

# 12.215 Modern Navigation

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# Summary of last class

- Ionospheric delay effects in GPS
  - Look at theoretical development from Maxwell's equations
  - Refractive index of a low-density plasma such as the Earth's ionosphere.
  - Most important part of today's class: Dual frequency ionospheric delay correction formula using measurements at two different frequencies
- Effects of ionospheric delay are large on GPS (10's of meters in point positioning); 1-10ppm for differential positioning
- Largely eliminated with a dual frequency correction (most important thing to remember from this class) at the expense of additional noise (and multipath)
- Residual errors due to neglected terms are small but can reach a few centimeters when ionospheric delay is large.

# Satellite Orbits

- Treat the basic description and dynamics of satellite orbits
- Major perturbations on GPS satellite orbits
- Sources of orbit information:
  - SP3 format from the International GPS service
  - Broadcast ephemeris message
- Accuracy of orbits and health of satellites

# Dynamics of satellite orbits

- Basic dynamics is described by  $F=Ma$  where the force,  $F$ , is composed of gravitational forces, radiation pressure (drag is negligible for GPS), and thruster firings (not directly modeled).
- Basic orbit behavior is given by

$$\ddot{\mathbf{W}} = -\frac{GM_e}{r^3} \mathbf{r}$$

# Simple dynamics

- $GM_e = \mu = 3986006 \times 10^8 \text{ m}^3\text{s}^{-2}$
- The analytical solution to the central force model is a Keplerian orbit. For GPS these are elliptical orbits.
- Mean motion,  $n$ , in terms of period  $P$  is given by

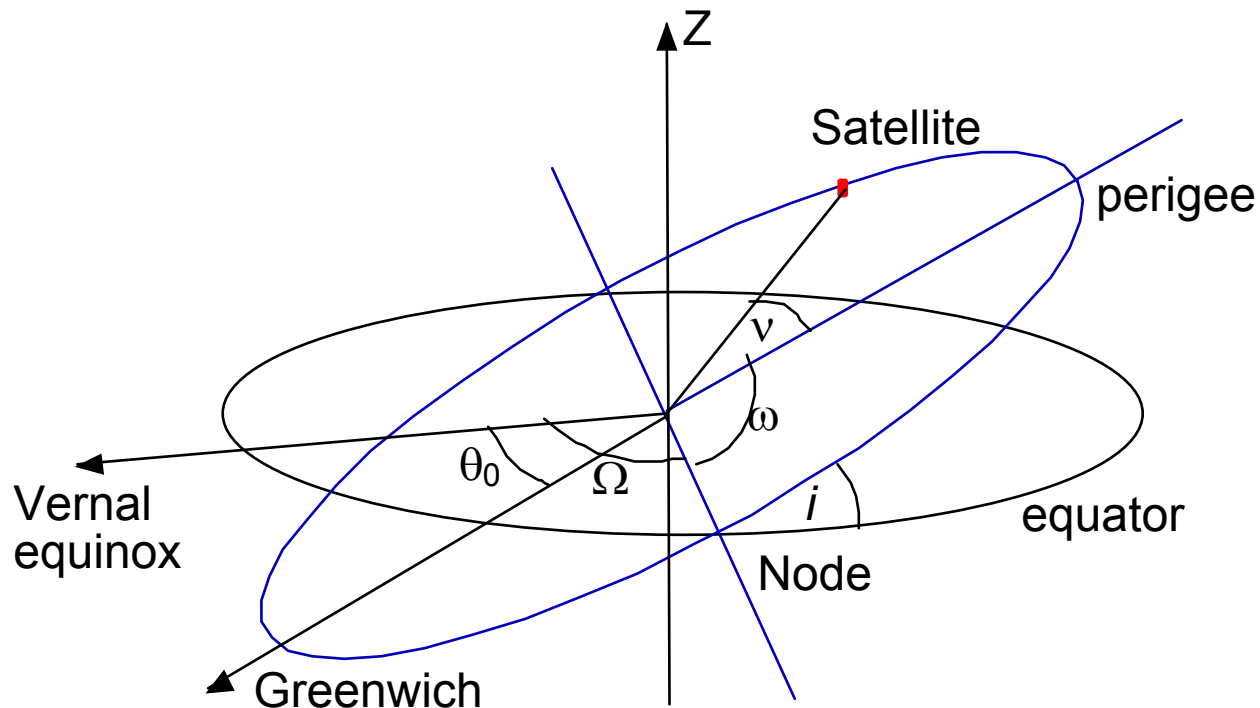
$$n = \frac{2\pi}{P} = \sqrt{\frac{\mu}{a^3}}$$

