16.901: Probabilistic Methods Project
Heat Transfer in a Turbine Blade
Due Date: May 9, 2pm

1 Background

In the first stages of a turbine, the blades are subjected to a high temperature flow due to the hot gas produced in the combustor. As a result, turbine blades are often internally cooled by pumping low temperature air through the blades in passages. In this project, we will quantify the impact of uncertainty on the temperature in the turbine blade. We will utilize the finite element heat transfer analysis developed from Project #3. The blade and the problem we are considering is actually representative of a large turbine used in power generation.

The heat transfer equation is a diffusion equation for the temperature,

$$\nabla^2 T = 0,$$

Recall that the heat transfer rate is given by,

$$\vec{q} = -k\nabla T.$$

where k is the thermal conductivity of the blade material. The boundary conditions will all be convective heat transfer conditions. For the blade surface, the heat flux out of the blade will be given by,

$$\vec{q} \cdot \vec{n} = h_{ext}\left(T - T_{ext}\right),$$

where h_{ext} is the convective heat transfer coefficient and T_{ext} is the temperature outside the blade. Note, $\vec{n}$ is a normal pointing out of the blade so that $\vec{q} \cdot \vec{n}$ represents the heat flux out of the blade. Similarly for the heat flux at a cooling passage,

$$\vec{q} \cdot \vec{n} = h_{int}\left(T - T_{int}\right).$$

Again, $\vec{n}$ is out of the blade (into the cooling passage) so that $\vec{q} \cdot \vec{n}$ represents the heat flux out of the blade.

For this probabilistic analysis, we will only use the coarse grid (**g0012coarse.mat**) as the temperature on this mesh was only a few degrees from the results using the finest grid. Note, the dimensions of the blade have been non-dimensionalized by the chord length, L. So, the coordinate values are actually x/L and y/L. Though, in this project, you will need to know the actual chord, specifically,

$$L = 0.1\,m.$$

The remaining input parameters all have uncertainty. As this probabilistic analysis is being performed early in the design of the turbine, only limited information is available, so we will use triangular probability density functions. For each of the parameters, Table 1 gives the minimum and maximum values. We will assume the most-likely point for all of these parameters is at the midpoint between the minimum and maximum values.

2 Tasks

2.1 CDF's and percentiles for symmetric triangular distributions

In this task, we will derive the CDF's and the percentiles (i.e. the inverse of the CDF's) required to perform a Monte Carlo simulation for the project. The input distributions are all symmetric triangular distributions where the minimum (i.e. $x_{\min}$) and maximum (i.e. $x_{\max}$) values are known for the random input variable $\mathbf{x}$. Specifically:

1. Determine the $CDF(x)$ for symmetric triangular distributions.
2. Detemine the u-percentiles, x_u, for the symmetric triangular distribution. Recall, the u-percentiles are defined as $u = CDF(x_u)$.

Parameter	Minimum	Maximum
$T_{ext}\,(C)$	1200	1400
$h_{ext}\,(W/m^2C)$	2000	4000
$T_{int}\,(C)$	150	250
$h_{int}\,(W/m^2C)$	500	1500
$k\,(W/mC)$	20.0	23.0

Table 1: Input uncertainty specification. Most-likely value assumed to be at the midpoint between the minimum and maximum values.

2.2 Nominal Simulation

Using the heat transfer Matlab® source code distributed on-line (or use your own if you wish), calculate and plot the temperature in the blade at the nominal (i.e. the most-likely) parameter values. Determine the minimum and maximum temperature in the blade at nominal conditions.

2.3 Monte Carlo Simulation

Develop a probabilistic method using a Monte Carlo simulation including all of the random input as independent triangular-distributed random variables. The code needs to have the following capabilities:

1. Calculate and plot the distribution of the mean temperature, $\mu_T(x, y)$.
2. Calculate and plot the distribution of the standard deviation of the temperature, $\sigma_T(x, y)$.
3. For every iteration (i.e. trial) of the Monte Carlo simulation, calculate the minimum (i.e. $T^n_{\min}$) and maximum temperature (i.e. $T^n_{\max}$) in the **entire** blade where n is the current iteration number.
4. Plot histograms (use the Matlab® function **hist** with 20 bins) of $T_{\min}$ and $T_{\max}$.
5. Calculate and plot the history versus iteration for the mean and standard deviation of $T_{\min}$ and $T_{\max}$.

As documentation for this task, turn in a copy of all Matlab® code you have written or modified. Also, run a Monte Carlo simulation with 1000 trials and plot all of the required results. How do the mean values of $T_{\min}$ and $T_{\max}$ compare to the nominal values predicted above?

2.4 Distribution of $T_{\min}$ and $T_{\max}$

You should find that the distributions (i.e. histograms) of $T_{\min}$ and $T_{\max}$ have different shapes. Explain why this is happening (use concrete results/data to demonstrate your argument).

2.5 Error Estimates

Estimate the error in the predicted values of mean and variance for both $T_{\min}$ and $T_{\max}$ based on the 1000 trial simulation. Estimate many iterations would be required so that the mean temperatures would be predicted to within $1°C$ at 95% confidence?