

# 12.215 Modern Navigation

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# Review of last class

- Fundamentals of GPS
- Method of encoding GPS signals (bi-phase, quadrature modulation)
- Fundamentals of correlation methods used (Matlab code)
- Specifics of the GPS system
  - Frequencies
  - Chip rates
  - Data rates and message content
- Homework number 4 is due **Monday November 18.**

# Today's Class

- GPS measurements
  - Tracking methods used in GPS (“codeless” tracking)
  - Basic geometry of orbits (discuss more later)
  - Specific details of the GPS signal structure
  - Phase measurements (allow millimeter level position with GPS and cm in realtime)
  - Examine some GPS data.

# “Codeless” tracking

- For security reasons (call anti-spoofing which stops GPS military receivers being faked out by false transmissions), the PRN code on the second GPS frequency (L2) is not fully known.
- On the L1 GPS signal both C/A and P(Y)-codes are written, but at L2 on P(Y) code is written. How to track L2 since without code, signal will average to zero?
- Three basic methods:
  - Codeless tracking: First method. Square the L2 signal. This removes the  $\pi$  phase changes but halves the wavelength. Used until the early 1990's. Not used anymore. Requires Doppler shift to separate satellites and can have problems when Doppler shift is the same for two satellites

# “Codeless” tracking

- Second method (still in use):
  - Cross-correlation of L1 (C/A removed) with L2. Since the C/A is known, it can be digitally removed from L1 signal leaving just the P(Y) code plus noise behind. Since the P(Y) code also is present in the L2 signal, cross correlation allows the time and phase differences to be determined. For L2 tracking, to correct ionospheric delay (discussed in future classes), this difference is all that is needed.
  - Since GPS signal is below noise level (spread spectrum), this is noisy method (but not as bad as squaring since L1 signal is stronger than L2)
  - Averaging times to get signal can be as long as 10 seconds (compared to 10-100 ms for L1)

# “Reduced” code tracking

- The third class of methods: Reduced code tracking:
  - Several patented (actually codeless methods are patented) are used here and go under names like Z-tracking (Ashtech).
  - These all assume the P-code is known and that that the Y-code is a slowly modulated version of P-code (~20 KHz rates)
  - Basically pieces of the L2 signal are correlated using the P-code, but at the rate of Y-code additional modulation (called W-code), incoherent averages are formed because the code may have changed sign.
  - Because of low W-code rate, these tracking loops can be quite sensitive (averaging intervals of 100 ms)
  - Latest generation of receivers use these methods.

# Data message

- GPS satellites need to transmit information about their location and clock errors. This done in a messages made of up “frames” and transmitted at 50 bits/second.
- The 5 frames take 6 seconds each to broadcast so 30 seconds to get complete message (broadcast ephemeris for satellite plus almanac for one other satellite)
- Complete almanac takes 12.5 minutes to receive.
- Once one satellite is locked, the almanac is used to search for other satellites

# Receiver start up

- The length of time a GPS receiver takes before it outputs good data and positions depends on starting condition
  - Hot start: Only recently turned off and back on again. Since all the satellite ephemerides are good and do not need to be re-read, receiver clock and position known, start can be a few seconds.
  - Warm start: Turned for a day or so. Clock probably OK, if position still OK (receiver can tell be how long it takes to acquire satellites), generally start-up takes a few minutes
  - Cold start: Turned off for some time: Can take tens of minutes to half an hour to start. May go into auto-locate mode where it assumes nothing about position



# Receiver start up

- General procedure is to:
  - Acquire one satellite to get time and almanacs
  - Acquire two more satellites to get 2-D position (assumes height is last value or value given by receiver)
  - Acquire 4th satellite to get 3-D position (talk about how this is done in later classes)
  - Acquire any other satellites that should be visible
- Most receivers use a type Kalman filter and so position and velocity states are estimated

