Chapters 21 & 22

Modern Portfolio Theory

&

Equilibrium Asset Pricing

"MODERN PORTFOLIO THEORY"

(aka "Mean-Variance Portfolio Theory", or "Markowitz Portfolio Theory" – Either way: "MPT" for short)

- **DEVELOPED IN 1950s (by MARKOWITZ, SHARPE, LINTNER)**

(Won Nobel Prize in Economics in 1990.)

- **WIDELY USED AMONG PROFESSIONAL INVESTORS**

- **FUNDAMENTAL DISCIPLINE OF PORTFOLIO-LEVEL INVESTMENT STRATEGIC DECISION MAKING.**

I. *REVIEW OF STATISTICS ABOUT PERIODIC TOTAL RETURNS:*
(Note: these are all "time-series" statistics: measured across time, not across assets within a single point in time.)

"1st Moment" Across Time (measures "central tendency"):
"MEAN", used to measure:

- ☞ Expected Performance ("ex ante", usually arithmetic mean: used in portf ana.)
- ☞ Achieved Performance ("ex post", usually geometric mean)

"2nd Moments" Across Time (measure characteristics of the deviation around the central tendancy). They include…

1) "STANDARD DEVIATION" (aka "volatility"), which measures:

- ☞ Square root of variance of returns across time.
- ☞ "Total Risk" (of exposure to asset if investor not diversified)

2) "COVARIANCE", which measures "Co-Movement", aka:

- ☞ "Systematic Risk" (component of total risk which cannot be "diversified away")
- ☞ Covariance with investor's portfolio measures asset contribution to portfolio total risk.

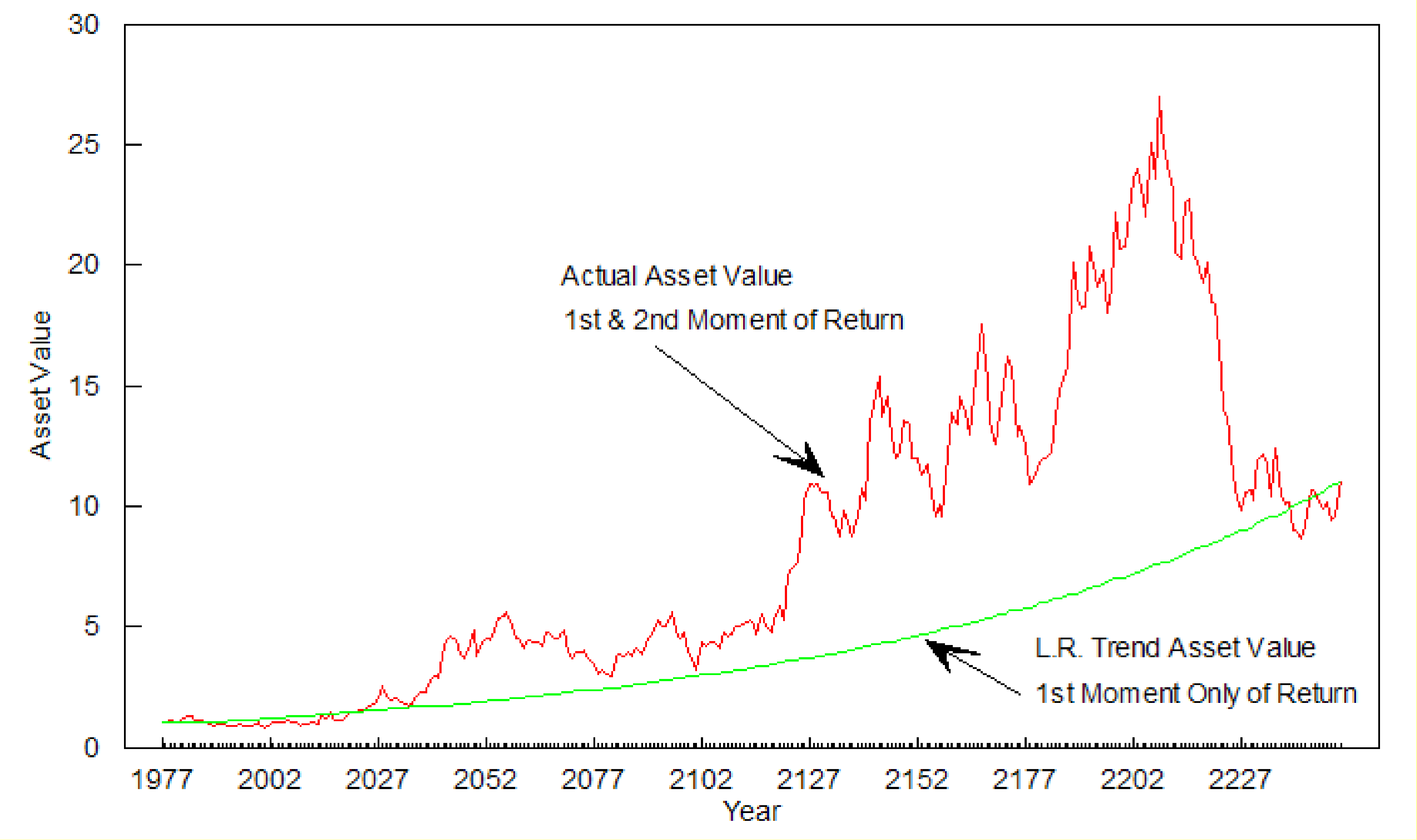
3) "CROSS-CORRELATION" (just "correlation" for short). Based on contemporaneous covariance between two assets or asset classes. Measures how two assets "move together":

- ☞ important for Portfolio Analysis.

4) "AUTOCORRELATION" (or "serial correlation": Correlation with itself across time), which reflects the nature of the "Informational Efficiency" in the Asset Market; e.g.:

- ☞ Zero ➔ "Efficient" Market (prices quickly reflect full information; returns lack predictability) ➔ Like securities markets (approximately).
- ☞ Positive ➔ "Sluggish" (inertia, inefficient) Market (prices only gradually incorporate new info.) ➔ Like private real estate markets.
- ☞ Negative ➔ "Noisy" Mkt (excessive s.r. volatility, price "overreactions") ➔ Like securities markets (to some extent).

"Picture" of **1st** and **2nd** Moments . . .



First Moment is "Trend“. Second Moment is "Deviation" around trend.

***Food for Thought Question*:**

IF THE TWO LINES ABOVE WERE TWO DIFFERENT ASSETS, WHICH WOULD YOU PREFER TO INVEST IN, OTHER THINGS BEING EQUAL? . . .

Historical statistics, annual periodic total returns:
Stocks, Bonds, Real Estate, 1970-2001…

	S&P500	*LTG Bonds*	*Private Real Estate*
Mean (arith)	13.30%	9.75%	9.65%
Std.Deviation	16.67%	11.95%	9.67%
Correlations:			
S&P500	100%	36.61%	11.83%
LTG Bonds		100%	-18.34%
Priv. Real Estate			100%

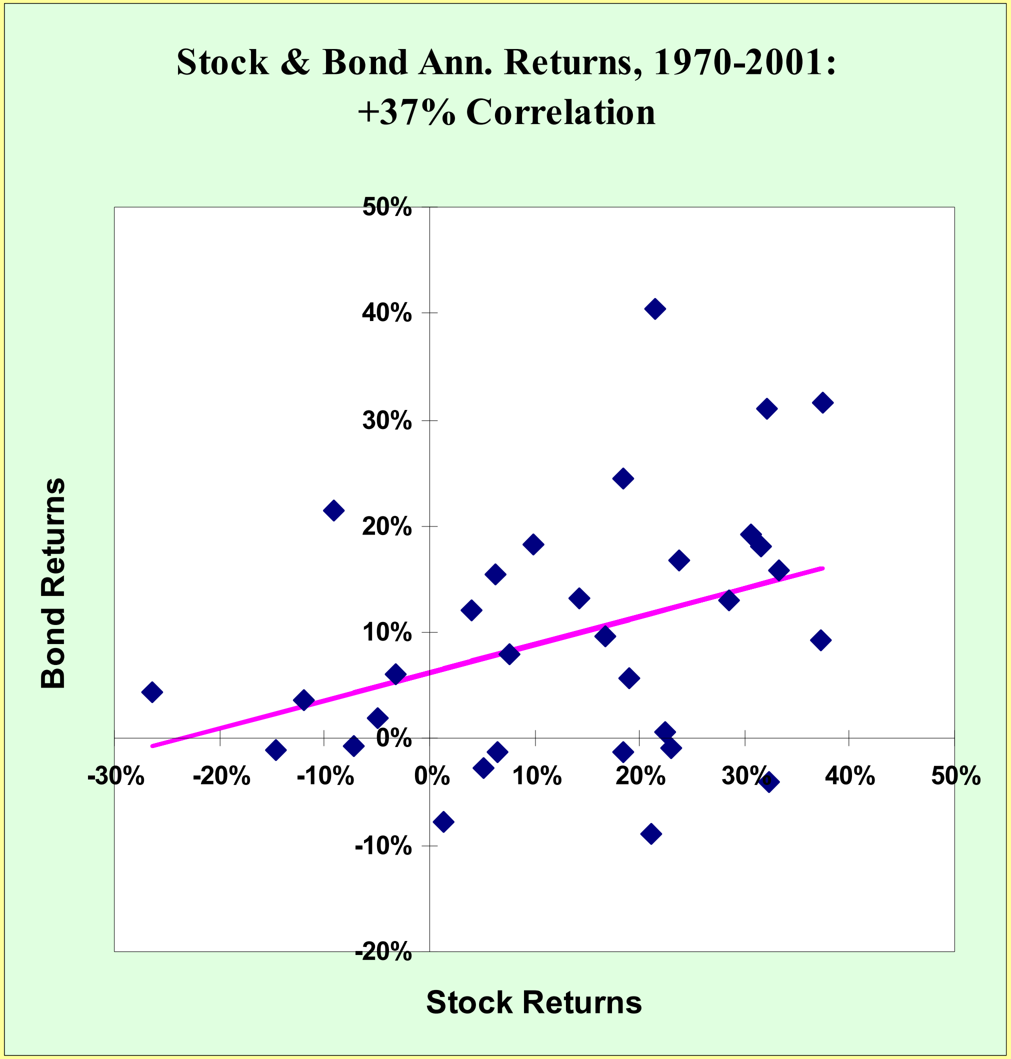
1st Moments

2nd Moments

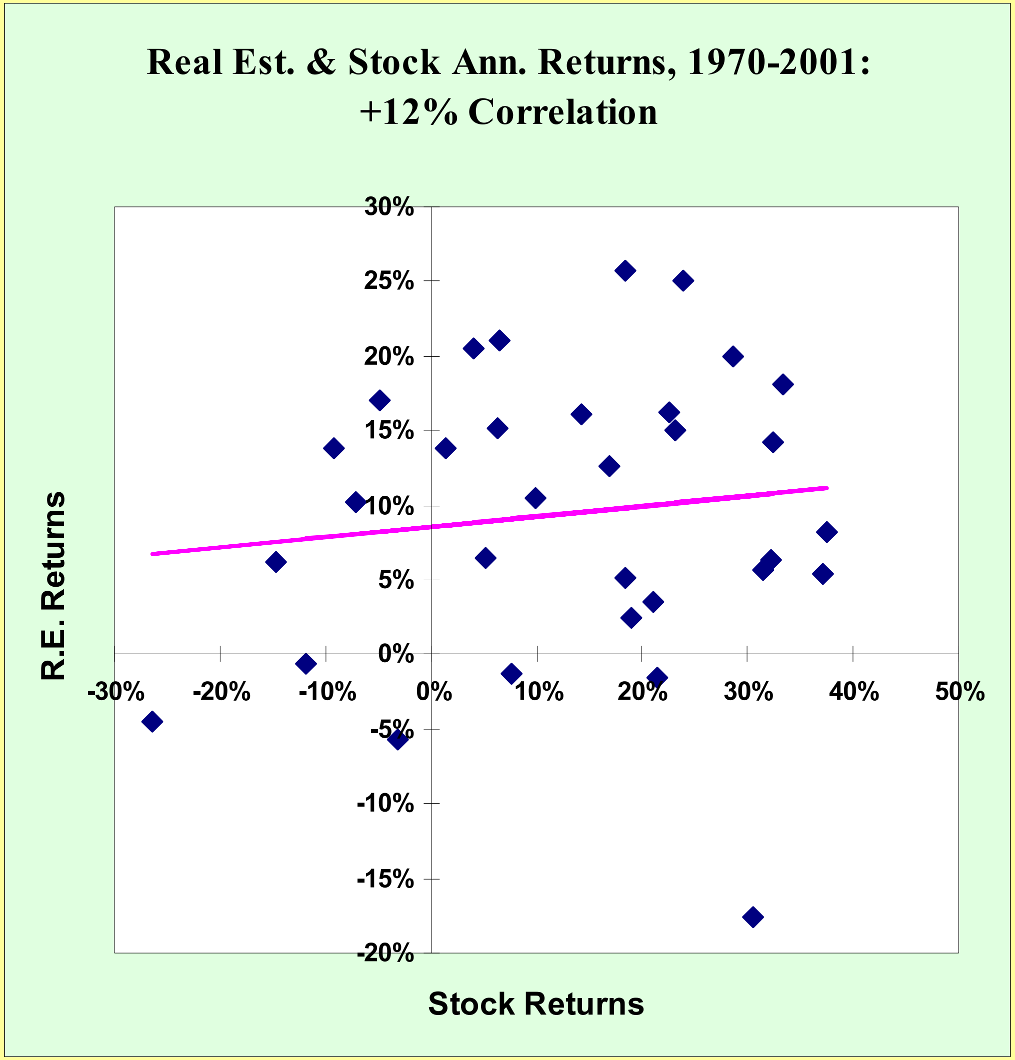
PORTFOLIO THEORY IS A WAY TO CONSIDER BOTH THE 1ST & 2ND MOMENTS (& INTEGRATE THE TWO) IN INVESTMENT ANALYSIS.

What do these historical 2nd moments (esp. the correlations) "look like"? . . .

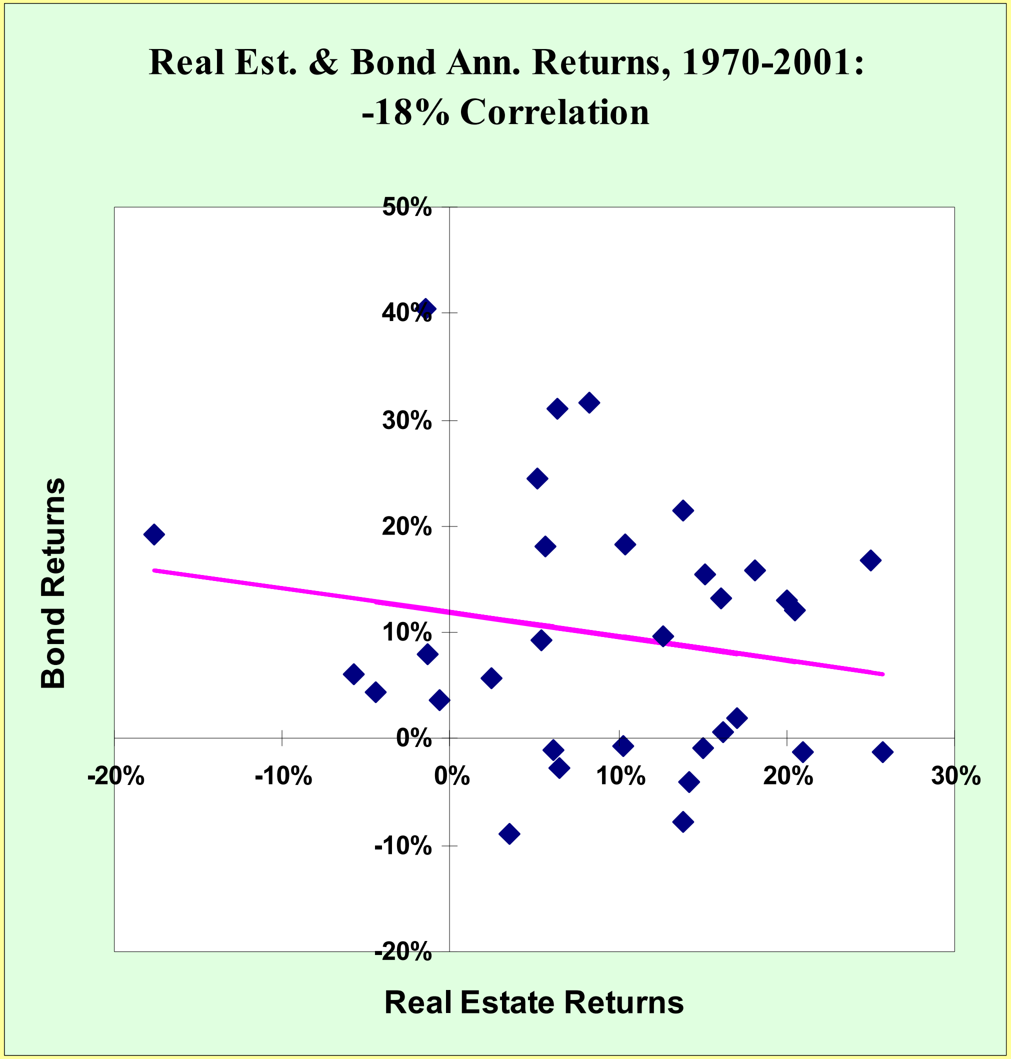
Stocks & bonds (+37% correlation): Each dot is one year's returns.



Stocks & real estate (+12% correlation): Each dot is one year's returns.



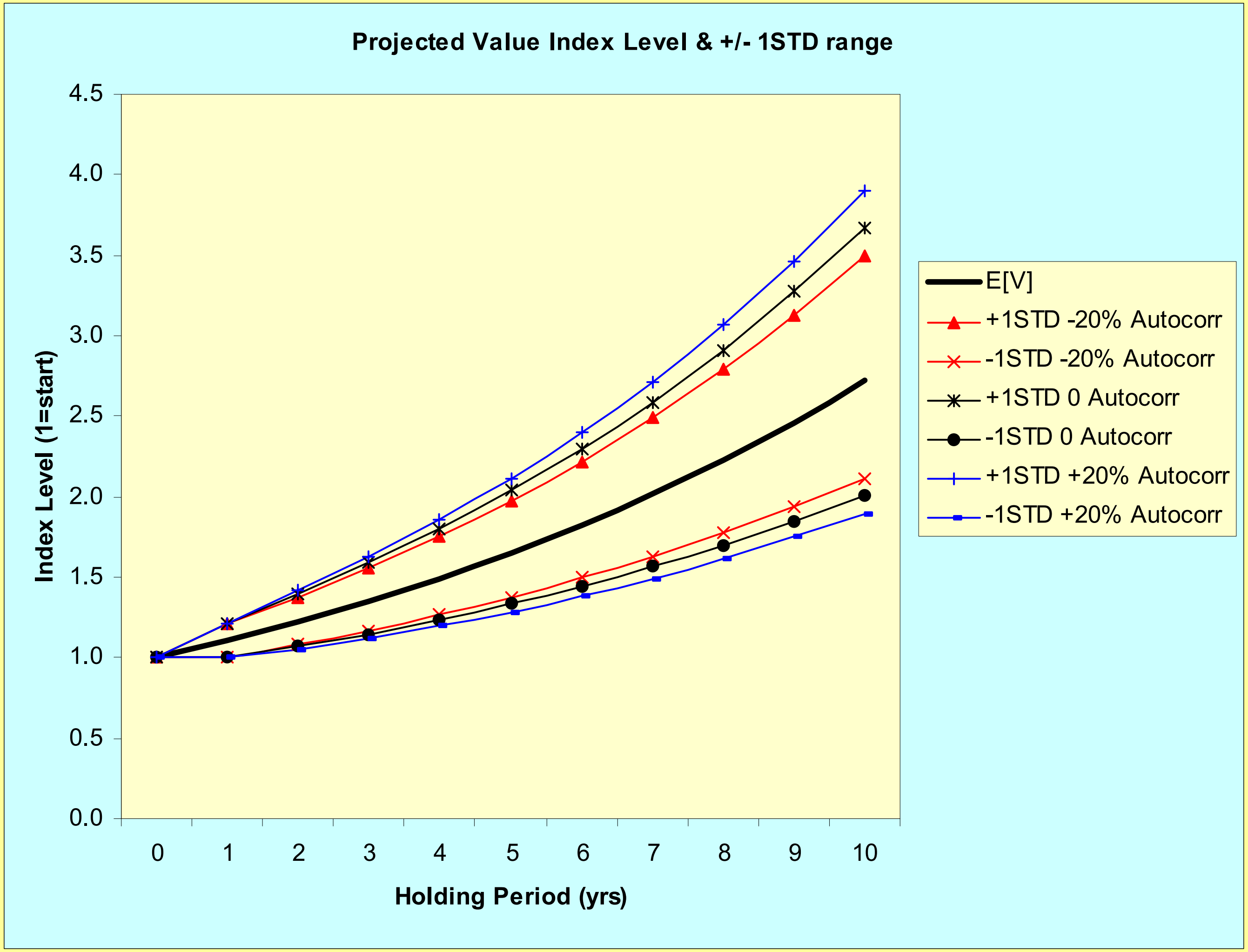
Bonds & real estate (-18% correlation): Each dot is one year's returns.