- For GPS semimajor axis  $a \sim 26400\text{km}$

# Solution for central force model

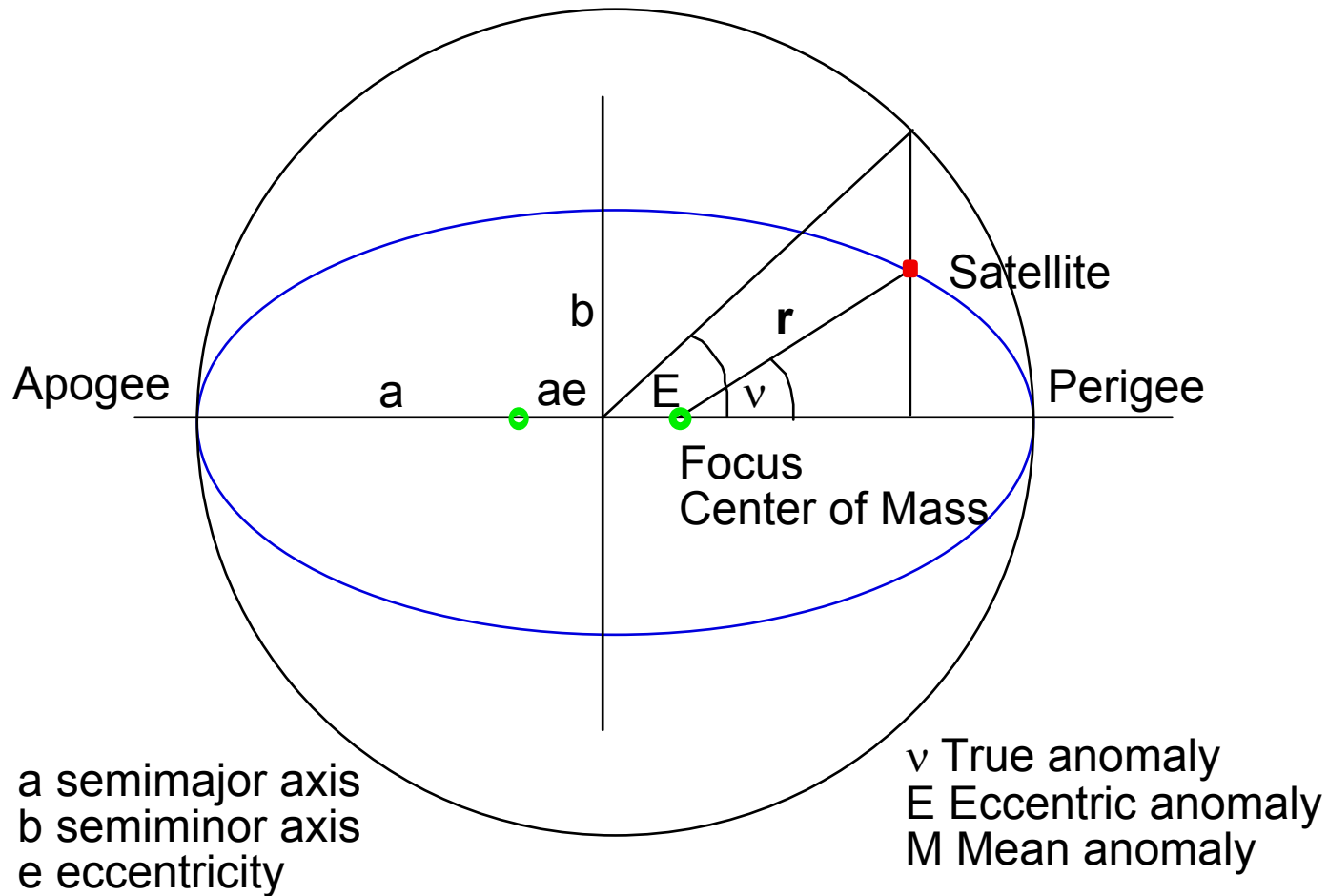
- This class of force model generates orbits that are conic sections. We will deal only with closed elliptical orbits.
- The orbit plane stays fixed in space
- One of the foci of the ellipse is the center of mass of the body
- These orbits are described Keplerian elements