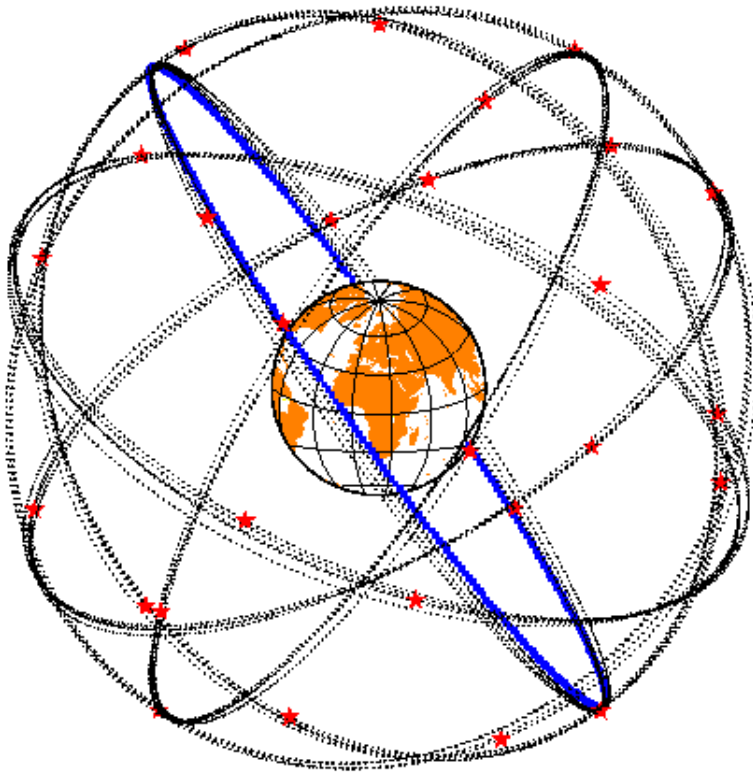
# Some guidelines for Receiver Use

- If you are going to use a GPS receiver in poor viewing conditions (e.g. downtown area or heavy foliage), best to lock on satellites before entering area – tracking will be much more complete in bad conditions
- Some receivers make selection of satellites to get first position based on first seen ones. If these satellites become obstructed before full positioning is started, they will continue to wait for these satellites and not operate until they are reacquired. In these cases, best to turn receiver off and start again.

# Basics on GPS Orbits

- Orbit characteristics are
  - Semimajor axis 26400 km (12 sidereal hour period)
  - Inclination 55.5 degrees
  - Eccentricity near 0 (largest 0.02)
  - 6 orbital planes with 4-5 satellites per plan
- Design lifetime is 6 years, average lifetime 10 years
- Generations: Block II/IIA 9729 kg, Block IIR 11000 kg

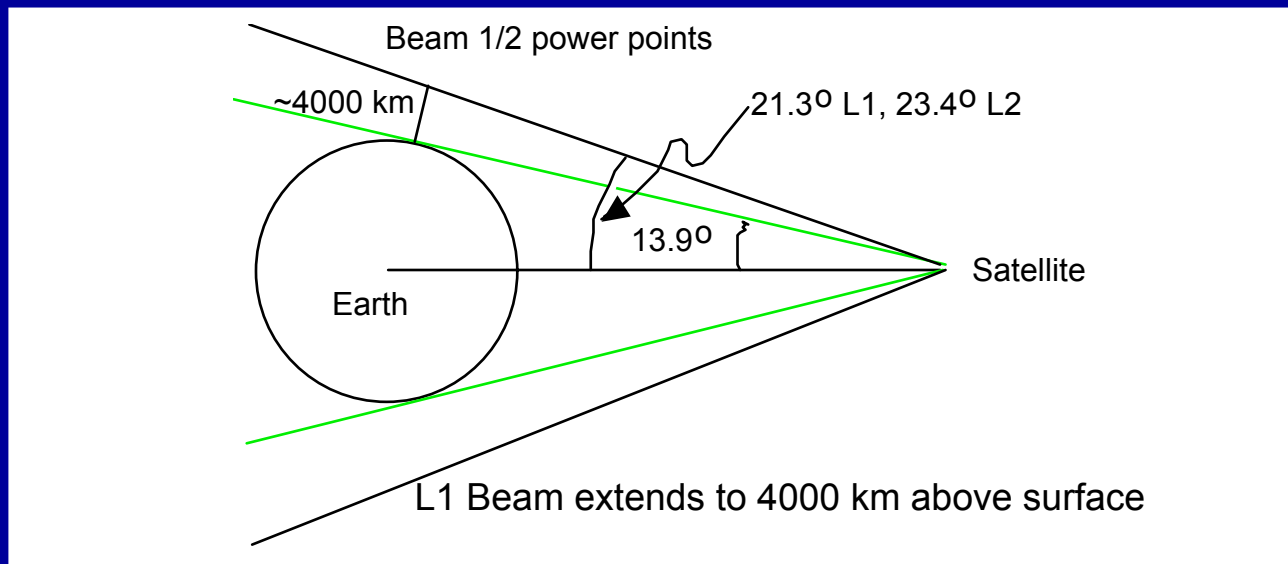
# Basic Constellation



Orbits shown in inertial space and size relative to Earth is correct

# Satellite Coverage

- The GPS satellites point their transmission antenna to the center of the Earth.
- Main beam at L1 is  $21.3^\circ$  half width, at L2  $23.4^\circ$  (due to lower frequency)



