

II. MICROWAVE GASEOUS DISCHARGES*

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RESEARCH OBJECTIVES

The aim of this group continues to be to study some of the fundamental properties of gas discharge physics by means of microwave plasmas, with more and more emphasis on high-density plasmas and plasmas in magnetic fields. To carry out this general objective, we have spent a great deal of effort on the production of plasmas of high-percentage ionization at low pressures under steady-state conditions, the achievement of which will allow us to carry on the fundamental studies in which we are most interested. At the present time, we have not produced plasmas with as high percentage of ionization as we had hoped to, but we have several schemes underway for producing them.

We are also studying ways of determining the characteristics of the plasmas by means of microwaves, spectroscopic methods, and the diamagnetic effect of the electrons. Along with these production and diagnostic studies, we are continuing measurements on fundamental physics studies of collision cross sections, loss mechanisms, and gain mechanisms of the electrons in the plasmas.

S. C. Brown

A. "MULTIPACTING" IN A MAGNETIC FIELD

In the course of experiments with high-density plasmas – experiments that were carried out with the equipment described in the Quarterly Progress Report of January 15, 1957, page 16 – the following phenomenon was observed. At pressures below 10^{-5} mm Hg, with the magnetic field around 550 gauss, with the frequency of the magnetron carefully tuned to the resonant frequency of the TE_{111} mode of the empty cavity, and with the amplitude of the microwave E field at the center of the cavity at approximately 2000 volts/cm, there appeared a bluish-violet-reddish glow in the region around the center of the quartz tube. The light seemed to emanate from the region near the radial walls of the quartz tube, and especially from those parts of the wall at which the microwave E field was perpendicular to the wall. The glow did not fill the whole length of the tube but terminated at some distance from the end walls of the tube in what appeared as reddish bands or rings.

In order to determine the nature of the origin of this light, the magnetic field was varied while all other pertinent parameters were kept constant. The light intensity (as observed with a photocell) peaked at a magnetic field of approximately 550 gauss, the resonance being the sharper the smaller the diameter of the quartz tube. The

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curious feature of this resonance is that it appears at a value of the magnetic field that is approximately one-half the value required for cyclotron resonance at the frequency of the S-band microwave field. With the largest tube that was used (diameter, 3.5 cm), at times there was evidence of an additional resonance at approximately 250 gauss, but this is still uncertain. There was no evidence of a resonance at 1100 gauss.

The light intensity and its distribution along the axis of the tube did not vary appreciably with pressure for pressures below 10^{-5} mm Hg. The glow was still present when the system was opened to pumps when the ionization-gauge pressure readings were as low as 3×10^{-8} mm Hg. In a closed, previously evacuated system, the onset of the glow always caused the pressure, as measured by the ionization gauge, in the system to rise. This rise was at times as large as one order of magnitude, indicating that large quantities of gas were liberated from the walls of the quartz tube (the volume of the tube was approximately 10 cm^3 ; the volume of the vacuum system was approximately 2 liters).

Preliminary spectroscopic examination of the light showed no pronounced line or band spectra. The electron density associated with the glow was low. It was barely capable of perturbing even the most sensitive mode of the microwave cavity which can be used to measure plasma densities.

It is thought that the characteristics which have been described are not those of a gas discharge but of a "multipacting" discharge modified by the presence of a static magnetic field. The basic characteristics of a "multipacting" discharge in the absence of a magnetic field are the following. An electron starting with small initial velocity at one wall of the discharge vessel is accelerated during an odd number of half-cycles of an alternating electric field, so that it reaches the opposite wall of the vessel with sufficient energy to cause the emission of more than one secondary electron. The requirements on the initial and final phases of the electric field, and on its strength and frequency, are such that the secondary electron must be capable of repeating the multiplication process started by the primary electron.

In a magnetic field sufficiently strong to bend the path of the electron and to return it to its surface of origin it may be possible to have a "multipacting" discharge at a single wall of a vessel. However, the B-field must be of such strength that when the electron returns to its surface of origin, it is sufficiently energetic to cause the emission of a secondary electron.

It is this mechanism which is thought to be responsible for the glow observed at the low pressures in the quartz tube. Although a precise quantitative theory that takes into account all of the experimental conditions is still lacking, the following considerations will account for at least the main features of the phenomenon.