# Keplerian elements: Orbit plane



- $i$  Inclination
- $\Omega$  Right Ascension of ascending node
- $\omega$  Argument of perigee
- $v$  True anomaly

# Keplerian elements in plane





# Satellite motion

- The motion of the satellite in its orbit is given by

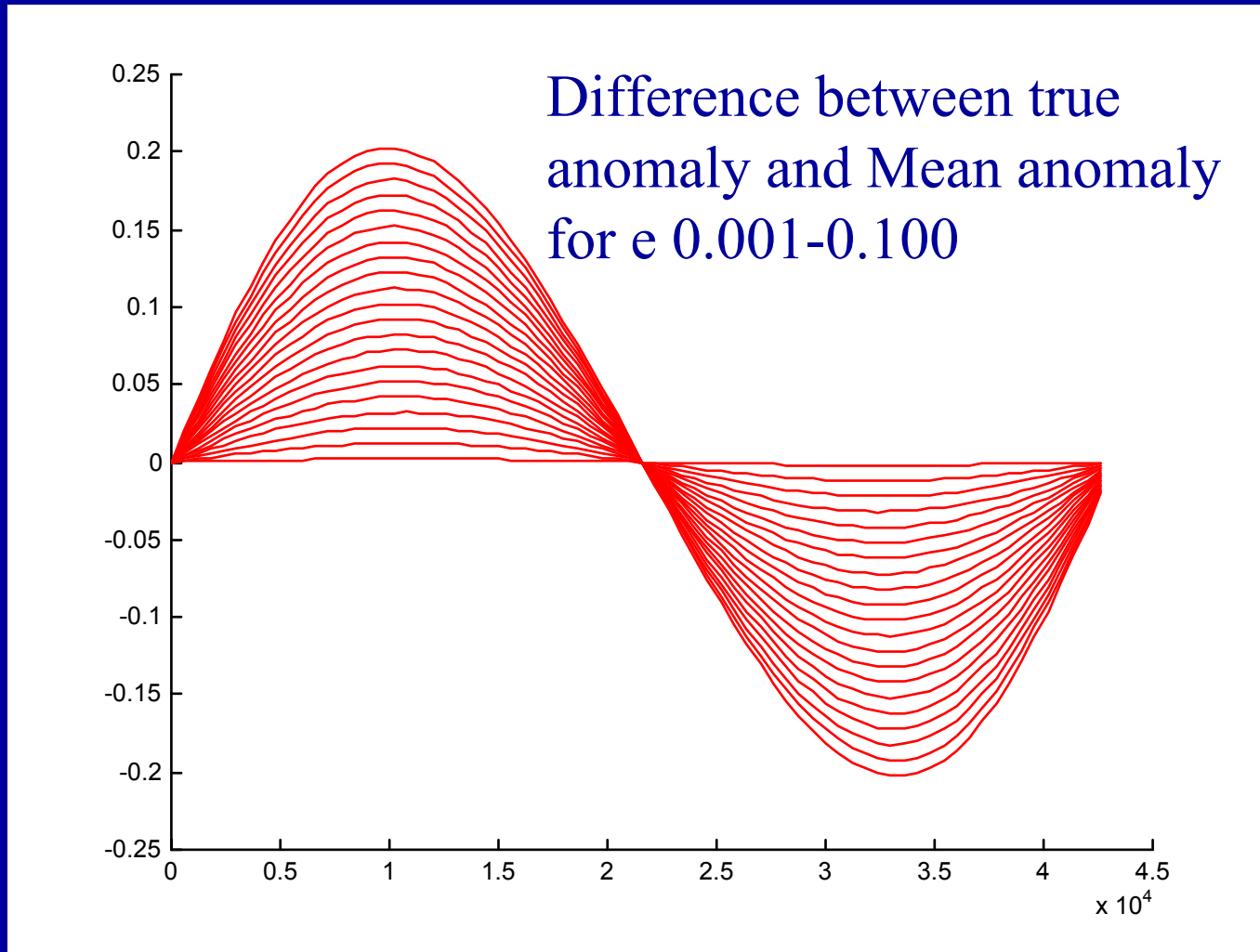
$$M(t) = n(t - T_0)$$

$$E(t) = M(t) + e \sin E(t)$$

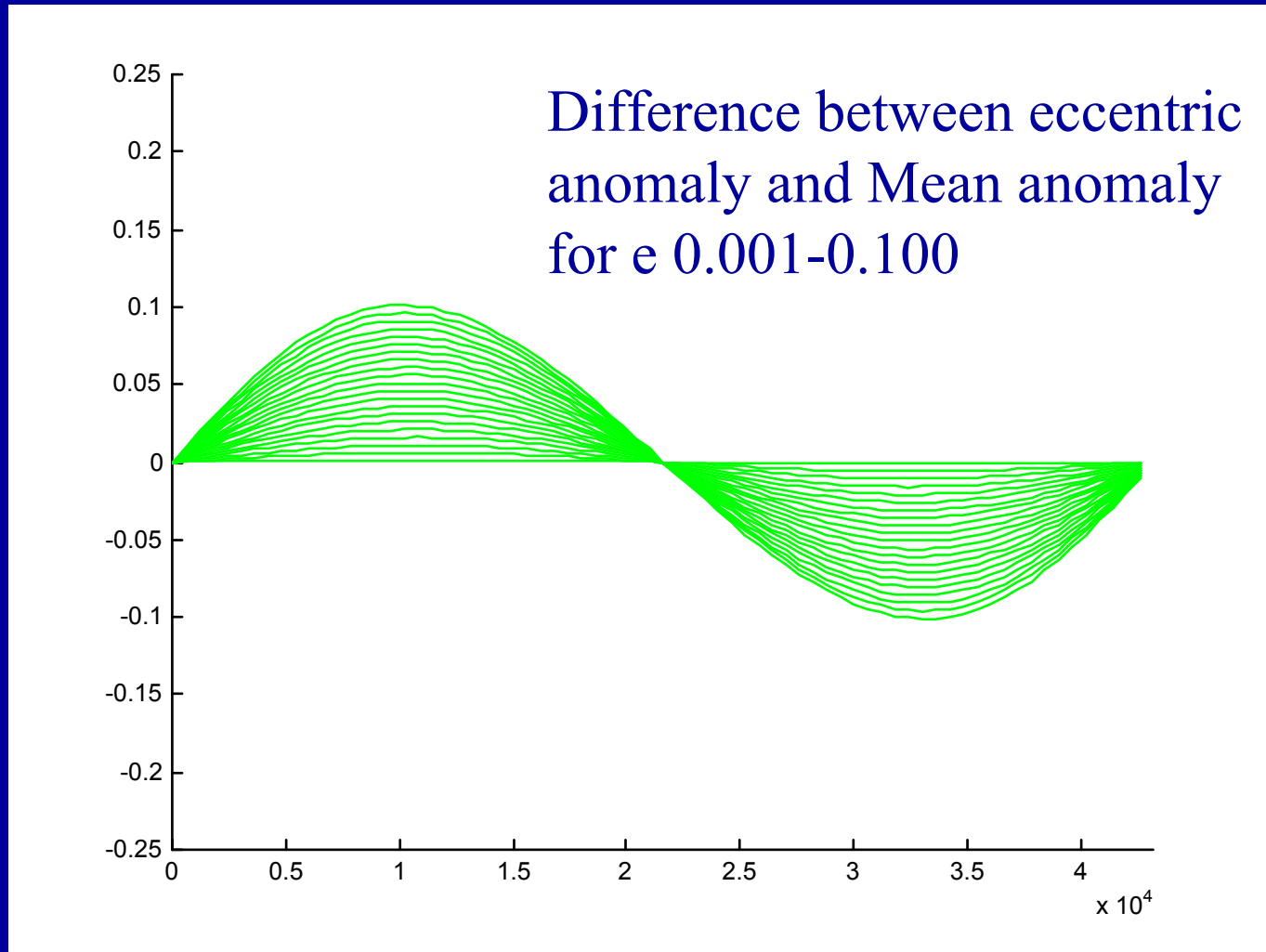
$$\nu(t) = \tan^{-1} \left[ \frac{\sqrt{1 - e^2} \sin E(t) / (1 - e \cos E(t))}{(\cos E(t) - e) / (1 - e \cos E(t))} \right]$$

