Chapter 18 (Section 18.1):

Expected Returns vs. Stated Yields

Measuring the Impact of Default Risk

"Expected Returns" versus *"Stated Yields"* . . .

In a bond or mortgage (capital asset with contractual cash flows):

Stated Yield (aka "Contractual Yield") = YTM based on *contractual obligation*.

Expected Return (aka "Expected Yield" or "Ex Ante Yield") = *E*[*r*] = Mean of probability distribution of future total return on the bond or mortgage investment.

- **Quoted yields are always *stated yields*.**
- **Contract yields are used in mortgage design and evaluation.**

- **Expected return is more *fundamental* measure for mortgage investors.**
- **For making investment decisions.**

Difference:

Stated Yield – Expected Return

←→ Impact of *Default Risk* in *ex ante* return investor cares about.

18.1.1 Yield Degradation & Conditional Cash Flows…

"Credit Losses" **= Shortfalls to the lender (mortgage investor) as a result of** ***default*** **and** ***foreclosure*****.**

"Realized Yield" **= What the lender (investor) actually receives (as an IRR).**

"Yield Degradation" **= Impact of** ***credit losses*** **on the lender's realized yield as compared to the contractual yield (expressed in IRR units).**

Contractual Yield

- Yield Degradation ← Due to ***Credit Losses***

= Realized Yield

Yield Degradation **(*"YDEGR"*) = Lender's losses measured as a multi-period lifetime return on the original investment (IRR impact).**

Numerical example of *Yield Degradation*:

- **$100 loan.**
- **3 years, annual payments in arrears.**
- **10% interest rate.**
- **Interest-only loan.**

Here are the *contractual* terms of the loan as an NPV equation:

$$0 = -\$100 + \frac{\$10}{1+(0.10)} + \frac{\$10}{(1+(0.10))^2} + \frac{\$110}{(1+(0.10))^3}$$

Contractual YTM = 10.00%.

Suppose:

- **Loan defaults in 3rd year.**
- **Bank takes property & sells in foreclosure, but**
- **Bank only gets 70% of OLB: $77.**

- $33 = "Credit Losses".
- 70% = "Recovery Rate".
- 30% = "Loss Severity".

Here are the *realized* cash flows of the loan as an NPV equation:

$$0 = -\$100 + \frac{\$10}{1+(-0.0112)} + \frac{\$10}{(1+(-0.0112))^2} + \frac{\$77}{(1+(-0.0112))^3}$$

Realized IRR = -1.12%
Yield Degradation = 11.12%:
Contract.YTM – Yld Degrad = Realized Yld:
10.00%. – 11.12% = -1.12%.

From an *ex ante* perspective, this 11.12% yield degradation is a "conditional" yield degradation.

It is the yield degradation that will occur *if* the loan defaults in the third year, and *if* the lender gets 70% of the OLB at that time.

(Also, 70% is a conditional recovery rate.)

Suppose the default occurred in the 2nd year instead of the 3rd:

$$0 = -\$100 + \frac{\$10}{1+(-0.0711)} + \frac{\$77}{\left(1+(-0.0711)\right)^2}$$

Yield Degradation = -17.11%.

Other things being equal (in particular, the conditional recovery rate), *the conditional yield degradation is greater, the earlier the default occurs in the loan life*.

Lenders are hit worse when default occurs early in the life of a mortgage.

Note: "YDEGR" as defined in the previous example was:

- **The reduction in the IRR (yield to maturity) below the contract rate,**
- ***Conditional* on default occurring (in the 3rd year), and**
- **Based on a specified conditional *recovery rate* (or *loss severity*) in the event that default occurs.**

$$YDEGR_t = YTM - YLD|DEF_t = YTM - IRR(loss\ severity_t)|DEF_t$$

For example, if the loss severity were 20% instead of 30%, then the conditional yield degradation would be 7.13% instead of 11.12%:

$$0 = -\$100 + \frac{\$10}{1+(0.0287)} + \frac{\$10}{(1+(0.0287))^2} + \frac{\$88}{(1+(0.0287))^3}$$

$YDEGR_3$ = 10% - 2.87% = 7.13%.