We assume that at any point the wall of the quartz tube can be considered a plane located at the position $x = 0$ of a Cartesian coordinate system. The microwave field E is along the x -axis, and the magnetic field is along the z -axis. An electron appears

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with negligible initial velocity at time $t = 0$ at the position $\mathbf{x} = 0$. Its subsequent velocity and position in the crossed microwave electric field, $\vec{E} = \vec{a}_x E_0 \cos(\omega t + \phi)$, and static magnetic field, $B = \vec{a}_z B_0$, is given by

$$m \frac{d\vec{v}}{dt} = -e[\vec{E} + \vec{v} \times \vec{B}] \quad (1)$$

From Eq. 1 the kinetic energy of the electron can be obtained as

$$u = \frac{1}{2} m |\mathbf{v}|^2 = \frac{e^2 E_0^2}{2m} \left[\frac{\sin^2 \left[\frac{1}{2}(\omega + \omega_b) t \right]}{(\omega + \omega_b)^2} + \frac{\sin^2 \left[\frac{1}{2}(\omega - \omega_b) t \right]}{(\omega - \omega_b)^2} + 2 \frac{\cos(\omega t + \phi) \sin \left[\frac{1}{2}(\omega + \omega_b) t \right] \sin \left[\frac{1}{2}(\omega - \omega_b) t \right]}{(\omega + \omega_b)(\omega - \omega_b)} \right] \quad (2)$$

as well as the electron position along the x-axis,

$$x = \frac{eE_0}{2m} \left[\frac{\cos(\omega t + \phi)}{\omega(\omega + \omega_b)} + \frac{\cos(\omega_b t - \phi)}{\omega_b(\omega + \omega_b)} + \frac{\cos(\omega t + \phi)}{\omega(\omega - \omega_b)} - \frac{\cos(\omega_b t + \phi)}{\omega_b(\omega - \omega_b)} \right] \quad (3)$$

where $\omega_b = \frac{eB}{m}$ is the cyclotron frequency. Equation 2 exhibits the well-known cyclotron resonance effect at $\omega = \omega_b$, at which the electron gains energy continuously from the microwave field at the rate given, to a good approximation, by the equation

$$u = \frac{e^2 E_0^2}{8m\omega^2} t^2 \quad (4)$$

We must now determine from Eq. 3 those times t for which $x = 0$, consistent with the requirement $u \geq u_{\min}$, where u_{\min} is the energy for which the secondary emission coefficient is unity. We can do this by inspection. From Eq. 2 we notice that, in order to make u largest for a given ω and ω_b , all three numerators on its right-hand side ought to be unity. This and the condition $x = 0$ are satisfied if $\phi = -(\pi/2)$ (we neglect $\phi = +(\pi/2)$ because it corresponds to an electron that is accelerated initially in the negative x-direction), $\omega_b t = \pi$ and $\omega t = 2n\pi$, where $n = 1/2$ or n is an integer. For $n = 1/2$, which corresponds to cyclotron resonance, we have

$$u_{\omega=\omega_b, \omega t=\pi} = \frac{\pi}{8} \frac{e^2 E_0^2}{m\omega^2} \quad (5)$$

For $n = 1$, which corresponds to the experimentally observed glow at $\omega = 2\omega_b$, we have

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$$u_{\substack{\omega=2\omega_b \\ \omega t=2\pi}} = \frac{32}{9} \frac{e^2 E_0^2}{m\omega^2} \quad (6)$$

We see that the energy as given by Eq. 6 is approximately three times as large as that given by Eq. 5. Physically, there is a good reason for this: Under the conditions of Eq. 6, $[\omega = 2\omega_b, \omega t = 2\pi, \phi = -\pi/2]$, the electron has been accelerated twice as long as it was under conditions of Eq. 5, $[\omega = \omega_b, \omega t = \pi, \phi = -\pi/2]$.

For $n = 2$, which corresponds to a possible resonance at $\omega = 4\omega_b$, we have

$$u_{\substack{\omega=4\omega_b \\ \omega t=4\pi}} = 0.82 \frac{e^2 E_0^2}{m\omega^2} \quad (7)$$

which is somewhat smaller than the energy at cyclotron resonance. For $n = 3$ (or more), the energies are smaller still. Note, also, that since the secondary electron is emitted almost instantaneously (emission times are less than 10^{-13} sec), it enters the microwave field when its phase ϕ is $-\pi/2$ so that it can continue the multiplication process.

We must now determine whether or not the energy, as given by Eq. 6, is sufficient to emit more than one secondary electron from quartz. For the microwave cavity that was used, the field amplitude E_0 at the center of the cavity is given by $E_0 = 340 P^{1/2}$ volts/cm, where P is the power dissipated in the cavity in watts. For $P \approx 30$ watts, $E_0 \approx 2000$ volts/cm, so that $u_{\omega=2\omega_b} \approx 75$ ev.

Reliable data on the secondary emission coefficient, δ , in quartz are scarce, but most observers agree that δ crosses the value unity at approximately 40 ev. Surface contamination or adsorbed gas layers may lower this value considerably. Experimentally, the multipacting discharge was observed for P as low as a few watts, especially in the large quartz tube.

S. J. Buchsbaum

B. PROBE MEASUREMENTS IN COAXIAL GEOMETRY

A method is presented for the determination of the electron density and temperature of a plasma situated in an infinite coaxial geometry. These parameters can be determined as a function of time. We shall be limited to those plasmas for which the Debye length and mean free path are much shorter than the distance between the cylinders.