- $T_0$  is time of perigee

# True anomaly



# Eccentric anomaly



# Vector to satellite

- At a specific time past perigee; compute Mean anomaly; solve Kepler's equation to get Eccentric anomaly and then compute true anomaly.
- Vector  $\mathbf{r}$  in orbit frame is

$$\mathbf{r} = a \begin{bmatrix} \cos E - e \\ \sqrt{1 - e^2} \sin E \end{bmatrix} = r \begin{bmatrix} \cos \nu \\ \sin \nu \end{bmatrix}$$

$$r = a(1 - e \cos E) = \frac{a(1 - e^2)}{1 + e \cos \nu}$$

# Final conversion to Earth Fixed XYZ

- Vector  $\mathbf{r}$  is in satellite orbit frame
- To bring to inertial space coordinates or Earth fixed coordinates, use

$$\mathbf{r}_i = R_3(-\Omega)R_1(-i)R_3(-\omega)\mathbf{r}$$

$$\mathbf{r}_e = R_3(-\Omega + \theta)R_1(-i)R_3(-\omega)\mathbf{r}$$

- This basically the method used to compute positions from the broadcast ephemeris
- (see Lecture 9 for discussion of rotation matrices)

# Perturbed motions

- The central force is the main force acting on the GPS satellites, but there are other significant perturbations.
- Historically, there was a great deal of work on analytic expressions for these perturbations e.g. Lagrange planetary equations which gave expressions for rates of change of orbital elements as function of disturbing potential
- Today: Orbits are numerically integrated although some analytic work on form of disturbing forces.

# Perturbation from Flattening $J_2$

- The  $J_2$  perturbation can be computed from the Lagrange planetary equations

$$\dot{\Omega} = -\frac{3}{2} n a_e^2 \frac{\cos i}{a^2 (1-e^2)^2} J_2$$

$$\dot{\omega} = \frac{3}{4} n a_e^2 \frac{5 \cos^2 i - 1}{a^2 (1-e^2)^2} J_2$$

$$\dot{M} = n + \frac{3}{4} n a_e^2 \frac{3 \cos^2 i - 1}{a^2 \sqrt{(1-e^2)^3}} J_2$$

# $J_2$ Perturbations

- Notice that only  $\Omega$ ,  $\omega$  and  $n$  are effected and so this perturbation results in a secular perturbation
- The node of the orbit precesses, the argument of perigee rotates around the orbit plane, and the satellite moves with a slightly different mean motion
- For the Earth,  $J_2 = 1.08284 \times 10^{-3}$



# Gravitational perturbation styles

Parameter	Secular	Long period	Short period
a	No	No	Yes
e	No	Yes	Yes
i	No	Yes	Yes
$\Omega$	Yes	Yes	Yes
$\omega$	Yes	Yes	Yes
M	Yes	Yes	Yes

