Optimal Design of Transonic Fan Blade Leading Edge Shape Using CFD and Simultaneous Perturbation Stochastic Approximation Method

X.Q.Xing and M.Damodaran

***Abstract*—Simultaneous Perturbation Stochastic Approximation method has attracted considerable application in many different areas such as statistical parameter estimation, feedback control, simulation-based optimization, signal & image processing, and experimental design. In this paper, its performance as a viable optimization tool is demonstrated by applying it first to a simple wing geometry design problem for which the objective function is described by an empirical formula from aircraft design practice and then it is used in a transonic fan blade design problem in which the objective function is not represented by any explicit function but is estimated at each design iteration by a computational fluid dynamics algorithm for solving the Navier-Stokes equations**

***Keywords*—global optimization; simultaneous perturbation stochastic approximation method, simulated annealing, transonic fan design**

I. Introduction

The need for solving multivariate optimization problems is pervasive in engineering, the physical and social sciences. The characteristic features of such problems are the presence of a large number of design variables, complex constraints, and even discrete design parameter values. A number of optimization algorithms, both local and global optimization algorithms have been developed for optimizing such problems. Besides deterministic methods, stochastic methods such as genetic algorithm (GA) and simulated annealing (SA) algorithm etc have recently found applications in practical engineering design optimization problems. These algorithms are all stochastic in nature and easily implemented in robust computer codes as compared with deterministic methods. However, SA and GA methods require large number of function evaluations and relative long computation time especially in the case of complex design problems. One approach to reduce computational time would be to use parallel GA and parallel SA as outlined in Wang and Damodaran[1]. An attractive alternative to SA and GA could be the Simultaneous Perturbation Stochastic Approximation (SPSA) method described by Spall[2-4] and which has been applied to difficult multivariate optimization problems. The SPSA method has attracted considerable application in many different areas such as statistical parameter estimation, feedback control, simulation-based optimization, signal & image processing, and experimental design. The essential feature of SPSA, which accounts for its power and relative ease of implementation, is the underlying gradient approximation which requires only two measurements of the objective function regardless of the dimension of the optimization problem. This feature allows for a significant decrease in the cost of optimization, especially in problems with a large number of variables to be optimized.

SPSA methods are briefly outlined and its performance as a viable optimization tool is demonstrated by applying it first to a simple wing geometry design problem for which the objective function is described by an empirical formula from aircraft design practice and then it is used in a transonic fan blade design problem in which the objective function is not represented by any explicit function but is estimated at each design iteration by a computational fluid dynamics algorithm for solving the Navier-Stokes equations.

II. SPSA Method

SPSA is relatively easy to implement and does not require gradient information. It is a fairly robust method and has the ability to find a global minimum in the presence of multiple minima. SPSA is an algorithm that is based on a “simultaneous perturbation” gradient approximation. The “simultaneous perturbation” approximation uses only two function measurements independent of the number of parameters (say, *p*) being optimized. The SPSA algorithm works by iterating from an initial guess of the optimal vector X_0. First, the counter index *k* is initialized to a value of *0*, an initial guess of the design variable vector X_k and non-negative empirical coefficients are set. Next a *p*-dimensional random

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simultaneous perturbation vector Δ_k is constructed and two measurements of the objective function, namely $y(X_k + c_k\Delta_k)$ and $y(X_k - c_k\Delta_k)$ are obtained based on the simultaneous perturbation around the given vector X_k. Then generate the simultaneous perturbation approximation to the gradient $g(X_k)$. The parameter $c_k = c_0/(k^m)$ where c_0 is a small positive number taken as 0.01 in this study, *k* is the loop index and *m* is a coefficient taken as 1/6 in this study. The term Δ_k represents the random perturbation vector generated by Monte-Carlo approaches and the components of this perturbation are independently generated from a zero-mean probability distribution and a simple distribution that has been used in this study is the Bernoulli ± 1 distribution with probability of ½ for each ± 1 outcome. This is followed immediately by the calculation of the gradient approximation based on two measurements of the function based on the simultaneous perturbation around the current value of the design variable vector and the updating of the design vector X_k to a new value X_{k+1} using standard SA form, i.e. $X_{k+1} = X_k - a * g(X_k)$. The value of a can be chosen to ensure effective practical performance of the algorithm. Finally the algorithm is terminated if there are insignificant changes in several successive iterations or if the maximum allowable number of iterations has been reached. The details of the step-by-step implementation of the SPSA algorithm are outlined in Spall[2,4].