Why do you suppose there has been this negative correlation?

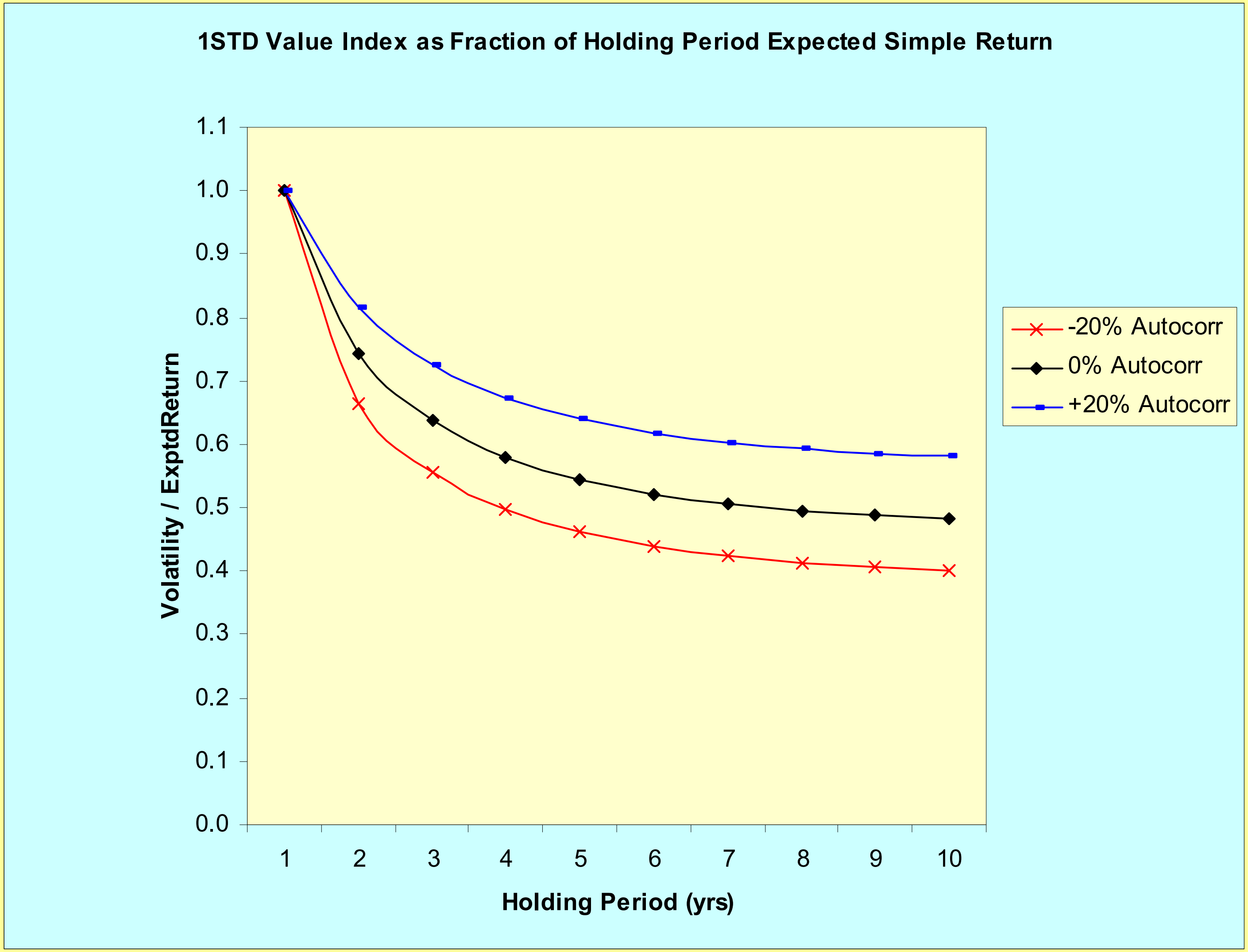
An important mathematical fact about investment risk & return . . .

"Normal" risk (volatility) accumulates roughly with the *SQUARE ROOT* of time (holding period)



An important mathematical fact about investment risk & return . . .

➔ *"Normal"* risk (volatility) as a *proportion of expected return* diminishes with the length of the expected holding period.



Thus, as far as "normal" risk is concerned:

- **The longer your investment holding horizon, the less important risk is to you, i.e.,**
- **You can afford to be more "aggressive" in your investments (less "risk averse"),**
- ***Other things being equal* (in particular, holding your fundamental risk preferences the same).**

What is "normal" risk? . . .

***"Normal"* risk is the regular, ordinary type of risk that always exists, every day, in the investment world, due to the fact that the future is uncertain and *"news"* is continuously arriving about the unfolding future.**

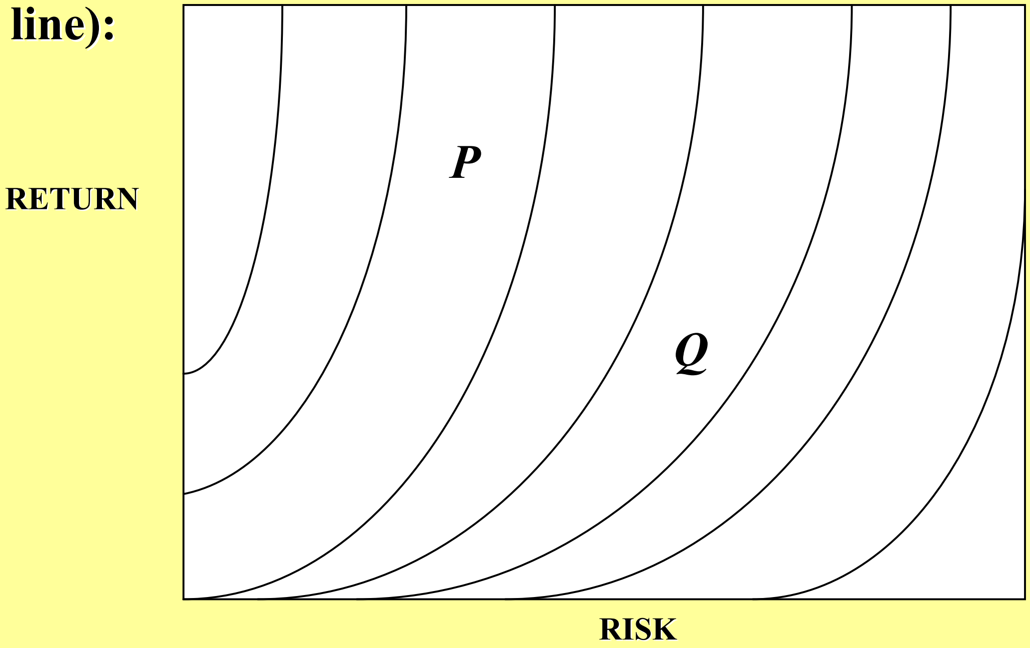
***"Normal"* risk is the dominant type of risk in modern, developed economies such as the U.S.**

***"Normal"* risk is the subject of MPT, and is well modeled statistically by the Normal probability distribution, by continuous time, and by periodic return time-series 2^{nd}-moment statistics such as variance, volatility (std.dev.), covariance, and *"beta"*.**

II. *WHAT IS PORTFOLIO THEORY?...*

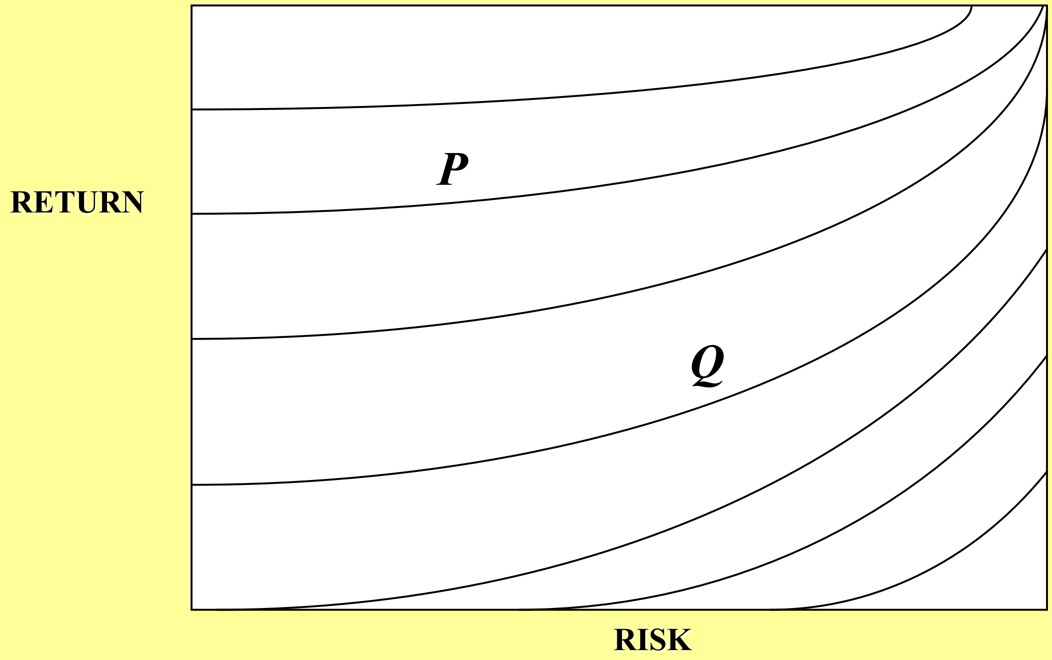
SUPPOSE WE DRAW A 2-DIMENSIONAL SPACE WITH RISK (2ND-MOMENT) ON HORIZONTAL AXIS AND EXPECTED RETURN (1ST MOMENT) ON VERTICAL AXIS.

A RISK-AVERSE INVESTOR MIGHT HAVE A UTILITY (PREFERENCE) SURFACE INDICATED BY CONTOUR LINES LIKE THESE (investor is indifferent along a given contour line):



THE CONTOUR LINES ARE STEEPLY RISING AS THE RISK-AVERSE INVESTOR WANTS MUCH MORE RETURN TO COMPENSATE FOR A LITTLE MORE RISK.

A MORE AGGRESSIVE INVESTOR MIGHT HAVE A UTILITY (PREFERENCE) SURFACE INDICATED BY CONTOUR LINES LIKE THESE.



THE SHALLOW CONTOUR LINES INDICATE THE INVESTOR DOES NOT NEED MUCH ADDITIONAL RETURN TO COMPENSATE FOR MORE RISK.

BUT BOTH INVESTORS WOULD AGREE THEY PREFER POINTS TO THE "NORTH" AND "WEST" IN THE RISK/RETURN SPACE. THEY BOTH PREFER POINT "P" TO POINT "Q".

FOR ANY TWO PORTFOLIOS "P" AND "Q" SUCH THAT:

EXPECTED RETURN "P" ≥ EXPECTED RETURN "Q"

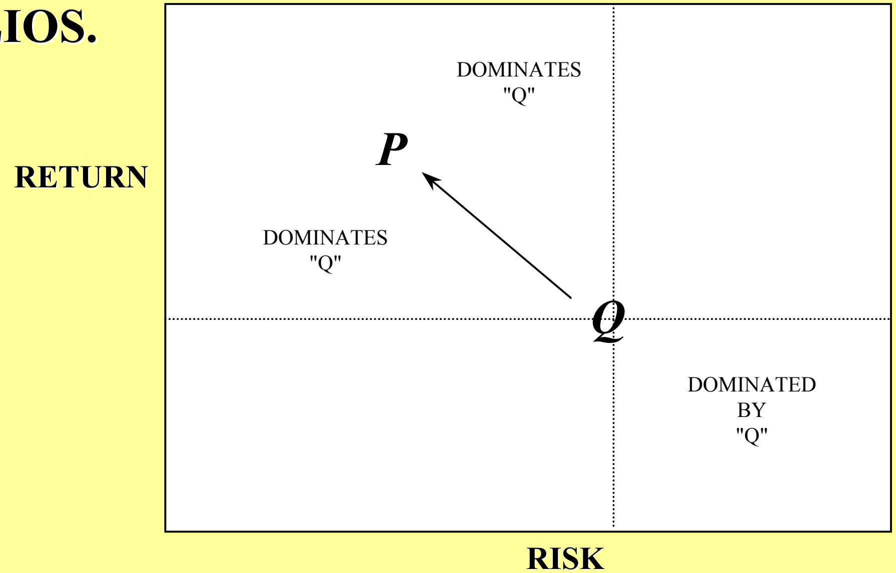
AND (SIMULTANEOUSLY): **RISK "P" ≤ RISK "Q"**

IT IS SAID THAT: "Q" IS *DOMINATED* BY "P".

THIS IS INDEPENDENT OF RISK PREFERENCES.

➔ BOTH CONSERVATIVE AND AGGRESSIVE INVESTORS WOULD AGREE ABOUT THIS.

IN ESSENCE, PORTFOLIO THEORY IS ABOUT HOW TO AVOID INVESTING IN DOMINATED PORTFOLIOS.



PORTFOLIO THEORY TRIES TO MOVE INVESTORS FROM POINTS LIKE "Q" TO POINTS LIKE "P".

III. *PORTFOLIO THEORY AND <u>DIVERSIFICATION</u>...*

"PORTFOLIOS" ARE "COMBINATIONS OF ASSETS".

PORTFOLIO THEORY FOR (or *from*) YOUR GRANDMOTHER:

"DON'T PUT ALL YOUR EGGS IN ONE BASKET!"

WHAT MORE THAN THIS CAN WE SAY? . . .

(e.g., <u>How many</u> "eggs" should we put in <u>which</u> "baskets".)

In other words,
GIVEN YOUR OVERALL INVESTABLE WEALTH, PORTFOLIO THEORY TELLS YOU HOW MUCH YOU SHOULD INVEST IN DIFFERENT TYPES OF ASSETS. FOR EXAMPLE:

WHAT % SHOULD YOU PUT IN REAL ESTATE?
WHAT % SHOULD YOU PUT IN STOCKS?

TO BEGIN TO RIGOROUSLY ANSWER THIS QUESTION, CONSIDER...

AT THE HEART OF PORTFOLIO THEORY ARE TWO BASIC MATHEMATICAL FACTS:

1) PORTFOLIO RETURN IS A LINEAR FUNCTION OF THE ASSET WEIGHTS:

$$r_P = \sum_{n=1}^{N} w_n r_n$$

IN PARTICULAR, THE PORTFOLIO EXPECTED RETURN IS A WEIGHTED AVERAGE OF THE EXPECTED RETURNS TO THE INDIVIDUAL ASSETS. E.G., WITH TWO ASSETS ("i" & "j"):

$$r_p = \omega r_i + (1-\omega) r_j$$

WHERE ω_i IS THE SHARE OF PORTFOLIO TOTAL VALUE INVESTED IN ASSET i.

e.g., If Asset A has $E[r_A]$=5% and Asset B has $E[r_B]$=10%, then a 50/50 Portfolio (50% A + 50% B) will have $E[r_P]$=7.5%.

THE 2ND FACT:

2) PORTFOLIO VOLATILITY IS A NON-LINEAR FUNCTION OF THE ASSET WEIGHTS:

$$VAR_P = \sum_{I=1}^{N}\sum_{J=1}^{N} w_i w_j COV_{ij}$$

SUCH THAT THE PORTFOLIO VOLATILITY IS <u>LESS THAN</u> A WEIGHTED AVERAGE OF THE VOLATILITIES OF THE INDIVIDUAL ASSETS. E.G., WITH TWO ASSETS:

$$s_P = \sqrt{[\,\omega^2(s_i)^2 + (1-\omega)^2(s_j)^2 + 2\omega(1-\omega)s_i s_j C_{ij}\,]}$$

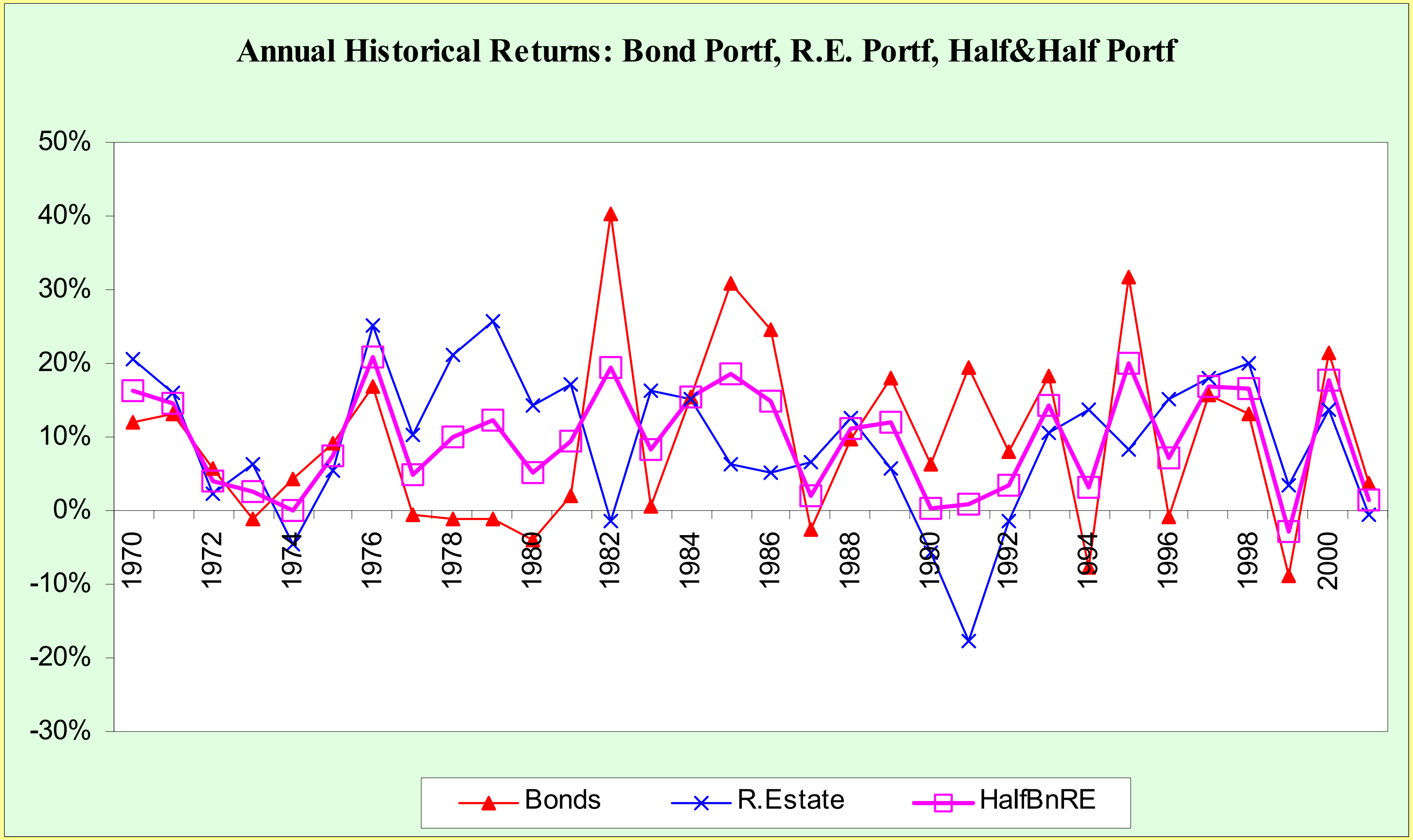
$$\leq \omega s_i + (1-\omega)s_j$$

WHERE s_i IS THE RISK (MEASURED BY STD.DEV.) OF ASSET i.

e.g., If Asset A has StdDev[r_A]=5% and Asset B has StdDev[r_B]=10%, then a 50/50 Portfolio (50% A + 50% B) will have StdDev[r_P] < 7.5% (conceivably even < 5%).

