\mu$m, tilt $\theta = 30°$ with respect to the aperture, and edge lengths $a = 5$mm, $b = 3$mm. *Hint:* First calculate the Fourier transforms of the titled grating and the aperture individually. Then use the convolution theorem.



3. **Image processing.** Download an image from your favorite image-intensive website (e.g., `imdb.com`), convert it to grayscale by adding the R, G, B color values at each pixel, and crop its central portion $g(x, y)$ so that it have square shape (e.g., 128×128.) Please do not use any images that might be considered offensive.

   3.a) Plot your image next to the amplitude $|G(u, v)|$ of its Fourier transform $G(u, v)$ (more details may be visible if instead you plot $\log_{10} |G(u, v)|$.)

   3.b) Select a 5×5 square region $\mathcal{S}$ around the origin of the Fourier transform domain and define the new function $G_1(u, v)$ such that

$$G_1(u, v) = \begin{cases} G(u, v) & \text{outside } \mathcal{S} \\ 0 & \text{inside } \mathcal{S} \end{cases}$$

   Plot the inverse Fourier transform $g_1(x, y)$ of $G_1(u, v)$.

   3.c) Define the new function $G_2(u, v)$ such that

$$G_2(u, v) = \begin{cases} G(u, v) & \text{inside } \mathcal{S} \\ 0 & \text{outside } \mathcal{S} \end{cases}$$

   Plot the inverse Fourier transform $g_2(x, y)$ of $G_2(u, v)$.

   3.d) Comment on the appearances of $g_1(x, y)$, $g_2(x, y)$ and how these appearances are affected by the size of the region $\mathcal{S}$.

   If you use MATLAB to solve this problem, you will find the following functions useful: (i) `fft2` computes the 2D Fourier transform of an image and returns it with some quadrants swapped, (ii) `fftshift` rearranges the quadrants of the Fourier transform in their proper order, (iii) `ifft2` computes the inverse 2D Fourier transform, (iv) `imagesc; colormap gray` displays a real grayscale image, (v) `print -dps [filename]` prints a figure into a postscript file which you can then print at any Athena printer using the `lpr` command.