# Specifics of GPS signal

- GPS transmits at two frequencies
  - L1 1575.42 MHz ( $2 \times 77 \times 10.23$  MHz)
  - L2 1227.60 MHz ( $2 \times 60 \times 10.23$  MHz)
  - Wavelengths L1 ~190 mm; L2 ~244 mm
- Codes:
  - Course acquisition code (C/A) Chip rate (rate at which phase might change) 1.023 MHz
  - Precise positioning code (P code) 10.23 MHz
  - Y-code (Antispoofing code) also 10.23 MHz derived by multiplying P-code by ~20KHz code (highly classified)

# Specifics of GPS

- Code lengths:
  - C/A code is 1023 bits long
  - P-code is 37 weeks long ( $2 \times 10^{14}$  bits in code)
  - Only one P-code, satellites use different weeks from same code (P-code repeats each week)
  - As far as we know Y-code never repeats (again classified)
- Data message: Implemented by changing sign of code at rate of 50 bits/second (low data rate)

# Specifics of GPS

- 10.23 MHz is fundamental frequency in GPS
- All radiofrequencies and codes generated from the same 10.23MHz crystal whose long term stability is controlled by Cesium or Rubidium clock (older satellites)
- The following graphics show schematically the construction of the GPS signal



# Basic measurement types

- Pseudorange:

$$P_k^p = (t_k - t^p) \cdot c$$

Where  $P_k^p$  is the pseudorange between receiver  $k$  and satellite  $p$ ;  $t_k$  is the receiver clock time,  $t^p$  is the satellite transmit time; and  $c$  is the speed of light

This expression can be related to the true range by introducing corrections to the clock times

$$t_k = \tau_k + \Delta t_k \quad t^p = \tau^p + \Delta t^p$$

$\tau_k$  and  $\tau^p$  are true times;  $\Delta t_k$  and  $\Delta t^p$  are clock corrections

# Basic measurement types

- Substituting into the equation of the pseudorange yields

$$P_k^p = [(\tau_k - \tau^p) + (\Delta t_k - \Delta t^p)] \cdot c$$

$$P_k^p = \rho_k^p + (\Delta t_k - \Delta t^p) \cdot c + \underbrace{I_k^p}_{\text{Ionospheric delay}} + \underbrace{A_k^p}_{\text{Atmospheric delay}}$$

- $\rho_k^p$  is true range, and the ionospheric and atmospheric terms are introduced because the propagation velocity is not  $c$ .

# Basic measurement types

- The equation for the pseudorange uses the true range and corrections applied for propagation delays because the propagation velocity is not the in-vacuum value,  $c$ ,  $2.99792458 \times 10^8$  m/s
- To convert times to distance  $c$  is used and then corrections applied for the actual velocity not equaling  $c$ . (Discussed in later lectures)
- The true range is related to the positions of the ground receiver and satellite.
- We also need to account for noise in measurements

# Pseudorange noise

- Pseudorange noise (random and not so random errors in measurements) contributions:
  - **Correlation function width:** The width of the correlation is inversely proportional to the bandwidth of the signal. Therefore the 1MHz bandwidth of C/A produces a peak 1  $\mu$ sec wide (300m) compared to the P(Y) code 10MHz bandwidth which produces 0.1  $\mu$ sec peak (30 m)  
Rough rule is that peak of correlation function can be determined to 1% of width (with care). Therefore 3 m for C/A code and 0.3 m for P(Y) code.

# Pseudorange noise

- More noise sources
  - **Thermal noise:** Effects of other random radio noise in the GPS bands  
Black body radiation:  $I=2kT/\lambda^2$  where  $I$  is the specific intensity in, for example, watts/(m<sup>2</sup>Hz ster),  $k$  is Boltzman's constant,  $1.380 \times 10^{-23}$  watts/Hz/K and  $\lambda$  is wavelength.  
Depends on area of antenna, area of sky seen (ster=steradians), temperature  $T$  (Kelvin) and frequency. Since P(Y) code has narrower bandwidth, tracking it in theory has 10 times less thermal noise power (cut by factor of 2 because less transmission power)  
Thermal noise is general smallest effect
  - **Multipath:** Reflected signals (discussed later)

# Pseudorange noise

- The main noise sources are related to reflected signals and tracking approximations.
- High quality receiver: noise about 10 cm
- Low cost receiver (\$200): noise is a few meters (depends on surroundings and antenna)
- In general: C/A code pseudoranges are of similar quality to P(Y) code ranges. C/A can use narrowband tracking which reduces amount of thermal noise
- Precise positioning (P-) code is not really the case.