➔ This is the beauty of <u>Diversification</u>. It is at the core of Portfolio Theory. It is perhaps the only place in economics where you get a "free lunch": In this case, less risk without necessarily reducing your expected return!

For example, a portfolio of 50% bonds & 50% real estate would have had less volatility than either asset class alone during 1970-2001, but a very similar return:



Returns:	Bonds	R.Estate	Half&Half
Mean	9.7%	9.7%	9.7%
Std.Dev.	12.0%	9.7%	7.0%

This "Diversification Effect" is greater, the lower is the correlation among the assets in the portfolio.

NUMERICAL EXAMPLE . . .

SUPPOSE REAL ESTATE HAS:
EXPECTED RETURN = 8%
RISK (STD.DEV) = 10%

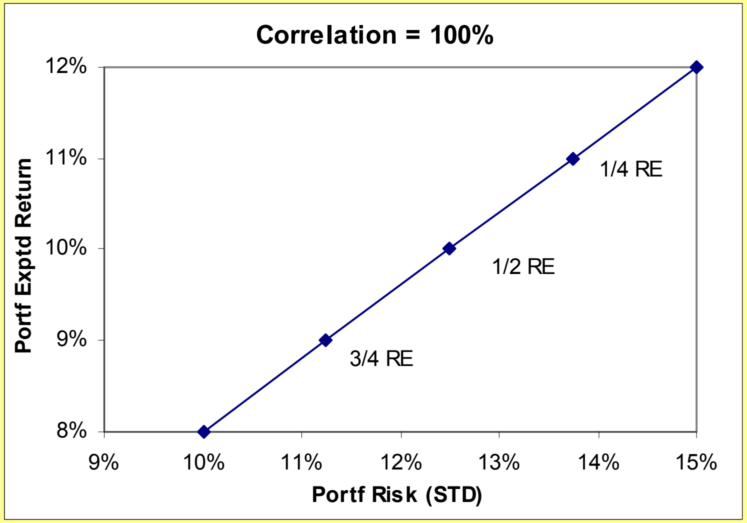
SUPPOSE STOCKS HAVE:
EXPECTED RETURN = 12%
RISK (STD.DEV) = 15%

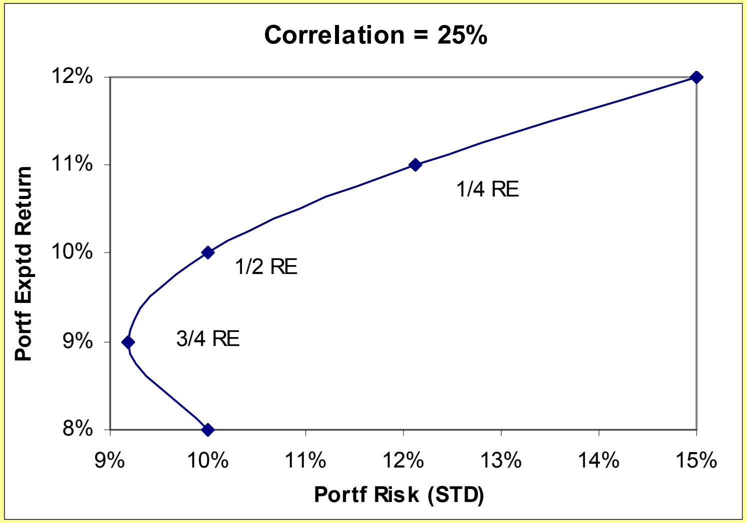
THEN A PORTFOLIO WITH ω SHARE IN REAL ESTATE & (1-ω) SHARE IN STOCKS WILL RESULT IN THESE RISK/RETURN COMBINATIONS, DEPENDING ON THE CORRELATION BETWEEN THE REAL ESTATE AND STOCK RETURNS:

	C = 100%		C = 25%		C = 0%		C = -50%	
ω	r_P	s_P	rP	sP	r_P	s_P	r_P	s_P
0%	12.0%	15.0%	12.0%	15.0%	12.0%	15.0%	12.0%	15.0%
25%	11.0%	13.8%	11.0%	12.1%	11.0%	11.5%	11.0%	10.2%
50%	10.0%	12.5%	10.0%	10.0%	10.0%	9.0%	10.0%	6.6%
75%	9.0%	11.3%	9.0%	9.2%	9.0%	8.4%	9.0%	6.5%
100%	8.0%	10.0%	8.0%	10.0%	8.0%	10.0%	8.0%	10.0%

where: C = Correlation Coefficient between Stocks & Real Estate.
(This table was simply computed using the formulas noted previously.)

This "Diversification Effect" is greater, the lower is the correlation among the assets in the portfolio.





IN ESSENCE,

PORTFOLIO THEORY ASSUMES:

YOUR OBJECTIVE FOR YOUR OVERALL WEALTH PORTFOLIO IS:

- **➔ MAXIMIZE EXPECTED FUTURE RETURN**
- **➔ MINIMIZE RISK IN THE FUTURE RETURN**

GIVEN THIS BASIC ASSUMPTION, AND THE EFFECT OF DIVERSIFICATION, WE ARRIVE AT THE FIRST MAJOR RESULT OF PORTFOLIO THEORY. . .

To the investor, the risk that matters in an investment is that investment's contribution to the risk in the investor's overall portfolio, not the risk in the investment by itself. This means that *covariance* (correlation and variance) may be as important as (or more important than) *variance* (or volatility) in the investment alone. *(e.g., if the investor's portfolio is primarily in stocks & bonds, and real estate has a low correlation with stocks & bonds, then the volatility in real estate may not matter much to the investor, because it will not contribute much to the volatility in the investor's portfolio. Indeed, it may allow a reduction in the portfolio's risk.)*

THIS IS A MAJOR SIGNPOST ON THE WAY TO FIGURING OUT "HOW MANY EGGS" WE SHOULD PUT IN WHICH "BASKETS".

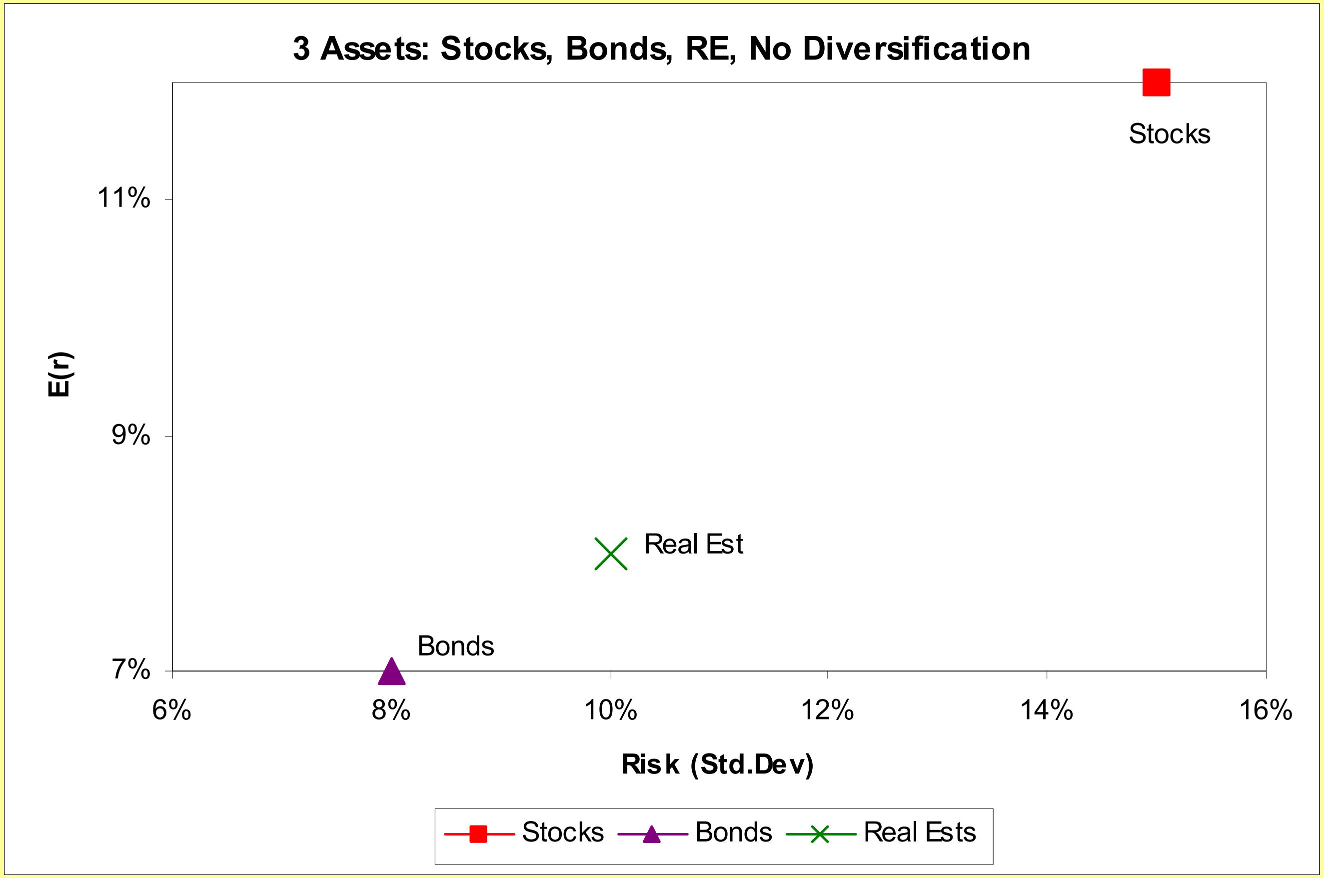
IV. *QUANTIFYING OPTIMAL PORTFOLIOS: STEP 1: FINDING THE "EFFICIENT FRONTIER". . .*

SUPPOSE WE HAVE THE FOLLOWING RISK & RETURN EXPECTATIONS (INCUDING CORRELATIONS):

	Stocks	Bonds	RE
Mean	12.00%	7.00%	8.00%
STD	15.00%	8.00%	10.00%
Corr			
Stocks	100.00%	40.00%	25.00%
Bonds		100.00%	0.00%
RE			100.00%

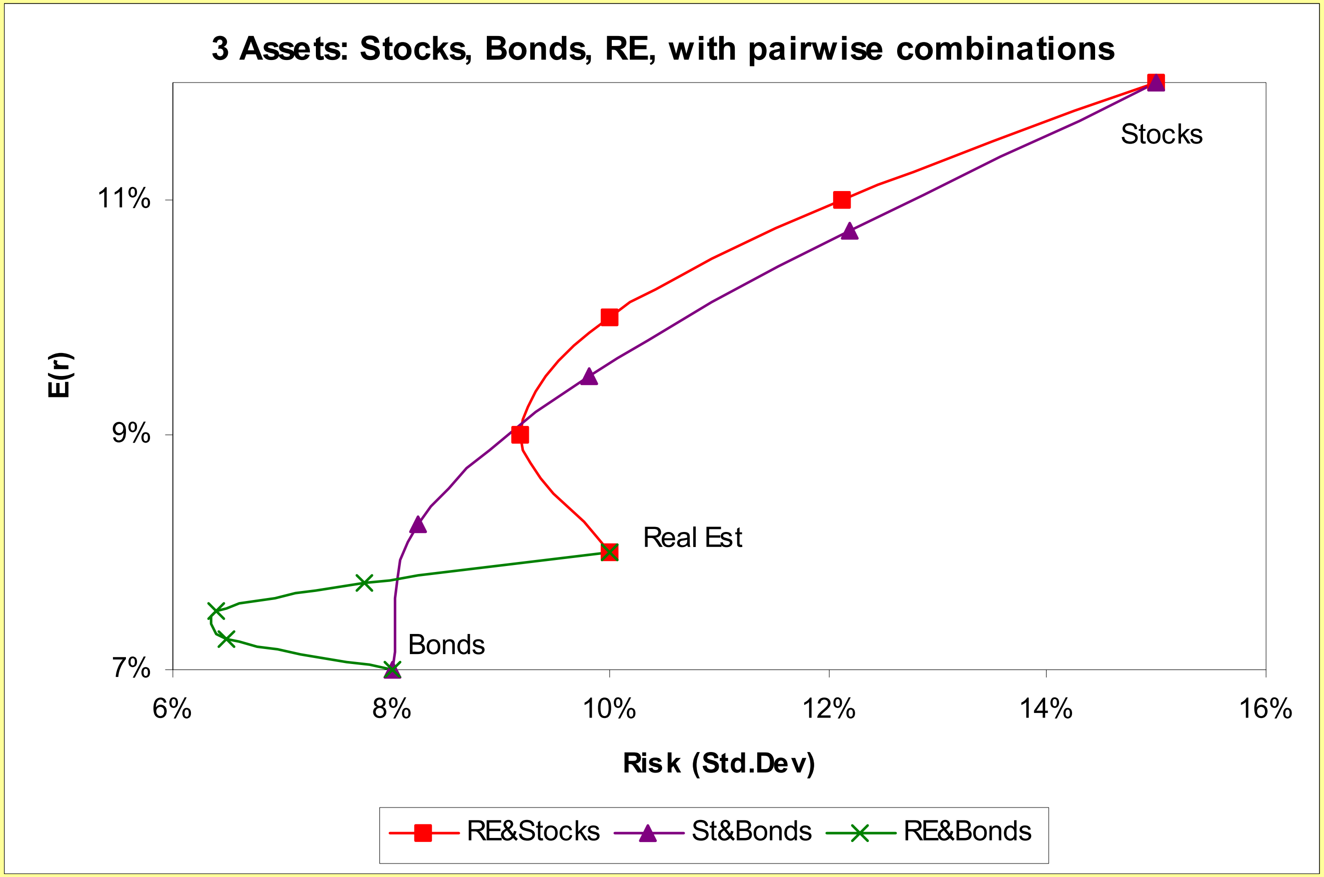
INVESTING IN ANY ONE OF THE THREE ASSET CLASSES WITHOUT DIVERSIFICATION ALLOWS THE INVESTOR TO ACHIEVE ONLY ONE OF THREE POSSIBLE RISK/RETURN POINTS…

INVESTING IN ANY ONE OF THE THREE ASSET CLASSES WITHOUT DIVERSIFICATION ALLOWS THE INVESTOR TO ACHIEVE ONLY ONE OF THE THREE POSSIBLE RISK/RETURN POINTS DEPICTED IN THE GRAPH BELOW…

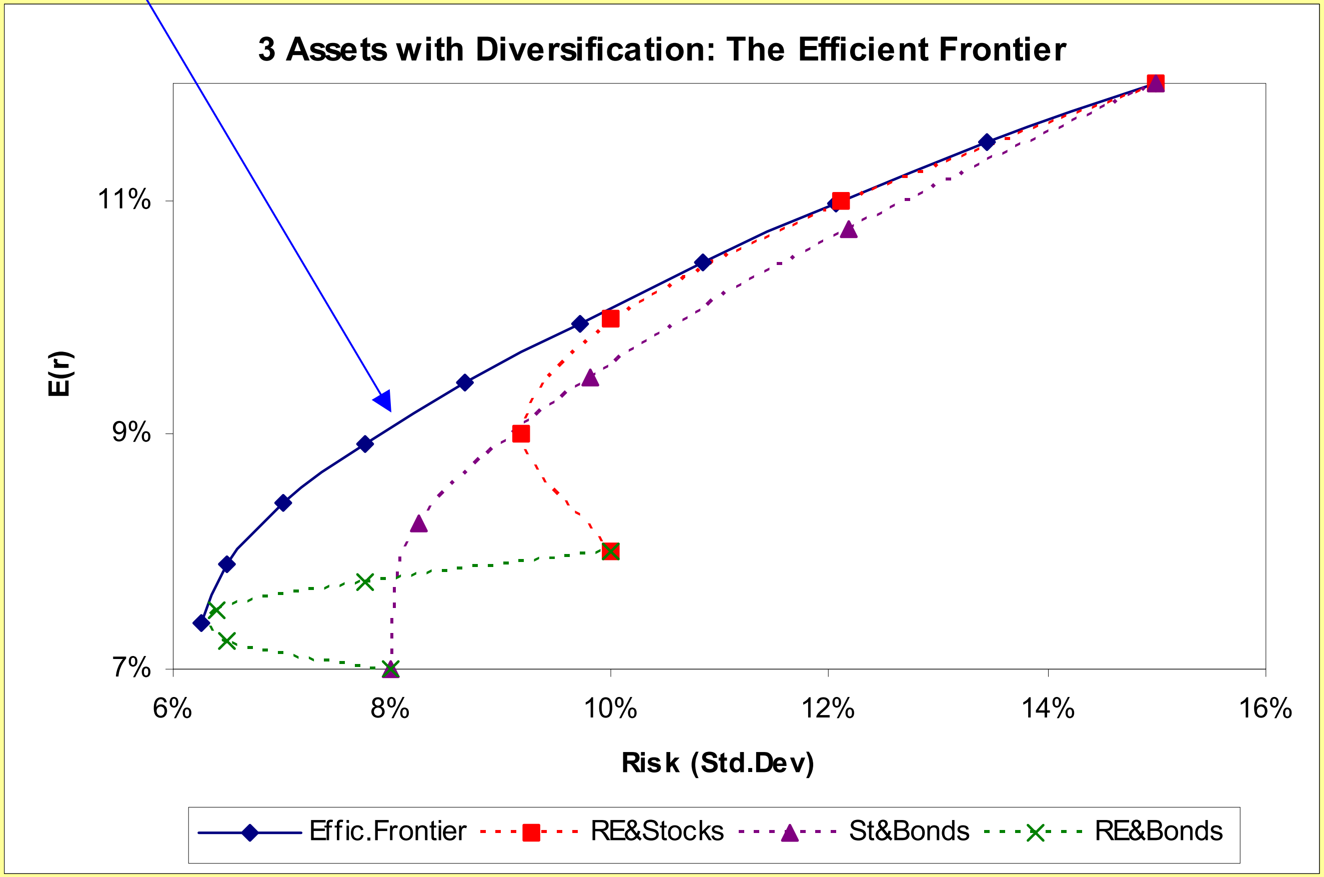


IN A RISK/RETURN CHART LIKE THIS, ONE WANTS TO BE ABLE TO GET AS MANY RISK/RETURN COMBINATIONS AS POSSIBLE, ***AS FAR TO THE "NORTH" AND "WEST" AS POSSIBLE.***

ALLOWING PAIRWISE COMBINATIONS (AS WITH OUR PREVIOUS STOCKS & REAL ESTATE EXAMPLE), INCREASES THE RISK/RETURN POSSIBILITIES TO THESE…



FINALLY, IF WE ALLOW UNLIMITED DIVERSIFICATION AMONG ALL THREE ASSET CLASSES, WE ENABLE AN INFINITE NUMBER OF COMBINATIONS, THE "BEST" (I.E., MOST "NORTH" AND "WEST") OF WHICH ARE SHOWN BY THE OUTSIDE (ENVELOPING) CURVE.