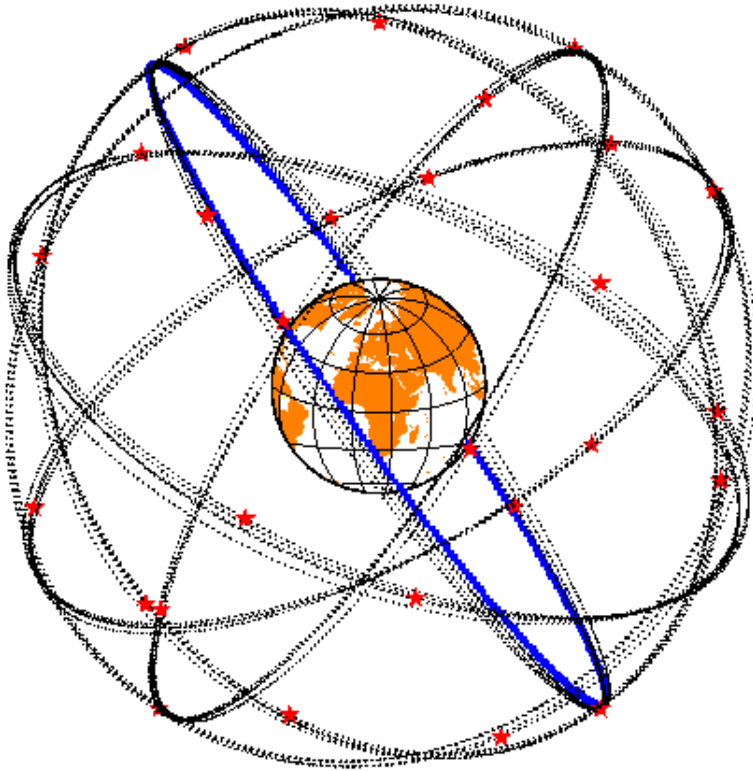
# Other perturbation on orbits and approximate size

Term	Acceleration (m/sec <sup>2</sup> )
Central	0.6
J <sub>2</sub>	5x10 <sup>-5</sup>
Other gravity	3x10 <sup>-7</sup>
Third body	5x10 <sup>-6</sup>
Earth tides	10 <sup>-9</sup>
Ocean tides	10 <sup>-10</sup>
Drag	~0
Solar radiation	10 <sup>-7</sup>
Albedo radiation	10 <sup>-9</sup>

# GPS Orbits

- Orbit characteristics are
  - Semimajor axis 26400 km (12 sidereal hour period)
  - Inclination 55.5 degrees
  - Eccentricity near 0 (largest 0.02)
  - 6 orbital planes with 4-5 satellites per plan
- Design lifetime is 6 years, average lifetime 10 years
- Generations: Block II/IIA 9729 kg, Block IIR 11000 kg

# Basic Constellation



Orbits shown in inertial space and size relative to Earth is correct

# Broadcast Ephemeris

- Satellites transmit as part of their data message the elements of the orbit
- These are Keplerian elements with periodic terms added to account for solar radiation and gravity perturbations
- Periodic terms are added for argument of perigee, geocentric distance and inclination
- The message and its use are described in the ICD-GPS-200 [icd200c123.pdf](#) (page 105 in PDF)

# Distribution of Ephemerides

- The broadcast ephemeris is decoded by all GPS receivers and for geodetic receivers the software that converts the receiver binary to an exchange format outputs an ASCII version
- The exchange format: Receiver Independent Exchange format (RINEX) has a standard for the broadcast ephemeris.
- Form [4-char][Day of year][Session].[yy]n  
e.g. brdc0120.02n

# RINEX and SP3 standard

- Description of RINEX standard can be found at <ftp://igscb.jpl.nasa.gov/igscb/data/format/rinex2.txt>
- The RINEX standard is an ASCII format that is used internationally to exchange GPS data and broadcast ephemeris information.
- Recent additions to the format also include metrological and ionospheric data records.
- Precise orbit information in the form of tabulation of the positions of the satellites (usually at 15 minute intervals) is provided in the SP3 format (3rd version of Satellite Position format). Latest version SP3c contains clock information for the satellites as well. (These can be used for point positioning at a few millimeters several days after real-time).

# Accuracy of orbits

- Broadcast ephemeris is transmitted real time and must be a prediction of the motion of the satellites. Its current accuracy is typically better than a 1-meter (except after maneuvers).
- International GPS service (IGS) ultra-rapid orbits (available in real-time--predictions), typically 25 cm.
- IGS rapid orbits (17-hour delay) about 5 cm
- IGS final orbits (13 day delay) better than 5 cm
- Quality differences due to amount of data used and ability to check the quality of the solutions and data.  
<http://igscb.jpl.nasa.gov/components/prods.html>



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