# Phase measurements

- Carrier phase measurements are similar to pseudorange in that they are the difference in phase between the transmitting and receiving oscillators. Integration of the oscillator frequency gives the clock time.
- Basic notion in carrier phase:  $\phi = f\Delta t$  where  $\phi$  is phase and  $f$  is frequency
- “Big” problem is know the number of cycles in the phase measurements

# Phase measurements

$$\phi_k^p(t_r) = \phi_k(t_r) - \phi_r^p(t_r) + N_k^p(1)$$

- The carrier phase is the difference between phase of receiver oscillator and signal received plus the number of cycles at the initial start of tracking
- The received phase is related to the transmitted phase and propagation time by

$$\phi_r^p(t_r) = \phi_t^p(t_t) = \phi_t^p(t_r - \rho_k^p / c) = \phi_t^p(t_r) - \dot{\phi}^p(t_r) \cdot \rho_k^p / c$$



# Phase measurements

- The rate of change of phase is frequency. Notice that the phase difference changes as  $\rho/c$  changes. If clocks perfect and nothing moving then would be constant.
- Subtle effects in phase equation
  - Phase received at time  $t$  = phase transmitted at  $t-\tau$  (riding the wave)
  - Transmitter phase referred to ground time (used later). Also possible to formulate as transmit time.

# Phase measurements

- When phase is used it is converted to distance using the standard L1 and L2 frequencies and vacuum speed of light.
- Clock terms are introduced to account for difference between true frequencies and nominal frequencies. As with range ionospheric and atmospheric delays account for propagation velocity

