Option Theory and Land Development

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Outline of Resuscitation

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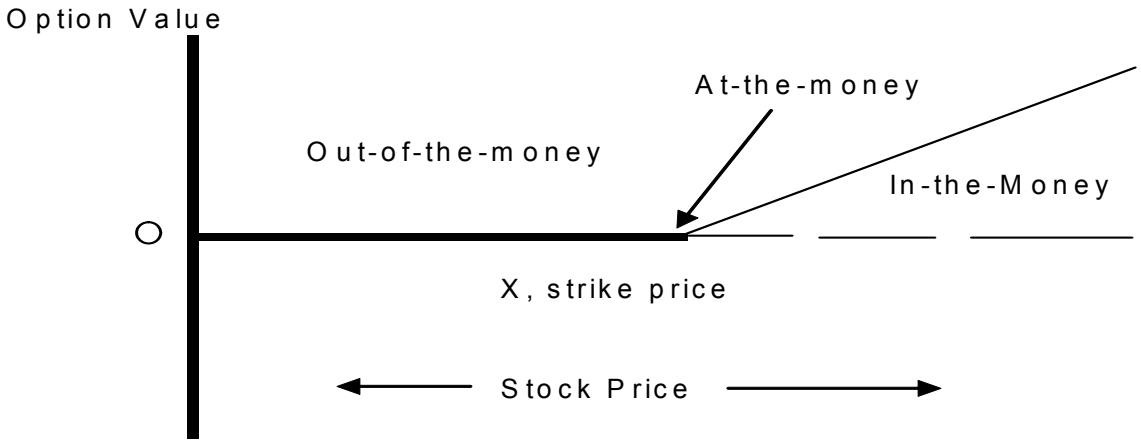
Option Theory Definitions

- Options give the holder the right to buy (or sell) an asset during a time period for a specific price.
  - Call options are the right to buy the asset at a set price.
  - Put options are the right to sell the asset at a set price.
  - The option writer is the person selling the option.
- An option's exercise or strike price is the price at which the underlying stock can be bought or sold for by exercising the option.
- An option's premium is what traders have to pay for the option.

‘Moneyness’ of Options



The ‘moneyness’ chart:	Calls	Puts
In-the-money	S>X	S<X
At-the-money	S=X	S=X
Out-of-the-money	S<X	S>X

Basic Option Notation

- S_t = the price of the underlying stock at time t.
- X = the exercise price of the option.
- T = the expiration date of the option.
- C_t = the price of the call option at time t.
- P_t = the price of the put option at time t.

Valuing Call Options – Discrete Case

- Example: The common shares of XYZ, Inc. are trading at 24. Determine the fair market value of a call option on the XYZ shares under the following conditions:
  - Call on XYZ:

Exercise Price	$20
Expiration	3 months
Risk-Free Rate	8% (Annualized)
Range of Possible Prices of XYZ stock in 3 months.	Current price (+-) $6

Necessary Steps

- Determine the Hedge Ratio: To do this, determine a hedged position of one share of stock, plus a certain number of options (X) on a share that will produce the same wealth position at the end of the period, whether the stock is at its possible low or high value (18 or 30).
- Discount the hedged ending wealth position to its present value using the risk-free rate as the discount rate (either the high or the low ending price possibility may be used, since they both lead to identical answers).
- Determine the value of the call option based upon the current price of the stock.

Mathematical Specifics

- The hedge ratio (X) is determined as follows:
  - (Ending Low Stock Price + (Ending Low Option Price) X = Ending High Stock Price + (Ending High Options Price) X)
  - 18 + 0 X = 30 + 10 X; X = -1.2

Ending Low Stock Price	$18
Ending Low Option Price (if the stock is $18 at expiration, the option will have no value)	$0
Ending High Stock Price	$30
Ending High Option Price ($30-$20) = $10	$10