III. Wing Design Optimization

A. *Wing Design Problem*

The design problem concerns the design of wing shape such that the aerodynamic efficiency of the wing reaches a maximum value during cruise with the wing weight acting as a constraint, i.e. the goal is to determine the shape of the wing for minimizing D/L (drag-to-lift) or maximizing L/D with the wing weight as a constraint. Explicit empirical function for D/L which form the objective function for the optimization problem and empirical expressions used here are defined in Raymer[6].

The objective function to be optimized is defined as follows:

$$Minimize\ F(x) = D/L \qquad (1)$$

Subject to six constraints on the design variables defined as follow:

$$1.0^\circ \le \alpha \le 10.0^\circ,\quad 10.0 \le b \le 50.0,$$
$$3.5 \le c \le 10.0,\quad 0.0^\circ \le \lambda \le 35.0^\circ,$$
$$0.5 \le A_R \le 15.0,\quad W_{wing} \le 2473(lb)$$

Where α is the angle of attack, *b* is the wing span, *c* is the mean aerodynamic chord, λ is the wing sweep, A_R is the wing aspect ratio and W_{wing} is the wing weight. The constraints are incorporated in into a composite objective function by way of penalty functions.

B. *Optimization Results and Comparison*

Table 1 shows the optimum values of the objective function and design variables reached by different optimization algorithms. It can be seen that the results of objective function and variables from SPSA are very similar to those of Simulated Annealing (SA). Fig. 1 compares the evaluations of function to reach the optimal values based on the specified convergence criterion using SPSA with those attained by SA method. It can be seen that SPSA reaches optimal values about 9 times faster than SA thereby suggesting that SPSA is a viable and a more efficient stochastic design method.

TABLE I
COMPARING OPTIMAL DESIGN RESULTS USING SPSA AND SA OPTIMIZATION METHODS

Algorithm	α (°)	b (ft)	c (ft)	λ (°)	Wing Weight (lb)	objective Function D/L
SPSA	3.432 °	44.994	5.986	18.115 °	2443.93	0.0319
SA	3.199 °	44.987	5.947	18.002 °	2443.93	0.0319

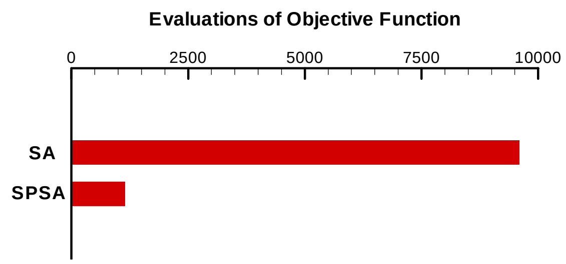


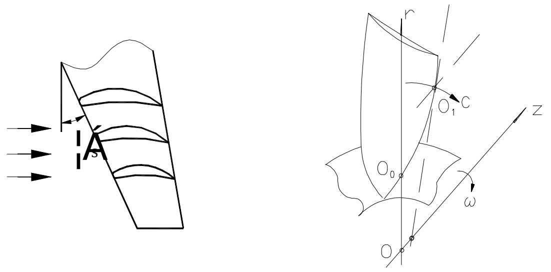
Fig. 1. Evaluations of Objective Functions

IV. Transonic Fan Blade Leading Edge Design Problem

Transonic fan blade design problems include the design of the blade airfoil sections at different radial positions and the stacking of these airfoils to form a three dimensional blade. Many studies have focused on the airfoil optimization using the Navier-Stokes and adjoint equations as in optimized design of shock-free airfoils and wings reported in Sung and Kwon[8]. Catalano[9] presented an inverse design of a three-dimensional turbo-machinery blade in which objective functions were based on inviscid transonic flow model and Tong[10] proposed a multi-objective simulated annealing algorithm to improve the initial design of turbine blade.

