### IX. MISCELLANEOUS PROBLEMS

### A. ELECTRONIC DIFFERENTIAL ANALYZER

# Prof. Henry Wallman R. Maartmann-Moe

The problem of synchronizing a pulsed magnetron with a continuous-wave oscillator was studied on the machine by Dr. E. E. David. The problem required the solution to the Van der Pol equation with a sinusoidal driving function

$$
x'' + \mu(x'^2 - 1)x' + \omega_0^2 x = E\omega^2 \sin \omega t
$$

It was desired to show about 30 cycles of the solution. Difficulties were encountered because of the frequency response of the adders and integrators. When more than about ten cycles of the solution to the equation

$$
y''' = -\omega^2 y
$$

are shown during the solution time of the machine, an exponentially growing sine wave rather than a constant amplitude sine wave results. It was therefore found necessary to introduce compensation, or add a damping term to the auxiliary equation  $y'' = -\omega^2 y$  used to produce a driving term sin wt of constant amplitude. A similar damping term had to be added to compensate for the same effect in the solution of the Van der Pol equation. A study of the adders and integrators shows that the cause of the difficulty lies in the frequency response of the integrators. For stability reasons, the feedback amplifier used in the integrators cannot employ high gain at high frequencies; therefore the integrators have a phase shift in excess of the ideal **900** at frequencies above about 1000 cps, corresponding to 10 cycles during the solution time.

Mr. Leif Arnesen and Mr. Lawson Harris of the Mechanical Engineering department studied a problem in connection with a Geneva mechanism. This problem required the solution to the equation

$$
x'' + ax' + \omega^2 x = cy'' + dy'
$$

The driving function was given by a specified y". About one hundred runs were made for different types of driving functions and damping coefficients a. Analytical solution of the equation would have been rather tedious, as the driving function was of a fairly complex nature.

Mr. W. P. Schneider investigated the properties of the equation

$$
\theta' + K \sin \theta = M \sin \omega t
$$

in connection with a method of magnetron modulation.

The negative, -110-volt, power supplies have been replaced in part by a -275-volt power supply, and consequent improvements in stability of the multipliers and improved

#### (IX. MISCELLANEOUS PROBLEMS)

designs of some of the amplifiers used in the function-generator-multiplier units have been made. The output circuit of the unit has been improved by the use of 5687 tubes, enabling the function-generator-multiplier unit to handle heavier loads without overloading. A simple compensating network has been added in the magnetic channel of the multiplier, materially reducing the phase shift in this channel of the multiplier. It is believed that this reduction in the phase shift will reduce the errors reported by Dr. Macnee in Technical Report No. 136 on Fourier transform work.

Difficulty which was experienced due to burning of the cathode-ray-tube screens in the function generators has been reduced by a new, regulated high-voltage supply, increasing the voltage on the 921-A photomultiplier tubes, and resulting in higher gains and reduced spot intensity required for tracking.

It has been found necessary to use some kind of adjustable suspension on the multiplier coil, as the magnetic shields around the cathode-ray tube distort the field, making it necessary to adjust the coil axis with respect to the tube axis to obtain coincidence of the tube axis and the magnetic field.

## B. ANALOG DEVICES FOR NETWORK PROBLEMS



## **1.** Automatic Impedance Function Analyzer

The original machine is available to solve approximation problems in network synthesis. No major changes are contemplated until the work on the dipole analog is completed. Eventually it is hoped that a machine will be built combining the two analogs.

R. E. Scott

2. The Dipole Potential Analog

The dipole potential analog was first conceived by Professor E. A. Guillemin and is discussed by him in his book "The Mathematics of Circuit Analysis. " The advantages of this form of potential analog are: 1. The potential in the conducting medium is everywhere proportional to the real part (or the imaginary part) of the impedance function. 2. The impulse response may be obtained by an analog integration along the imaginary axis (Quarterly Progress Report, April 15, 1950).

Instrumentation has proceeded far enough to show that the dipole analog is practicable but the impulse response has not yet been obtained. A double-layer semicircular representation of the complex frequency plane has been constructed. Teledeltos paper and Pliotherm rubber have been investigated for the conducting medium. Both materials have a current capacity of about 15 ma per linear inch. The resistivity of the paper is about 4000 ohms per square; that of the rubber is only 600. The higher voltage levels obtained when the paper is used are a distinct advantage.

The double layer tank removes the finite-size errors, and leaves the frequency scale linear in form. In return for these advantages it has been necessary to take extreme precautions in joining the upper and the lower layers at the circumference. Small nonuniformities at the boundary cause disproportionately large errors in the field pattern which is to be measured.

Alternating currents are used to offset the disadvantages of the low voltage levels in the device. Each dipole is driven from a separate transformer in the plate of a pentode tube. This system provides a floating current source, free to assume the average potential of the field in which it is placed. The angle of dipole rotation is obtained electrically rather than mechanically. Two dipoles at right angles are used. They are inserted in the plane parallel to the grid lines and the angle is obtained by varying the relative strengths of the currents in the dipoles.

