Robust Cooperative Strategy for Contour Matching Using Epipolar Geometry

Miaolong Yuan, Ming Xie, Xiaoming Yin

Abstract- **Feature matching in images plays an important role in computer vision such as for 3D reconstruction, motion analysis, object recognition, target tracking and dynamic scene analysis. In this paper, we present a robust cooperative strategy to establish the correspondence of the contours between two uncalibrated images based on the recovered epipolar geometry. We take into account two representations of contours in image as contour points and contour chains. The method proposed in the paper is composed of the following two consecutive steps: (1) The first step uses the LMedS method to estimate the fundamental matrix based on Hartley's 8-point algorithm, (2) The second step uses a new robust cooperative strategy to match contours. The presented approach has been tested with various real images and experimental results show that our method can produce more accurate contour correspondences.**

1. INTRODUCTION

In computer vision, feature matching between different images of a single scene is one of the fundamental problems and has many applications such as 3D reconstruction, motion analysis, object recognition and dynamic scene analysis. A large amount of related work has been carried out during the last twenty years [3][5][11][12][14]. However, the results by whichever method are not satisfactory because the correspondence problem is not straightforward to find one-to-one mapping between two images.

Miaolong Yuan, is with the Innovation in Manufacturing Systems and Technology(IMST), Singapore-MIT Alliance (SMA), N2-B2c-15, Nanyang Technological University, Nanyang Avenue, Singapore 639798, email: mlyuan@ntu.edu.sg

Ming Xie, is with the Innovation in Manufacturing Systems and Technology(IMST), Singapore-MIT Alliance (SMA), N2-B2c-15, Nanyang Technological University, Nanyang Avenue, Singapore 639798, Email: mmxie@ntu.edu.sg

Xiaoming Yin, is with the School of Mechanical and Production Engineering, Nanyang Technological University, Nanyang Avenue, Singapore 639798

In this area, one distinctive correspondence problem is contour matching. It aims at establishing the correspondence between image contours observed at different positions by one or more cameras. A contour in an image is defined by two representations as contour points and contour chains. Contours are more general descriptions in an image and contain more geometric information about the image. Matching is then usually achieved as follows: First, contour segments are extracted from two images by detecting edge maps, linking them and determining a set of chained points. Then, correspondence between two sets of contours is sought for by using varies methods.

In the literature, there are many investigations devoted to the problem of either stereo or motion correspondence of contours within two or more frames [3][7][11-18]. In [15], the correspondence is obtained by using a relaxation operation which tries to find the most consistent matches preserving the geometric constraints between features. Relaxation process is an attractive mean of inexact matching because it has ability to infer consistent interpretation from incomplete input. However, this kind of method suffers from high computational cost. Another method of facing this problem is to introduce a temporal recursive filter to track contour segments [16][17]. Matching becomes a cyclic process composed of three steps: predict, match and update. The correspondence between observed and predicted segments is obtained through the use of similarity function based on Mahalanobis distance between attributes of the segments. These methods can limit the search area and handle ambiguities more effectively since they produce a prediction of the displacement from frame to frame. Other typical methods form matching have been proposed, graphing matching methods [13] and cooperative strategy for multi-level edge primitive matching [8].

Unlike previous methods, the correspondence points on the contours are investigated using the smoothness constraints or curvature variations or the combination of epipolar geometry and correlation_based technique for contour matching [3][11][12]. As more information is added, these methods tend to be more robust. However, these methods will be more subjected to the noise disturbance and the complexity of the real images.

In this paper, we propose an approach for contour matching based on recovering epipolar geometry and a robust cooperative strategy. Our approach can handle the correspondence problems robustly between different related images with different complexity. Several real images have been tested by our approach and good results are obtained.

We assume that two images are obtained from two cameras at different positions or a single camera at different time instants and the intensity value of a region does not change too much.