Studies of swept rotors are motivated by the fact that swept wings reduced shock wave strength, and the shape of the leading edge of a swept blade can be altered to control the shock structure and the migration of the low-momentum flow fluid, which consequently decrease the losses induced by the interaction between shock and boundary layer as well as the interaction between shock and secondary flow. This motivates the current application problem dealing with the design of the leading edge curve to meet design objectives.

The flow fields around swept wings can be assumed as two-dimensional in which case it is sufficient to describe the wing leading edge using a sweep angle α_s as shown in Fig. 2(a). However, the leading edges of transonic fan blades are three-dimensional as shown in Fig. 2(b).



(a) (b)

Fig.2. Difference between (a) wing and (b)blade leading edge

The design problem concerns the design of the leading edge curve such that the adiabatic efficiency of a transonic fan's rotor reaches a maximum value during 100% of the design speed, with the rotor's stall margin acting as a constraint. At the same time, the pressure ratio must satisfy a given value.

As the stall margin and the pressure ratio cannot be expressed by the design variables, these were not integrated into the design problem as constraints but were only checked through numerical simulation for the present study.

A. Blade design problem

The rotor's leading edge curve defined here is same to that of reference 11 by a cubic three-dimensional curve as follows:

$$\vec{R}(t) = \bar{a} + \bar{b}t + \bar{c}t^2 + \bar{d}t^3 \quad (2)$$

Where,

$$\bar{a} = [a_1, a_2, a_3],\ \bar{b} = [b_1, b_2, b_3],$$
$$\bar{c} = [c_1, c_2, c_3],\ \bar{d} = [d_1, d_2, d_3] \quad (3)$$

If the starting point vector $\bar{R}(0)$, the corresponding tangent vector $\bar{R}'(0)$, the end point vector $\bar{R}(1)$ and the corresponding tangent vector $\bar{R}'(1)$ are given, the curve can be determined. The equation of the cubic curve defined in cartesian coordinates is as follows:

$$[x(t), y(t), z(t)] = [1, t, t^2, t^3] * M * \begin{bmatrix} x(0) & y(0) & z(0) \\ x(1) & y(1) & z(1) \\ x'(0) & y'(0) & z'(0) \\ x'(1) & y'(1) & z'(1) \end{bmatrix} \quad (4)$$

Where,

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -3 & 3 & -2 & -1 \\ 2 & -2 & 1 & 1 \end{bmatrix} \quad (5)$$

In order to reduce design variables and to control the evolving shape, the ends of the leading edge i.e. *x(0), y (0), z(0), x(1), y(1), z(1)* are fixed according to a baseline design configuration which is chosen to start this design optimization study. The gradients at the ends of the leading edge curve, i.e.. $x'(0)$, $y'(0)$, $z'(0)$, $x'(1)$, $y'(1)$, $z'(1)$ are altered during design iteration to form new leading edge curves so that there are the six optimization variables.

The objective function to be maximized is defined as:

$$F = \text{Max}\ (\eta_r) \quad (6)$$

Or the problem can be cast as a minimization problem as follows:

$$\bar{F} = Min(1./\eta_r) \quad (7)$$

Where η_r is the rotor's adiabatic efficiency which is estimated using a compressible computational fluid dynamics solver solving the Navier-Stokes equations and this is briefly outlined in the next section.