The flow equations are given by

$$\vec{\Gamma}_+ = -D_+ \text{grad } n_+ + \mu_+ n_+ \vec{E} \quad (1)$$

$$\vec{\Gamma}_- = -D_- \text{grad } n_- - \mu_- n_- \vec{E} \quad (2)$$

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in which D and μ are the diffusion coefficient and mobility, and E is the electric field that exists as a result of space charge and any voltage that is placed across the inner and outer conducting cylinders. The continuity equation can be written

$$\text{div } \vec{\Gamma}_- = \text{div } \vec{\Gamma}_+ = \gamma n_- \quad (3)$$

in which γ represents either the ionization coefficient or the rate of decay of particle density. Since $\text{div} (\vec{\Gamma}_+ - \vec{\Gamma}_-) = 0$, we can write generally

$$\Gamma_+ - \Gamma_- = \frac{k}{r} \quad (4)$$

where k is a constant that will be determined. Multiplying Eq. 1 by $\mu_- n_-$, Eq. 2 by $\mu_+ n_+$, adding, and using Eq. 4, we obtain

$$\Gamma_- = -D_s \text{grad } n_- - \mu_- \left[\mu_- + \mu_+ + \mu_+ \left(\frac{n_+ - n_-}{n_-} \right) \right]^{-1} \frac{k}{r} \quad (5)$$

$$D_s = \left(\frac{D_- \mu_+ + D_+ \mu_-}{\mu_+ + \mu_-} \right) \left(1 + \frac{\mu_- e (n_+ - n_-)}{e (\mu_+ n_+ + \mu_- n_-)} \right) = D_a \left(1 + \frac{\mu_- \rho}{\sigma} \right)$$

where D_a is the ambipolar diffusion coefficient, σ is the plasma conductivity, and $\rho = (n_+ - n_-)e$. In Eq. 5 it was assumed that $n_+/\text{grad } n_+ = n_-/\text{grad } n_-$. Allis (1) has shown that this assumption is valid for sufficiently high densities. If we equate Eq. 5 with Eq. 2 and assume that $(n_+ - n_-)/n_- \rightarrow 0$, so that $D_s \rightarrow D_a$, we obtain

$$\frac{k}{rn_-} = (\mu_+ + \mu_-) \left[\left(\frac{D_- - D_a}{\mu_-} \right) \frac{\text{grad } n_-}{n_-} + E \right] \quad (6)$$

Integrating Eq. 6 from the inner cylinder of radius a to the outer cylinder of radius b , we obtain

$$k = \frac{\left(\frac{D_- - D_a}{\mu_-} \right) \ln \frac{n(b)}{n(a)} + V_{ab}}{\frac{1}{\mu_+ + \mu_-} \int_a^b \frac{dr}{rn}} \quad (7)$$

in which V_{ab} is the voltage between the inner and outer cylinders. The current per unit length flowing between the inner and outer cylinders is given by

$$I = e(\Gamma_+ - \Gamma_-) 2\pi r = 2\pi e k \quad (8)$$

Therefore, we can say that

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$$\left(\frac{D_- - D_a}{\mu_-}\right) \ln \frac{n(b)}{n(a)} \quad \text{and} \quad \frac{1}{2\pi e(\mu_+ + \mu_-)} \int_a^b \frac{dr}{rn}$$

represent the electromotive force and internal resistance per unit length of the plasma which is considered as being a battery. Thus, in principle, a measurement of the open-circuit voltage between the coaxial cylinders measures the ratio $(D_- - D_a)/\mu_- \approx kT/e$, which is the average electron energy in volts, and a measurement of the current with no applied voltage measures the electron density.

From Eq. 5 we can see that the constant k does not enter into the continuity equation, since $\text{div } k/r = 0$. In the absence of recombination the differential equation for the electron density has the usual Bessel-function solution. Thus we can write

$$n = n_o \left(\frac{J_o\left(\frac{r}{\Lambda}\right) - \frac{J_o\left(\frac{a}{\Lambda}\right)}{N_o\left(\frac{a}{\Lambda}\right)} N_o\left(\frac{r}{\Lambda}\right)}{J_o\left(\frac{r}{\Lambda}\right) - \frac{J_o\left(\frac{b}{\Lambda}\right)}{N_o\left(\frac{b}{\Lambda}\right)} N_o\left(\frac{r}{\Lambda}\right)} \right) \quad (9)$$

in which $1/\Lambda = \sqrt{D_a}$ is defined by

$$J_o\left(\frac{a}{\Lambda}\right)/N_o\left(\frac{a}{\Lambda}\right) = J_o\left(\frac{b}{\Lambda}\right)/N_o\left(\frac{b}{\Lambda}\right) \quad (10)$$

If the actual radii of the cylinders are given by a_o and b_o , then a and b are given by $a = a_o - \delta$, $b = b_o + \delta$, and δ is approximately equal to the mean free path.

