VII. PHYSICAL ACOUSTICS*

Prof. K. U. Ingard
Dr. L. W. Dean III
Dr. G. C. Maling, Jr.
Dr. H. L. Willke, Jr.
P. A. Fleury V
K. W. Gentle

A. SOUND EMISSION FROM KARMAN VORTICES

As reported previously,[1] the intensity of the Karman vortex street produced behind a cylinder in a flow is amplified to a high degree when the frequency of the vortex shedding is coincident with one of the transverse cross resonances in the pipe in which the cylinder is placed. This strong reaction of a sound field on the flow field about the cylinder at the Karman vortex frequency seems to give rise to an increase in effective flow resistence of the region of the pipe which contains the cylinder. This has led us to believe that the presence of the transverse sound field at the Karman vortex frequency will produce an increase in the drag force on the cylinder produced by the flow. To study this, a drag-force balance and associated wind-tunnel test section have been built. The completed arrangement is shown schematically in Fig. VII-1.

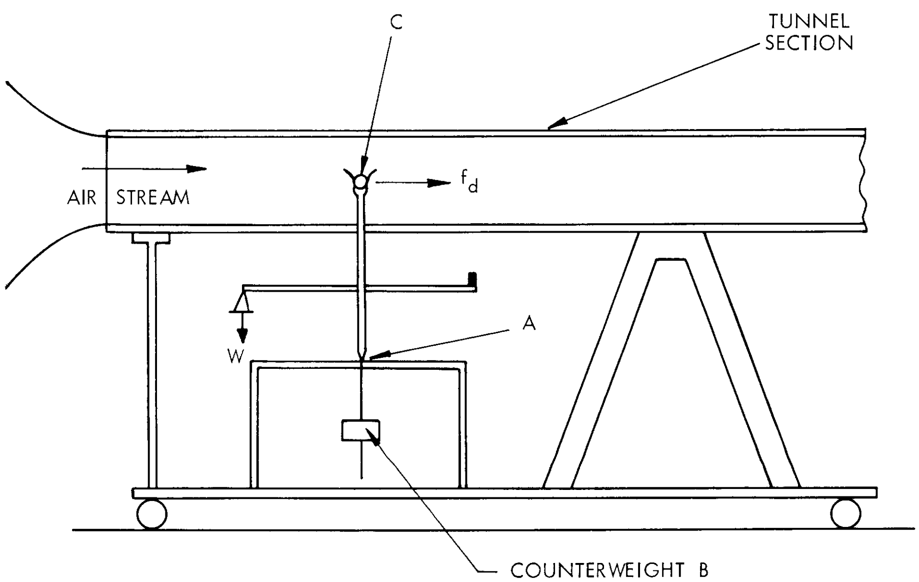

Fig. VII-1. Schematic diagram of the experimental arrangement.

The duct test section is built from 0.5-in. Lucite plates so that the entire duct is transparent, enabling us to make visual observations of the flow by means of smoke streamers. The cross-section area of the tunnel is 12 in. × 12 in. and the length is 48 inches. This test section is to be attached to an existing low-turbulence wind tunnel.

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The balance is arranged to measure horizontal drag forces, and consists of a framework supporting the cylinder C and resting on a knife-edge at point A. The sensitivity of the balance can be changed by adjusting the vertical position of the counterbalance B, and the drag force f_d on the cylinder can then be directly measured by a known weight W on the pan. Two types of measurements will be made. In the first set, the cylinder and the balance will be placed at the exit of the test section, and the cylinder will be exposed to a variable transverse field produced by an external sound source. The Karman vortex frequency, measured by a hot-wire anemometer, as well as the drag force, will be measured as functions of the frequency and amplitude of the external sound field.

In the second set of experiments, the cylinder will be placed inside the duct, and the drag of the cylinder will be measured in the range of flow speeds for which the coincidence between Karman vortex frequencies and cross resonance occurs.

K. U. Ingard, G. C. Maling, Jr.

References

1. K. U. Ingard and W. M. Manheimer, Sound emission from Karman vortices, Quarterly Progress Report No. 70, Research Laboratory of Electronics, M.I.T., July 15, 1963, pp. 74-75.

B. STABILITY OF PARALLEL FLOWS

The three-dimensional Navier-Stokes equations yield time-stationary parallel-flow solutions in the simple form

$$V_1 = V_2 = 0, \qquad V_3 = W(x_1, x_2), \qquad P = x_3 P_o,$$

where $W(x_1, x_2)$ is a polynomial of degree two, at most, in x_1 and x_2. The stability problem for parallel flows consists in discovering whether those solutions of the Navier-Stokes equations which at some time are "close" to time-stationary ones will approach boundedly such a stationary solution as t tends to infinity.