B. Numerical Simulation and Optimization

The numerical simulation code that was used to test the adiabatic efficiency of the rotor, pressure ratio and stall margin is outlined in Wang and Zhao[12]. For the current study, the Navier-Stokes equations are solved with a finite-volume, four-step Runge-Kutta, time marching method, without considering tip-clearance effects. The turbulence model used here is a simple algebraic model and the grid consists of 62 nodes in axial direction, 25 nodes in the radial direction and 25 nodes in the blade-to-blade direction. The whole design process is shown in Fig. 3.

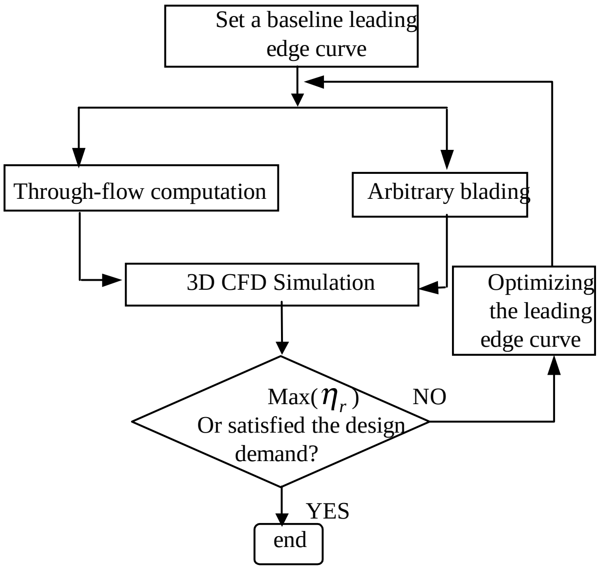


Fig. 3. Design Process Flow Chart

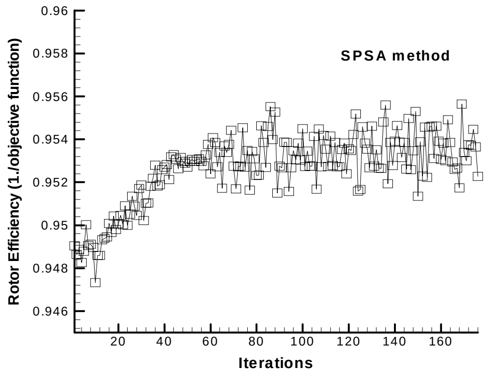
Based on an initial guess of a curve defining the shape of the leading edge, it is possible to design a three-dimensional blade profile using the semi-empirical through-flow analysis method of Wennerstrom and Puterhaugh[13] and the arbitrary blading method to determine the local blade camber in the spanwise direction and to distribute the thickness around the camber. The local thickness distribution at the spanwise stations of the blade is assumed to remain fixed for the current study. After obtaining the approximate blade geometry with local spanwise camber and thickness the flow field of the rotor can be estimated using CFD. Based on the estimated values of total temperature and total pressure from CFD analysis, the efficiency η_r of the rotor defined as

$$\eta_r = \frac{(P^*_{out}/P^*_{in})^{\frac{\gamma-1}{\gamma}} - 1}{T^*_{out}/T^*_{in} - 1} \quad (8)$$

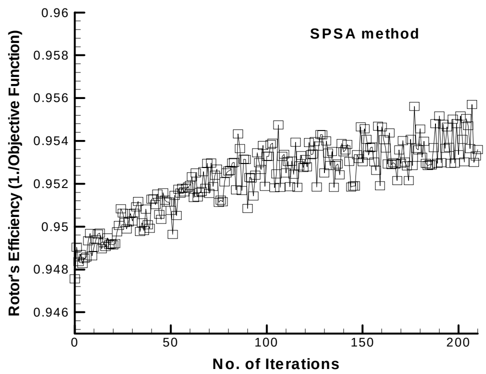
where P^*_{in} and P^*_{out} are the total pressures and T^*_{in} and T^*_{out} are the total temperatures at the inflow and outflow boundaries of the rotor flow field computational domain and γ is the ratio specific heat of the gas, can be computed. This is followed by the optimization steps to find a new blade leading edge curve using the optimization method and the process is continued till desired convergence is achieved.