The solution of Eq. 10 yields

$$1/\Lambda = \beta/(b-a) \quad (11)$$

in which β is a function of $a/(b-a)$, as plotted in Fig. II-1. An approximate expression for β when $a/(b-a) \ll 1$ can be shown to be

$$\beta = 1.54 \ln(.468 b/a)^{-1} + 2.405 \quad (12)$$

For $a/(b-a) > 2$, β can be taken as equal to π . It can be shown that

$$n(b_o)/n(a_o) = \frac{a_o}{b_o} \frac{J_o\left(\frac{a_o}{\Lambda}\right)}{J_o\left(\frac{b_o}{\Lambda}\right)}$$

For $a = 1/2b$, $\ln n(b)/n(a) = 0.36$. The electromotive force of the plasma is approximately one-third of the plasma temperature and goes to zero for $a(b-a) \gg 1$.

The resistance of the plasma can be computed for $a/(b-a) \gg 1$. In hydrogen the resistance allows a current density of approximately 10^{-6} amp cm^{-2} per volt, for a pressure of 1 mm Hg and density of 10^{10} cm^{-3} . The drawback is that the greatest

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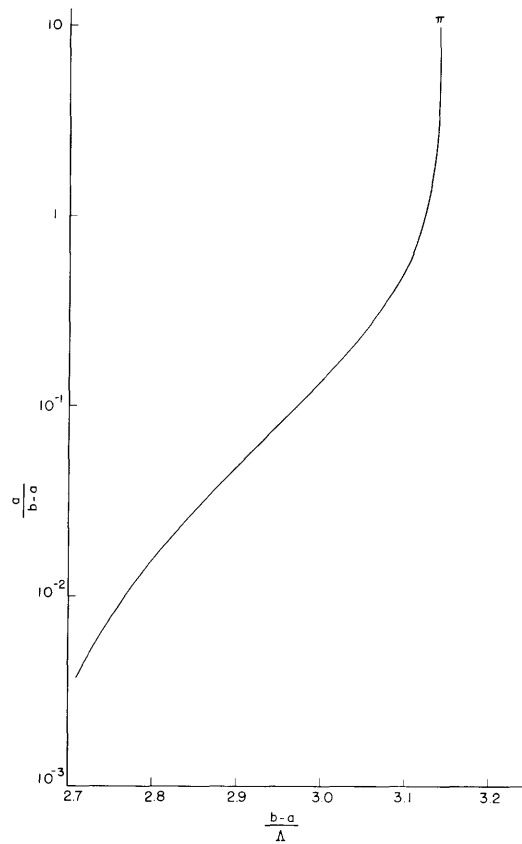


Fig. II-1. β as a function of $a/(b-a)$.

contribution to the plasma resistance arises from the part of the plasma that is near the wall, and it is here that the greatest error can be made in the shape of the density distribution. However, relative density measurements can be made. This technique is well suited to the determination of electron energy and temperature in a decaying plasma, since it is not subject to the high-density limitation imposed upon microwave measurements of electron density.

E. I. Gordon

References

1. W. P. Allis, Motions of ions and electrons, Technical Report 299, Research Laboratory of Electronics, M.I.T., June 13, 1956.

C. MEASUREMENT OF THE COLLISION CROSS SECTION AND OF THE ENERGY LOSS OF SLOW ELECTRONS IN HYDROGEN

In the Quarterly Progress Report of October 15, 1957, page 6, a measurement of the electron collision cross section, P_m , in hydrogen was discussed. The technique

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(hereafter designated method I) consisted of determining the complex rf conductivity as a function of the gas pressure in the afterglow of a pulsed microwave discharge. The value of P_m was given as $28 (v/v_0)^{0.6} (\text{cm-mm Hg})^{-1}$, with v the electron velocity, and v_0 the most probable velocity at the ambient temperature; it is assumed that v departs slightly from v_0 . This result agrees within 5 per cent with the shower experiments of Crompton and Sutton (1) over electron temperatures from 300° K to 1000° K.

Studies of P_m can be made over much greater temperature ranges by exciting the electrons with a microwave field of known magnitude and measuring the plasma conductivity. This method has been used successfully in measurements of the collision cross sections in monatomic gases (2, 3). The electron temperature T_e is then given in terms of the gas temperature T_g , and the heating field E_h by

$$kT_e = kT_g + e^2 E_h^2 / 3m \omega^2 \lambda \quad (1)$$

which is valid for low gas pressures at which the collision frequency is small compared with the radian frequency ω of the heating field; m and e are the mass and charge of the electron, and λ is its fractional loss of energy per collision with the molecule. In monatomic gases the collisions are elastic, and λ equals $2m/M$, where M is the molecular mass. In polyatomic gases, λ is not known and is generally a function of the electron velocity. This further complicates the problem in that Eq. 1 is not valid because the distribution function from which it is derived is no longer Maxwellian.