Relation between *Contract Yield*, *Conditional Yield Degradation*, & the *Expected Return* on the mortgage…

Expected return is an *ex ante* measure.

To compute it we must specify:

- ***Ex ante probability of default*, &**
- **C*onditional recovery rate* (or the *conditional loss severity*) that will occur in the event of default.**

Suppose that at the time the mortgage is issued, there is:

- **10% chance of default in 3rd year.**
- **70% conditional recovery rate for such default.**
- **No chance of any other default event.**

Then at the time of mortgage issuance, the *expected return* is:

$$E[r] = 8.89\% = (0.9)10.00\% + (0.1)(-1.12\%)$$
$$= (0.9)10.00\% + (0.1)(10.00\%-11.12\%)$$
$$= 10.00\% - (0.1)(11.12\%) = 8.89\%.$$

In general: *Expected Return* = *Contract Yield* – Prob. of Default * *Yield Degradation*.

$$E[r] = \text{YTM} - (\text{PrDEF})(\text{YDEGR})$$

What would be the expected return if the ex ante default probability and conditional credit loss expectations were:

- **80% chance of no default;**
- **10% chance of default in 2nd year with 70% conditional recovery;**
- **10% chance of default in 3rd year with 70% conditional recovery.**

?

Answer:

$$E[r] = \text{YTM} - \Sigma(\text{PrDEF})(\text{YDEGR})$$

$$E[r] = 10\% - (.1)(11.12\%) - (.1)(17.11\%) = 10\% - 2.82\% = 7.18\%.$$

Note: The probabilities we were working with in the previous example:

- **80% chance of no default;**
- **10% chance of default in 2nd year;**
- **10% chance of default in 3rd year.**

Were "*unconditional probabilities*" as of the time of mortgage issuance:

- **They did not depend on any pre-conditioning event;**
- **They describe an exhaustive and mutually-exclusive set of possible outcomes for the mortgage, i.e.,:**
- **The probabilities sum to 100% across all the eventualities.**

18.1.2 Hazard Functions and the Timing of Default…

More realistic and detailed analysis of mortgage (or bond) default probability (and the resulting impact of credit losses on expected returns) usually works with *conditional probabilities* of default, what is known as a:

Hazard Function

The hazard function tells the *conditional probability* of default at each point in time *given that* default has not already occurred before then.

Example: Suppose this is the *hazard function* for the previous 3-yr loan:

Year:	Hazard:
1	1%
2	2%
3	3%

i.e., There is:

- **1% chance loan will default in the 1st year (i.e., at the time of the first payment);**
- **2% chance loan will default in 2nd year *if it has not already defaulted in the 1st year*; &**
- **3% chance loan will default in 3rd year *given that it has not already defaulted by then*.**

Given the hazard function for a mortgage, we can compute the cumulative and unconditional default and survival probabilities.

Example: Suppose this is the *hazard function* for the previous 3-yr loan:

Year:	Hazard:
1	1%
2	2%
3	3%

Then the table below computes the unconditional and cumulative default probabilities for this loan:

Year	Hazard	Conditional Survival	Cumulative Survival	Unconditional PrDEF	Cumulative PrDEF
1	0.01	1-.01 = 0.9900	0.99*1.0000 = 0.9900	.01*1.0000 = 0.0100	0.0100
2	0.02	1-.02 = 0.9800	0.98*0.9900 = 0.9702	.02*0.9900 = 0.0198	.0100+.0198 = 0.0298
3	0.03	1-.03 = 0.9700	0.97*0.9702 = 0.9411	.03*0.9702 = 0.0291	.0298+.0291 = 0.0589

- *"Conditional Survival Probability"* (for year t) = 1 – *Hazard* for year t.
- *"Cumulative Survival Prob."* (for year t) = Probability loan survives through that yr.
- *"Unconditional Default Prob."* (for year t) = Prob.(as of time of loan origination) that loan will default in the given year (t) = *Hazard * Cumulative Survival (t-1) = Cumulative Survival (t) – Cumulative Survival (t-1)*.
- *"Cumulative Default Prob."* (yr.t) = Prob.(as of time of loan origination) that loan will default any time up through year t.

