XIII. CIRCUIT THEORY

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A. A PREDETECTION DIVERSITY COMBINER

Bandpass multipliers and narrow-band crystal filters that are needed in the combiner were constructed and investigated. Figure XIII-1 is a simplified version of a multiplier and filter as they might be arranged in the combiner.

The first grid of the 6AS6 multiplier may be biased to produce plate and screen currents compatible with the ratings of the tube; the third grid should be biased almost to the value that maximizes the transconductance from grid No. 3 to plate.

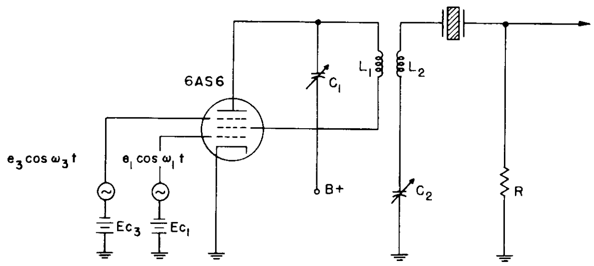

Fig. XIII-1. Multiplier and filter.

If we plot the multiplier output Ke_1e_3 against one of the inputs on log-log paper, we expect straight lines with a unit slope. If $e_1 = e_3$, we expect straight lines with slope two. Figure XIII-2 shows the experimental curves (dotted lines) compared with the desired curves (solid lines).

The difference-frequency component is selected from the output of the multiplier by means of a narrow-band crystal filter. This extremely selective filter is needed in the combiner to obtain a short-time average measure of the gain of the path from the transmitter to one of the receiving antennas. The filter represented in Fig. XIII-1 has a bandwidth of approximately 100 cps and a center frequency of 3.6 mc which may be varied ±120 cps by adjusting C_2.

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B. SYNTHESIS OF TWO TERMINAL-PAIR RESISTOR NETWORKS

Given a two terminal-pair resistor network, consider the effect of short-circuiting one pair as measured at the other pair. If the effect is large, then the two pairs are closely coupled and have driving-point resistances of the same order of magnitude. Conversely, if the driving-point resistances of the pairs are of different orders, then the pairs are loosely coupled, and short-circuiting one pair will not greatly affect the other. More precisely, let

$$k = z_{11}y_{11} \quad \text{and} \quad \ell = \max\left(\frac{z_{11}}{z_{22}}, \frac{z_{22}}{z_{11}}\right)$$