C. Results and Discussion

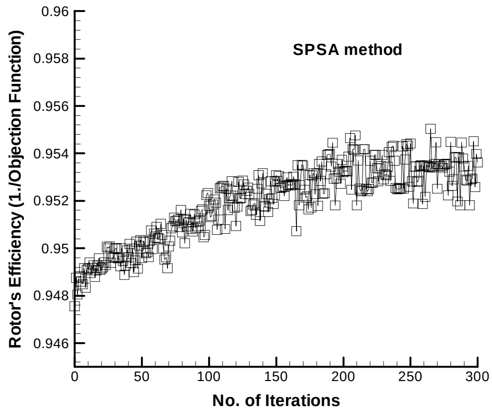
For purpose of illustrating the application of SPSA method a swept 3D blade shape with a straight wing leading edge is considered as the baseline configuration to initiate the optimization process. The effect of the parameter *a* appearing in the formulation of the SPSA method as discussed in Section II on the convergence of the objective function i.e. the computed/optimized rotor adiabatic efficiency vs. number of design iterations, is first considered. Figures 4(a)-(c) shows the convergence of the design iterations corresponding to values of the parameter a set to 30, 50 and 100 respectively. The results show that the rate of convergence is the most fastest when a =30. When the value of a is increased, the rate of convergence is reduced, while if the value of a is decreased to less than 30, there is a possibility that the design iterations may diverge during the initial stages of the optimization. For the case when *a*=30 the optimal value is reached in about 60 iterations which requires about 180 function evaluations. For the case when *a*=100 optimal result is reached in about 200 iterations requiring about 600 function evaluation. The optimal value of rotor's efficiency η_r attained can be bracketed between 0.952 and 0.955 for all values of a in view of the oscillatory manner in which the optimal value is reached by the SPSA method.



(a) a=30



(b) a=50



(c) a=100

Fig. 4. Effect of SPSA parameter a on convergence of objective function

From Fig. 4, it can be seen that the adiabatic efficiency of the rotor does not converge to a fixed value in view of the oscillatory nature of the variation of the objective function to the optimal value. Based on the trends of the objective function variation, it is possible to approximately fit a mean curve through the band of the oscillating region. One reason for this kind of oscillatory behavior could be the manner in which the treatment of the design parameters is done. It is well known that during the design process of a turbo-machinery blade, distribution of losses and other design parameters within the framework of the empirical method for arbitrary blading distribution should be adjusted carefully to get a reasonably good design. In the present study the distribution of the losses and other design parameters had been limited for all blade shapes to produce the automatic search process. The impact of this will be analyzed in a separate study as the focus of this exercise is to gauge the feasibility of SPSA method in shape design.

Figures 5 and 6 compare the blade shape and the leading edge curve shape corresponding to baseline and optimal shape obtained by the SPSA method.

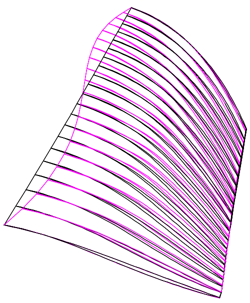
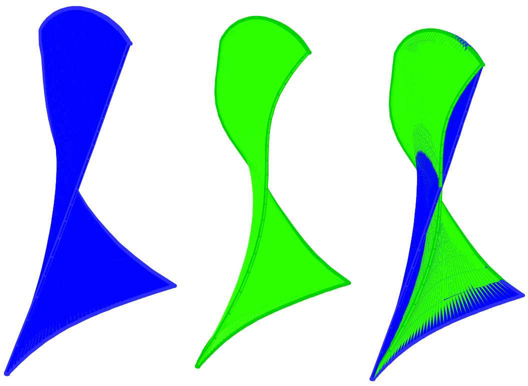