2. EPIPOLAR GEOMETRY FOR CONTOUR MATCHING

As we know, the only available geometric constraint in the correspondence problem between two images is the epipolar geometry [1][2][4][6][8]. Considering the case of two cameras as shown in Fig.1. Let X_k be a 3D point, x_{ik} and x_{jk} be its projections to image I and image J, respectively and O_1 and O_2 be the optical centers of the first and second cameras, respectively. The plane defined by X_k, O_1 and O_2 is known as epipolar plane. The intersection of the epipolar plane with the Image J is called the epipolar line and denoted by l_{jk}. So the correspondence of x_{ik} is constrained to lie on a line l_{jk}. This is because the point x_{ik} may correspond to an arbitrary point on the semi-line O_2X_k and the projection of O_2X_k on image J is l_{jk}

It is well known that at least eight point matches are needed for computing an essential matrix [2]. The essential matrix is called a fundamental matrix in the case of two uncalibrated images. The fundamental matrix between image I and image J is a 3×3 matrix satisfying

$$x_{jk}^T F_{ij} x_{ik} = 0, k = 1,...,n \quad (1)$$
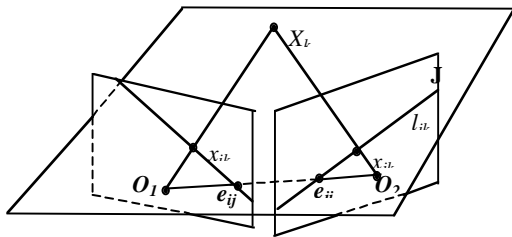

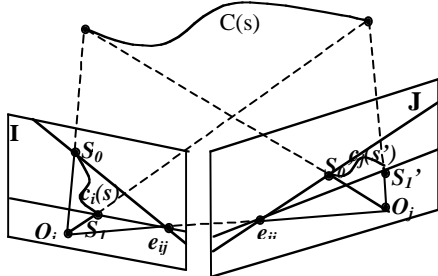**Fig.1 The Epipolar geometry of two images**

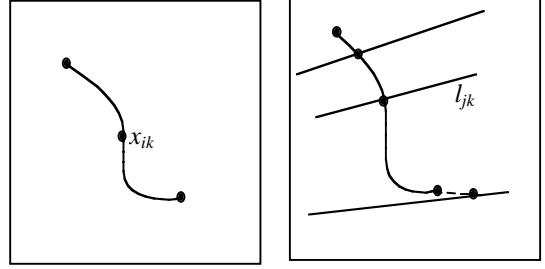**Fig. 2 The geometry of a contour**
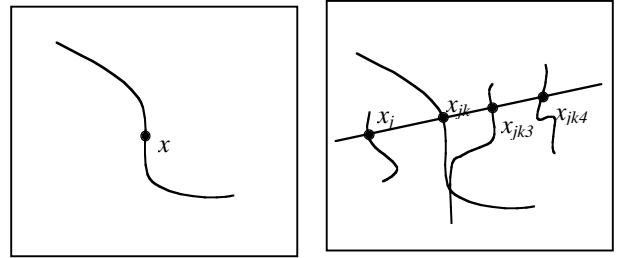

Fig.3 Two corresponding contours

Fig. 4 Contour points along epipolar line

for any match pair (x_{ik}, x_{jk}) in the two image I and image J. The fundamental matrix F_{ij} indicates the epipolar geometry of the two cameras. Therefore, if the epipolar geometry is recovered, the correspondence problem is reduced from a 2D search (the whole image) to a 1D search problem (along the epipolar line).

In our paper, we use a classical robust approach named the Least Median of Squares (LMedS) based on the technique described in [1] and [2] to estimate the fundamental matrix.

Now we discuss the epipolar geometry for contour matching. Let $c_i(s)$ and $c_j(s')$ be the projected image curves of a parameterized space curve $C(S)$ in image I and image J, respectively, as shown in Fig.2. For a given $c_i(s)$ in image I, as shown in Fig.3, we need to find its corresponding contour $c_j(s')$ in image J.

For every point x_{ik} of a contour $c_i(s)$, its epipolar l_{jk} in image J is defined as:

$$l_{jk} = F_{ij} * x_{ik}.$$

If there is no error between the two images, we will find a one-to-one mapping between the points of $c_i(s)$ and the points of a contour $c_j(s')$ in image J. Unfortunately, the ideal case does not exist because of noise perturbation. For example, the end-points and lengths of the correspondence contour chains may not be identical as shown in Fig.3. Therefore, the matching process should be able to handle inexact matching. We

have the fact that the correspondence of x_{ik} on $c_i(s)$ must be one of the intersections of l_{jk} with the contours in image J as shown in Fig.4. Out method for contour matching will be based on the fact. In the later sections, we assume that the epipolar geometry between two images is recovered and will enforce the recovered epipolar geometry when we use cooperative strategy in the matching process.