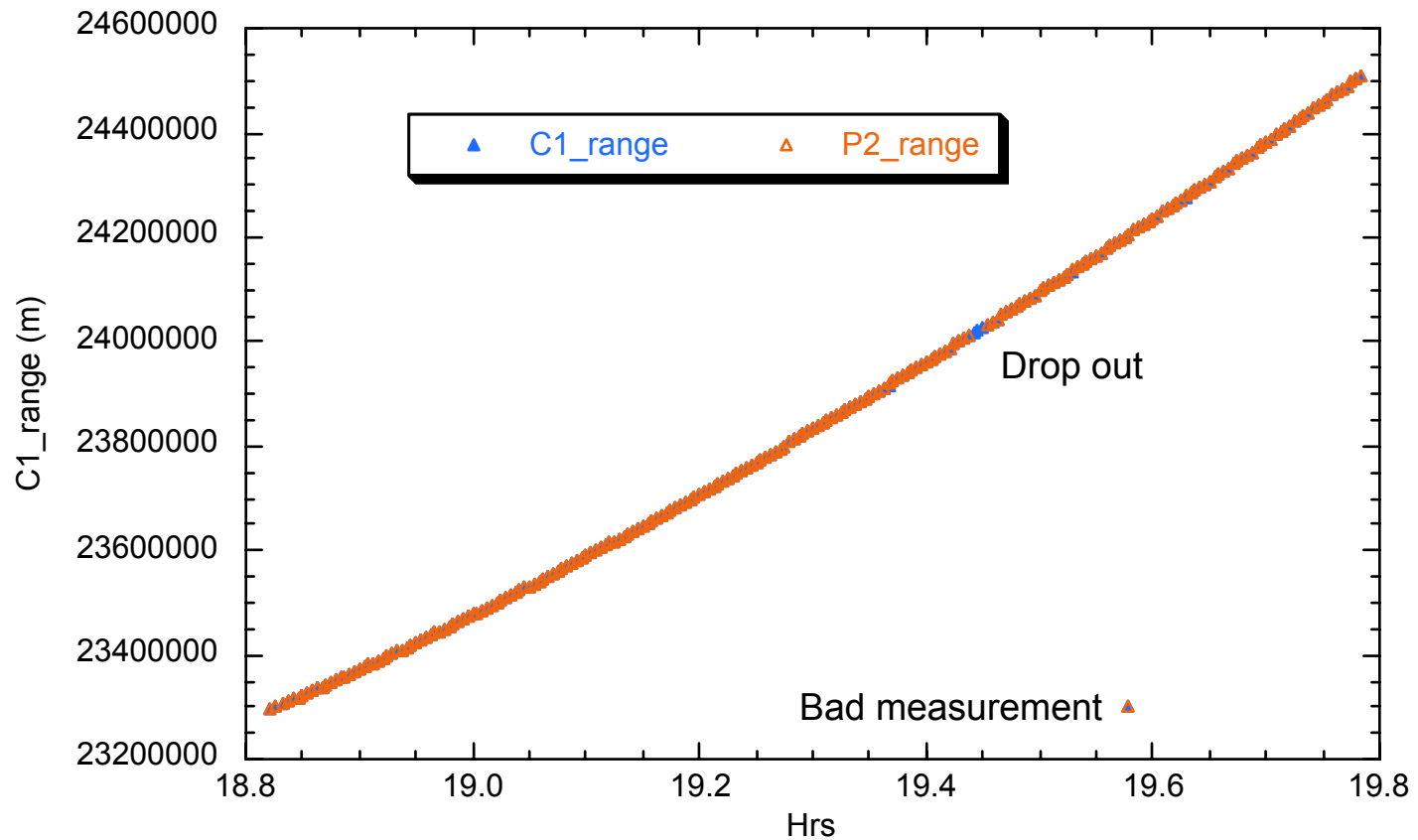
# Precision of phase measurements

- Nominally phase can be measured to 1% of wavelength ( $\sim 2\text{mm}$  L1 and  $\sim 2.4\text{ mm}$  L2)
- Again effected by multipath, ionospheric delays ( $\sim 30\text{m}$ ), atmospheric delays (3-30m).
- Since phase is more precise than range, more effects need to be carefully accounted for with phase.
- Precise and consistent definition of time of events is one the most critical areas
- In general, phase can be treated like range measurement with unknown offset due to cycles and offsets of oscillator phases.