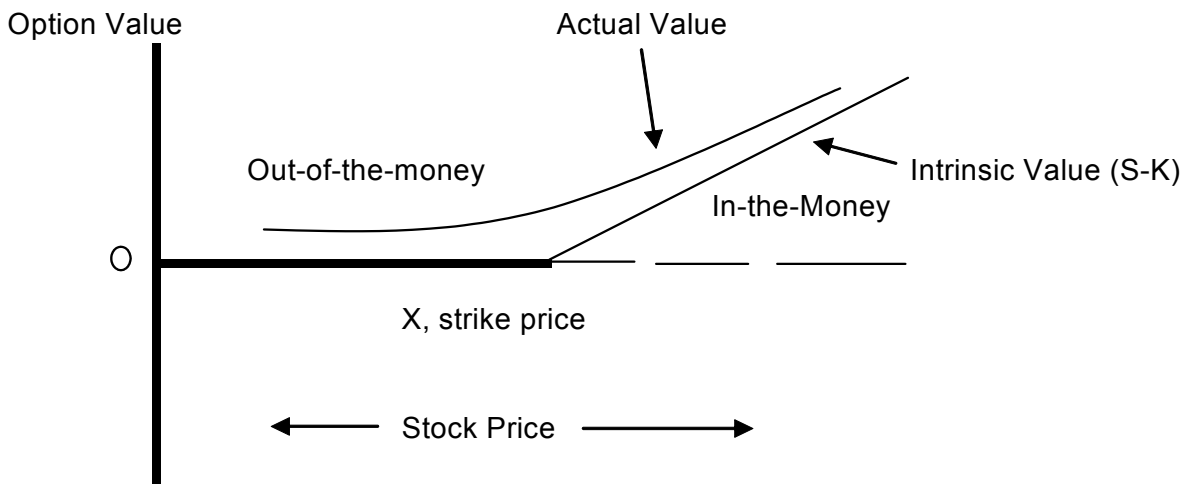
Mathematical Specifics

- Discounting the hedged ending wealth position to its present value is determined as follows:
  - Present value of ending wealth = (Ending stock price – (Ending option price)(# Options shorted)) / $(1 + r_f)^t$; where t = the fraction of a year that the option is "alive".
  - Present value of ending wealth = $(18 – 0(1.2)) / (1.08)^{0.25}$ = \$17.66
- Determine the value of the call option based upon the current price of the stock:
  - Present value of ending wealth = Current price of the stock – No. of calls shorted x c
  - $17.66 = 24 – 1.2c$; $c = \$5.28$

Factors Affecting the Value of Options

- Exercise price of the option: All other factors held constant, the higher the exercise price, the less calls are worth and the more puts are worth.
- The price of the underlying asset: For a given exercise price, the value of a call option increases as the price of the underlying asset rises.
- Volatility: The greater the volatility of the underlying asset, the more a put or call option will be worth, all other factors being equal.
- Time until expiration: The longer the time until expiration, the higher will be the value of an option, all other factors being equal.
- The level of interest rates: The higher the level of interest rates, the higher will be the value of a call option.

Asset Value to Option Value Relationship



- The intrinsic value of a put option (p) is the difference between the exercise price of the option (K) and the price of the underlying asset (S).
- The difference between the actual price of a call option and its intrinsic value is the time component in a call option's overall price structure.

The Call Option Model of Land Value

- Option theory provides a framework from which to examine the link between land value and real estate development.
- The physical investment decision yields the call option characteristic. The holder has the right without obligation to undertake the construction project.
- The exercise price represents the construction cost. As in financial options, the option is given up at the time of exercise. The construction process is irreversible.

Land Option versus Financial Option

- Perpetual Option: There is no expiration/maturity date. There is no time upon which the landowner looses his/her ability to build.
- Time to Build: Exercise of the land option is not immediate. It take time to build. Much uncertainty can exist between the decision to develop and completion of that development.
- Accuracy of the current value of the underlying asset. It is not directly observable contrary to the equity market.

Application of Option Theory to Land Value

- Option premium in land value and the value of waiting to build.

The value of waiting:	Year 1	Year 2
Valuation of built property	$1000	$1100
Construction Cost	800	840
NPV (immediate construction)	$200	$260

- NPV today of construction next year, @ 20% discount rate: 260/1.2 = $217
  - Land is worth $217
  - Current HBU = Hold undeveloped;
  - Option premium = 217-200 = 17

Application of Option Theory to Land Value

- Option premium in land value and future uncertainty in build property value.

	Year 0	Year 1 Don't Build	Year 1 Build
Probability	100%	60%	40%
Value of Developed Property	$1,000	$600	$1,600
Development cost (excluding land)	$800	$900	$900
NPV of exercise	$200	-$300	$700
Future values		0	$700

- Expected values (Probability x outcome)
  - Year 0 = $200 // Year 1 = $280
  - PV (today) of alternatives @ 20% discount rate; Year 0 = $200 // Year 1= $233
  - Land value today Max(200,233) = $233

Samuelson-McKean Formula and land option valuation

- The simplest option valuation formula
- Based on a perpetual American warrant.
- It requires the following assumptions:
  - Frictionless markets;
  - "Random Walk" market value of underlying asset;
  - Normally distributed returns to underlying asset;
  - Known parameter values (e.g., volatility of underlying asset).