3. COOPERATIVE MATCHING STRATEGY

Since a contour point belongs to a contour, a pair of matched contour points indicates a possible correspondence of the two contours to which they belong. Therefore, the output of the matching process of contour points can be used as input of the matching process of contours. The initial contour correspondences can be obtained from the matching process of contours. Then, the obtained initial contour correspondences are used as the input of the next matching process of contour points and the matching process of contours. This cooperative strategy can be explained as the following three ordered processes:
(1) The matching of contour points: This matching process aims at establishing the contour point correspondences. In order to solve the contour point correspondences, a classical method based on similarity criterion is used along with the recovered epipolar geometry.
(2) The initial matching of contours: This process makes use of the results of the first matching process. Since a pair of matched contour points indicates a possible correspondence of two contours to which they belong, we can identify a set of candidates in image J to be matched to a contour $c_i(s)$ in image I if points on $c_i(s)$ correspond to a set of points on different contours in image J.
(3) Final matching of contours: The matching process aims at establishing the final correspondences using the initial contour correspondences obtained from the second step. We use the matching process of contour points by using another different similarity criterion to get contour point correspondences of $c_i(s)$ among its initial contour correspondences. Then another contour candidates corresponding to $c_i(s)$ will be identified by using the matching process of contours. Repeat the three processes iteratively. Finally a contour to be matched with $c_i(s)$ will be found until the set of the contour candidates consists of only one contour.

Based on the above discussion, we developed a matching algorithm based on the cooperation between the matching process of contour point and the matching process of contours. In the following section, we will explain in detail the cooperative method.

3.1 Similarity function

As mentioned above, we first aim at establishing the correspondences of contour points by using a similarity criterion along with epipolar geometry. Without loss of generality, we set up a general similarity function for the correspondence problem. For each point x_{ik} in a contour, we define its l normalized measures as $\{m_{i,k} k=1,...,l\}$. For examples, the measures can be defined the classical normalized correlation coefficient, the norm of gradients, or the value of convolution with a directional mask at this point.

Let $S_k(x_{ik}, x_{jk})$ be the kth measure between a pair of points (x_{ik}, x_{jk}). If $S_k(x_{ik}, x_{jk})$ is equal to 1, we consider the two points (x_{ik}, x_{jk}) are matched under the kth measure. Therefore, by combining the different measures, we can define a measure function that depends on two contour points to be matched.

$$F(x_{ik}, x_{jk}) = \prod_{k=1}^{l} S_k(x_{ik}, x_{jk})^{\lambda_k} \quad (2)$$

where λ_k ($\lambda_k \geq 0$) is a weighted exponent applied to $S_k(x_{ik}, x_{jk})$ and can be set to 1. Obviously, the weighted $F(x_{ik}, x_{jk})$ will always be in the [0,1]. In an ideal case, if $F(x_{ik}, x_{jk})$ is equal to 1, (x_{ik}, x_{jk}) is a contour point correspondence. In this paper, (x_{ik}, x_{jk}) is considered to be a pair of matched points if they satisfy the following condition:

$$\begin{cases} F(x_{ik}, x_{jk}) = \max\{ \prod_{k=1}^{l} S_k(x_{ik}, x_{jk})^{\lambda_k}, \quad \forall x_{jk} \in SP_{ik} \} \\ F(x_{ik}, x_{jk}) > \Omega \end{cases} \quad (3)$$

where Ω is an threshold in the range (0,1) for validating the local maxima and SP_{ik} is the intersections of l_{jk} with the all contours in image J. This operation can be explained as attempting to find a contour point correspondence x_{jk} in SP_{i1} that maximizes the $F(x_{ik}, x_{jk})$. In this paper, two measures (i.e. $l=2$) are used to identify a contour point:
(1) The norm of gradients: This measure reflects the image contrast near a point. Let G_{i1} and G_{j1} be the norm of gradients of point x_{ik} and x_{jk} in image I and image J respectively. $S_k(x_{ik}, x_{jk})$ can be defined as the follow formation:

$$S_k(x_{ik}, x_{jk}) = \min(\| G_{ik} \|, \| G_{jk} \|) / \max(\| G_{ik} \|, \| G_{jk} \|)$$

(2) The correlation_based measure: The definition of this measure can been seen in [1]. In this definition, if the value is smaller than 0, we will regard it as 0.