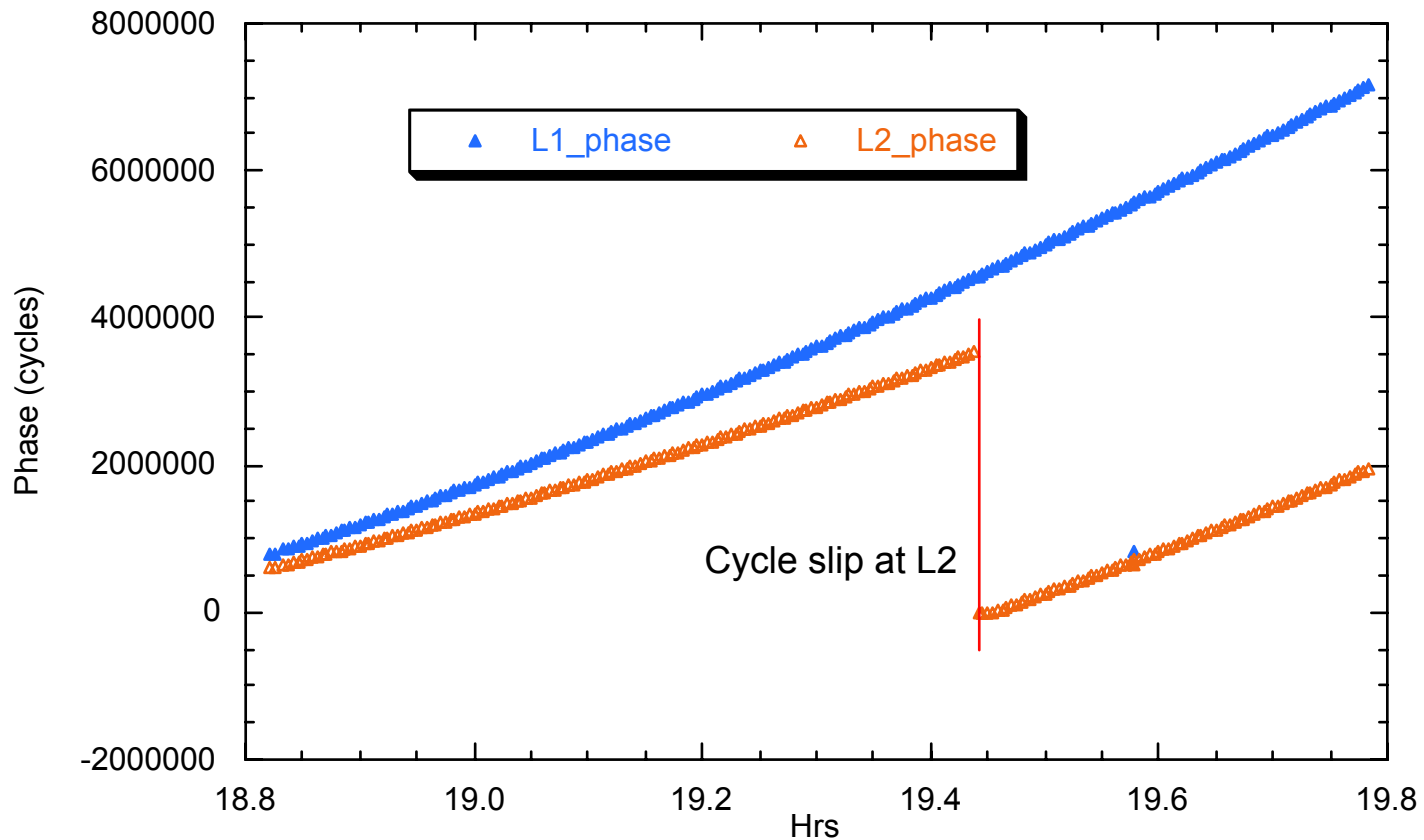
# GPS data

- Next set of plots will look at the GPS data
- Examples for one satellite over about 1 hour interval:
  - Raw range data
  - Raw phase data
  - Differences between data