The effects of the λ factor are very pronounced in hydrogen, as will be seen from Fig. II-2. Here the conductivity ratio ($\sigma_r/p_0\sigma_i$) is plotted against the temperature, as obtained from the measurements of P_m by method I, and with the range somewhat extended by using Crompton and Sutton's values for P_m . It should be noted that method I does not require a knowledge of λ , so that T_e represents the true electron temperature. In the same figure measurements of the conductivity ratio, which were obtained by rf heating of the electrons (hereafter designated method II), are shown. The temperature scale that corresponds with method II was calculated from Eq. 1 with the assumption that λ equals $2m/M$. If λ had this value in hydrogen, the two curves would coincide; if it were different from $2m/M$ but constant, the two curves would differ by a multiplicative factor. Neither case holds in hydrogen over the range of temperatures that was considered. However, the difference between the two curves allows λ to be computed as a function of the electron energy (see Fig. II-3). Comparison of the present results with those of Crompton and Sutton (1) and of Townsend and Bailey (4) are fairly satisfactory. In making the comparison it should be noted that these authors assume, for purposes of calculation, a distribution of electron velocities; consequently, their final results are not uniquely determined.

The measurements of Fig. II-2 also permit the evaluation of the mean electron

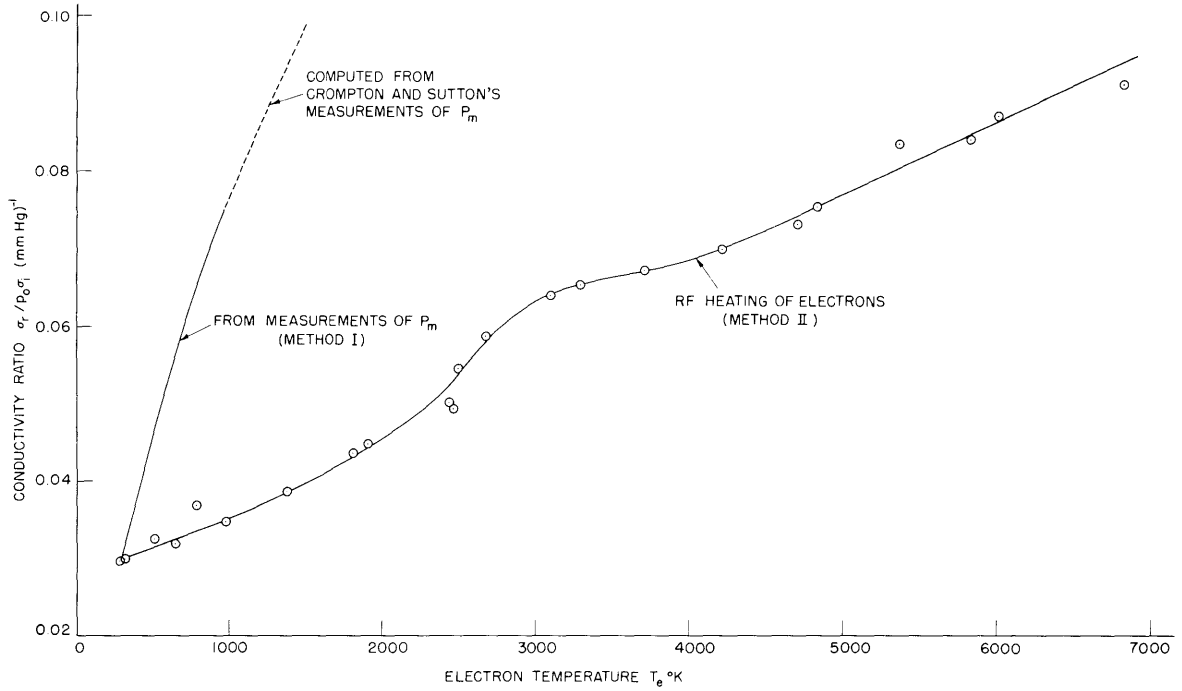


Fig. II-2. Conductivity ratio as a function of the electron temperature.