3.2 Contour matching algorithm

Given a set of points { x_{ik}} of a contour $c_i(s)$ in image I and {x_{jk}} of a contour in image J, we need to seek the correspondences between these two sets of points. As we know, a point x_{ik} on $c_i(s)$ is constrained to lie on its epipolar line l_{jk}. So, after we computed the fundamental matrix F_{ij}, we can use epiploar geometry in finding the correspondences between the two sets of contour points. In the other words, for a point x_{ik} on $c_i(s)$, its corresponding point has only be searched in SP_{ik} in image J, where SP_{ik} is the intersections of l_{jk} with the contours in image J, as shown in Fig.4. The point x_{jk} satisfying (3) is found as the match of x_{ik}. If x_{jk} on a contour $c_j(s)$ in image J is found, we consider that x_{ik} is matched to $c_j(s)$ and then $c_j(s)$ will be identified a candidate corresponding to $c_i(s)$. If points on $c_i(s)$ are matched to several contours, then several contours are identified as initial contour correspondences. This means that there are false matches in the initial contour correspondences, as shown in Fig.5 (1)-(2). So the next step is to remove the false matches. In order to remove these false matches, we used an iterative method based on cooperative strategy.
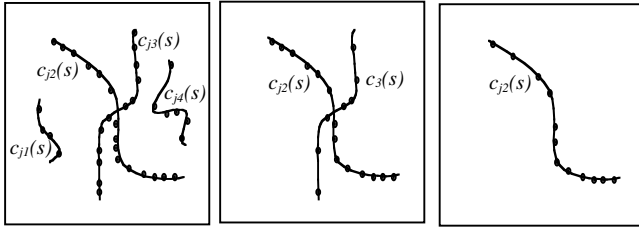

Fig. 5 Contour candidates

Let { c_{j1}, c_{j2}, …,c_{jn}} be the set of contours in image J that match to at least one point on contour c_i, $n_{c_{jk}}$ be the number of points on c_{jk} that match to points on c_i and $n_{j\max}$ be maximal value of $n_{c_{jl}}$ for all , $1 \le l \le n$. For convenience, the set of { $n_{c_{jk}}$ } ($1 \le l \le n$) is denoted by N_j. Then, the set of {c_{j1}, c_{j2}, …,c_{jn}} will be considered initial correspondences to $c_i(s)$ in image I. The contour c_{jk} in {c_{j1}, c_{j2}, …,c_{jn}} will be considered to be a candidate match corresponding to $c_i(s)$ if its $n_{c_{jk}}$ satisfies one of the following forms:

$$n_{c_{jk}} > n_{j\max} - D_1 \quad (4)$$

$$n_{c_{jk}} > n_{j\max}/D_2 \quad (5)$$

where D_1 and D_2 are initial numbers, such as 10, 3, respectively. The main reason to do so is to try to ensure that two matched contour chains will have similar lengths according to an initial similarity threshold Ω .

As the above discussion, we can get a set of possible contours from {c_{j1}, c_{j2}, …,c_{jn}}to be matched with $c_i(s)$ after the first matching process of contours. For convenience, this set of contour is defined as {c_{j1}, c_{j2}, …,c_{jm}}, where $m \le n$. So some false matches in {c_{j1}, c_{j2}, …,c_{jn}} will be removed if $m<n$. Then the next problem is how to remove the other false matches in {c_{j1}, c_{j2}, …,c_{jm}}. In practice, if the similarity threshold Ω is too big, such as 0.95, the corresponding $n_{c_{jl}}$ will be often very small, even equal to 0. If $n_{c_{jl}}$ is equal to 0, c_{jl} is not the corresponding contour of $c_i(s)$ and is discarded. If $n_{c_{jl}}$ is a very small number, contour c_{jl} in image J is lack of confidence to be the correspondence candidate of $c_i(s)$ because only a very few points are matched with the points on $c_i(s)$. On the other hand, if Ω is set to a relative small value, such as 0.6, there may be several contours that are all larger than a relative big number. For example, $n_{c_{j2}}$ and $n_{c_{j3}}$ are equal to 45, 47, respectively, it is not clear to determine which contour in {$c_{j2}(s)$, $c_{j3}(s)$} is the right correspondence of $c_i(s)$. Therefore, Ω is set adaptively and in a robust manner during each iteration. The above observation is the key why we propose an iterative method based on cooperative strategy for contour matching.