Experimental results for particular configurations show that the voltages measured along the imaginary axis agree with the calculated values within 3 or 4 percent. The primary limitation of the present device is the low voltage level associated with the use of paper as the conducting medium. It is felt that a resistance matrix would solve this problem, and at the same time eliminate the contact resistance errors in the present device. D. **D.** Holmes

## 3. The Electronic Isograph

The isograph for finding the roots of polynomials to the sixth degree has been completed. The unknown variable  $x = r(\cos \theta + i \sin \theta)$  is represented by voltages at right angles on a cathode-ray tube. Powers of the variable are produced as multiple frequency sine and cosine waves whose frequency is controlled from a master oscillator through counting circuits, multivibrators, and filters. The cosine waves, modified by the appropriate coefficients, are summed and applied to the X plates of the cathode-ray tube; the sum of the sine waves, to the Y plates. The resultant trace on the face of the cathoderay tube represents the polynomial for a particular value of r and for a continuously varying phase **0.**

The operator varies r by means of a single ganged potentiometer until the trace passes through the origin on the cathode-ray tube. The value of r which produces this result is a root of the equation. The corresponding value of  $\theta$  is obtained by moving a blanking pulse along the trace until it, too, is at the origin. The phase through which it is necessary to move the blanking pulse is the required value of **0.** The coefficients of the polynomial are applied to the amplitudes of the outputs from the multivibrators before they are filtered. Since the filters are linear the output sine and cosine waves are

### (IX. MISCELLANEOUS PROBLEMS)

likewise proportional to the settings of the coefficient potentiometers.

The accuracy of the machine has been checked on a number of problems. It is better than 5 percent unless the roots become very much less than one. More accurate results can be obtained by a method of successive refinement.

R. H. Pantell

## 4. A Panoramic Display for the Electronic Isograph

A panoramic display for the electronic isograph has both an immediate and a longrange purpose. The immediate purpose lies in connection with the theoretical work of Dr. M. V. Cerrillo's group. In the course of the approximation technique which they have developed for a large class of integrals, it becomes necessary to know the regions in which a polynomial has a positive real part. It is hoped that the electronic isograph can be made to yield this information. In a long-range view it is hoped that the roots of a polynomial may be displayed simultaneously and instantaneously upon the face of a cathode-ray tube. This procedure would eliminate a great deal of the work required to solve a problem on the present isograph.

The general procedure is to vary r continuously, meanwhile applying r cos  $\theta$  to the  $X$  plates, and r sin  $\theta$  to the Y plates of the oscilloscope. The value of the polynomial is computed as before, and the real part is applied to the intensity grid of the cathode-ray tube. The trace is thus blanked out when the real part of the polynomial is negative, and bright when the real part is positive. The original plan was to vary the magnitude of the square waves fed into the filters of the present isograph. The narrow band of these filters prevented this method from being successful. As an alternative it was decided to modulate the sine waves with a sawtooth for the first power, a parabola for the second, etc. Several balanced modulators were built with multigrid 6SA7 and 6BE6 tubes, but the requirements of the problem were so stringent with respect to the linearity of the envelope and the magnitude and phase of the carrier that no completely successful solution was attained. (The problem is essentially that of building a multiplying device.)

The difficulties encountered have suggested the value of another approach. Exponentially damped sine waves could be used with damping constants in multiple ratios. The sine waves could be obtained by shock-exiting passive tuned circuits. This method would result in the cathode-ray tube being scanned at an exponential rate from the outside to the inside; with a long persistence screen this should cause no trouble. A more serious difficulty is the frequency stability of the tuned circuits.

R. P. Talambiras

## 5. An Electronic Commutator

An electronic commutator will ultimately be needed for both the impedance function analyzer and the dipole potential analog. The unit must be able to handle low-level  $d-c$  signals and it must be capable of commutating at least 100 lines.

A diode tree commutator was conceived as a possible solution to the problem, and an experimental eight-line unit was constructed. The unit uses diodes and resistors in the form of a tree as the basic switching element. The tree is driven by a binary counting string. The present unit is naturally suited to low-level d-c work, and its repetition rate is high (1100 cps). It uses a smaller number of tubes than the standard crystal matrix: for N lines it requires  $2(N - 1)$  diodes; the matrix requires  $2^N$  crystals and N gate tubes.

The principal disadvantages of the device are a bias current of about 5 ma which must be passed through the source, and cross-talk which may amount to as much as 5 percent of the signal. A possible method of overcoming these difficulties would be to use triodes instead of diodes in the tree. The triodes, however, require transformer-coupling to the counting string in order to isolate the various tubes, and additional problems are introduced.

An interesting sidelight is that the present diode tree can be inverted and operated as a sequential pulse generator. The pulses are of very short rise time and the number of tubes used is considerably less than the number used in the conventional crystal matrix when the number of lines is large. The set of the set of