Fig.5. Comparison of the leading edge shape corresponding to baseline (straight leading edge)and optimal design (curved leading edge)



(a) (b) (c)

Fig.6. (a) Initial blade shape (b) Optimized blade shape (c) Comparison between (a) and (b)

A deterministic method based on the Broyden-Fletcher-Goldfarb-Shanno variable metric method and a stochastic method based on Simulated Annealing (SA) were also used to arrive at optimal shape designs of the blade under the same conditions considered using the SPSA method for purposes of comparing the outcomes with that of the SPSA method. Fig. 7 shows the convergence of the objective function with design iterations obtained by using deterministic method while Fig. 8 shows the convergence of the objective function using the simulated annealing optimization method.

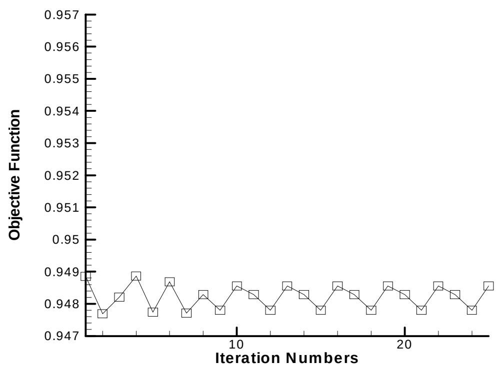


Fig. 7. Convergence of Objective function Using Deterministic Method

The termination criterion used to stop the design optimization process is

$$\left| f(x_{k+1}) - f(x_k) \right| \le 10^{-6}.$$

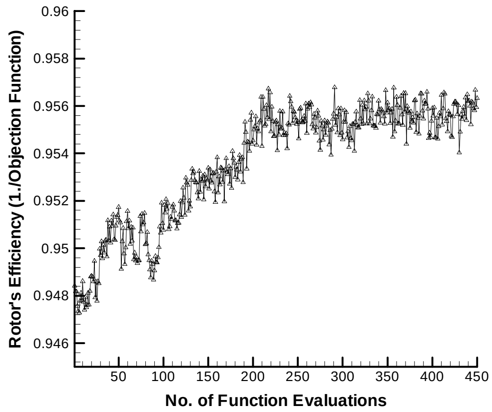


Fig.8. Results of SA method

From Fig. 7 it appears that after 25 design iterations the deterministic optimization method appears to have been trapped in a local minimum as there does not appear to be any significant variations in the objective function. From Fig. 8, it can be seen that convergence of the objective function using SA method is very similar to that of SPSA but the SA method requires about 320 to 400 function evaluations to achieve a value of η_r lying between 0.954 and 0.956.

It is very clear from this preliminary investigation that the SPSA method is a robust method for obtaining global optimization. It is certainly faster than the SA method while the gradient-based method GB method seems to be trapped in a local optima.

V. CONCLUSIONS

From this study focusing on the design optimization of the blade leading edge curve, it can be seen that it quite difficult to attain global optimum using deterministic method. Both SA and SPSA method are able to reach optimal values and that SPSA has the potential of achieving optimal results in much less computational effort than SA method. It can be concluded that SPSA method is a suitable feasible optimization method which can be used to handle complex design problems such as the fan blade shape designs. It is also relatively easy to implement. SPSA requires only two measurements of the objective function regardless of the dimensions of the design space corresponding to the optimization problem and the cost of optimization decreases. The present investigation forms the basis for further study to look at various aspects such as replacing the arbitrary blading step with a more sophisticated surface geometry representation incorporating blade twist, sweep etc thereby increasing the number of design variables using parameteric representation of blade surface patches, the impact of the various possible alternative methods for generating the random perturbation vector in the SPSA method and the impact of that on the design of complex 3D shape designs.

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