16.901: Optimization Project
Design of Film Cooling for Combustor Liners
Due Date: May 10, 2pm

1 Background

The temperatures within the primary zone of a combustor are significantly higher than the temperatures which most materials can withstand without significant deterioration. Thus, a critical aspect of the design of a combustor is the development of a method to cool the liner walls of a combustor such that the temperatures which the liner temperatures are well below the limit of the material. A typical method to cool a combustor liner is through film cooling. Film cooling consists of diverting air from the main flow path prior to combustion and then re-introducing this air along the liner surface to provide a film of cooler air to protect the liner.

In this project, you will consider the optimization of a liner film cooling design. The numerical simulation of the liner cooling is based on the finite-difference method developed in Project Two for 16.901. The specific model problem is shown in Figure 1. In this project, the cooling air velocity, U_{cool}, and the height of the cooling film, h, will be the two design variables. To make certain that the design is realistic, we will require that the design variables are bounded:

$$\begin{aligned} 50 \text{ m/s} \leq \; & U_{cool} \leq 250 \text{ m/s} \\ 0.001 \text{ m} \leq \; & h \leq 0.005 \text{ m} \end{aligned}$$

The values (or range of values) of all the parameters are given in Table 1.

Parameter	Definition	Value
k_g	air conductivity	$0.1\ W/(m\,K)$
k_w	wall conductivity	$26.0\ W/(m\,K)$
h	height of cooling passage	0.001 - 0.005 m
L	axial length between cooling passages	0.3 m
U_{hot}	velocity of hot flow	100 m/sec
U_{cool}	velocity of cool flow	50 - 250 m/sec
T_{hot}	temperature of hot flow	2200 K
T_{cool}	temperature of cool flow	800 K
t_w	thickness of liner wall	0.0015 m

Table 1: Parameter definitions and values

2 Tasks

2.1 Minimization of liner temperature (20%)

An initial set of working Matlab source codes for this project are available on the 16.901 webpage. In particular, **condif.m** is the main function call which takes as its arguments the design vector and will return the maximum temperature, $T_{\max}$. Note, $T_{\max}$ is assumed to occur on the upper surface at the outlet of the computational domain. This function should be used to minimize $T_{\max}$ within the allowable range for the design parameters. To do this, complete the following steps:

1. Map the design parameters into a non-dimensional set such that they both range from -1 to $+1$ as follows:

$$x_1 = -1 + 2\frac{U_{cool} - \min U_{cool}}{\max U_{cool} - \min U_{cool}} \qquad x_2 = -1 + 2\frac{h - \min h}{\max h - \min h}$$