Samuelson-McKean Formula and land option valuation

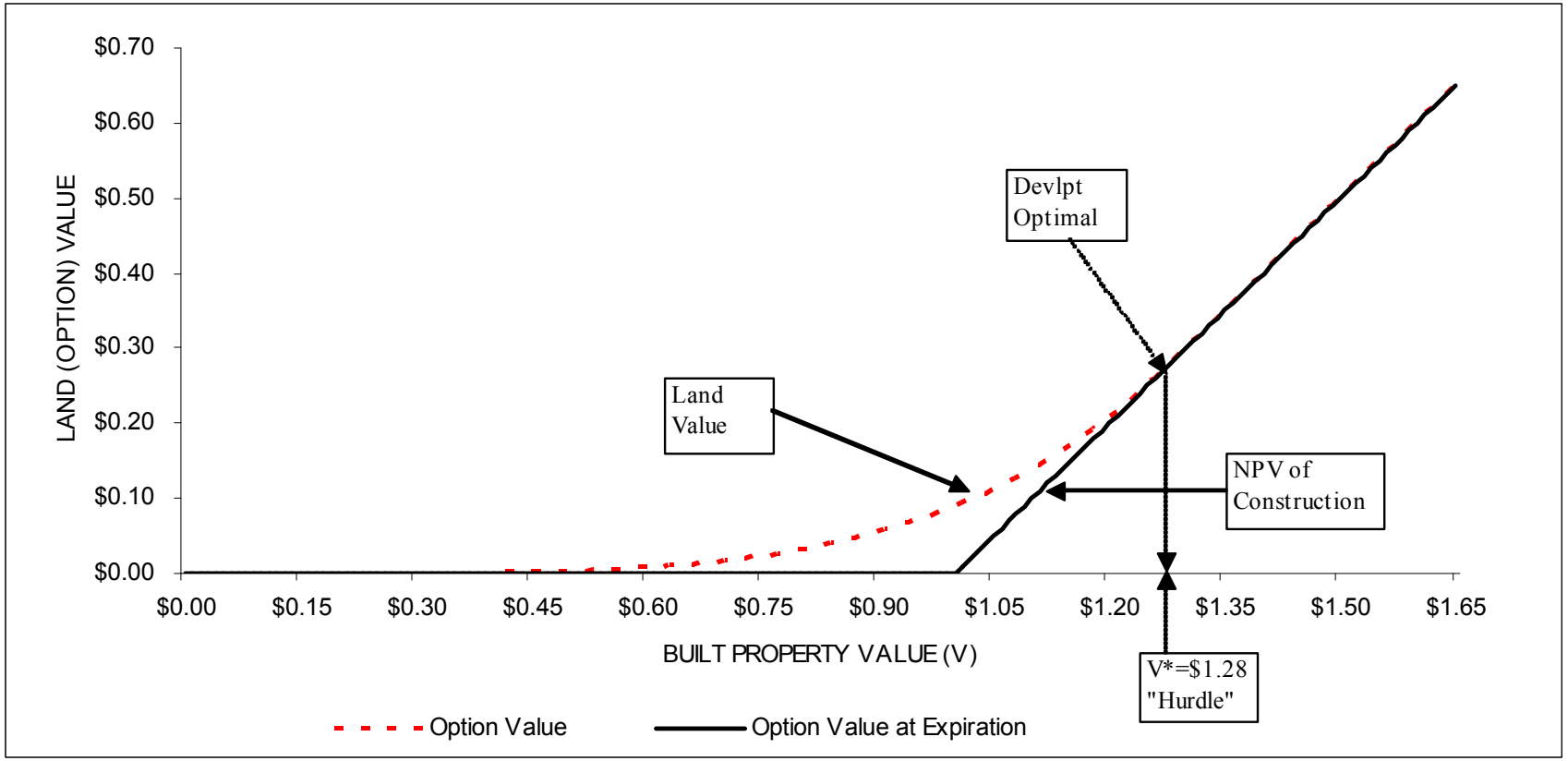
- Notation Basics:
  - V = Value of built property
  - S = Volatility of (Std.Dev. of return to unlevered) individual built properties. Notes that such a value includes idiosyncratic risk
  - y = Payout ratio of the built property. This represents the current cash yield rate.
  - r_f = Riskfree interest rate (e.g., short-term T-bill yield, typically 3% to 6%).

- The "option elasticity" [(dLAND/LAND)/(dV/V)], is given by:
  - $\eta = \{y-r_f+S2/2 + [(r_f-y-S2/2)2 + 2r_fS2]1/2\}/S2$
- The option (land) value is given by:

$$LAND = (V^*-K)\left(\frac{V}{V^*}\right)^{\eta}$$

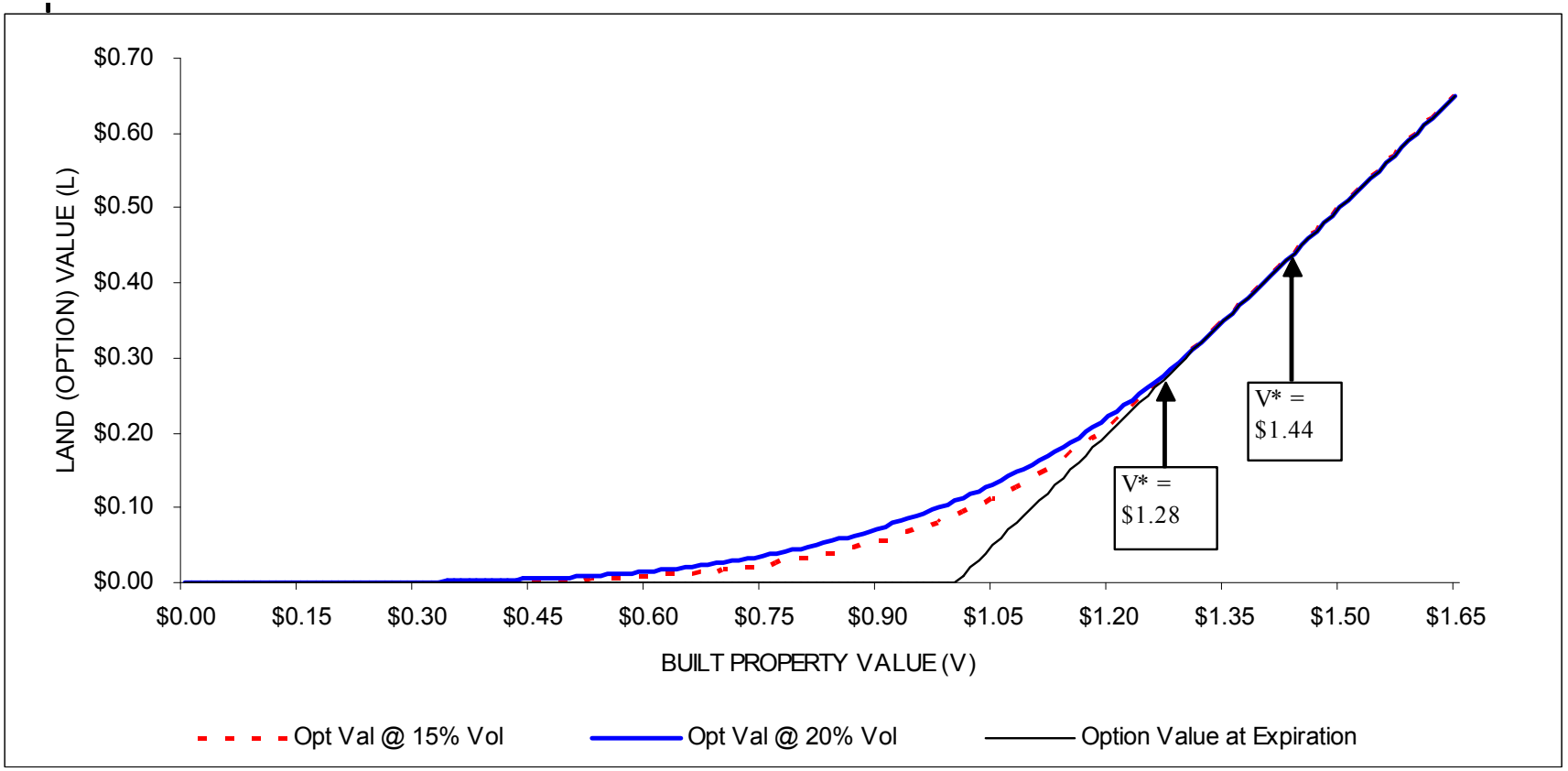
- V* represents the hurdle rate upon which below such a value the land should be left undeveloped.

Graphical Representation of Samuelson-McKean



- As in a financial call option, V (the land value) is a monotonically increasing, convex function of the current HBU built property value (underlying asset value).

Comparative statics on the land hurdle rate (V*)



- It can be shown mathematically that the option value, and the hurdle *V/K* ratio, are *increasing* functions of the volatility (*S*) and *decreasing* functions of the payout ratio (*y*).

Useful metrics from the Samuelson-McKean Formula

- The hurdle benefit/cost ratio (V*/K):
  - It represents the ratio of built property value divided by construction cost exclusive of land cost, which triggers immediate optimal development.
- The hurdle benefit/cost ratio (V*/K) is:
  - Increasing with the risk-free rate.
  - Decreasing with the built property current cash yield.
  - Increasing with the volatility of the built property asset value.
- The hurdle benefit/cost ratio (V*/K) is independent of the size of the project.

Useful metrics from the Samuelson-McKean Formula

- The relationship of the risk premium of the vacant land to the that of built properties in the underlying real estate asset market: ($RP_{LAND} = \eta RP_V$):
  - The risk premium of the vacant land is proportional to the option elasticity. The elasticity represents the percentage change in the vacant land value associated with a 1% change in the values of built properties in the underlying real estate market.
- Now to some blackboard examples.