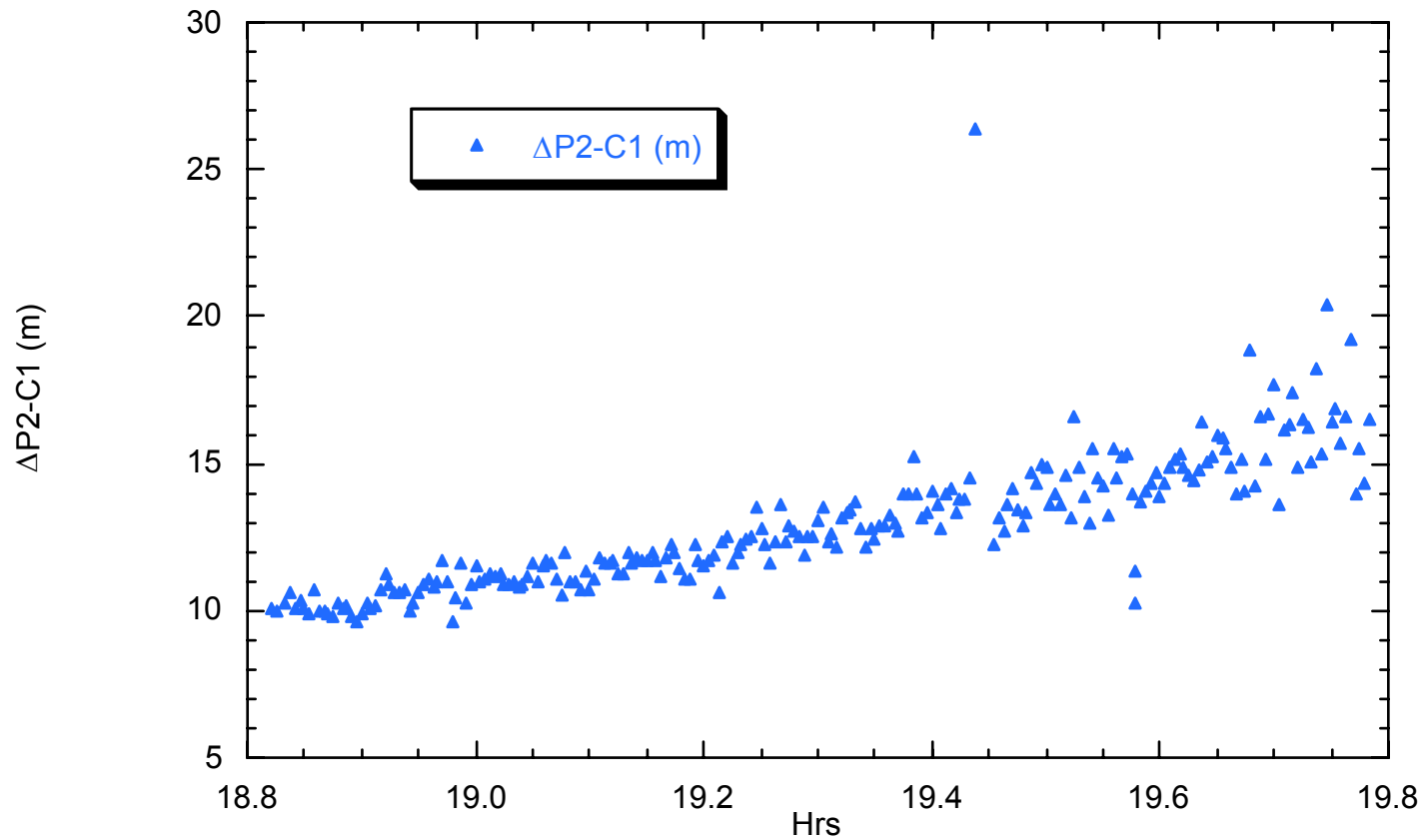
# Raw range data



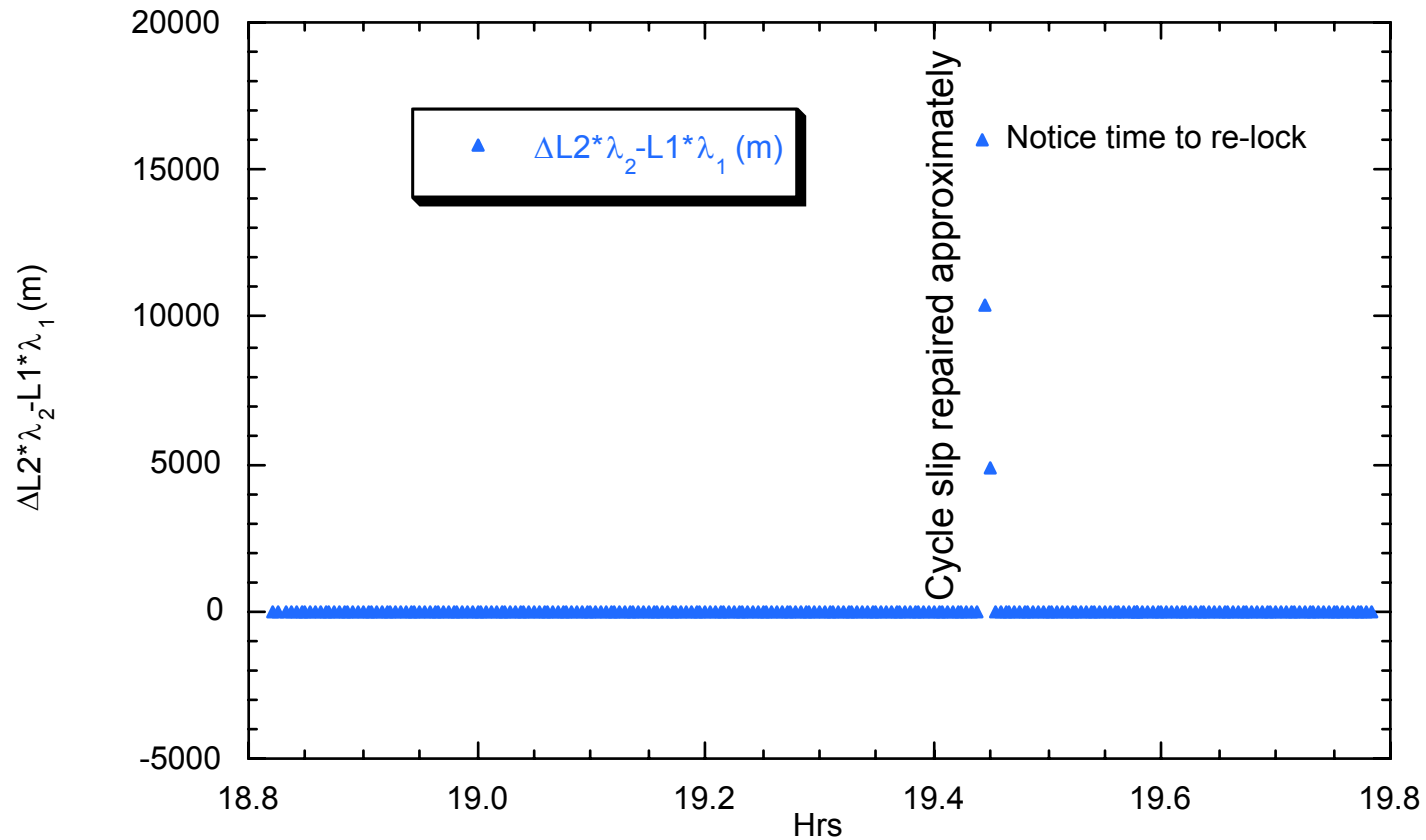
# Raw phase data (Note: sign)