So after the first matching process, we get the candidates {c_{j1}, c_{j2}, …,c_{jm}} corresponding to $c_i(s)$. By adding a small positive value δ to Ω , we use the matching process of contour points applied to {c_{j1}, c_{j2}, …,c_{jm}}. Then we will obtain a subset {c_{j1}, c_{j2}, …,c_{jl}}of {c_{j1}, c_{j2}, …,c_{jm}} that match to at least one point on contour c_i, and get corresponding n_{ck} and $n_{j\max}$. The c_{jk} in {c_{j1}, c_{j2}, …,c_{jl}} will be considered to be a candidate corresponding to c_i if its n_{ck} satisfies (4) or (5). So a subset of possible contour correspondences from {c_{j1}, c_{j2}, …,c_{jl}} will be obtained by using the matching process of contours again. This means we remove other false matches in {c_{j1}, c_{j2}, …,c_{jl}} again. Do the cooperative matching process iteratively until all false matches are removed. The only existed contour in the set of possible contour correspondences is the correct contour corresponding to $c_i(s)$, as shown in Fig. 5 (3).

In this algorithm, we use a cooperative method to solve contour correspondence problem between the two sets of contours in image I and image J. The method is robust because it can deal with different contours with different

similarities by adjusting the similarity threshold automatically.

4. EXPERIMENTS

We have implemented the presented robust cooperative matching by using C++. We applied this algorithm to two arbitrarily chosen images of one static scene. In our system, we use Harris corner detection algorithm [19] to extract corners and Deriche edge detection algorithm [20] to extract edge points, then we use our own edge linker algorithm to link edges. The first step of robustly estimating epipolar geometry is used the methods described in [1][2]. In the matching process of contours, the matching speed is quite quick because we use two data structures to index the related edge map directly. For the validation of the local maxima of the similarity, an initial threshold Ω, the incremental value δ, D_1 and D_2 are set to 0.7, 0.4, 10, 3, respectively in our experiments.

In the first experiment, the two images are all size of 320×240. There are 833 points and 899 points in image I and image J, respectively, and 168 contours and 202 contours in image I and image J, respectively. We first use the method described in [1,2] to get the following fundamental matrix:

$$F = \begin{pmatrix} 0.000000058 & 0.000001888 & -0.000347827 \\ -0.000011258 & 0.000003366 & 0.037711369 \\ -0.003494273 & -0.036521928 & 1.000000000 \end{pmatrix}$$

Then our robust cooperative method is used to establish contour correspondences. We give an example to explain the key in our algorithm. We would find the correspondence of contour c_{147}. We first use our method to find the possible contours {c_{127}, c_{128}, c_{131}, c_{147}, c_{162}, c_{173}, c_{174}, c_{184}} in image J and get N_j={17, 13, 7, 29, 31, 27, 6, 7}, respectively. According to (4) and (5), the false matches {c_{131}, c_{174}, c_{184}} are removed. The possible contours now become {c_{127}, c_{128}, c_{147}, c_{162}, c_{173}}. Now add 0.4 to Ω, then do the same matching process of contour points and the matching process of contours, we get N_j={13, 7, 22, 28, 13} and discard the contour c_{128}. When Ω =0.78, we get N_j ={11,22, 27,6} and discard the contour c_{173}. When Ω =0.82, we get N_j = {7, 16, 26} and discard the contour c_{127}. When Ω =0.86, we can get N_j= {6, 25} and discard the contour c_{147}. Finally the contour c_{162} in image J is found to be matched with the contour c_{147} in image I. For all contours in image I, we finally get 121 contour matches, as shown in Fig.6. The elapsed time to match all contours is only 2 second.

In the second experiment, the two images are all size of 384×288. There are 910, 933 corners in image I, J respectively and 168, 202 contours in image I, J respectively. We first use the method described in [1,2] to get the following fundamental matrix:

$$F = \begin{pmatrix} 0.000000877 & 0.000138304 & -0.018824929 \\ -0.000134127 & 0.000006929 & -0.038778013 \\ 0.018575354 & 0.030732459 & 1.000000000 \end{pmatrix}$$

Here we give an another example for finding correspondence of the contour c_{22} to explain the key in our algorithm. The process can been explain in the **Table 1.1** in reference to the explanation in the first experiment, here a, b, c represent the contour No. in image J, the corresponding $N_{c_{jk}}$ and Ω respectively. c_{46} in image J is the contour to be matched with the contour c_{22} in image I. The last result is 129 contour matches to be found as shown in Fig.7. The elapsed time to match all contours is about 2.5 second.