In this case: 5.89% unconditional probability (as of time of origination) that this loan will default (at some point in its life). 5.89% = 1.00% + 1.98% + 2.91% = 1 – 0.9411.

For each year in the life of the loan, a conditional yield degradation can be computed, conditional on default occurring in that year, and given an assumption about the conditional recovery rate in that year.

For example, we saw that with previous 3-yr loan the conditional yield degradation was 11.12% if default occurs in year 3, and 17.11% if default occurred in year 2, in both cases assuming a 70% recovery rate.

*Similar calculations reveal that the conditional yield degradation would be 22.00% if default occurs in year 1 with an 80% recovery rate.**

Defaults in each year of a loan's life and no default at all in the life of the loan represent mutually-exclusive events that together exhaust all of the possible default timing occurrences for any loan.

For example, with the three-year loan, Bob will either default in year 1, year 2, year 3, or never.

Thus, the expected return on the loan can be computed as the contractual yield minus the sum across all the years of the products of the <u>unconditional</u> default probabilities times the conditional yield degradations.

$$E[r] = YTM \quad - \quad \sum_{t=1}^{T} \left(\Pr DEF_t\right)\left(YDEGR_t\right)$$

Example:

- **Given previous hazard function (1%, 2%, and 3% for the successive years);**
- **Given conditional recovery rates (80%, 70%, and 70% for the successive years);**
- **Expected return on Bob's 10% mortgage at the time it is issued would be:**

$$E[r] = 10.00\% - ((.0100)(22.00\%) + (.0198)(17.11\%) + (.0291)(11.12\%))$$
$$= 10.00\% - 0.88\%$$
$$= 9.12\%.$$

The 88 basis-point shortfall of the expected return below the contractual yield is the "<u>ex ante yield degradation</u>" (aka: "<u>unconditional yield degradation</u>").

It reflects the ex ante credit loss expectation in the mortgage as of the time of its issuance.

Two alternative ways to compute the *expected return* . . .

"***Method 1***" "Return-based" (as previously described) ***E*[*IRR(CF)*]** :

Take the expectation over the conditional returns… ← Most commonly used.

$$E[r] = YTM - \sum_{t=1}^{T} (\Pr DEF_t)(YDEGR_t)$$

$$= YTM - \sum_{t=1}^{T} (\Pr DEF_t)(YTM - YLD|DEF_t)$$

$$= (\Pr NODEF)YTM + \sum_{t=1}^{T} (\Pr DEF_t)(YLD|DEF_t)$$

$$= \sum_{i=1}^{N} (\Pr SCEN_i)(YLD_i) = \sum_{i=1}^{N} (\Pr SCEN_i)(IRR(CF_i))$$

Makes sense if investor preferences are based on the return achieved.

