X. ANALOG COMPUTER RESEARCH

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A. THE OPERATION OF PRESENT COMPUTERS

1. The Macnee Differential Analyzer

During the past quarter the Macnee differential analyzer has been used to implement an investigation of the stability of nonlinear systems. For a linear system, stability is determined by the location of poles in the complex plane. For a quasi-linear system, the poles may be considered to move around as the variables change. For a really nonlinear system, the pole positions are not directly related to the stability. Nevertheless for these systems a linear incremental analysis yields a considerable amount of information. An equation which has been studied on the machine is

$$\dot{y} + ay + by^2 = 0.$$

For this equation the linear incremental analysis gave stability which was more conservative than the machine solution indicated was necessary. An analytic check verified the machine solution and indicated the unreliability of the linear incremental analysis. It is believed, however, that if the solution is known to possess a limit state in which the derivatives vanish, the stability of this state can be determined by the linear incremental analysis.

2. The Use of Computing Elements in Nonlinear Time Domain Filters

The concept of a derivative-limiting time domain filter has been generalized to some extent. Originally an open-ended system was proposed in which the signal and noise would be differentiated, clipped at some level, and reintegrated. An alternative scheme is the use of a feedback system as shown in Fig. X-1. In the feedback loop the gain A is made large so that ϵ, the difference between S and S', is made arbitrarily small. Because of the integrators in the loop, the signals at a and b are the first two derivatives of S. Diode limiters at these points can be used. When they are, S' will be a

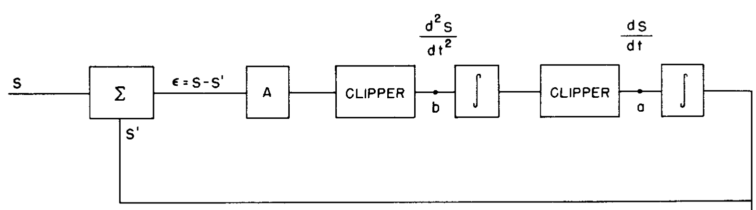

Fig. X-1
Feedback method of realizing nonlinear filter.

signal as nearly like S as possible, but it will have derivatives that remain within given bounds. If a priori knowledge is available about limits on the derivatives, noise which exceeds these values can be eliminated.

The loop would be unstable, of course, unless there were a derivative or lead network in A. Thus the system is seen to be similar to the open-loop, derivative-clipper-integrator scheme described in the Quarterly Progress Report, January 15, 1952.*

R. E. Scott

B. INTEGRAL EQUATION COMPUTER

The machine is now completed and is in use for the evaluation of integral transforms and for the solution of integral equations. The accuracy and repeatability realized is within one percent of the maximum modulus of the solution.

The most significant operation performed by the machine is that of multiplying a matrix P by a vector α. A 40×40 matrix can be multiplied by a 40-vector in 40 seconds. The matrix P may be that of a set of linear equations or a numerical equivalent for a continuous kernel function. The matrix P and vector α are inserted into the machine digitally on 70-mm paper tapes. The tape is standard Eastman Kodak film backing tape with edge sprocket punchings. Numbers in the range from -250 to +250 are recorded in binary code as rows of punchings across the tapes. A hole represents zero, no hole represents one. There are eleven columns across a tape: the first indicates sign, the next eight are digit columns, the remaining two provide control information for machine operation. The input tapes are driven mechanically past photoelectric reading heads. The input tapes are prepared manually on a keyboard punch which serves the additional function of automatically punching out the results of machine computation. The machine operations which intervene between the scanning of input data and the automatic punching of results are in part digital, in part analog. By employing hybrid digital-analog equipment, a compact machine occupying a single 72-inch relay rack has been realized. The accuracy obtained is nevertheless significant for many research purposes. All machine operations are self-sequencing from a single start button.

A typical application of the machine is that of convolving two functions. For this purpose, two tapes recording discrete sets of values of the functions to be convolved are punched and made endless. One tape which is to serve the function of a shifting matrix is made slightly shorter than the other so that after each complete revolution the shorter tape advances one interval of function tabulation with respect to the other. Since the tapes have sprocket punchings, positive initial synchronization is readily assured and the shifting operation characteristic of convolution is automatically

*G. A. Philbrick has independently arrived at a similar result.

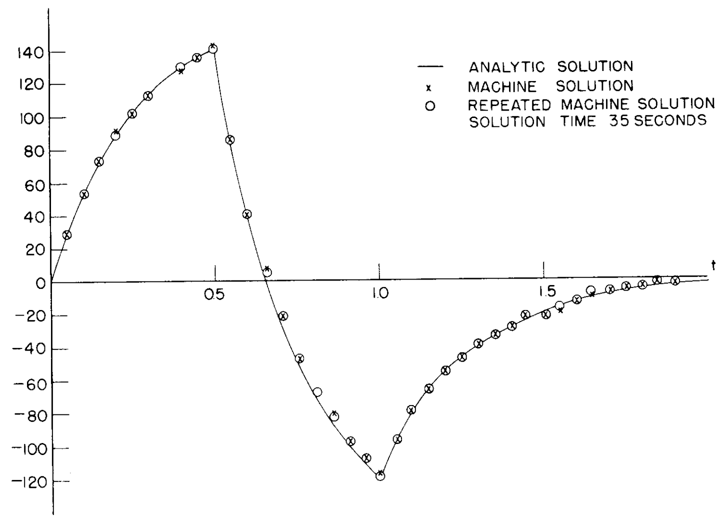

Fig. X-2
Convolution of a pulse doublet with an exponential.

performed. Figure X-2 shows a typical convolution computed by the machine. The solid curve is the exact convolution of a certain pulse doublet with an exponential. The crosses represent one machine solution; the dots, a solution repeated three days later.

In a general matrix product operation, the matrix P is punched on a long tape, each row of the matrix forming a block of punchings. The vector tape a is made endless and of such a length that it rotates successively in synchronism with the "block" rows of the matrix P. A library of standard kernel tapes is being prepared to effect Fourier and Hilbert transforms and to permit the expansion of arbitrary functions in orthonormal sets such as Bessel and Legendre functions.

The machine was designed primarily to effect the solution of integral equations by the method of safe descent (Von Eduard Stiefel: Über Einige Methoden der Relaxations-rechnung, Z. angew. Math.-Physik, Vol. III, 1-33, 1952) in the more or less intimate environment of laboratory research. Hence it is anticipated that the person desiring to solve an integral equation will himself use the machine. Since the machine may be shifted from one problem to another in less than five minutes, it is anticipated that it will be used by several research workers who at the same time are pursuing the solution of a problem by the method of successive approximations.

A technical report describing the design, construction, operation and application of the machine is in preparation.

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