MAS 622J: Pattern Recognition and Analysis

Problem Set 1
Date: Lecture 1 **Due:** Lecture 4

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1. Describe an application of pattern recognition in your life outside of work. What are the features? What is the decision to be made? Speculate on how one might solve the problem. Limit your answer to a page.

2. Let x and y be two Gaussian distributions

$$p(y) \sim \mathcal{N}(\mu, \alpha)$$
$$p(x|y) \sim \mathcal{N}(y, \sigma)$$

   (a) Find the posterior probability $p(y|x)$ in the form of a Gaussian. (Hint: You can use the fact that $\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}} dx = 1$)

   (b) Suppose $\mu = 0$, $\alpha = 50$, and $\sigma = 2$. Use MATLAB® to plot $p(y)$ and $p(y|x)$ at $x = 5$ for a reasonable range of y. Could you explain the difference between $p(y)$ and $p(y|x)$?

3. Let x and y be independent identically distributed random variables with common density function

$$p(x) = p(y) = \begin{cases} 1 & 0 \leq x, y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

   Let $s = x + y$.

   (a) Find and use MATLAB® to plot $p_s(s)$ for some reasonable range of s.

   (b) Find and use MATLAB® to plot $p_{x|s}(x|s)$ vs. x with s viewed as a known parameter.

   (c) Let z be a random variable with probability density function:

$$p(z) = \tfrac{1}{2}\delta(z+1) + \tfrac{1}{2}\delta(z-1)$$

   Where, $\delta(z)$ is the Dirac delta function

   Let $q = x + z$. Find and use MATLAB® to plot $p_q(q)$ for some reasonable range of q.

4. Let $\Lambda_x = \begin{bmatrix} 24 & -16 \\ -16 & 24 \end{bmatrix}$

   (a) Verify that Λ_x is a valid covariance matrix.

   (b) Compute the eigenvalues and eigenvectors of Λ_x.

   (c) We provide 200 data points sampled from the distribution $\mathcal{N}(0, \Lambda_x)$ (Download the dataset from the course website). Use MATLAB® to plot the datapoints. Project the data onto the eigenvectors and plot the transformed data. What is the difference between the two plots?

5. Dice difference: Dice difference is a game that involves two players. Two dice are thrown and the larger number is subtracted from the smaller number. Player A wins whenever the dice difference is 0, 1 or 2. Player B wins whenever the dice difference is 3, 4 or 5. What is the probability that player A wins? Is the game fair? Explain how you could change the rules and make it fair. Justify your rule changes.

6. Below is a variation on the classic "Monty Hall" problem. To get credit for this problem, you must pose it to at least one other person who have not understood it before, and help them find the correct answer. Have them sign your problem set with a statement that indicates you have succeeded in this goal.

   Problem:

   Suppose I hide the solution to the current problem set in one of three identical boxes while you aren't looking. Then I ask you to guess which box it's in. I know which one it's in, and after you guess, I deliberately open the lid of an empty box, which is one of the boxes you have *not* chosen. Thus, the solutions are either in the box you guessed, or in the box (you didn't pick) that I didn't open.

   I then offer you the chance to change your mind: you can either keep the box you originally guessed, or choose the other unopened box. To maximize your chances of getting the problem set solution, what choice should you make? You must explain your answer.