THIS IS THE "**EFFICIENT FRONTIER**" IN THIS CASE (OF THREE ASSET CLASSES).

IN PORTFOLIO THEORY THE *"EFFICIENT FRONTIER"* CONSISTS OF ALL ASSET COMBINATIONS (PORTFOLIOS) WHICH MAXIMIZE RETURN AND MINIMIZE RISK.

THE EFFICIENT FRONTIER IS AS FAR "NORTH" AND "WEST" AS YOU CAN POSSIBLY GET IN THE RISK/RETURN GRAPH.

A PORTFOLIO IS SAID TO BE *"EFFICIENT"* (i.e., represents one point on the efficient frontier) IF IT HAS THE MINIMUM POSSIBLE VOLATILITY FOR A GIVEN EXPECTED RETURN, AND/OR THE MAXIMUM EXPECTED RETURN FOR A GIVEN LEVEL OF VOLATILITY.

(Terminology note: This is a different definition of "efficiency" than the concept of informational efficiency applied to asset markets and asset prices.)

SUMMARY UP TO HERE:

DIVERSIFICATION AMONG RISKY ASSETS ALLOWS:

- **GREATER EXPECTED RETURN TO BE OBTAINED FOR ANY GIVEN RISK EXPOSURE, &/OR;**
- **LESS RISK TO BE INCURRED FOR ANY GIVEN EXPECTED RETURN TARGET.**

(This is called getting on the "efficient frontier".)

PORTFOLIO THEORY ALLOWS US TO:

- **QUANTIFY THIS EFFECT OF DIVERSIFICATION**
- **IDENTIFY THE "OPTIMAL" (BEST) MIXTURE OF RISKY ASSETS**

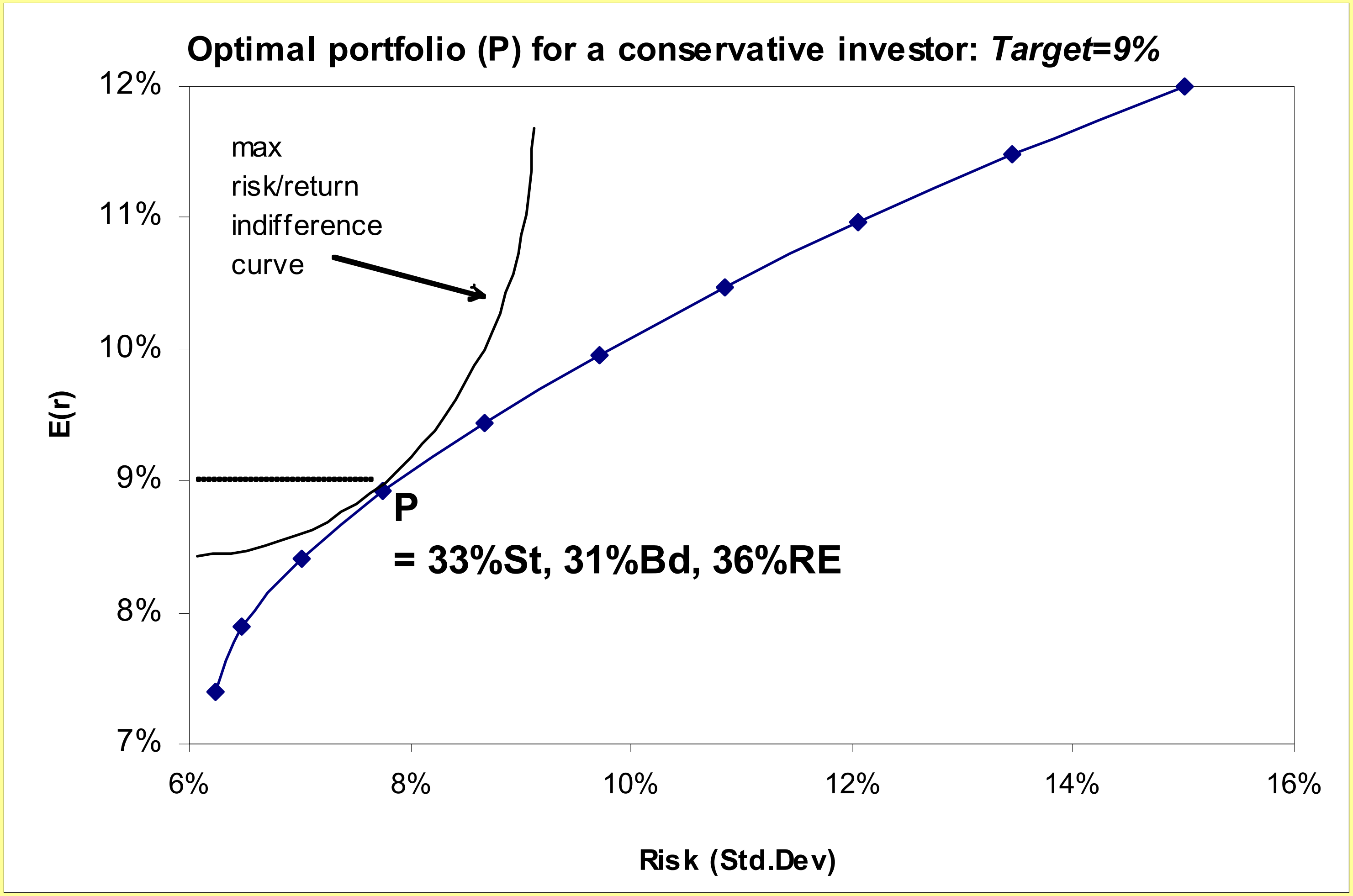
MATHEMATICALLY, THIS IS A "CONSTRAINED OPTIMIZATION" PROBLEM

==> Algebraic solution using calculus

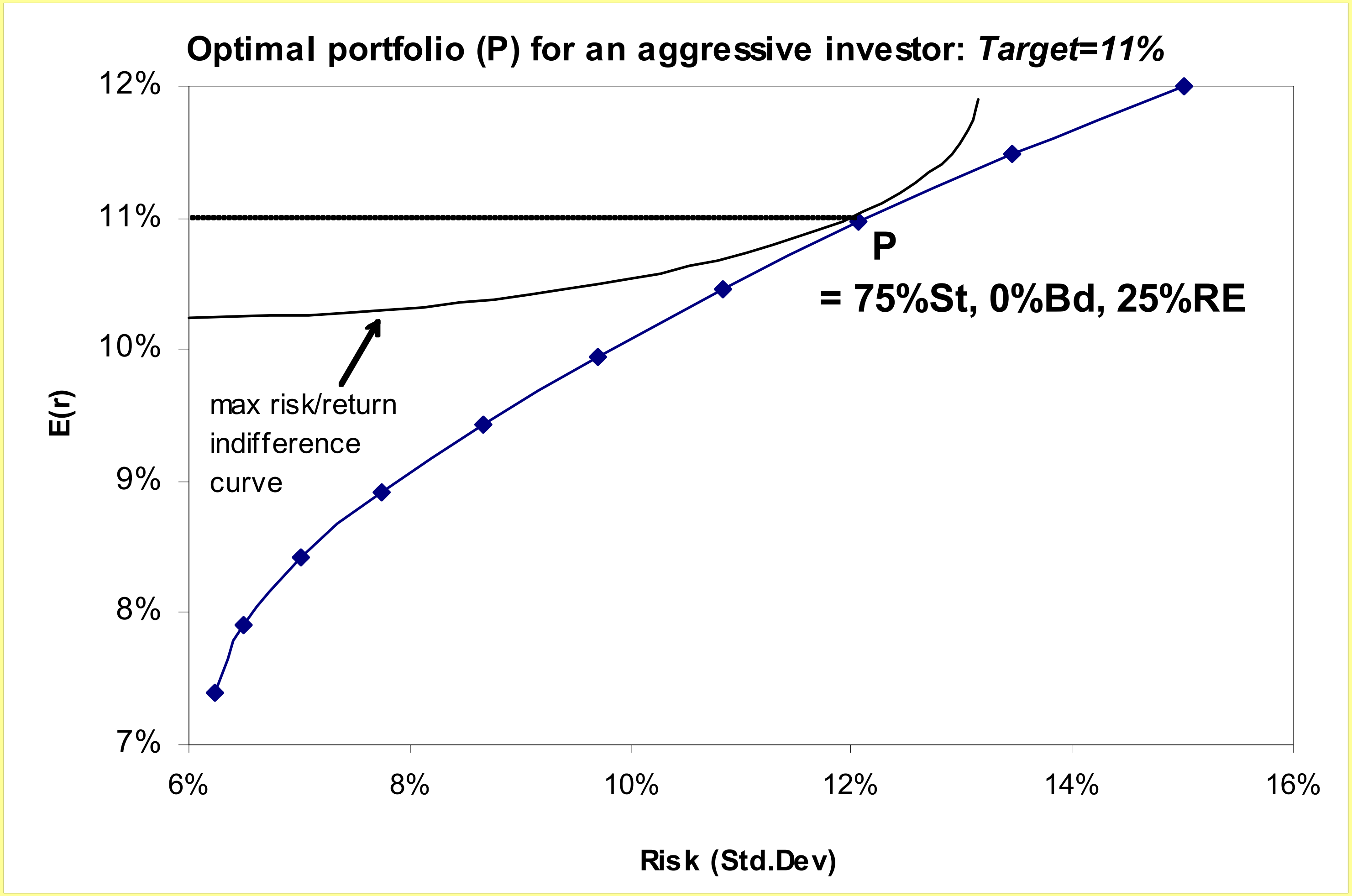
==> Numerical solution using computer and "quadratic programming". Spreadsheets such as Excel include "Solvers" that can find optimal portfolios this way.

STEP 2) PICK A RETURN TARGET FOR YOUR OVERALL WEALTH THAT REFLECTS YOUR RISK PREFERENCES...

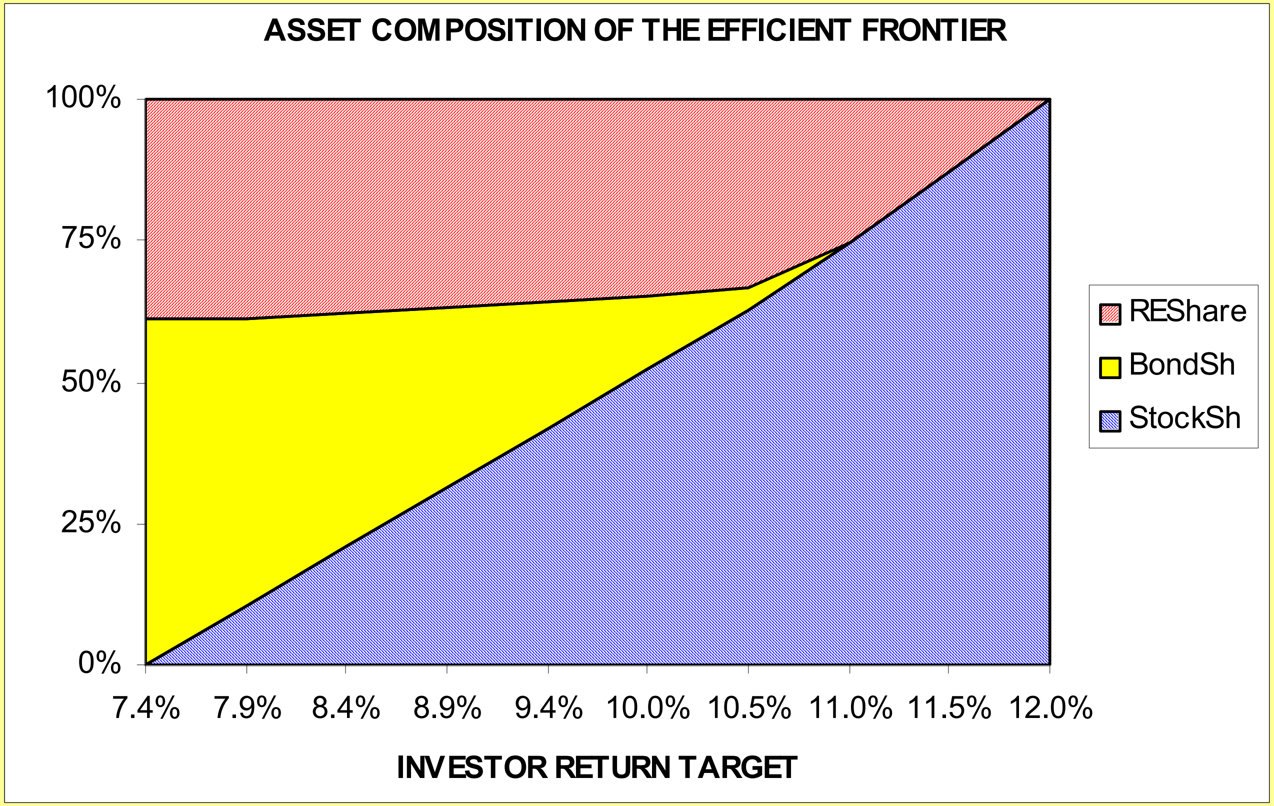
E.G., ARE YOU HERE (9%)?...



OR ARE YOU HERE (11%)?...



HERE IS A GRAPH OF THE OPTIMAL PORTFOLIO SHARES AS A FUNCTION OF THE INVESTOR'S RETURN TARGET:



CONSERVATIVE INVESTORS (E.G., PENSION FUNDS) WOULD TYPICALLY PICK A RETURN TARGET (HORIZONTAL AXIS) THAT WOULD PUT THEM IN OR AROUND THE MIDDLE OR LEFT HALF OF THIS GRAPH.

V. *GENERAL QUALITATIVE RESULTS OF PORTFOLIO THEORY*

1) THE OPTIMAL REAL ESTATE SHARE DEPENDS ON HOW CONSERVATIVE OR AGGRESSIVE IS THE INVESTOR;

2) FOR MOST OF THE RANGE OF RETURN TARGETS, REAL ESTATE IS A SIGNIFICANT SHARE. (COMPARE THESE SHARES TO THE AVERAGE PENSION FUND REAL ESTATE ALLOCATION WHICH IS LESS THAN 5%. THIS IS WHY PORTFOLIO THEORY HAS BEEN USED TO TRY TO GET INCREASED PF ALLOCATION TO REAL ESTATE.)

3) THE ROBUSTNESS OF REAL ESTATE'S INVESTMENT APPEAL IS DUE TO ITS LOW CORRELATION WITH BOTH STOCKS & BONDS, THAT IS, WITH ALL OF THE REST OF THE PORTFOLIO. (NOTE IN PARTICULAR THAT OUR INPUT ASSUMPTIONS IN THE ABOVE EXAMPLE NUMBERS DID *NOT* INCLUDE A PARTICULARLY HIGH RETURN OR PARTICULARLY LOW VOLATILITY FOR THE REAL ESTATE ASSET CLASS. THUS, THE LARGE REAL ESTATE SHARE IN THE OPTIMAL PORTFOLIO MUST NOT BE DUE TO SUCH ASSUMPTIONS.)

VI. Technical aside…

Opening the "black box": Nuts & bolts of Mean-Variance Portfolio Theory...

THE THREE STEPS IN CALCULATING EFFICIENT PORTFOLIOS:

A) INPUT INVESTOR EXPECTATIONS:

We need the following input information:

1) **Mean** (i.e., expected) return for each asset;
2) **Volatility** (i.e., Standard Deviation of Returns across time) for each asset class;
3) **Correlation** coefficients between each pair of asset classes.

B) ENTER COMPUTATION FORMULAS INTO THE SPREADSHEET:

We need the following mathematical formulas and tools . . .

(These are the same formulas we have previously noted.)

1) The formula for the return of a portfolio (& for portfolio expected return as a function of constituent assets expected returns):

$$r_P = \sum_{n=1}^{N} w_n r_n$$

(The weighted avg of the constituent returns, where the weights, w_n, sum to 1.)

2) The formula for the variance (volatility squared) of a portfolio:

$$VAR_P = \sum_{I=1}^{N} \sum_{J=1}^{N} w_i w_j COV_{ij}$$

where:

VAR_P = PORTFOLIO RETURN VARIANCE OF A PORTFOLIO WITH N ASSETS,

w_J = WEIGHT (PORTFOLIO VALUE SHARE) IN ASSET "j",

COV_{ij} = COVARIANCE BETWEEN THE RETURNS TO ASSETS "i" AND "j".

Note that:

$COV_{ij} = s_i s_j C_{ij}$, where s_i is STDev of i and C_{ij} is Correlation Coefficient between i and j.

$COV_{ii} = VAR_I = s_i^2$.

C) INVOKE THE COMPUTER'S "SOLVER" ROUTINE.

(See "portfo1.xls", downloadable from the course web site. This spreadsheet will solve portfolio problems for up to 7 different assets or asset classes.)

EXAMPLE:

SAME RISK & RETURN ASSUMPTIONS AS BEFORE:

	Stocks	Bonds	RE
Mean	12.00%	7.00%	8.00%
STD	15.00%	8.00%	10.00%
Corr			
Stocks	100.00%	40.00%	25.00%
Bonds		100.00%	0.00%
RE			100.00%

SUPPOSE PORTFOLIO *TARGET RETURN* = 9%.

WHAT WEIGHTS IN STOCKS, BONDS, REAL ESTATE WILL MEET THIS TARGET WITH MINIMUM PORTFOLIO VOLATILITY (VARIANCE)?...

STEP 1: COMPUTE VARIANCE FOR A STARTING PORTFOLIO (SAY, EQUAL (1/3) WEIGHTS IN EACH ASSET CLASS)…

StdDevs for stocks, bonds, R.E.,(s_i):

15.00%	8.00%	10.00%

Correlation matrix (C_{ij}):

1.00	0.40	0.25
0.40	1.00	0.00
0.25	0.00	1.00

Covariance matrix ($COV_{ij}=C_{ij}s_is_j$):

0.02250	0.00480	0.00375
0.00480	0.00640	0.00000
0.00375	0.00000	0.01000

e.g., .0048 = (0.40)(0.15)(0.08).

Portfolio S,B,RE shares (w_i):

0.3333	0.3333	0.3333

Weighted covariance matrix $(w_iw_jCOV_{ij})$:

0.00250	0.00053	0.00042
0.00053	0.00071	0.00000
0.00042	0.00000	0.00111

e.g., .00053 = (.33)(.33)(.0048).

Portfolio variance is sum of all nine cells in this matrix:
.0025+.00053+.00042
+.00053+.00071+.0000
+.00042+.0000+.00111 = .0062

Portvolio volatility (STD) = SQRT(.0062) = .0789 = 7.89%

STEP 2: DETERMINE WHICH ASSET CLASS CONTRIBUTES MOST TO THIS VARIANCE, AND WHICH CONTRIBUTES LEAST, PER UNIT OF ITS WEIGHT IN THE PORTFOLIO…

Vertical sums down the columns (or the horizontal sums across the rows) of the weighted covariance matrix give covariances between each asset and portfolio.

e.g., covariance of stock investment with portfolio is:
0.00345 = .0025+.00053+.00042.
This is contribution of stock investment in portfolio variance.