# L2-L1 range differences

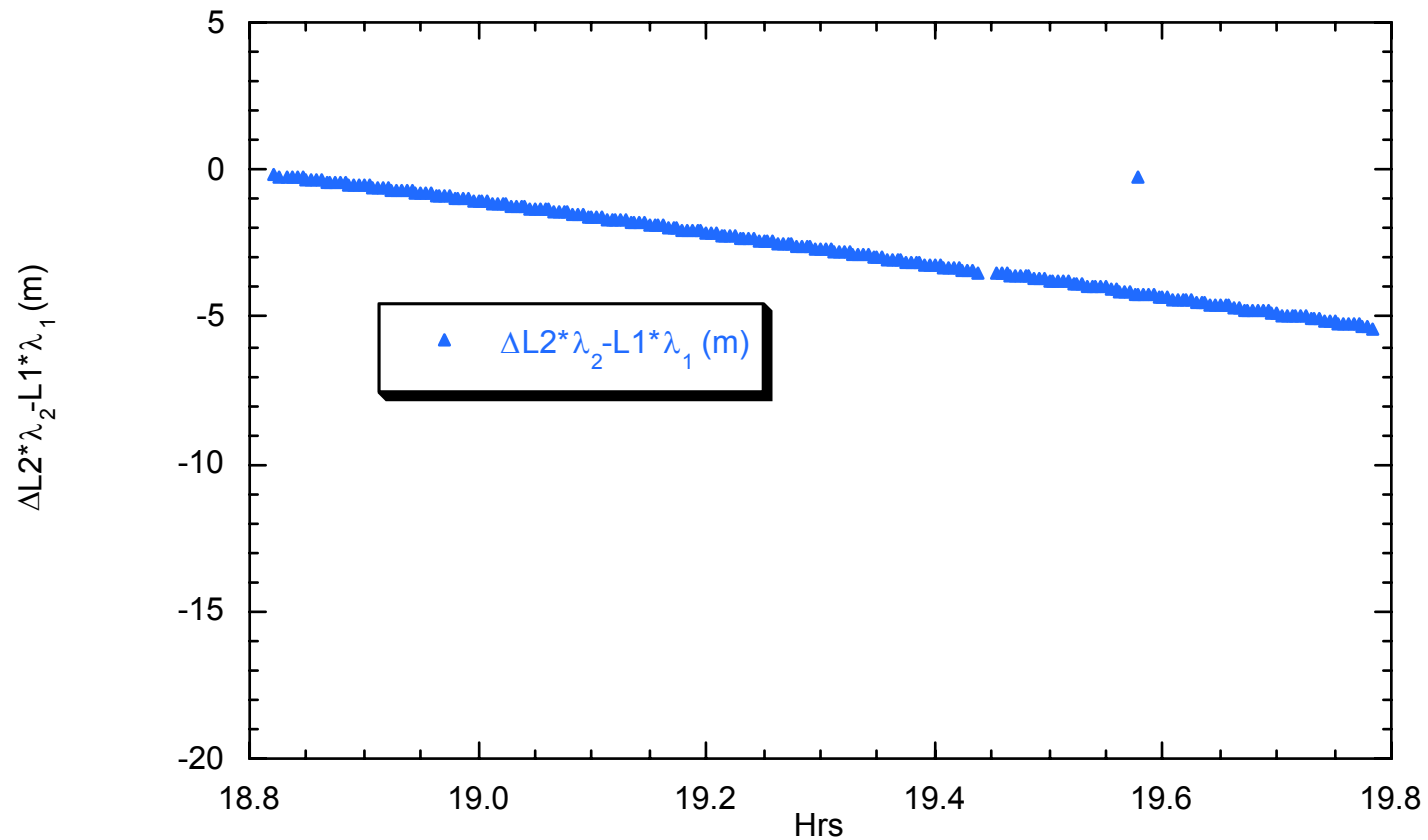


# L2-L1 phase differences





# Zoomed L2-L1 phase



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