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## RESEARCH OBJECTIVES

Our interest in the general noise-performance limitations of electron devices will continue along the lines we have followed during the past year. Much has been accomplished with respect to conventional tube and transistor amplifiers, and even with respect to the one terminal-pair maser as a negative-resistance device. Parametric amplification, in general, however, still presents puzzles. There is some evidence that no fundamental noise-performance limit is imposed by the device per se. Further study will be required to expose the actual limitations, if there are any.

We are still concerned with the question of the thermodynamic basis of noise and gain processes, although relatively little progress has been made in this direction. Certainly, the parametric amplifier again presents intriguing problems that tend to upset our earlier notions of the relationship between these two phenomena.

R. B. Adler

# A. NOISE PERFORMANCE OF PARAMETRIC, LONGITUDINAL-BEAM AMPLIFIERS

The generalization of the kinetic power theorem (see Section VI-C) shows that parametric electron-beam amplifiers do not possess the same basic limit on noise performance as the conventional longitudinal-beam amplifiers (1). A detailed mathematical formulation (given in ref. 1) shows that the lower limit to the noise figure of a parametric beam amplifier is unity even with large gain. The reason for this is best seen with the aid of an example. The general kinetic power theorem (Eq. 1 of Sec. VI-C) permits gain to be achieved through coupling to a fast wave in the electron beam. Indeed, under certain assumptions, such gain has been computed (2). But the noise in a fast wave can be removed. One way is by using the Kompfner-dip condition that occurs when the wave of a helix surrounding a beam, with a large value of Pierce's parameter QC, couples to the fast wave in the beam. The entire noise in the fast wave can be removed by this means. Coupling to the noise-free fast wave can then be used to provide gain in a parametric way. One means of accomplishing this is by coupling to another noise-free fast wave at another sideband (an upper sideband if the first fast wave pertains to a lower sideband, and a lower sideband if the first fast wave pertains to an upper sideband). The assumption is that there is no coupling to the noise of any other wave at any of the other sidebands. Elimination of this coupling, which, fundamentally, is not impossible, seems to be the most difficult requirement for

<sup>\*</sup> This research was supported in part by Purchase Order DDL-B222 with Lincoln Laboratory, which is supported by the Department of the Army, The Department of the Navy, and the Department of the Air Force under Contract AF19(122)-458 with M.I.T.

low-noise parametric amplification.

H. A. Haus

#### References

- 1. H. A. Haus and F. N. H. Robinson, The minimum noise figure of microwave beam amplifiers, Proc. IRE 43, 981-990 (1955).
- 2. C. F. Quate, private communication, 1957.

### B. NETWORK OPTIMIZATION OF MASER NOISE PERFORMANCE

One of the possible realizations of the maser – the only one that has actually been tested – leads to an equivalent circuit (Fig. XIV-1) containing a negative resistance in series with a noise voltage generator. It is interesting to find the best possible noise performance that can be achieved by appropriate network connections. The general theorems previously reported in the Quarterly Progress Reports (1) offer a simple method of accomplishing this.

Suppose we want to obtain a two terminal-pair amplifier with the maser in an appropriate network interconnection. Suppose, further, that a maser with the equivalent circuit of Fig. XIV-1 is to be employed with the exchangeable power

$$P_e = -k |T_m| \Delta f$$

(where  $T_m$  is the equivalent negative temperature of the maser) in conjunction with a positive resistance  $R_o$  at a temperature  $T_R$  with the exchangeable power (equal in this case to the available power)

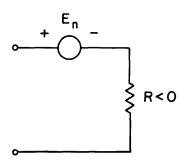


Fig. XIV-1. Equivalent circuit of maser amplifier.

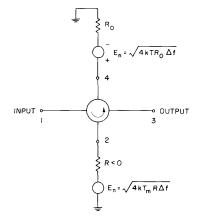


Fig. XIV-2. Maser amplifier with circulator.

$$P_e = +kT_R \Delta f$$

Then, the negative resistance of the maser and the additional positive resistance may be considered as the canonic form of a two terminal-pair network. The characteristic noise matrix of the network,

$$\underset{\sim}{\mathbb{N}} = -\frac{1}{2} \ \overbrace{\approx}^{\overline{\mathrm{EE}^{\dagger}}} (Z + Z^{\dagger})^{-1}$$

has the two eigenvalues  $\lambda_1 = k |T_m| \Delta f$  and  $\lambda_2 = -kT \Delta f$ . According to the general theorem previously stated (1) the best noise measure (identical with excess noise figure at high gain) that can be achieved with this network is given by the positive eigenvalue of N for the two terminal-pair network as

$$M_{opt} = \frac{F - 1}{1 - \frac{1}{G}} \bigg|_{opt} = \frac{\lambda_1}{kT \Delta f} = \frac{\left|T_m\right|}{T}$$

where T is the temperature of the source. But this noise measure can actually be accomplished by an imbedding of the maser and the other resistor in a lossless circulator with the scattering matrix

$$\mathbf{S} = \begin{pmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} \end{pmatrix}$$

where terminals 1 are used as the input and terminals 3 as the output, and the maser is connected to terminals 2 (see Fig. XIV-2). Indeed, the imbedding results in a matched, unilateral, amplifier with gain

$$\mathbf{G} = \left(\frac{\mathbf{R} - \mathbf{1}}{\mathbf{R} + \mathbf{1}}\right)^2$$

and a scattering matrix representation

$$b_1 = \delta_1$$
$$b_3 = \frac{R-1}{R+1} a_1 + \delta_3$$

where

$$\delta_1^2 = kT_R \Delta f$$

$$\overline{\delta_3^2} = \frac{\overline{|E_{n1}|^2}}{(R-1)^2}$$
$$\overline{\delta_1 \delta_3^*} = 0$$

It can be easily verified that the noise measure of this amplifier is, indeed, given by

$$M_{opt} = \frac{F - 1}{1 - \frac{1}{G}} = \frac{|T_m|}{T}$$

Note that the temperature  $T_R$  of the resistor  $R_o$  does not appear in the final result. The use of the resistor is an artifice designed to obtain a two terminal-pair device to which the theorems derived previously have thus been made applicable.

The present derivation shows that the circulator "imbedding" of the maser with the equivalent circuit of Fig. XIV-1 actually accomplishes the best noise performance that is theoretically achievable with the device.

R. B. Adler and H. A. Haus

## References

 Cf. H. A. Haus and R. B. Adler, Canonical form of a noisy linear n terminalpair network, Quarterly Progress Report, Research Laboratory of Electronics, Jan. 15, 1957, p. 117; An improved measure of noise performance, op. cit., April 15, 1956, p. 87.