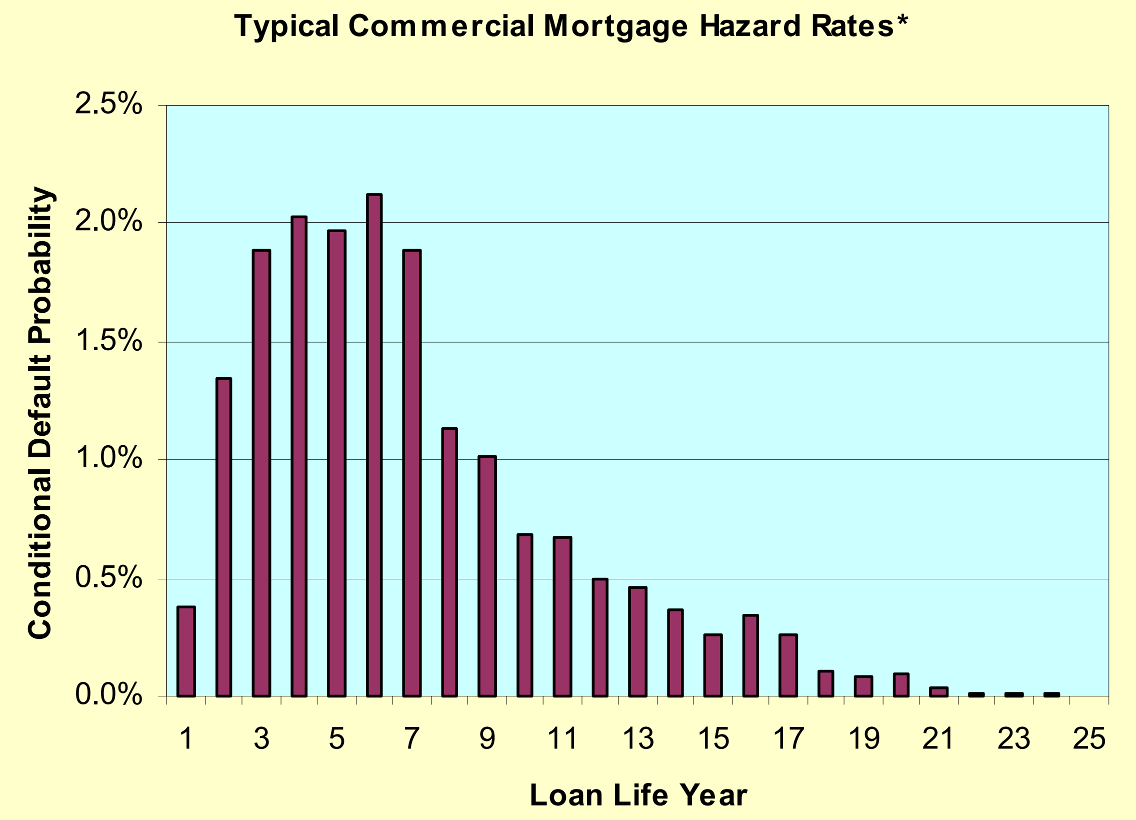
"***Method 2***" "Expected CF-based", or "Pooled CF-based", ***IRR(E*[*CF*]*)*** :

Take the expectation over the conditional cash flows and then compute the return on the expected cash flow stream: $E[r] = IRR\left(\sum_{i=1}^{N} (\Pr SCEN_i)(CF_i) \right)$

Makes sense if investor preferences are based on the cash flows achieved.

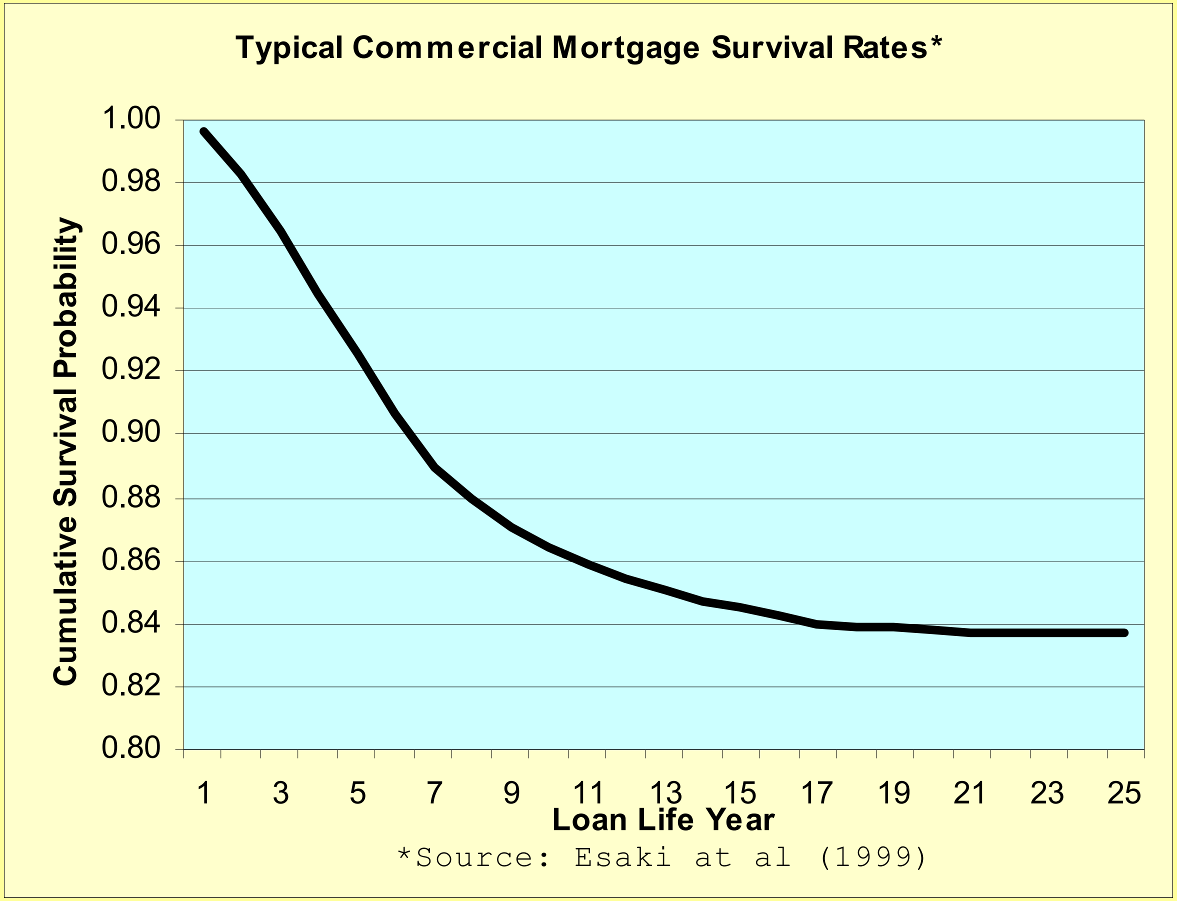
18.1.3 Yield Degradation in Typical Commercial Mortgages…