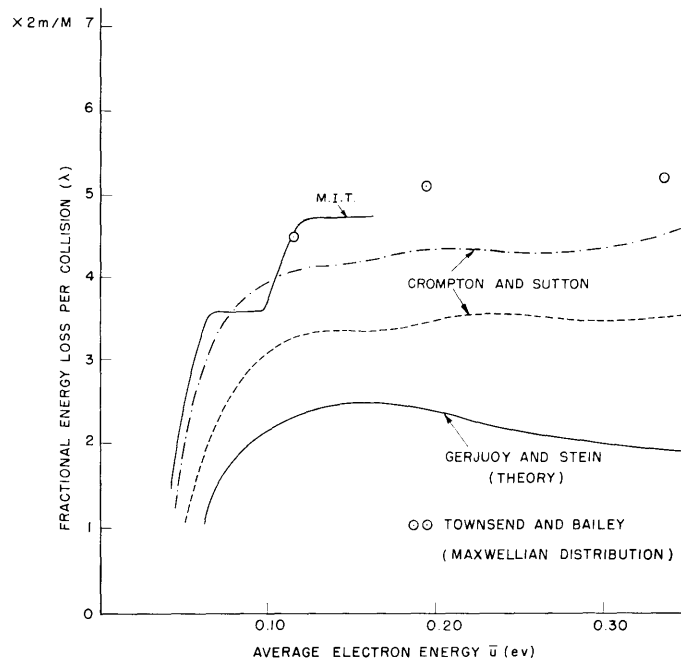


Fig. II-3. Fraction of energy lost by an electron upon collision with a hydrogen molecule.
 --- Assumed Maxwellian distribution of electron velocities.
 -.- Assumed Druyvesteyn distribution.

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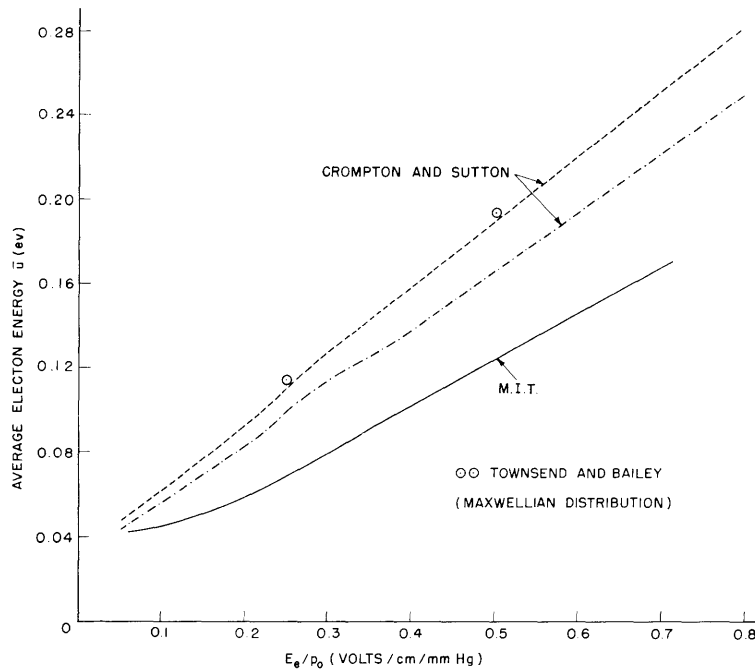


Fig. II-4. Average energy of electrons in hydrogen as a function of E_e/p_0 .
 --- Assumed Maxwellian distribution of electron velocities.
 -.- Assumed Druyvesteyn distribution.

energy for a given value of E_e/p_0 . Here E_e is the rf field which would cause the same transfer of energy as a steady field; p_0 is the gas pressure reduced to 0° C. The computation of E_e from the rf field E requires a knowledge of both P_m and the ac and dc distribution functions of electron velocities. For measurements taken at sufficiently low gas pressures, we can write symbolically

$$E_e^2 = \frac{E^2 h^2}{\omega^2} \bar{\nu} / \left(\frac{\bar{1}}{\bar{\nu}} \right)$$

where the single bar designates an averaging of the collision frequency ν over the ac distribution function, and the double bar an averaging over the dc function. The results of these calculations are shown in Fig. II-4. In this figure and in Fig. II-3 our results agree better with Crompton and Sutton's, subject to their assumption of a Druyvesteyn distribution. This is to be expected because the large variations of P_m and λ with velocity preclude the possibility of a Maxwellian distribution, except at very small values of E/p_0 .

At the present time, we are checking the value of P_m found by method I by thermally heating the whole plasma. Method II is being extended to much higher values of the

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heating field than those described here. Unfortunately, outside the range in which thermal heating is possible (and outside the range of method I) conductivity measurements alone cannot yield either of the two unknowns, P_m and λ , without recourse to an independent experiment. For this purpose, we propose to use dc measurements of the drift velocity versus E/p to give us the necessary information.

G. Bekefi

References

1. R. W. Crompton and D. J. Sutton, Proc. Roy. Soc. (London) 215A, 467 (1952).
2. L. Gould and S. C. Brown, Phys. Rev. 95, 897 (1954).
3. A. L. Gilardini and S. C. Brown, Phys. Rev. 105, 31 (1957).
4. S. J. Townsend and V. A. Bailey, Phil. Mag. 42, 873 (1921).
5. E. Gerjuoy and S. Stein, Phys. Rev. 98, 1848 (1955).