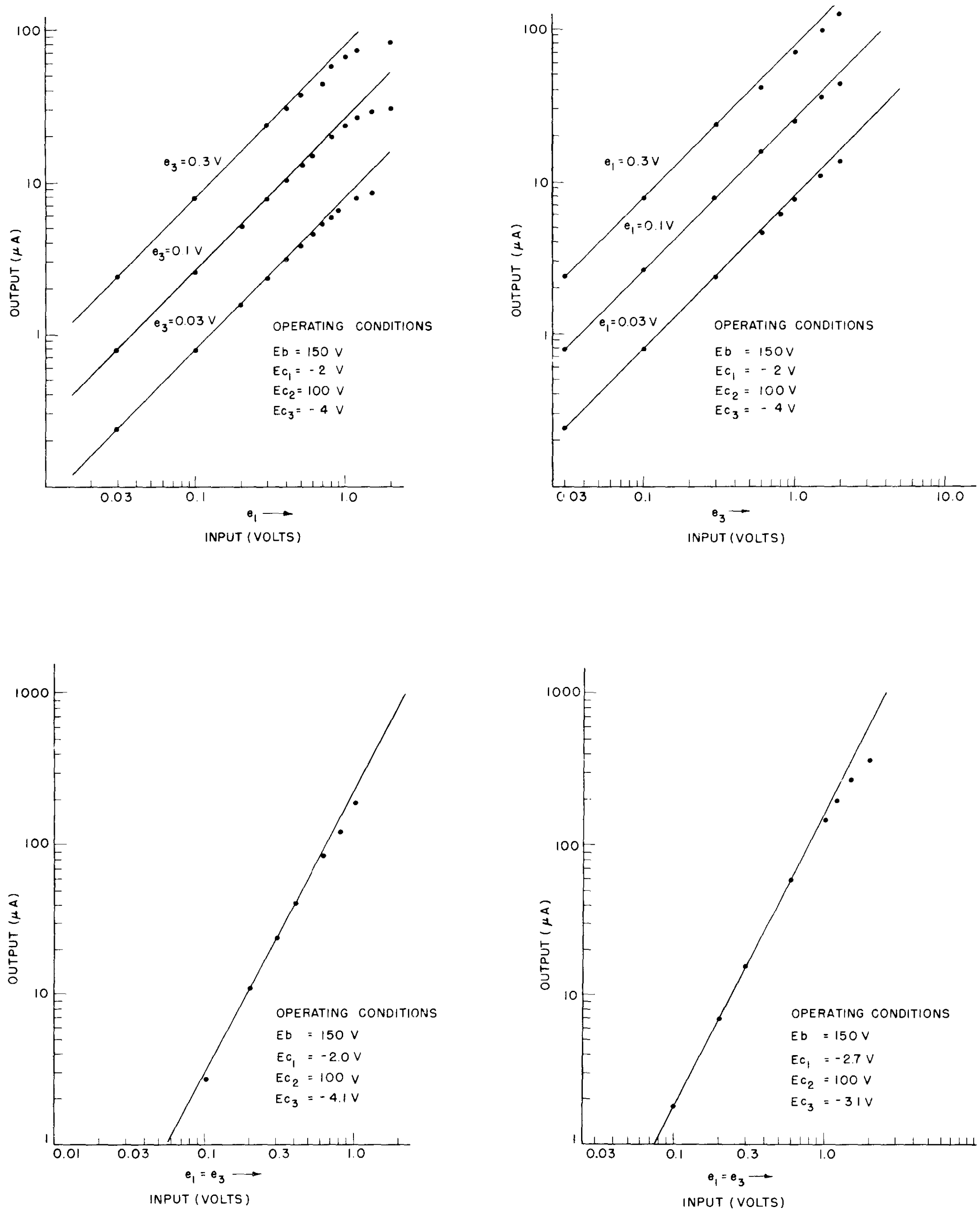


Fig. XIII-2. Curves of multiplier output versus signal input.

Then it can be shown that $k\ell \leq k + \ell$. Thus, if k or ℓ equals 2, the other cannot exceed 2.

THEOREM: A necessary and sufficient condition that four positive real numbers y_{11}, z_{11}^{-1}, y_{22}, z_{22}^{-1} be the four driving-point conductances of a two terminal-pair resistor network is that $y_{11}z_{11} = y_{22}z_{22}$ and $\left(z_{22} + y_{11}^{-1}\right)^{-1} \leq z_{11}^{-1} \leq y_{11} \leq z_{11}^{-1} + y_{22}$.

PROOF:

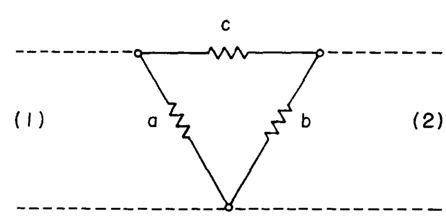
Necessity: The necessary conditions $y_{11}z_{11} = y_{22}z_{22}$ and $z_{11}^{-1} \leq y_{11}$ are well known (1). Consider the same two terminal-pair network with each resistor replaced by a switch. Let Z_1 be the switching function of the network viewed from terminal-pair 1, Z_2 the function as viewed from pair 2, Y_1 the function as viewed from pair 1 with pair 2 short-circuited, and Y_2 the corresponding function viewed from pair 2 with pair 1 short-circuited (where $\cap$ corresponds to a series connection and $\cup$ corresponds to a parallel connection). As each "path" in Y_1 is either a "path" in Z_1 or a "path" in Y_{22}, it follows that $Y_1 \subset Z_1 \cup Y_2$. Symmetrically, $Y_2 \subset Z_2 \cup Y_1$. If we make use of a theorem relating switching functions and resistance functions (2), these lattice (switching function) inequalities imply that $y_{11} \leq z_{11}^{-1} + y_{22}$ and $y_{22} \leq z_{22}^{-1} + y_{11}$. The latter inequality is equivalent to $\left(z_{22} + y_{11}^{-1}\right)^{-1} \leq z_{11}^{-1}$ under the assumption that $y_{11}z_{11} = y_{22}z_{22}$.

Sufficiency: The following network with conductances

$$a = y_{11} - \left(y_{11}y_{22} - z_{11}^{-1}\, y_{22}\right)^{1/2}$$

$$b = y_{22} - \left(y_{11}y_{22} - z_{11}^{-1}\, y_{22}\right)^{1/2}$$

$$c = \left(y_{11}y_{22} - z_{11}^{-1}\, y_{22}\right)^{1/2}$$



is sufficient.

The necessity proof, with proper symbolism, is applicable to some one-parameter network theories whose properties are quite different from those of resistor networks. For example, one such network theory admits neither Y-Δ transformations nor bridge-balancing. This type of proof can also be extended to n terminal-pair networks.

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References

1. For example, E. A. Guillemin, Introductory Circuit Theory (John Wiley and Sons, Inc., New York, 1953), pp. 153-161. (z_{11}^{-1} is the driving-point conductance measured across terminal-pair 1; z_{22}^{-1}, the conductance measured across terminal-pair 2; y_{11} the conductance measured across pair 1 with pair 2 short-circuited; and y_{22}, the corresponding conductance measured across pair 2 with pair 1 short-circuited.)

2. A. B. Lehman, Lattice order of two-terminal resistor networks, June 1956, page 5, Theorem 2 (unpublished).