Normalizing per unit of its weight in the portfolio, stock contribution to portfolio variance is:
0.00345/0.333 = 0.01035.
Normalized real estate contribution is:
(.00042+.00000+.00111)/0.333 = 0.00153/0.333 = 0.00458.

How does this suggest we could reduce the variance of this portfolio?

Answer: Reduce stock share and increase real estate share…

STEP 3: TRY VARIOUS COMBINATIONS OF ASSET CLASS WEIGHTS UNTIL MINIMUM-VARIANCE COMBINATION IS FOUND (SUBJECT TO TARGET RETURN CONSTRAINT)…

Repeat the above steps, modifying the asset weights according to an efficient algorithm, increasing asset classes that reduce variance and decreasing those that increase variance, in proportions so as to preserve the 9% portfolio return = $w_{ST}12\% + w_{BD}7\% + w_{RE}8\% = 9\%$ target.

Computer's *"Solver"* has an algorithm to do this efficiently, and can work *very fast*.

STEP 4: IF YOU WANT TO GENERATE THE ENTIRE "EFFICIENT FRONTIER", THEN REPEAT THE ABOVE STEPS FOR A SERIES OF DIFFERENT TARGET RETURNS…

THE EFFICIENT FRONTIER USING OUR PREVIOUS RISK/RETURN ASSUMPTIONS FOR THE THREE MAJOR ASSET CLASSES:

Three asset efficient frontier, given:				
Input data assumptions:				
	Stocks	Bonds	RE	
Mean Return	12.00%	7.00%	8.00%	
STD (vol.)	15.00%	8.00%	10.00%	
Correlation:				
Stocks	100.00%	40.00%	25.00%	
Bonds		100.00%	0.00%	
RE			100.00%	

Efficient Frontier:				
E(rP)	sP	StockSh	BondSh	REShare
7.39%	6.25%	0.00%	60.98%	39.02%
7.90%	6.48%	10.32%	51.02%	38.66%
8.41%	7.01%	20.76%	41.59%	37.64%
8.93%	7.76%	31.21%	32.16%	36.63%
9.44%	8.67%	41.66%	22.73%	35.61%
9.95%	9.71%	52.11%	13.30%	34.60%
10.46%	10.84%	62.55%	3.87%	33.58%
10.98%	12.06%	74.39%	0.00%	25.61%
11.49%	13.46%	87.20%	0.00%	12.80%
12.00%	15.00%	100.00%	0.00%	0.00%

(See if you can get the "portfo1.xls" spreadsheet to generate this efficient frontier using the Excel Solver...)

SOME NAGGING QUESTIONS ABOUT MPT . . .

HOW SENSITIVE ARE THE RESULTS TO OUR INPUT *ASSUMPTIONS* (RISK & RETURN EXPECTATIONS), AND HOW REALISTIC ARE THOSE EXPECTATIONS?

WHAT IS *LEFT OUT* OF THIS MODEL, AND HOW COULD YOU TRY TO INCORPORATE THESE OMISSIONS?

- **TRANSACTION COSTS?**
- **LIQUIDITY CONCERNS?**

CAN YOU *"GAME"* PORTFOLIO THEORY BY REDEFINING THE NUMBER AND DEFINITION OF "ASSET CLASSES"?

Watch out for "silly" results (e.g., putting conservative investors in poor performing investments). When applying portfolio theory, don't check your <u>common sense</u> at the door.

FOR EXAMPLE, DOES IT REALLY MAKE SENSE TO PUT SO LITTLE INTO STOCKS JUST BECAUSE YOU HAVE A CONSERVATIVE RETURN TARGET, EVEN THOUGH STOCKS PROVIDE A SUPERIOR RETURN RISK PREMIUM PER UNIT OF RISK?…

WHAT ABOUT LEVERAGE?...

SOME OF THESE QUESTIONS CAN BE ADDRESSED BY A NEAT TRICK, AN EXTENSION TO THE ABOVE-DESCRIBED PORTFOLIO THEORY...

VII. *INTRODUCING A "RISKLESS ASSET"...*

IN A COMBINATION OF A **RISKLESS** AND A RISKY ASSET, BOTH RISK AND RETURN ARE WEIGHTED AVERAGES OF RISK AND RETURN OF THE TWO ASSETS:

Recall:

$$s_P = \sqrt{[}\ \omega^2(s_i)^2 + (1-\omega)^2(s_j)^2 + 2\omega(1-\omega)s_i s_j C_{ij}\ ]$$

If s_j=0, this reduces to:

$$s_P = \sqrt{[}\ \omega^2(s_i)^2 = \omega s_i$$

SO THE RISK/RETURN COMBINATIONS OF A MIXTURE OF INVESTMENT IN A RISKLESS ASSET AND A RISKY ASSET LIE ON A **STRAIGHT LINE**, PASSING THROUGH THE TWO POINTS REPRESENTING THE RISK/RETURN COMBINATIONS OF THE RISKLESS ASSET AND THE RISKY ASSET.

➔ IN PORTFOLIO ANALYSIS, THE "RISKLESS ASSET" REPRESENTS **BORROWING** OR **LENDING** BY THE INVESTOR…

BORROWING IS LIKE "SELLING SHORT" OR HOLDING A NEGATIVE WEIGHT IN THE RISKLESS ASSET. *BORROWING IS "RISKLESS" BECAUSE YOU MUST PAY THE MONEY BACK "NO MATTER WHAT".*

LENDING IS LIKE BUYING A BOND OR HOLDING A POSITIVE WEIGHT IN THE RISKLESS ASSET. *LENDING IS "RISKLESS" BECAUSE YOU CAN INVEST IN GOVT BONDS AND HOLD TO MATURITY.*

SUPPOSE YOU COMBINE RISKLESS BORROWING OR LENDING WITH YOUR INVESTMENT IN THE RISKY PORTFOLIO OF STOCKS & REAL ESTATE.

YOUR OVERALL EXPECTED RETURN WILL BE:

$$r_W = vr_P + (1-v)r_f$$

AND YOUR OVERALL RISK WILL BE:

$$s_W = vs_P + (1-v)0 = vs_P$$

Where: v = Weight in risky portfolio
r_W, s_W = Return, Std.Dev., in overall wealth
r_P, s_P = Return, Std.Dev., in risky portfolio
r_f = Riskfree Interest Rate

v NEED NOT BE CONSTRAINED TO BE LESS THAN UNITY.
v CAN BE GREATER THAN 1 ("leverage" , "borrowing"), OR
v CAN BE LESS THAN 1 BUT POSITIVE ("lending", investing in bonds, in addition to investing in the risky portfolio).

THUS, USING BORROWING OR LENDING, IT IS POSSIBLE TO OBTAIN ANY RETURN TARGET OR ANY RISK TARGET. THE RISK/RETURN COMBINATIONS WILL LIE ON THE STRAIGHT LINE PASSING THROUGH POINTS r_f AND r_P.

NUMERICAL EXAMPLE

SUPPOSE:
RISKFREE INTEREST RATE = 5%
STOCK EXPECTED RETURN = 15%
STOCK STD.DEV. = 15%

IF RETURN TARGET = 20%,

BORROW $0.5
INVEST $1.5 IN STOCKS (v = 1.5).

EXPECTED RETURN WOULD BE:

$$(1.5)15\% + (-0.5)5\% = 20\%$$

RISK WOULD BE

$$(1.5)15\% + (-0.5)0\% = 22.5\%$$

IF RETURN TARGET = 10%,

LEND (INVEST IN BONDS) $0.5
INVEST $0.5 IN STOCKS (v = 0.5).

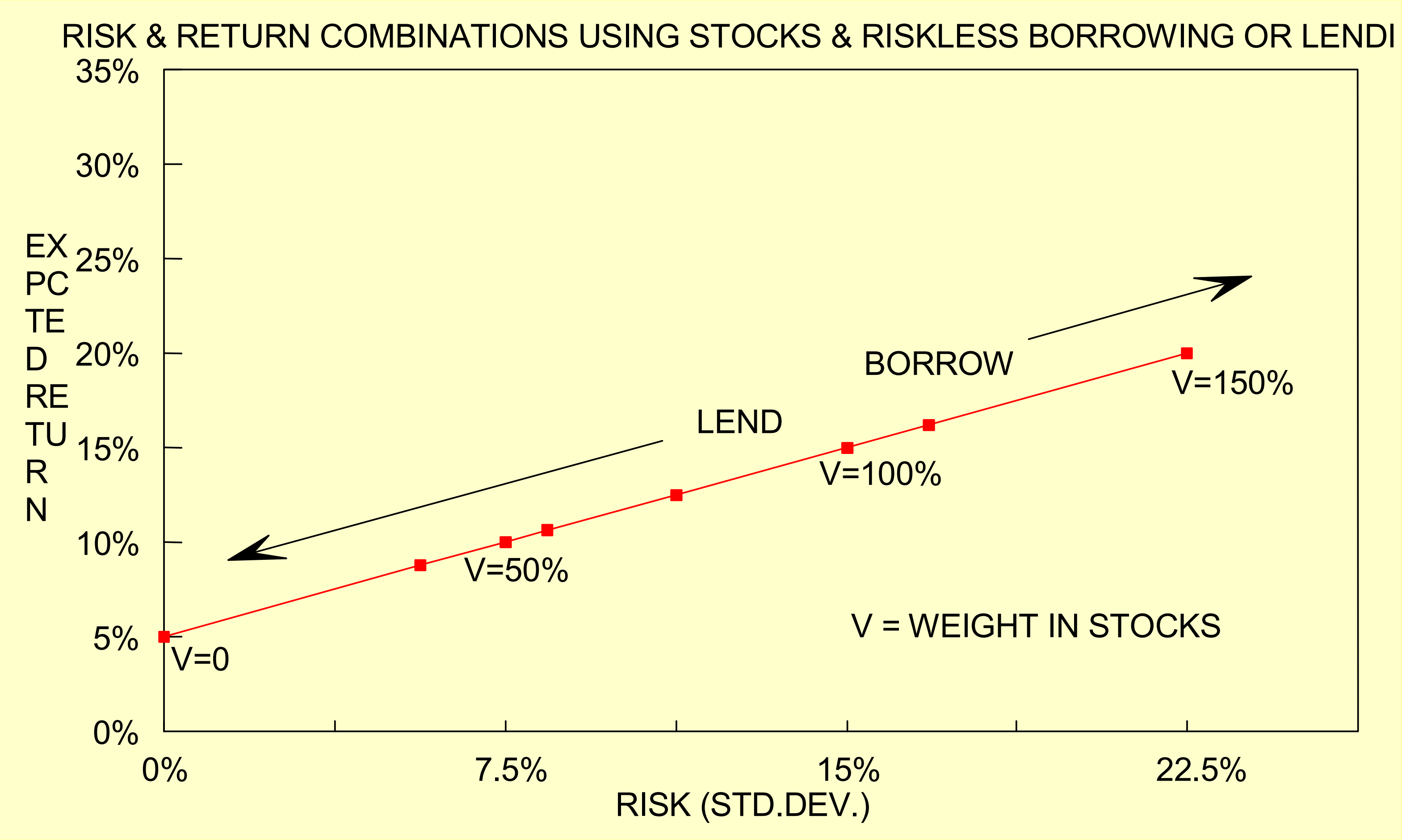
EXPECTED RETURN WOULD BE:

$$(0.5)15\% + (0.5)5\% = 10\%$$

RISK WOULD BE

$$(0.5)15\% + (0.5)0\% = 7.5\%$$

NOTICE THESE POSSIBILITIES LIE ON A STRAIGHT LINE IN RISK/RETURN SPACE . . .



BUT NO MATTER WHAT YOUR RETURN TARGET, YOU CAN DO BETTER BY PUTTING YOUR RISKY MONEY IN A DIVERSIFIED PORTFOLIO OF REAL ESTATE & STOCKS . . .

SUPPOSE:

REAL ESTATE EXPECTED RETURN = 10%
REAL ESTATE STD.DEV. = 10%
CORRELATION BETWEEN STOCKS & REAL ESTATE = 25%

THEN 50% R.E. / STOCKS MIXTURE WOULD PROVIDE:
EXPECTED RETURN = 12.5%; STD.DEV. = 10.0%

IF RETURN TARGET = 20%,

BORROW $1.0
INVEST $2.0 IN RISKY MIXED-ASSET PORTFOLIO (v = 2).

EXPECTED RETURN WOULD BE:

(2.0)12.5% + (-1.0)5% = 20%

RISK WOULD BE:

(2.0)10.0% + (-1.0)0% = 20% < 22.5%

IF RETURN TARGET = 10%,

LEND (INVEST IN BONDS) $0.33
INVEST $0.67 IN RISKY MIXED-ASSET PORTFOLIO (v = 0.67).

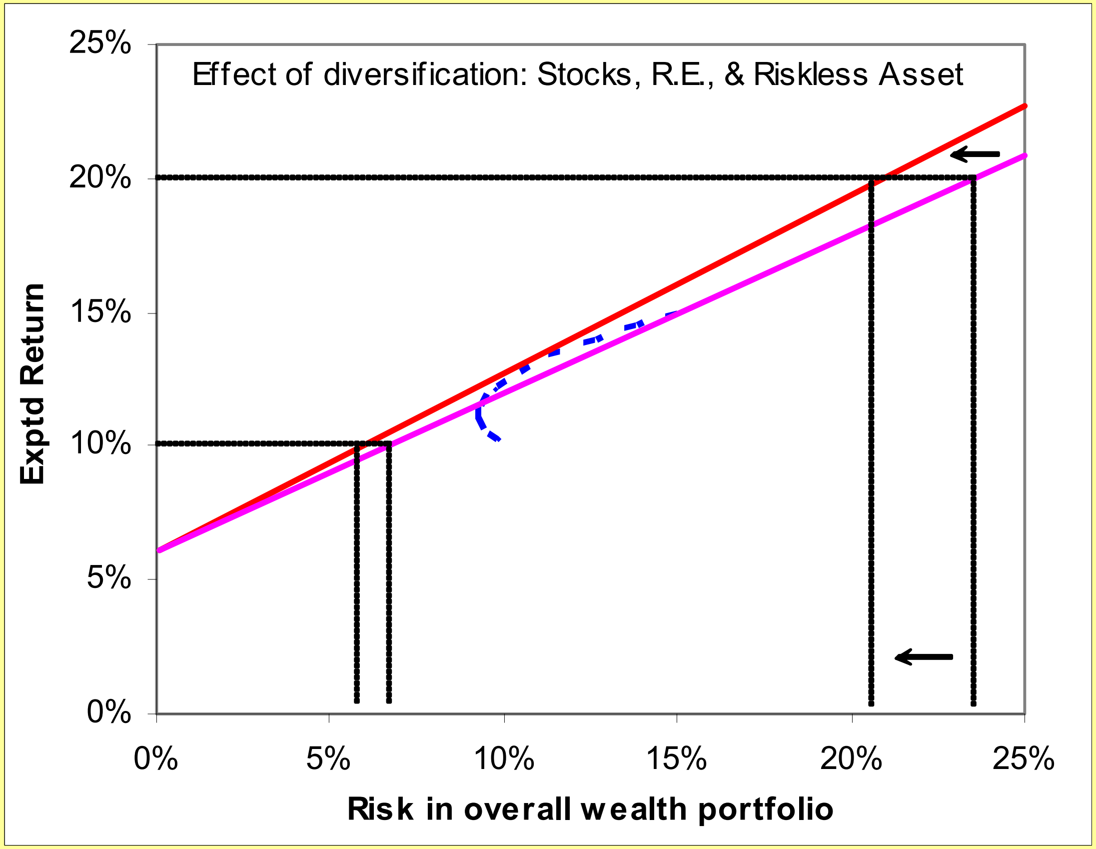
EXPECTED RETURN WOULD BE:

(0.67)12.5% + (0.33)5% = 10%

RISK WOULD BE:

(0.67)10.0% + (0.33)0% = 6.7% < 7.5%

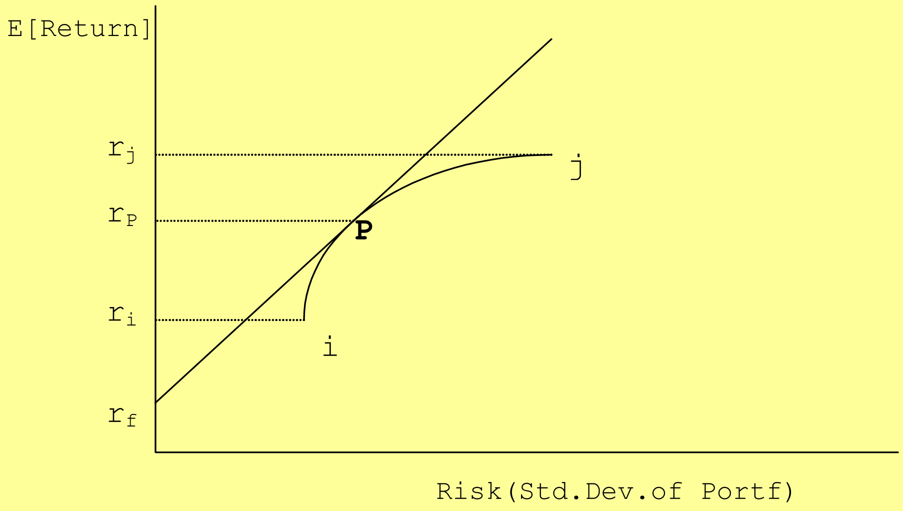
THE GRAPH BELOW SHOWS THE EFFECT DIVERSIFICATION IN THE RISKY PORTFOLIO HAS ON THE RISK/RETURN POSSIBILITY FRONTIER.



THE FRONTIER IS STILL A STRAIGHT LINE ANCHORED ON THE RISKFREE RATE, BUT THE LINE NOW HAS A GREATER "SLOPE", PROVIDING MORE RETURN FOR THE SAME AMOUNT OF RISK, ALLOWING LESS RISK FOR THE SAME EXPECTED RETURN.

THE "OPTIMAL" RISKY ASSET PORTFOLIO WITH A RISKLESS ASSET

(aka "TWO-FUND THEOREM")



CURVED LINE IS FRONTIER OBTAINABLE INVESTING ONLY IN RISKY ASSETS

STRAIGHT LINE PASSING THRU r_f AND PARABOLA IS OBTAINABLE BY MIXING RISKLESS ASSET (LONG OR SHORT) WITH RISKY ASSETS.

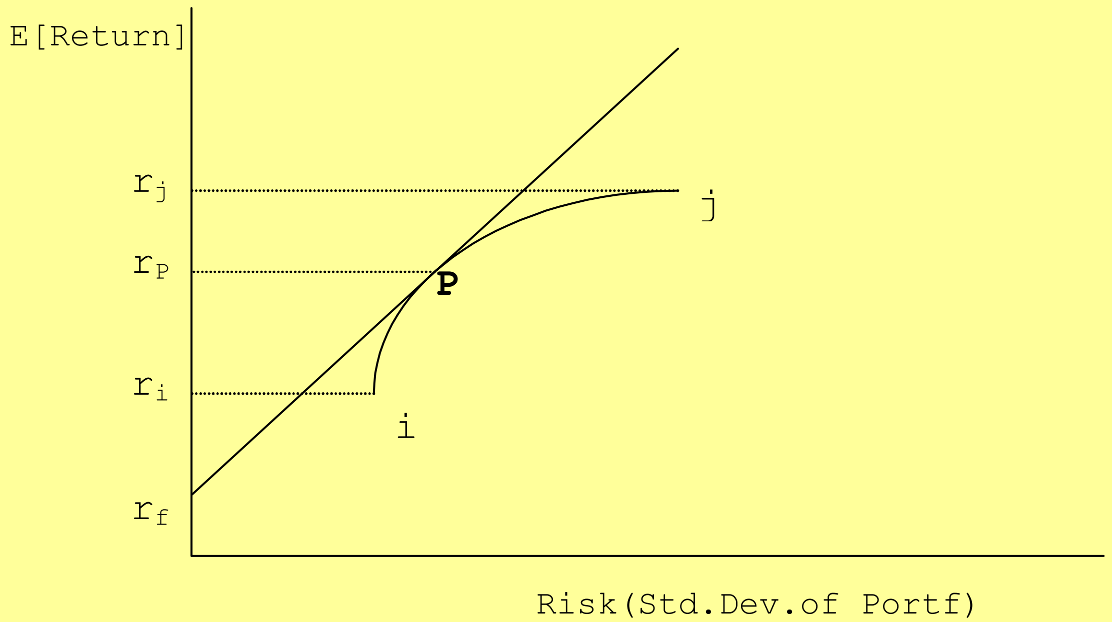
YOU WANT “HIGHEST” STRAIGHT LINE POSSIBLE (NO MATTER WHO YOU ARE!).