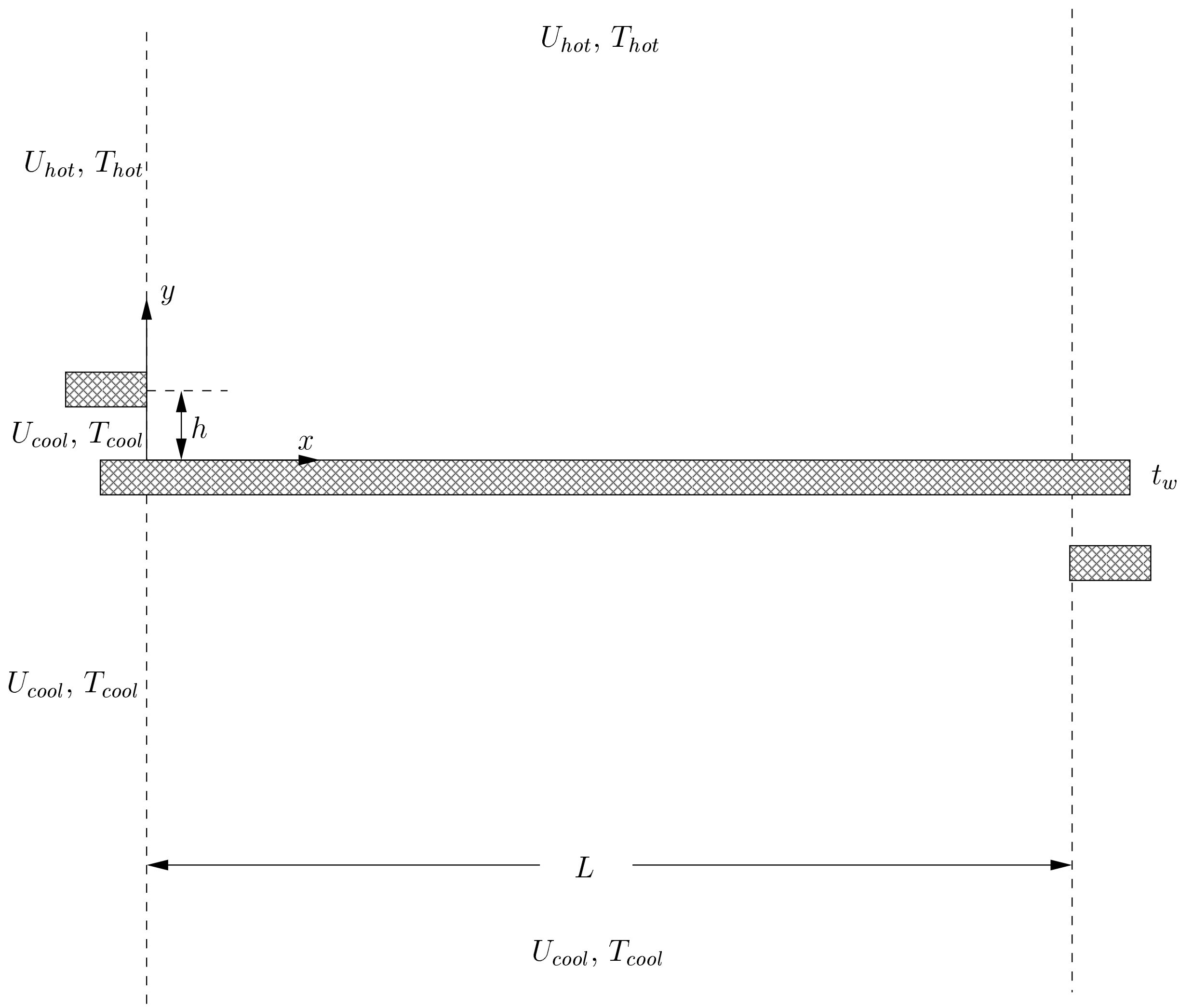

Figure 1: Combustor liner with film cooling

where $\min U_{cool}$ is the lower bound of the design range for U_{cool}, etc. This will help to condition the optimization problem since otherwise the numerical values of the design parameters would be several orders of magnitude different. Implement this mapping into **condif.m** so that the non-dimensional parameters (i.e. x_i) are passed into the function.

2. Then, use Matlab's **fmincon** optimization routine to minimize $T_{\max}$ over the design space. Let Matlab calculate the necessary derivatives through finite differencing for this task. When working properly, the optimizer should find the minimum value of $T_{\max} = 1180.4\,K$. Use **optimset** to set the 'Display' parameter to 'iter' and include in your write-up the resulting optimizaton history. Also, include what values of the design parameters gave the lowest $T_{\max}$. Note: you may want to consider setting 'TolX', 'TolCon', and 'TolFun' to terminate the optimization before the default settings for **fmincon** would. This is not essential but it may save some time later.

2.2 Minimization of cooling mass flow (20%)

In this version, we will now minimize the mass flow in the cooling film for a given maximum temperature limit. Specifically, solve the following problem:

$$\min \dot{m} = U_{cool} h \text{ such that } T_{\max} = T_{lim}.$$

Use the same design space range as given above for U_{cool} and h. For this task, set $T_{lim} = 1300\,K$. To do this, again use **fmincon** but this minimizing $\dot{m}$ while setting the $T_{\max}$ constraint using the nonlinear constraint capability of **fmincon**. To improve the conditioning of the optimization problem, set the temperature constraint in the following non-dimensional manner:

$$\frac{T_{\max}}{T_{lim}} - 1 = 0.$$

As in the previous case, use finite-difference derivatives. Include with your write-up: (a) the output of the optimization history, (b) the minimum value of $\dot{m}$ for $T_{lim} = 1300\,K$, and (c) the design parameters for this optimum solution.

2.3 Minimization of cooling mass flow with analytic derivatives (40%)

Next, calculate all of the derivatives of the objective function (i.e. $\dot{m}$) and the constraint equation with respect to the design variables. The derivatives of $\dot{m}$ are easy, but the derivatives of the constraint equation are difficult. In either case, you will need to modify your previous code to calculate and pass all derivatives back to the optimizer (warning: do not forget to turn on the gradients using optimset). Include the following in your write-up:

1. A thorough discussion of how to calculate the derivatives the constraint equation.
2. A hard-copy of the source code which you modified or added to original Matlab source. Note: this is the only version of the source code that you need to turn in with your assignment.
3. For the constraint $T_{lim} = 1300\,K$, include (a) the output of the optimization history, (b) the minimum value of $\dot{m}$, and (c) the design parameters for this optimum solution.
4. A concise discussion of the relative computational costs (i.e. run-time) between the finite-difference derivatives results from Section 2.2 and the analytic derivatives of this section.

2.4 Parametric study of cooling mass flow minimization (20%)

Using the constrained optimization with analytic derivatives developed in the previous section, perform a trade study by varying T_{lim} from $1200\,K$ to $1400\,K$. Perform as many optimizations as needed to demonstrate the trends and behavior of the film cooling without overkill. Specifically, perform the following:

1. Plot the variation of $\min \dot{m}$ versus T_{lim}.
2. Plot the location of the designs in the two-dimensional design space. Discuss the results including a discussion of which constraints are active for the different T_{lim} and explain the behavior observed as best possible.