The most widely used empirical evidence on commercial mortgage hazard rates in the U.S. is that of Snyderman and subsequent studies at Morgan-Stanley.*



*Source: Esaki et al (1999)

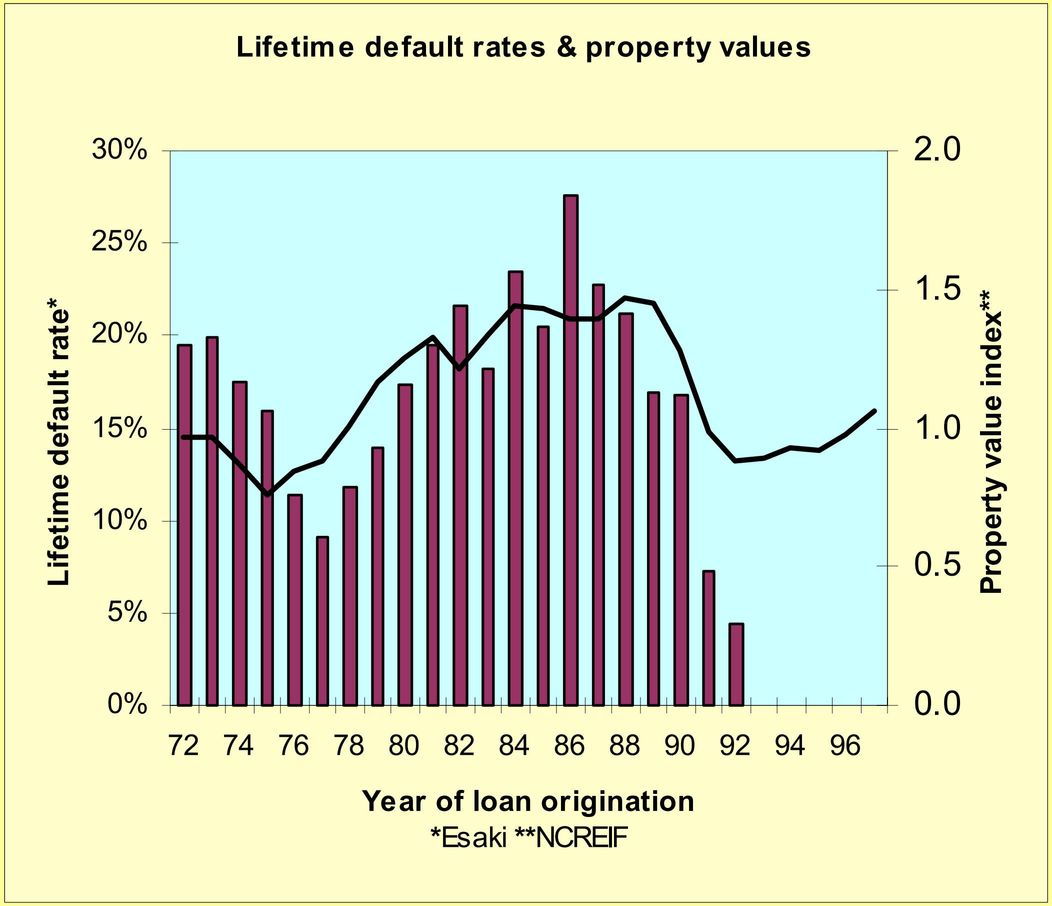
The implied survival function and cumulative default probability is shown here:



Overall Average Default Probability = 16%.

1 out of 6 commercial mortgages in the U.S. default at some point in their lives.

Loan lifetime default probabilities are strongly influenced by the time (phase of the real estate market cycle) at which the loan was originated:



Why do you suppose this is so?

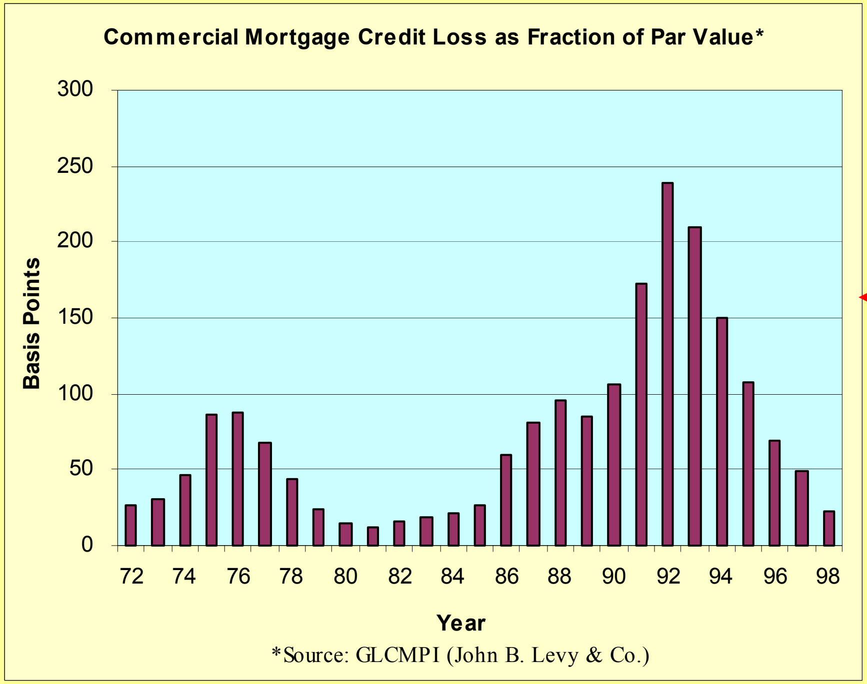
And what do you think about it?

Combining empirical data on conditional recovery rates (typically assumed to be between 60% and 70%), we can estimate the typical ex ante yield degradation in U.S. commercial mortgages…

➔Typical Yield Degradation:

60 to 120 basis points.

Similar results are observed in the *Giliberto-Levy Commercial Mortgage Index (GLCMI)*, the major index of commercial mortgage ("whole loan") periodic ex post returns (HPRs).



1972-98 Avg =

73 basis points*

Is 16% avg lifetime default probability surprisingly high? . . .