OPTIMAL STRAIGHT LINE IS THUS THE ONE PASSING THRU POINT "P".

IT IS THE STRAIGHT LINE ANCHORED IN r_f WITH THE MAXIMUM POSSIBLE SLOPE.

THUS, THE STRAIGHT LINE PASSING THROUGH “P” IS THE EFFICIENT FRONTIER. THE FRONTIER TOUCHES (AND INCLUDES) THE CURVED LINE AT ONLY ONE POINT: THE POINT "P".

THUS, THE *"2-FUND THEOREM"* TELLS US THAT THERE IS A SINGLE PARTICULAR COMBINATION OF RISKY ASSETS (THE PORTFOLIO "P") WHICH IS "OPTIMAL" *NO MATTER WHAT THE INVESTOR'S RISK PREFERENCES OR TARGET RETURN*.



THUS,
ALL EFFIC. PORTFS ARE COMBINATIONS OF JUST 2 FUNDS:

RISKLESS FUND (long or short position) + RISKY FUND "P" (long position).

HENCE THE NAME: "2-FUND THEOREM".

HOW DO WE KNOW WHICH COMBINATION OF RISKY ASSETS IS THE OPTIMAL ALL-RISKY PORTFOLIO "P"?

IT IS THE ONE THAT MAXIMIZES THE SLOPE OF THE STRAIGHT LINE FROM THE RISKFREE RETURN THROUGH "P". THE SLOPE OF THIS LINE IS GIVEN BY THE RATIO:

$$\text{Portfolio Sharpe Ratio} = (r_p - r_f) / s_P$$

MAXIMIZING THE **SHARPE RATIO** FINDS THE OPTIMAL RISKY ASSET COMBINATION. THE SHARPE RATIO IS ALSO A GOOD INTUITIVE MEASURE OF "RISK-ADJUSTED RETURN" FOR THE INVESTOR'S WEALTH, AS IT GIVES THE RISK PREMIUM PER UNIT OF RISK (MEASURED BY ST.DEV).

THUS, IF WE ASSUME THE EXISTENCE OF A RISKLESS ASSET, WE CAN USE THE 2-FUND THEOREM TO FIND THE OPTIMAL RISKY ASSET MIXTURE AS THAT PORTFOLIO WHICH HAS THE HIGHEST "SHARPE RATIO".

BACK TO PREVIOUS 2-ASSET NUMERICAL EXAMPLE...

USING OUR PREVIOUS EXAMPLE NUMBERS, THE OPTIMAL COMBINATION OF REAL ESTATE & STOCKS CAN BE FOUND BY EXAMINING THE SHARPE RATIO FOR EACH COMBINATION . . .

$\omega =$ RE share	r_P	r_p-r_f	s_P	Sharpe Ratio
0	15.0%	10.0%	15.0%	66.7%
0.1	14.5%	9.5%	13.8%	68.9%
0.2	14.0%	9.0%	12.6%	71.2%
0.3	13.5%	8.5%	11.6%	73.2%
0.4	13.0%	8.0%	10.7%	74.6%
0.5	12.5%	7.5%	10.0%	75.0%
0.6	12.0%	7.0%	9.5%	73.8%
0.7	11.5%	6.5%	9.2%	70.5%
0.8	11.0%	6.0%	9.2%	65.1%
0.9	10.5%	5.5%	9.5%	58.0%
1.0	10.0%	5.0%	10.0%	50.0%

OF THE 11 MIXTURES CONSIDERED ABOVE, THE 50% REAL ESTATE WOULD BE BEST BECAUSE IT HAS THE HIGHEST SHARPE MEASURE.

BUT SUPPOSE YOU ARE NOT SATISFIED WITH THE 12.5% Er THAT WILL GIVE YOU FOR YOUR OVERALL WEALTH? …
OR YOU DON'T WANT TO SUBJECT YOUR OVERALL WEALTH TO 10% VOLATILITY?...

THEN YOU CAN INVEST PROPORTIONATELY 50% IN REAL ESTATE AND 50% IN STOCKS, …

AND THEN ACHIEVE A GREATER RETURN THAN 12.5% BY BORROWING (LEVERAGE, v > 1),

OR YOU CAN INCUR LESS THAN 10.0% RISK BY LENDING (INVESTING IN GOVT BONDS, v<1)…

(BUT YOU CAN'T DO BOTH. THE "FREE LUNCH" OF PORTFOLIO THEORY ONLY GETS YOU SO FAR, THAT IS, TO THE EFFICIENT FRONTIER, BUT ON THAT FRONTIER THERE WILL BE A RISK/RETURN TRADEOFF. THAT TRADEOFF WILL BE DETERMINED BY THE MARKET…)

2-FUND THEOREM SUMMARY:

1. THE 2-FUND THEOREM ALLOWS AN ALTERNATIVE, INTUITIVELY APPEALING DEFINITION OF THE OPTIMAL RISKY PORTFOLIO: THE ONE WITH THE **MAXIMUM SHARPE RATIO**.

2. THIS CAN HELP **AVOID "SILLY"** OPTIMAL PORTFOLIOS THAT PUT TOO LITTLE WEIGHT IN HIGH-RETURN ASSETS JUST BECAUSE THE INVESTOR HAS A CONSERVATIVE TARGET RETURN. *(OR TOO LITTLE WEIGHT IN LOW-RETURN ASSETS JUST BECAUSE THE INVESTOR HAS AN AGGRESSIVE TARGET.)*

3. IT ALSO PROVIDES A GOOD **FRAMEWORK** FOR ACCOMMODATING THE POSSIBLE USE OF **LEVERAGE**, OR OF RISKLESS INVESTING (BY **HOLDING BONDS TO MATURITY**), BY THE INVESTOR.

Chapter 21 Summary: MPT & Real Estate . . .

- **The classical theory suggests a fairly robust, substantial role for the real estate asset class in the optimal portfolio (typically 25%-40% without any additional assumptions), either w or w/out riskless asset.**
- **This role tends to be greater for more conservative portfolios, less for very aggressive portfolios.**
- **Role is based primarily on *diversification benefits* of real estate, somewhat sensitive to R.E. correlation w stocks & bonds.**
- **Optimal real estate share roughly matches actual real estate proportion of all investable assets in the economy.**
- **Optimal real estate share in theory is substantially greater than actual pension fund allocations to real estate.**
- **Optimal R.E. share can be reduced by adding assumptions and extensions to the classical model:**
  - Extra transaction costs, illiquidity penalties;
  - Long-term horizon risk & returns;
  - Net Asset-Liability portfolio framework;
  - Investor constrained to over-invest in owner-occupied house as investment.
- **But even with such extensions, optimal R.E. share often substantially exceeds existing P.F. allocations to R.E. (approx. 3% on avg.*)**

VIII. *FROM PORTFOLIO THEORY TO EQUILIBRIUM ASSET PRICE MODELLING...*

➔ *HOW ASSET MARKET PRICES ARE DETERMINED.*

i.e.,

WHAT SHOULD BE **"E[r]"** FOR ANY GIVEN ASSET?…

RECALL RELATION BETW "PV" AND "E[r]".

e.g., for perpetutity: PV = CF / E[r]

(A model of price is a model of expected return,
and vice versa, a model of expected return is a model of price.)

THUS, ASSET PRICING MODEL CAN IDENTIFY "MISPRICED" ASSETS (ASSETS WHOSE "E[r]" IS ABOVE OR BELOW WHAT IT SHOULD BE, THAT IS, ASSETS WHOSE CURRENT "MVs" ARE "WRONG", AND WILL PRESUMABLY TEND TO "GET CORRECTED" IN THE MKT OVER TIME).

IF PRICE (HENCE E[r]) OF ANY ASSET DIFFERS FROM WHAT THE MODEL PREDICTS, THE IMPLICATION IS THAT THE PRICE OF THAT ASSET WILL TEND TO REVERT TOWARD WHAT THE MODEL PREDICTS, THEREBY ALLOWING PREDICTION OF SUPER-NORMAL OR SUB-NORMAL RETURNS FOR SPECIFIC ASSETS, WITH OBVIOUS INVESTMENT POLICY IMPLICATIONS.

Quick & simple example…

Suppose model predicts E[r] for $10 perpetuity asset should be 10%.

This means equilibrium price of this asset should be $100.

But you find an asset like this whose price is $83.

This means it is providing an E[r] of 12% (= 10 / 83).

Thus, if model is correct, you should buy this asset for $83.

Because at that price it is providing a “supernormal” return,

and because we would expect that as prices move toward equilibrium the value of this asset will move toward $100 from its current $83 price.

(i.e., You will get your supernormal return either by continuing to receive a 12% yield when the risk only warrants a 10% yield, or else by the asset price moving up in equilibrium providing a capital gain “pop”.)

THE **"SHARPE-LINTNER CAPM"** *(in 4 easy steps!)…*

(Nobel prize-winning stuff here – Show some respect!)

1ST) **2-FUND THEOREM** SUGGESTS THERE IS A SINGLE COMBINATION OF RISKY ASSETS THAT YOU SHOULD HOLD, NO MATTER WHAT YOUR RISK PREFERENCES.

THUS, ANY INVESTORS WITH THE SAME EXPECTATIONS ABOUT ASSET RETURNS WILL WANT TO HOLD THE SAME RISKY PORTFOLIO (SAME COMBINATION OR RELATIVE WEIGHTS).

2ND) GIVEN **INFORMATIONAL EFFICIENCY** IN SECURITIES MARKET, IT IS UNLIKELY ANY ONE INVESTOR CAN HAVE BETTER INFORMATION THAN THE MARKET AS A WHOLE, SO IT IS UNLIKELY THAT YOUR OWN PRIVATE EXPECTATIONS CAN BE SUPERIOR TO EVERY ONE ELSE'S. THUS, EVERYONE WILL CONVERGE TO HAVING THE SAME EXPECTATIONS, LEADING EVERYONE TO WANT TO HOLD THE **SAME PORTFOLIO**.

THAT PORTFOLIO WILL THEREFORE BE OBSERVABLE AS THE **"MARKET PORTFOLIO"**, THE COMBINATION OF ALL THE ASSETS IN THE MARKET, IN VALUE WEIGHTS PROPORTIONAL TO THEIR CURRENT CAPITALIZED VALUES IN THE MARKET.

3RD) SINCE EVERYBODY HOLDS THIS SAME PORTFOLIO, THE **ONLY RISK THAT MATTERS** TO INVESTORS, AND THEREFORE THE ONLY RISK THAT GETS REFLECTED IN EQUILIBRIUM MARKET PRICES, IS THE **COVARIANCE WITH THE MARKET** PORTFOLIO. ***(Recall that the contribution of an asset to the risk of a portfolio is the covariance betw that asset & the portf.)*** THIS COVARIANCE, NORMALIZED SO IT IS EXPRESSED PER UNIT OF VARIANCE IN THE MARKET PORTFOLIO, IS CALLED **"BETA"**.

4TH) THEREFORE, IN EQUILIBRIUM, ASSETS WILL REQUIRE AN EXPECTED RETURN EQUAL TO THE RISKFREE RATE PLUS THE MARKET'S RISK PREMIUM TIMES THE ASSET'S BETA:

$$E[r_i] = r_f + RP_i = r_f + \beta_i(Er_M - r_f)$$

THE CAPM IS OBVIOUSLY A SIMPLIFICATION (of reality)…

(Yes, I know that markets are not really perfectly efficient.

I know we don't all have the same expectations.

I know we do not all really hold the same portfolios.)

BUT IT IS A POWERFUL AND WIDELY-USED MODEL. IT CAPTURES AN IMPORTANT PART OF THE ESSENCE OF REALITY ABOUT ASSET MARKET PRICING…

Conceptually:

➔ Asset markets are "pretty efficient" (most of the time).

➔ Many investors (especially large institutions) hold very <u>similar</u> portfolios.

➔ Investors who determine market prices are those who are buying <u>and</u> selling in the asset market, and "<u>on average</u>" (in some vague sense) those investors "<u>ARE</u> the market". In other words, if there were just one giant investor, whose name was "the market", then the CAPM would explain the prices (and expected) returns that investor would pay (and require), if that giant investor were "rational".

*➔ Models ARE SUPPOSED TO **"<u>simplify</u>"** reality, enabling us to gain **<u>insight</u>** and **<u>understanding</u>** from the "jumble of too-many facts" that is reality.*

Empirically:

*➔ The **CAPM works** (pretty well, not perfectly) for explaining stock prices (stock average returns across time), using the stock market itself as a proxy for the "market portfolio".*

APPLYING THE CAPM TO REAL ESTATE…

(WE NEED TO CONSIDER REITs & “DIRECT” PRIVATE REAL ESTATE SEPARATELY…)

THE CAPM IS TRADITIONALLY APPLIED ONLY TO THE STOCK MARKET. THE "MARKET PORTFOLIO" (THE INDEX ON WHICH "BETA" IS DEFINED) IS TRADITIONALLY PROXIED BY THE STOCK MARKET.

(FIRST, CONSIDER REITs…)

➔ THIS TRADITIONAL APPLICATION WORKS ABOUT AS WELL FOR REITs AS IT DOES FOR OTHER STOCKS.

CAVEAT APPLYING TRADITIONAL CAPM TO REITs...

IN GENERAL, REITs ARE LOW-BETA STOCKS, AND MANY REITs ARE SMALL STOCKS.

THE CAPM TENDS TO UNDER-PREDICT THE AVERAGE RETURNS TO LOW-BETA STOCKS AND SMALL STOCKS, INCLUDING REITs.

- THE SMALL STOCK EFFECT MAY BE DUE TO GREATER SENSITIVITY OF SMALL STOCK RETURNS TO THE BUSINESS CYCLE, PARTICULARLY EXTREME DOWNSIDE RETURN SENSITIVITY TO RECESSIONS.

- INVESTORS CARE ABOUT BUSINESS CYCLE RISK BECAUSE THEIR OWN HUMAN CAPITAL VALUE AND CONSUMPTION IS POSITIVELY CORRELATED WITH THE BUSINESS CYCLE.

- A STOCK THAT IS SENSITIVE TO THE BUSINESS CYCLE WILL NOT HEDGE THAT RISK AND MAY IN FACT EXACERBATE IT.

- HOWEVER, IT IS NOT CLEAR THAT REITs ARE TYPICAL OF OTHER SMALL STOCKS IN THIS REGARD.

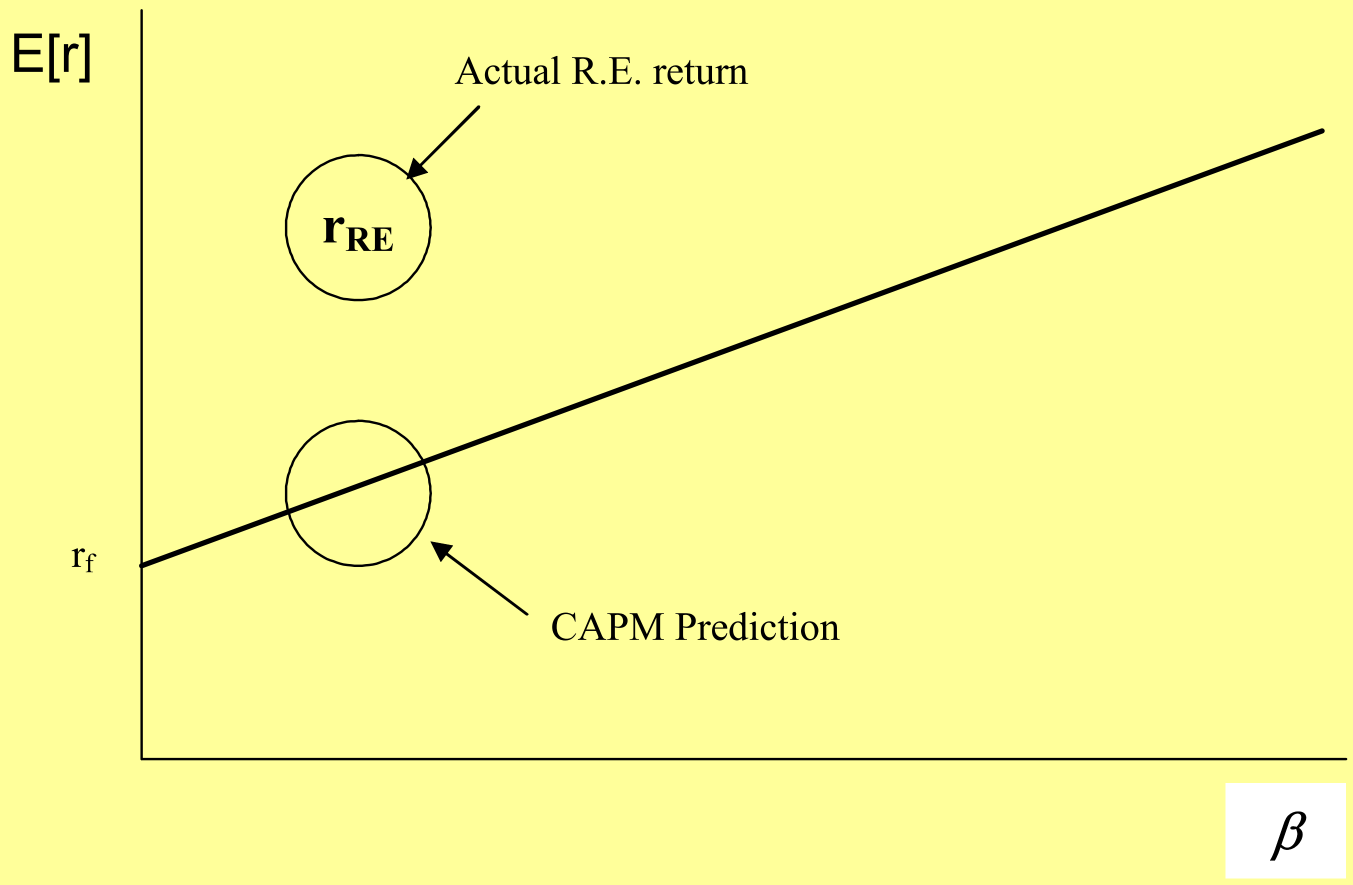
(NEXT, CONSIDER PRIVATE REAL ESTATE…)

TRADITIONAL CAPM, BASED ON THE STOCK MARKET AS THE "BETA" INDEX, DOES NOT WORK WELL FOR PRIVATE REAL ESTATE…

PRIVATE REAL ESTATE RETURNS ARE NOT HIGHLY CORRELATED WITH STOCK MARKET.

THIS GIVES REAL ESTATE A VERY LOW "BETA" (MEASURED WRT STOCK MARKET). YET REAL ESTATE IS GENERALLY VIEWED AS A “RISKY INVESTMENT” MERRITING (AND GETTING) A SUBSTANTIAL RISK PREMIUM IN ITS EX ANTE RETURN.