Another example is shown in Fig8. The test images are obtained from the following address: ftp://sunsite.unc.edu/pub/academic/computer-science/ virtual-reality/3d/

In this experiment, the two images are all size of 268×385. There are 907, 914 corners in image I, J respectively and 162, 159 contours in image I, J respectively. The estimated fundamental matrix is as following:

$$F = \begin{pmatrix} 0.000002132 & 0.000095598 & -0.030711505 \\ -0.000092629 & 0.000002949 & -0.262462471 \\ 0.035091203 & 0.255102678 & 1.000000000 \end{pmatrix}$$
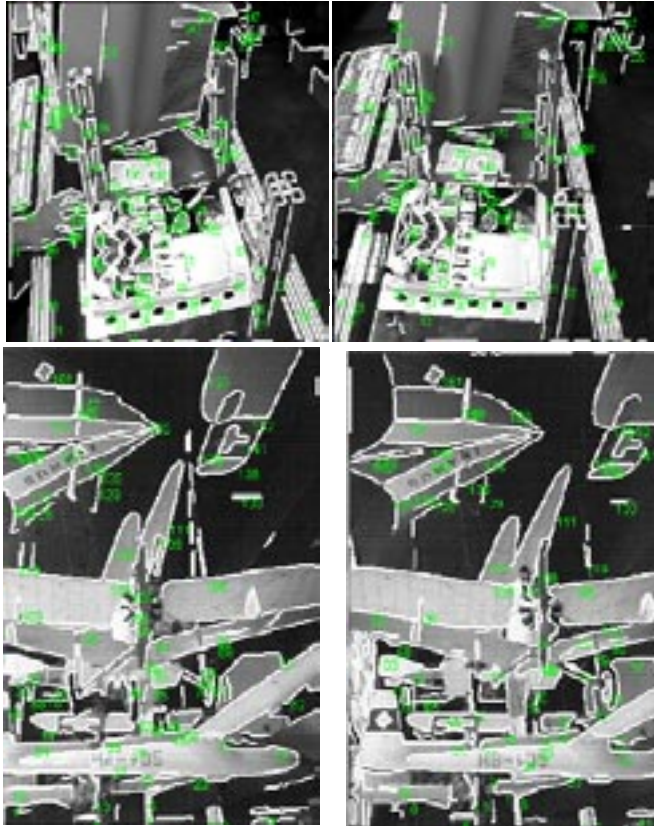
Finally, 112 contour correspondences are found as shown in Fig. 8. The elapsed time to match all contours is about 2.7 second.

Table 1.1

c\b a	0.7	0.74	0.78	0.82	0.86	0.9
0	77	74	67	58	37	13
2	25	24	24	23	14	
3	62	62	54	17		
4	22	17				
6	67	41	18	12		
9	50	32	23	17		
11	73	70	67	58	53	46
21	45	45	44	37	33	4
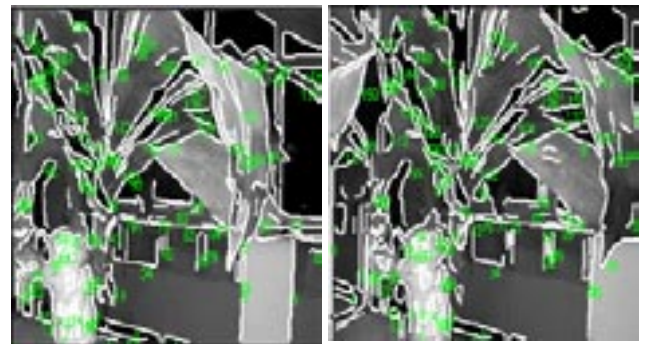
Fig.6 Matching result

Fig.8 Matching result

5. CONCLUSIONS

A robust cooperative strategy for matching contours has been presented in this paper. Based on the cooperative strategy, we have implemented a contour matching algorithm which allows us to match the contours robustly. The method is robust because it can deal with different contours with different similarities by adjusting the similarity threshold automatically. The performance of this matching algorithm strongly depends on the estimation of fundamental matrix and the success of the segmentation process of edge maps. The presented approach has been tested with various real images and the above experimental results show that our method can produce more accurate contour correspondences.

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