Consider relation between:

- **LTV,**
- **Property Risk (volatility),**
- **Loan Default Probability.**

A simplified example…

(Text box p. 447)

Suppose…

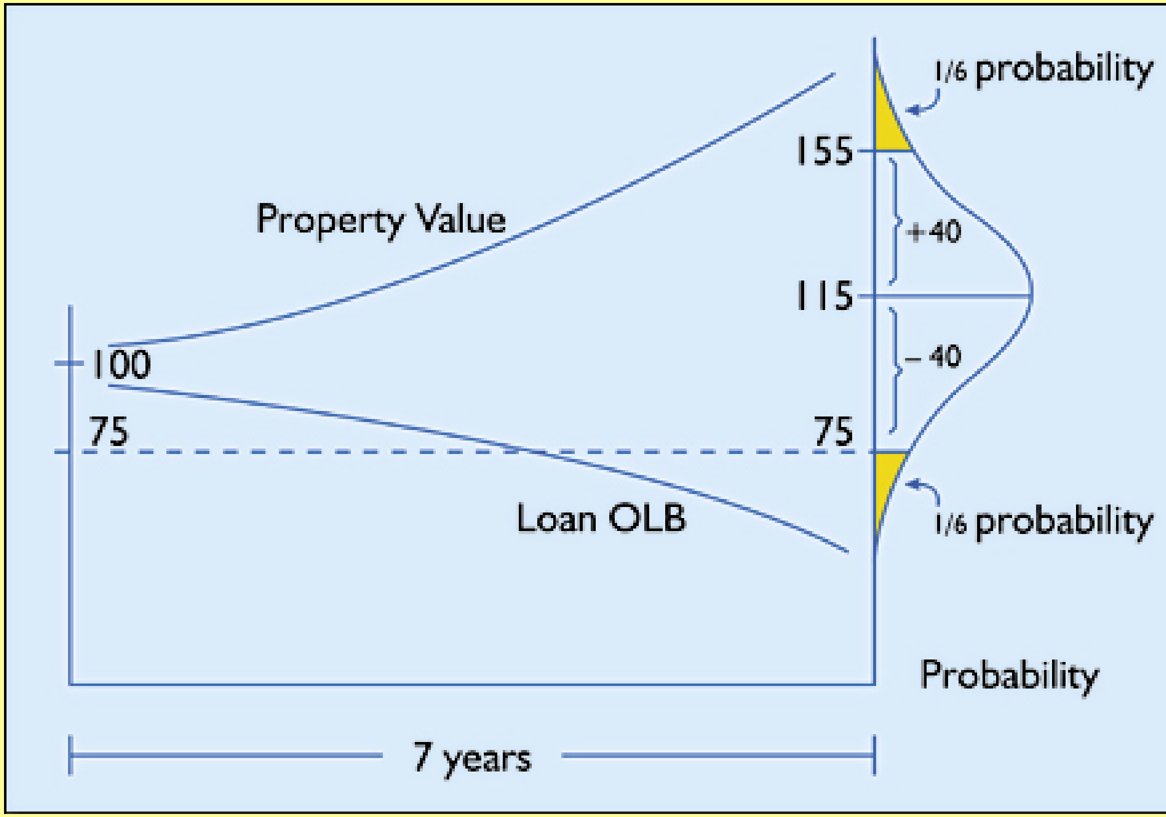
- **Initial Prop. Val = $100, E[g] = 2%/yr.**
- **75% LTV (No amort ➔ OLB = $75 constant).**
- **Average loan default occurs in year 7 of loan life** *(Esaki)*.
- ***Individ.* Prop. Ann. Volatility (Std.Dev[g]) = 15%.**
- **Prop. Val follows *random walk* (effic. mkt.).**
- ➔ $T\ yr\ Volatility = \sqrt{T}\left(Ann.Volatility\right)$

A simplified example…

Thus, After 7 years:

- **E[Val] = $1.02^7(100)$ = 115**
- **Std.Dev[Val] =**

$$\sqrt{7}\,(15\%)(100)$$
$$= 2.6 * 15\%(100)$$
$$= \pm 40\%(100) = \pm 40.$$



- **1 Std.Dev below E[Val] = $115 - $40 = $75.**
- **If Prob[Val] ~ *Normal*, ➔ 1/6 chance Val < OLB, ➔ Loan *"under water"* (large chance of default in that case).**