THUS, TRADITIONAL APPLICATION OF CAPM DOES NOT SEEM TO WORK FOR PRIVATE REAL ESTATE…



(ANYWAY, THIS IS THE TRADITIONAL "COMPLAINT" ABOUT THE CAPM AS IT RELATES TO PRIVATE REAL ESTATE.)

ASIDE:
IS THIS TRADITIONAL COMPLAINT REALLY BORN OUT BY THE EMPIRICAL EVIDENCE?…

➔ SO-CALLED "INSTITUTIONAL QUALITY" COMMERCIAL PROPERTY HAS PROVIDED ONLY A VERY SMALL RISK PREMIUM OVER THE PAST COUPLE OF DECADES, ABOUT THE SAME AS LONG-TERM BONDS, FOR EXAMPLE.

➔ MANY OF THE "INSTITUTIONS" WHO INVEST IN SUCH PROPERTY (SUCH AS PENSION FUNDS AND LIFE INSURANCE COMPANIES) HAVE OVERALL PORTFOLIOS THAT ARE DOMINATED BY STOCKS AND BONDS, ASSETS WITH WHICH PRIVATE REAL ESTATE HAS LOW CORRELATION.

THUS, THE TRADITIONAL CAPM MAY INDEED WORK WELL FOR "INSTITUTIONAL" REAL ESTATE…

➔ SUCH INVESTORS WOULD BE SATISFIED WITH LOW RISK PREMIUMS IN REAL ESTATE, BECAUSE OF THE DIVERSIFICATION ROLE REAL ESTATE PLAYS IN THEIR OVERALL PORTFOLIOS.

➔ ON THE OTHER HAND, NON-INSTITUTIONAL REAL ESTATE, INCLUDING HOUSING, SEEMS GENERALLY TO HAVE PROVIDED A SUBSTANTIAL RISK PREMIUM ON AVERAGE, THOUGH THIS IS DIFFICULT TO QUANTIFY RELIABLY.

➔ MUCH OF THIS NON-INSTITUTIONAL REAL ESTATE MAY BE OWNED BY INVESTORS WHO ARE NOT SO WELL DIVERSIFIED, AND MAY HAVE A SUBSTANTIAL FRACTION OF THEIR OVERALL WEALTH IN THEIR REAL ESTATE INVESTMENTS. THIS WOULD MAKE SUCH INVESTORS NEED A HIGH RISK PREMIUM FROM REAL ESTATE, BASED PURELY ON ITS VOLATILITY, AS ITS LOW CORRELATION WITH STOCKS AND BONDS WOULD NOT HELP THEM OUT.

SO, IT WOULD MAKE SENSE THAT THE TRADITIONAL CAPM WOULD NOT HOLD FOR NON-INSTITUTIONAL PRIVATE REAL ESTATE.

CAN THE CAPM BE APPLIED MORE BROADLY TO ENCOMPASS ALL PRIVATE REAL ESTATE AS WELL AS PUBLICLY-TRADED SECURITIES SUCH AS STOCKS AND REITs?…

ACCORDING TO THE CAPM THEORY, THE "MARKET PORTFOLIO" ON WHICH "BETA" (AND HENCE THE EXPECTED RETURN RISK PREMIUM) IS BASED SHOULD INCLUDE <u>ALL</u> THE ASSETS IN THE ECONOMY.

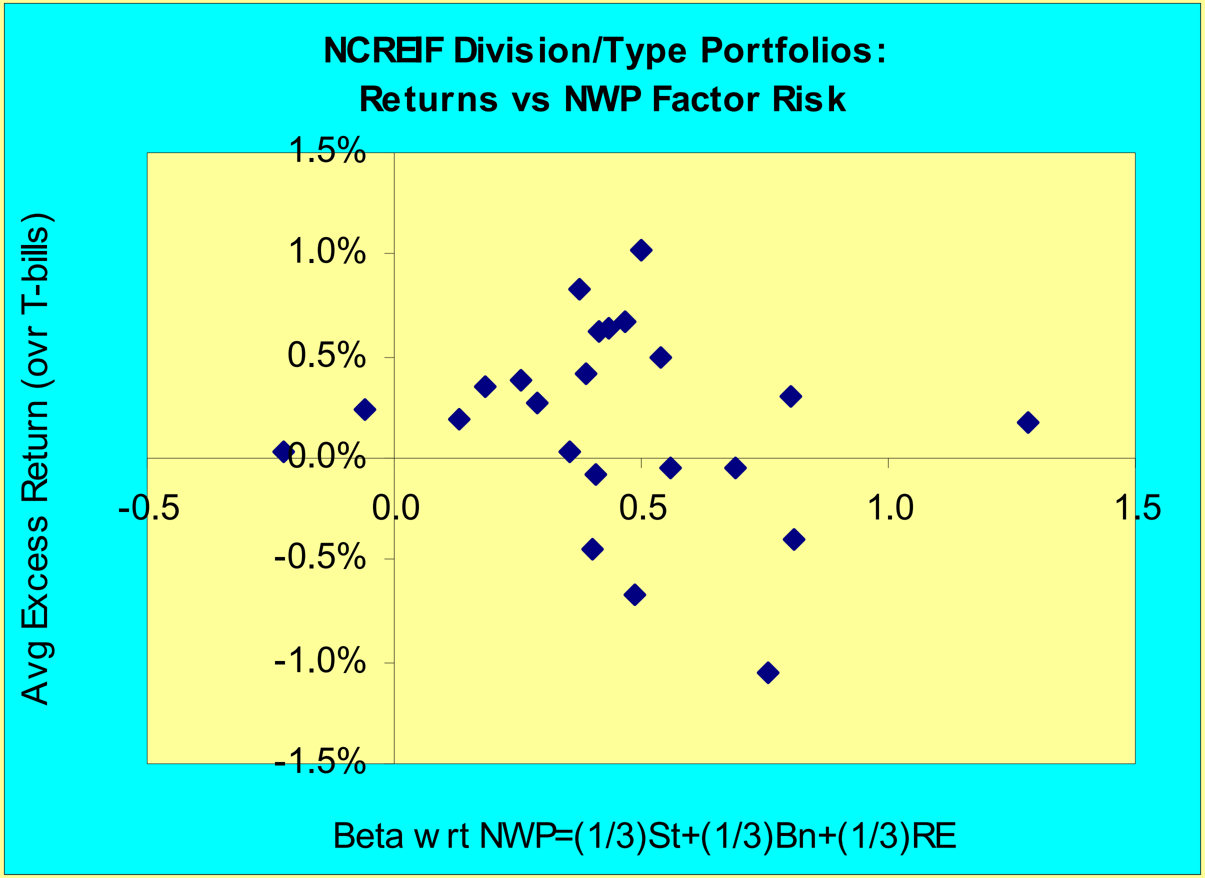
THIS SHOULD INCLUDE, IN ADDITION TO STOCKS AND BONDS, REAL ESTATE ITSELF, AS WELL AS INVESTORS' OWN "HUMAN CAPITAL", AND OTHER NON-TRADABLE ASSETS.

THERE IS SOME EVIDENCE THAT IF ONE MEASURES PRIVATE REAL ESTATE'S "BETA" IN THIS WAY, BASED ON A BROADER MARKET PORTFOLIO (OR BASED ON NATIONAL CONSUMPTION), THEN REAL ESTATE HAS A SUBSTANTIALLY POSITIVE BETA, PROBABLY AT LEAST HALF THAT OF THE STOCK MARKET.

THUS, A MORE BROADLY APPLIED CAPM WOULD SEEM TO SUGGEST THAT PRIVATE REAL ESTATE DOES REQUIRE A SUBSTANTIAL RISK PREMIUM IN ITS EXPECTED RETURN.

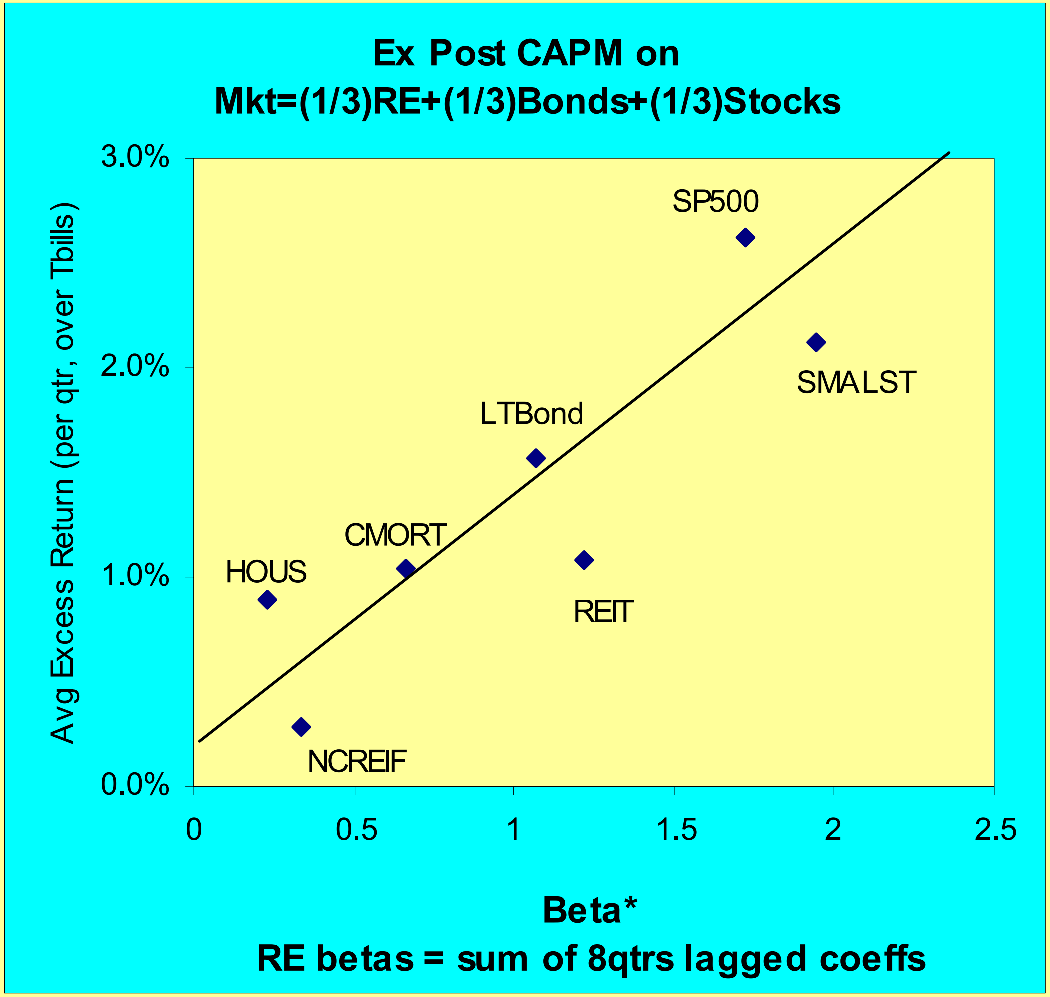
ON AVERAGE, INCLUDING BOTH INSTITUTIONAL AND NON-INSTITUTIONAL REAL ESTATE, PRIVATE REAL ESTATE PROBABLY DOES PROVIDE SUCH A RISK PREMIUM.

Another perspective on the relevance of the CAPM to real estate: Distinguish between applications *Within the institutional private R.E. asset class*, versus applications: *Across broad asset classes ("mixed asset portfolio" level)* . . .

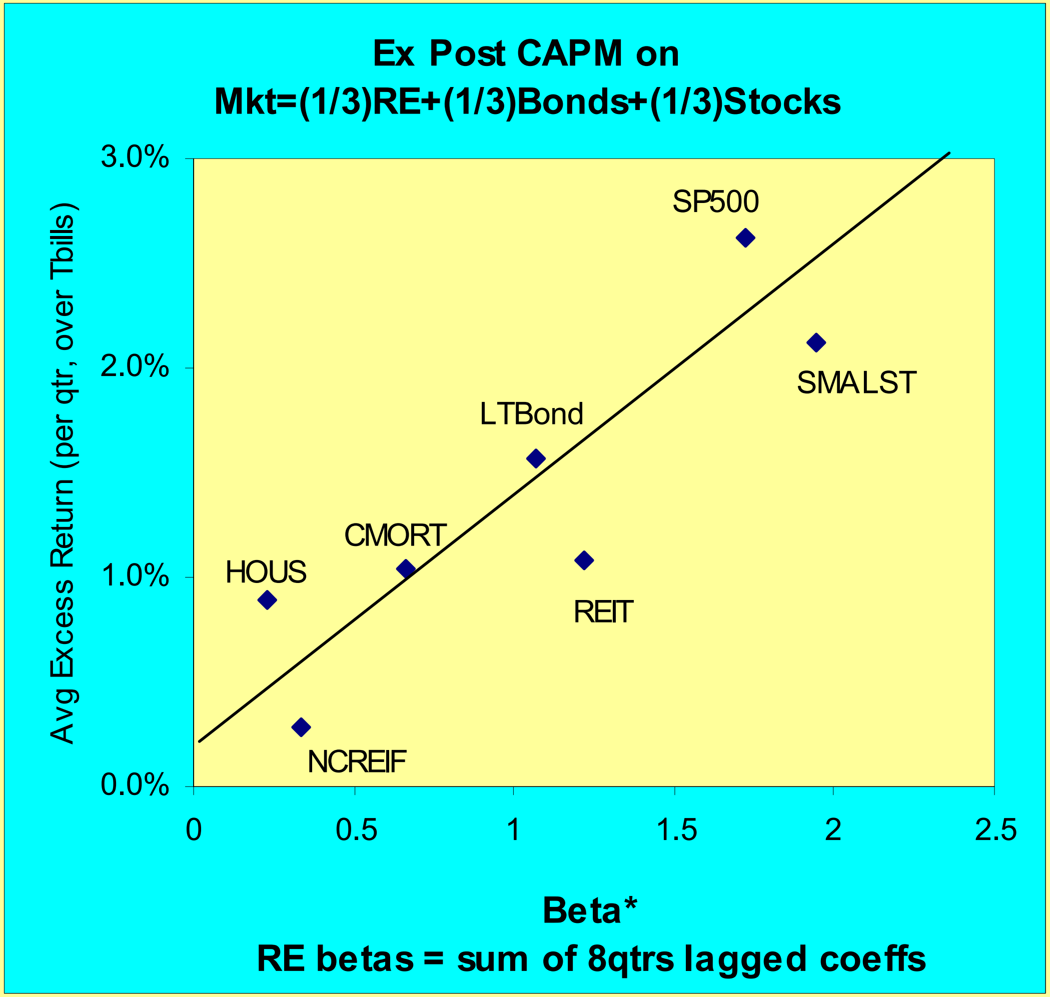


➔ No relationship between CAPM-defined risk and cross-section of ex post returns.

*But the CAPM appears to be more meaningful when we take a broader perspective **ACROSS** asset classes. . .*



*Regression statistics for historical returns **ACROSS** asset classes . . .*

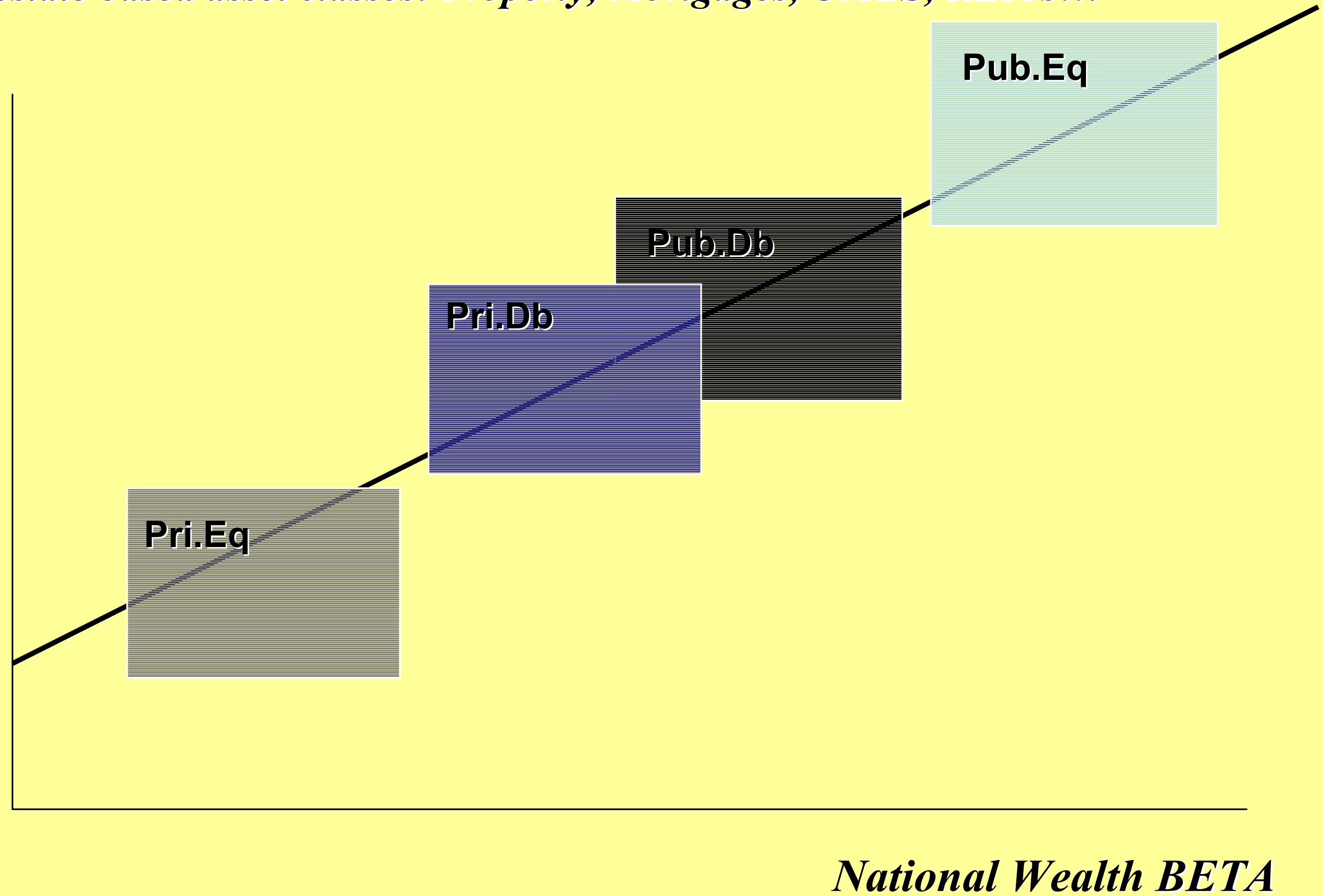


- **Adj. R^2 = 73%**
- **Intercept is Insignif.**
- **Coeff on Beta is Pos & Signif.**

"CAPM works..."

The Capital Market does perceive (and price) risk differences ACROSS asset classes . . .