D. ZERO-SHIFT METHOD FOR MEASURING THE FREQUENCY OF COLLISION OF ELECTRONS IN GASES

The microwave conductivity of an ionized gas, in the absence of a magnetic field, exhibits negative imaginary values, and measurements of electron density are made by observing the upward shift in the resonant frequency of a cavity containing the ionized gas (1). If, however, a dc magnetic field is applied transverse to the microwave electric field, the reactance of the plasma will change in character, depending upon whether the magnitude of the magnetic field is less than, or greater than, the value corresponding to cyclotron resonance, since the magnetic field itself tends to force the electron current either to lag or lead the electric field. Thus it is possible that a superimposed magnetic field could be adjusted so as to cancel the reactance of a plasma and bring any frequency shifts of the cavity to zero. That this should, indeed, be so is shown by the expression for the frequency shift for a cylindrical cavity that resonates in the TE_{011} mode, contains a plasma, and has its axis of symmetry along a uniform dc magnetic field (2).

$$\frac{\Delta\omega}{\omega_0} = \frac{1}{2} \langle \eta \rangle \int_0^\infty \left\{ \frac{\beta^2 + 1 - \gamma^2}{[\beta^2 + 1 + \gamma^2]^2 - 4\gamma^2} \right\} \frac{4\pi}{3} v^3 \frac{\partial f_0}{\partial v} dv \quad (1)$$

where ω_0 is the resonant frequency of the empty cavity; $\beta = v_m/\omega$, with v_m the collision frequency for momentum transfer; $\gamma = \omega_b/\omega$, with $\omega_b = eB/m$, where B is the magnetic field strength; f_0 is the first term in a spherical harmonic expansion of the electron

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velocity distribution; and $\langle \eta \rangle$, the square of the ratio of plasma frequency to radian frequency, as averaged over the electric field, $= \left(\frac{e^2}{m\epsilon_0 \omega_0^2} \right) \int n E^2 dV / \int E^2 dV$.

Plots of typical frequency shift and resulting cavity Q, for ν_m independent of velocity, are shown in Fig. II-5. At a particular magnetic field, which depends in general on ν_m and f_0 , the frequency shift passes from positive to negative values. We shall consider the case of constant ν_m for the remainder of this discussion; the gross features of the method are not altered when ν_m depends on velocity. From Eq. 1 we see that the point of "zero-shift" is $\gamma_0 = 1 + \beta^2$, from which the collision frequency is obtained directly. Notice that this point is independent of electron density.

Experimental verification of the existence of such a zero-shift point is shown in Fig. II-6. By sweeping the frequency of a probing signal at a rate that is high compared with the rate of pulsed rf breakdown, superposition of cavity transmission curves, each at a different time in the afterglow, is observed on the screen of an oscilloscope. Since the electron density is decaying with time in the afterglow, the resonant frequency of successive transmissions will shift toward the empty value, unless the magnetic field has been adjusted in the manner previously discussed. The amplitude of transmission decreases with increasing density because of the decreased cavity Q and the decoupling of the cavity caused by the plasma. Frequency is increasing to the right in this figure. Figure II-6a is for no magnetic field, and the shifts are positive; Fig. II-6b is for a magnetic field at the critical value, and all shifts are essentially zero, independent of

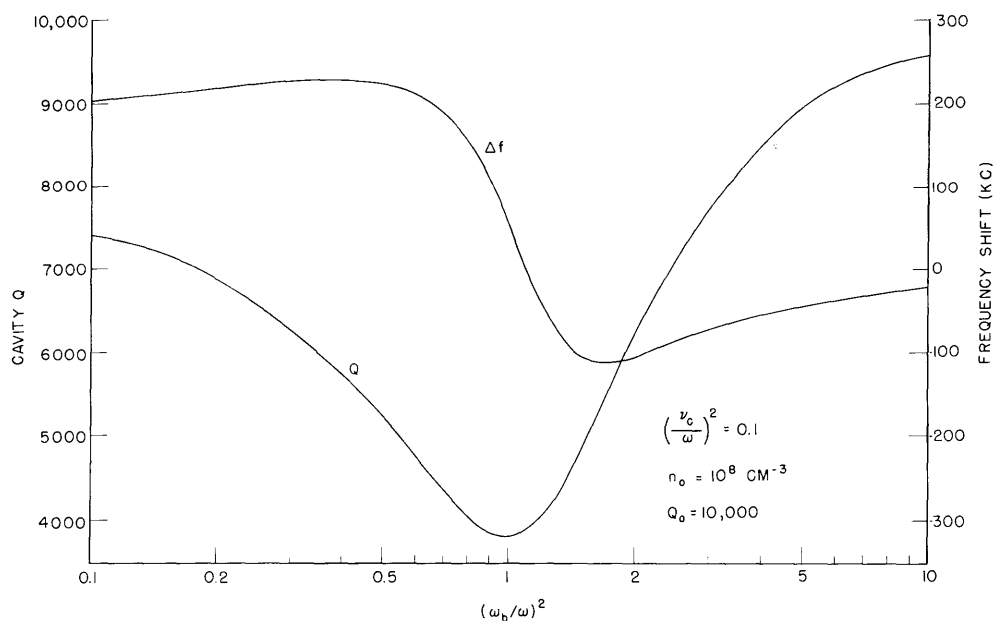


Fig. II-5. Typical frequency shift and cavity Q as a function of magnetic field.