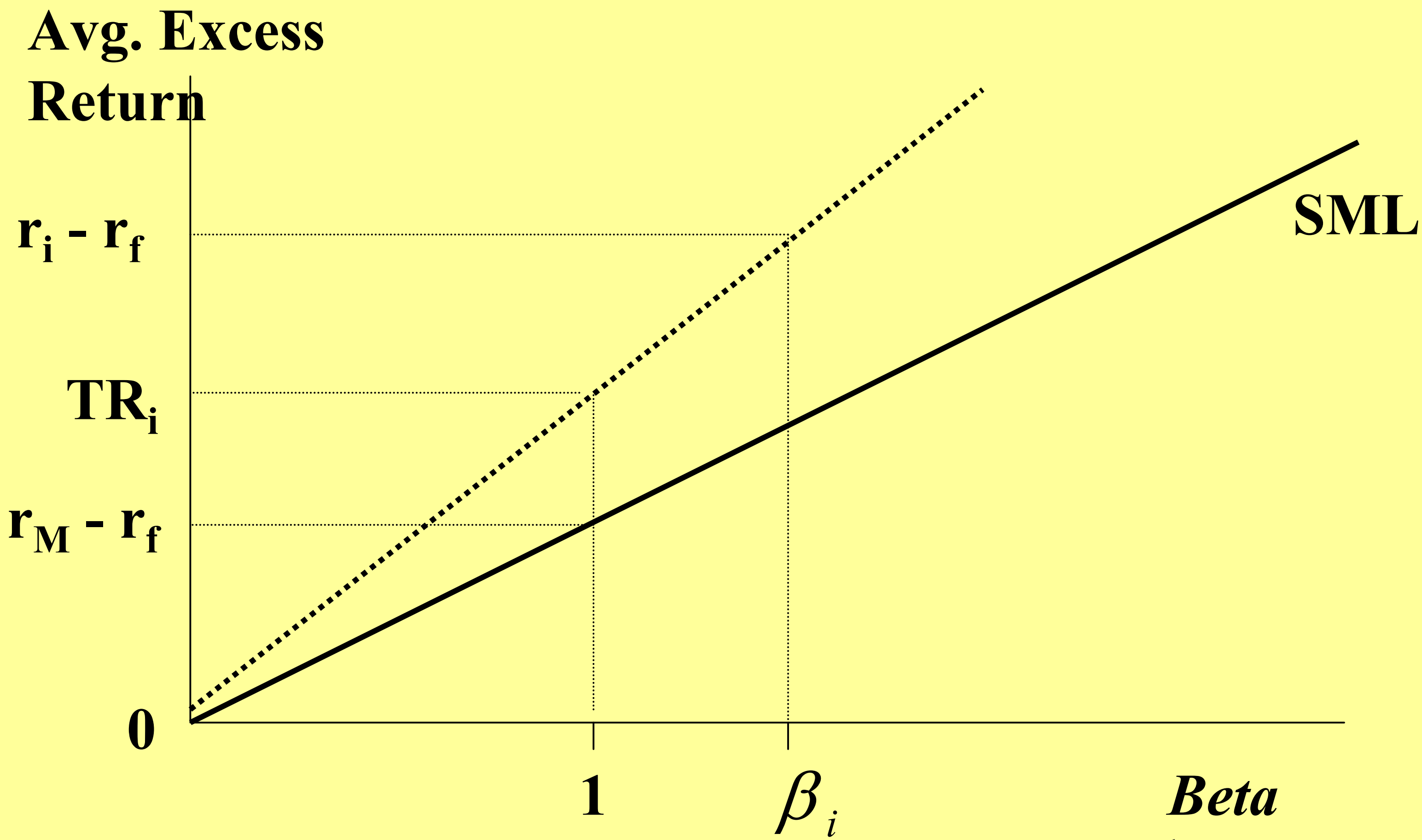
Real estate based asset classes: Property, Mortgages, CMBS, REITs…



Asset Class Ex Post Betas and Risk Premia (Per Annum, over T-bills, 1981-98)...

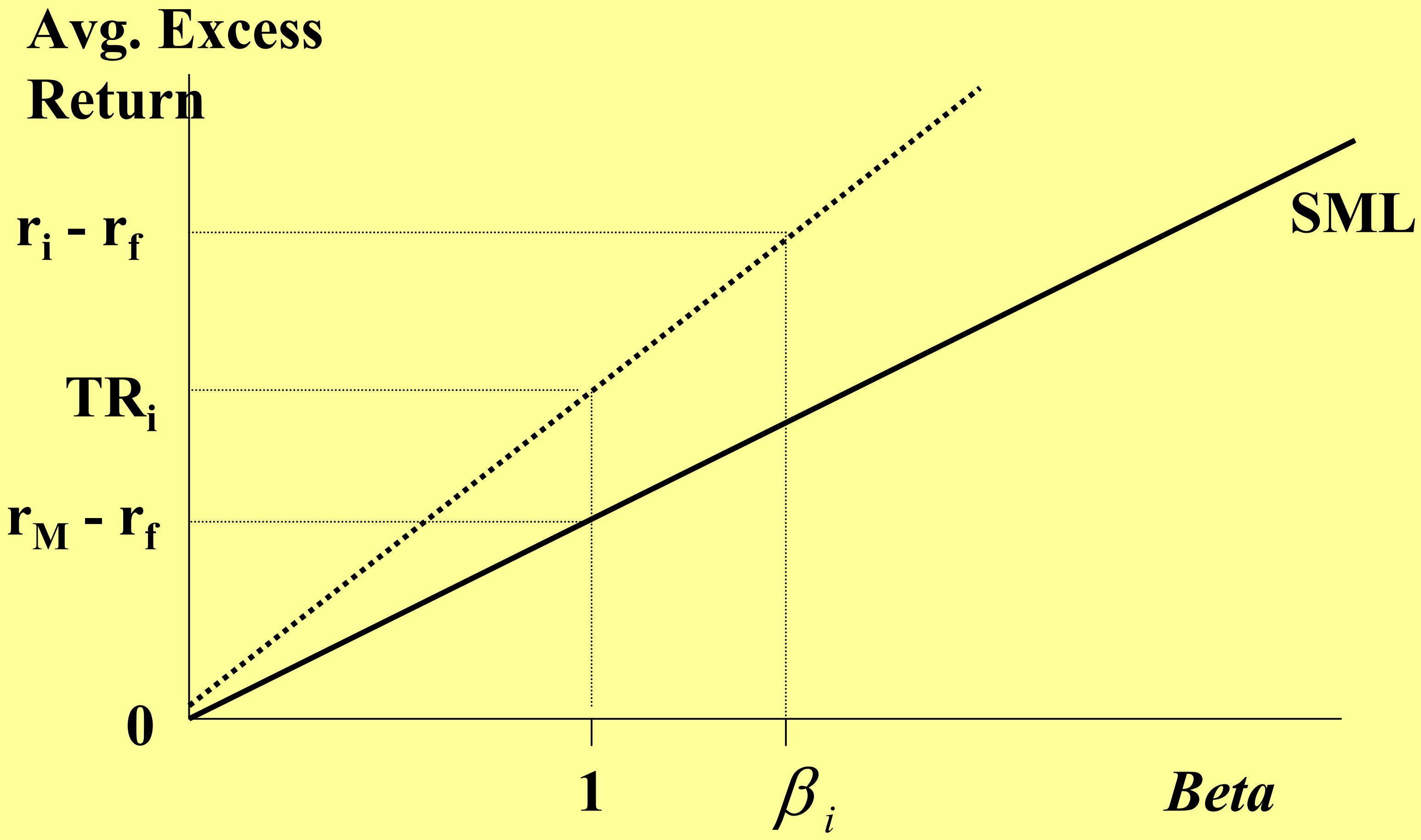
Asset Class:	Excess Return:	Beta:
Small Stocks	8.48%	1.94
S&P500	10.48%	1.72
REITs	4.32%	1.22
LT Bonds	6.24%	1.07
Com.Mortgs	4.15%	0.66
NCREIF	1.15%	0.34
Houses	3.59%	0.23

A CAPM-based method to adjust investment performance for risk: The Treynor Ratio...

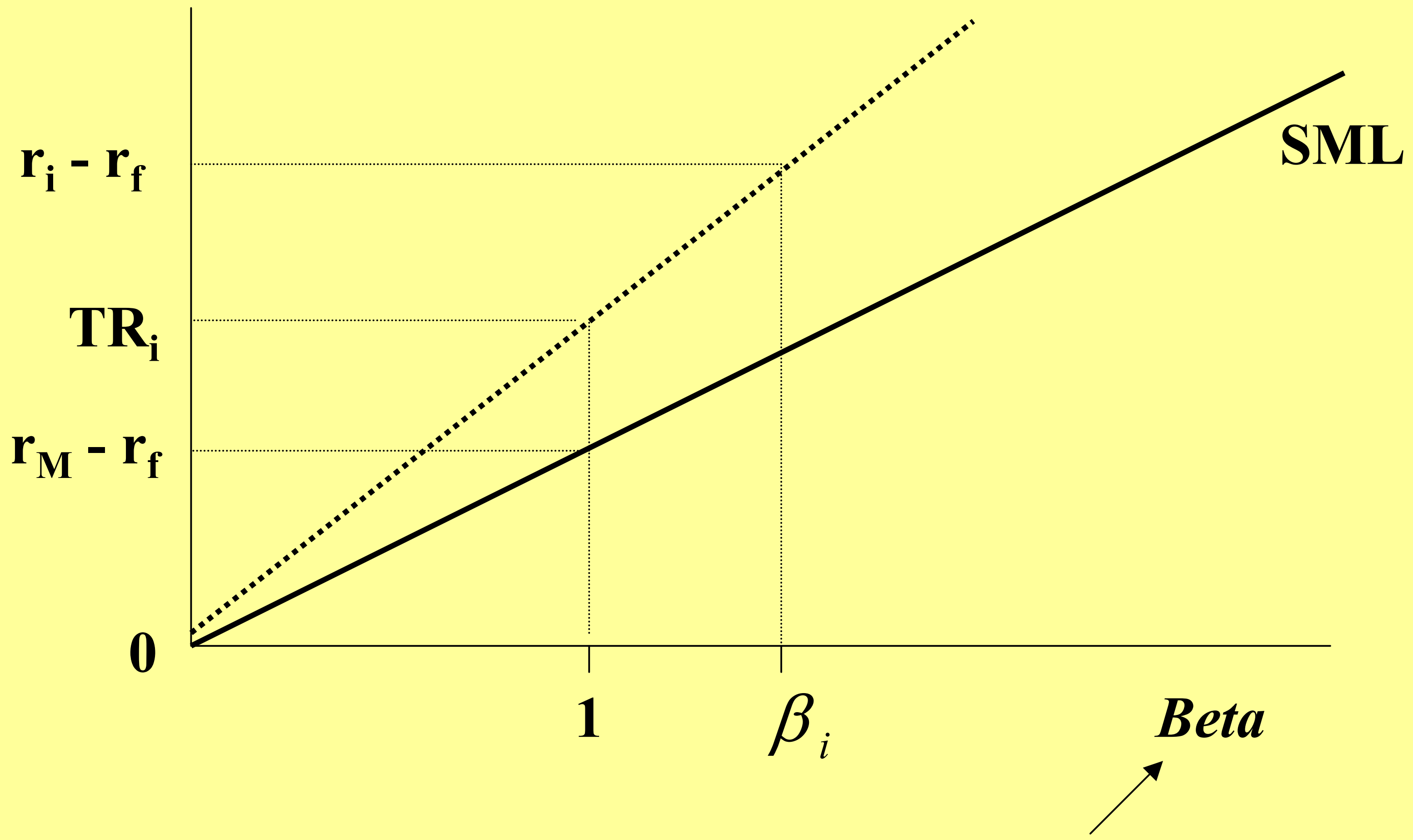


Based on *"Risk Benchmark"*

The Treynor Ratio (or something like it) could perhaps be applied to managers (portfolios) spanning the major asset classes...



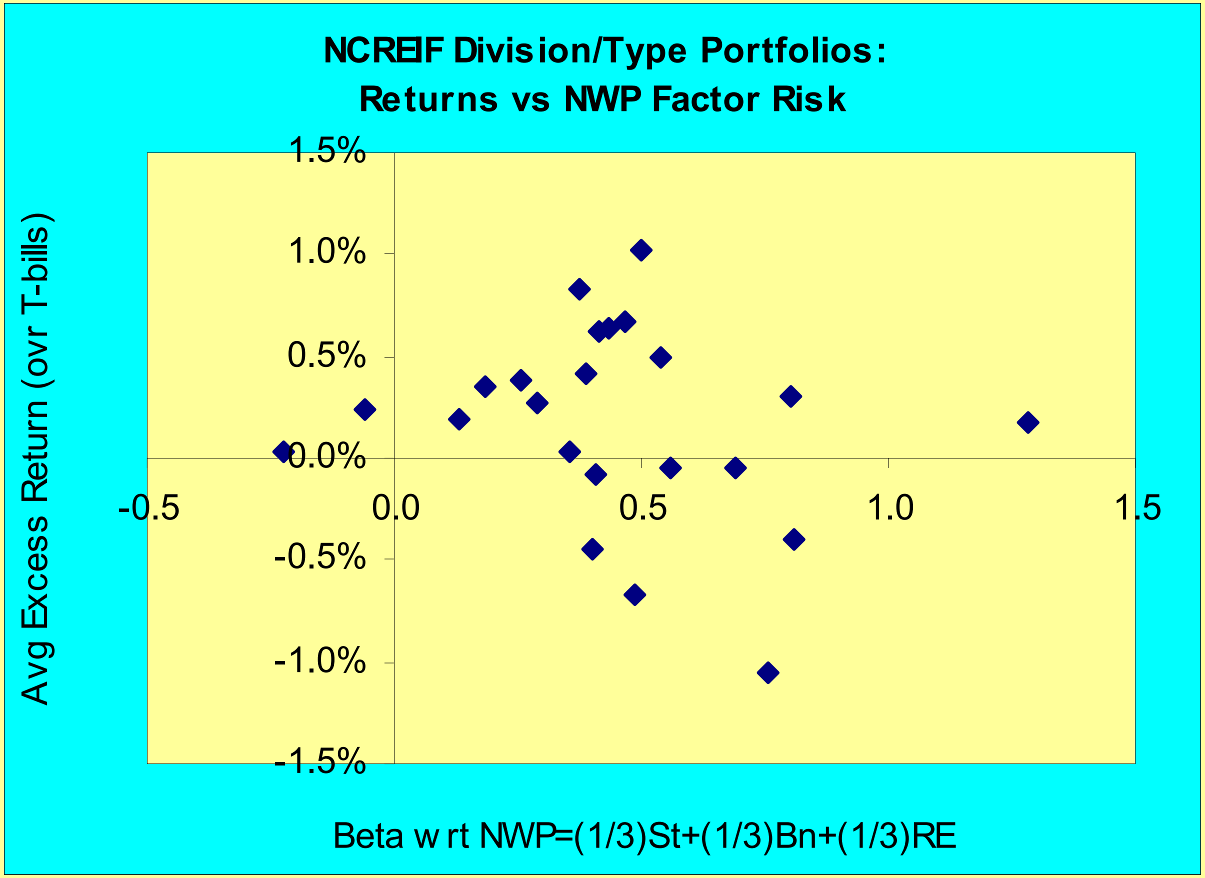
The Beta can be estimated based on the "National Wealth Portfolio" **(= (1/3)Stocks + (1/3)Bonds + (1/3)RE) as the mixed-asset "Risk Benchmark". . .**



Based on ***"National Wealth Portfolio"***

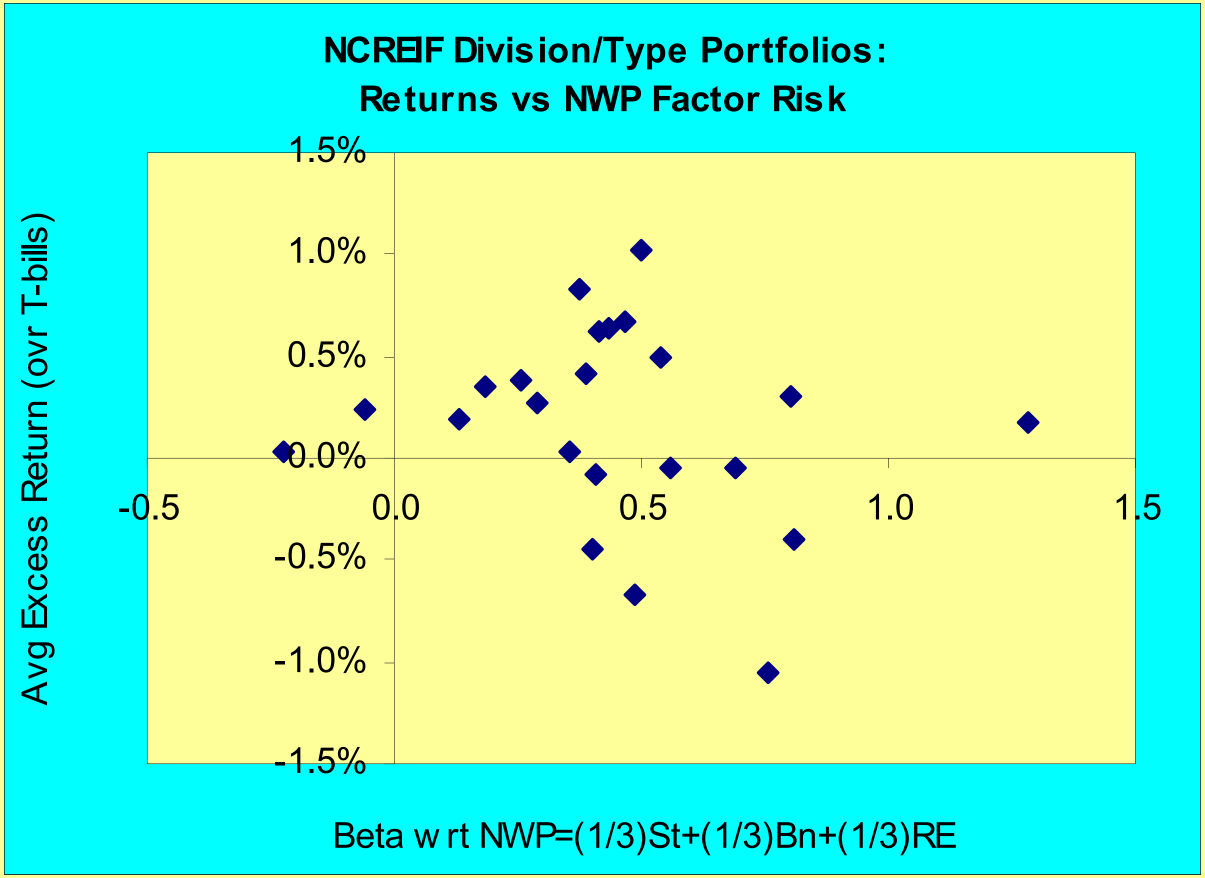
Go back to the *within the private real estate asset class* level of application of the CAPM…

Recall that we see little ability to systematically or rigorously distinguish between the risk and return expectations for different market segments *within* the asset class (e.g., Denver shopping ctrs vs Boston office bldgs):



This holds implications for portfolio-level tactical investment policy:

- ➔ ***If all mkt segments effectively present the same investment risk, then those that present the highest expected returns automatically look like "good investments" (bargains) from a risk-adjusted ex ante return perspective.***



- ➔ ***Search for markets where the combination of current asset yields (cap rates, "y") and rental growth prospects ("g") present higher expected total returns (Er = y + Eg).***

Summarizing Chapter 22: *Equilibrium Asset Price Modelling & Real Estate*

- **Like the MPT on which it is based, equilibrium asset price modelling (the CAPM in particular) has substantial relevance and applicability to real estate when applied at the broad-brush *across asset classes* level.**
- **At the property level (unlevered), real estate in general tends to be a low-beta, low-return asset class in equilibrium, but certainly not riskless, requiring (and providing) some positive risk premium (ex ante).**
- **CAPM type models can provide some guidance regarding the relative pricing of real estate as compared to other asset classes (*"Should it currently be over-weighted or under-weighted?"*), and…**
- **CAPM-based risk-adjusted return measures (such as the Treynor Ratio) may provide a basis for helping to judge the performance of multi-asset-class investment managers (who can allocate across asset classes).**
- ***Within* the private real estate asset class, the CAPM is less effective at distinguishing between the relative levels of risk among real estate market segments, implying (within the state of current knowledge) a generally *flat security market line*.**
- **This holds implications for tactical portfolio investment policy within the private real estate asset class: ➔ *Search for market segments with a combination of high asset yields and high rental growth opportunities: Such apparent "bargains" present favorable risk-adjusted ex ante returns.***