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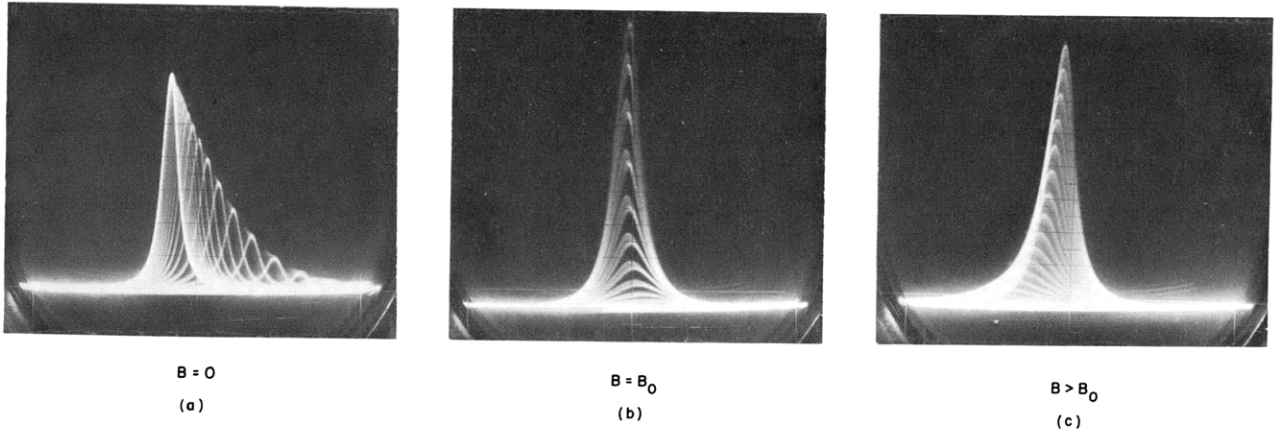


Fig. II-6. Superimposed cavity transmission curves for a decaying plasma in a magnetic field.

density; Fig. II-6c is for a magnetic field above the critical value, and the shifts are toward lower frequencies, the plasma behaving capacitively.

The precision in a measurement of collision frequency by such a technique will depend mainly upon the experimenter's ability to set the magnetic field in such a way that cavity and plasma resonate at the same frequency as the cavity alone. It depends upon the homogeneity and stability of the magnetic field. If we are able to reset the cavity with plasma to its resonance without plasma within a frequency interval $\Delta f = f/mQ$, the uncertainty in β can be shown to be

$$\frac{\Delta\beta}{\beta} = \left(\beta + \frac{1}{\beta}\right) \cdot \frac{2}{mQ\Delta\left(\frac{1}{Q}\right)} \quad (2)$$

where Q is the value with the plasma, and $\Delta\left(\frac{1}{Q}\right) = \frac{1}{Q} - \frac{1}{Q_0}$. For $\gamma^2 = 1 + \beta^2$, $\omega = 2.5 \times 10^{10} \text{ sec}^{-1}$, $Q_0 = 10,000$, $\Delta(1/Q) = 1.5 \times 10^{-4}$, $n_0 = 10^8 \text{ cm}^{-3}$, and $\beta^2 = 0.1$, we obtain $\Delta\beta/\beta = 9/m$. Sproull and Linder (3) have indicated that cavity-comparator techniques give m better than 100, so that there may be a possibility of making such a measurement within several per cent.

In order to estimate the effect of variations in plasma reactance across the volume which arise from magnetic field inhomogeneities, a field that varied as $B = B_0 \left(1 + \alpha \sin^2 \frac{\pi z}{L}\right)$ was assumed. The error made in β by calculating it on the basis of a uniform field of strength B_0 can be shown to be

$$\frac{\Delta\beta}{\beta} = \frac{\alpha}{5} \left(\frac{\gamma}{\beta}\right)^2 \quad (3)$$

Thus, for typical conditions with $(\beta/\gamma)^2 = 0.1$, α should be less than 0.005 to keep

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$\Delta\beta/\beta$ below 0.01. This same limit can be set on the time stability of the magnetic field.

Effort directed toward increasing the present sensitivity of the apparatus, as well as improving present magnetic fields, is planned. Theoretical attempts to extend the analysis to more realistic velocity variations of collision frequency by using correct non-Maxwellian distribution functions have been started.

J. L. Hirshfield

References

1. M. A. Biondi and S. C. Brown, Phys. Rev. 75, 1700 (1949).
2. W. P. Allis, Motions of ions and electrons, Technical Report 299, Research Laboratory of Electronics, M.I.T., June 13, 1956.
3. R. L. Sproull and E. G. Linder, Proc. IRE 34, 305 (1946).