If we write

$$V_1 = u_1, \quad V_2 = u_2, \quad V_3 = W(x_1, x_2) + u_3, \quad P = x_3 P_o + p,$$

then the (linearized) equations for the u_i take the form

$$\frac{\partial u_i}{\partial t} - \nu \nabla^2 u_i + W \frac{\partial u_i}{\partial x_3} = -\frac{\partial p}{\partial x_i} - \delta_{i3} \left[\frac{\partial W}{\partial x_1} u_1 + \frac{\partial W}{\partial x_2} u_2 \right] \tag{1a}$$

$$\frac{\partial u_i}{\partial x_i} = 0. \tag{1b}$$

The free-field propagating kernel for the left-hand side of (1a) has been found. We may without loss of generality write

$$W(x_1, x_2) = V_o + V_1 x_1 + U_1 x_1^2 + V_2 x_2 + U_2 x_2^2$$

in terms of which the propagator takes the form

$$K(\vec{x}, \vec{y}, t) = \int_{-\infty}^{\infty} dk \exp\left(ik(x_3-y_3-V_o t) - \nu k^2 t\right) \prod_{j=1,2} \sqrt{\frac{\omega_j}{4\pi\nu \sin \omega_j t}} \exp\left(-\frac{\omega_j (x_j-y_j)^2}{4\nu \sin \omega_j t}\right)$$

$$\exp -ik\left\{\frac{\tan \omega_j \frac{t}{2}}{\omega_j}\left[U_j\left(x_j^2+y_j^2\right) + V_j\left(x_j+y_2+\tfrac{1}{2}\right)\right] - \tfrac{1}{4}V_j t\right\}, \tag{2}$$

where

$$\omega_j = \sqrt{\frac{4\nu k U_j}{i}}.$$

Were it not for the constraint (1b), we could solve at once the free-field initial value problem for (1a), in the form

$$u_i(\vec{x}, t) = \int_{E^3} d^3y\, K(\vec{x}, \vec{y}, t)\, u_i^{(0)}(\vec{y}) - \int_0^t dt' \int_{E^3} d^3y\, K(\vec{x}, \vec{y}, t-t') \frac{\partial p(\vec{y}, t')}{\partial y_i} \qquad i = 1, 2 \tag{3a}$$

$$u_3(\vec{x}, t) = \int_{E^3} d^3y\, K(\vec{x}, \vec{y}, t)\, u_3^{(0)}(\vec{y}) - \int_0^t dt' \int_{E^3} d^3y\, K(\vec{x}, \vec{y}, t-t') \cdot \left[\frac{\partial p(\vec{y}, t')}{\partial y_3} + \frac{\partial W}{\partial y_1} u_1(\vec{y}, t') + \frac{\partial W}{\partial y_2} u_2(\vec{y}, t')\right]. \tag{3b}$$

The integrals in (3) will converge for $t > 0$ if the $u_i^{(0)}$ and $\frac{\partial p}{\partial x_i}$ are in any of the L_p classes, with $\left\|\frac{\partial p}{\partial x_i}(t)\right\|_p$ locally integrable in t. With similar restrictions on the $\frac{\partial^2 p}{\partial x_j \partial x_i}$ and $\frac{\partial^2 p}{\partial t \partial x_i}$, the claimed solutions (3) satisfy (1a) for $t > 0$ and converge pointwise almost everywhere to the given $u_i^{(0)}$ as t tends to zero.

Since $\frac{\partial}{\partial x_1}$ and $\frac{\partial}{\partial x_2}$ do not in general commute with the propagator (2), that is, $\frac{\partial}{\partial x_i} K(\vec{x}, \vec{y}, t) \neq K(\vec{x}, \vec{y}, t) \frac{\partial}{\partial y_i}$ $(i = 1, 2)$, the solution (3) cannot be expected to preserve the condition (1b), even in case the $u_i^{(0)}$ should be differentiable and satisfy (1b). The task of maintaining (1b) thus falls to the pressure terms in (3). An indication of the proper form for the pressure terms may be gained by formally taking the divergence of (1a) and applying (1b), with the well-known result

$$\nabla^2 p = -2\left[\frac{\partial W}{\partial x_1}\frac{\partial u_1}{\partial x_3} + \frac{\partial W}{\partial x_2}\frac{\partial u_2}{\partial x_3}\right]. \tag{4}$$

Equation 4 could be used to convert (3) into an integral equation in the u_i, but this procedure does not seem to lead quickly to solutions or estimates, except when the $\frac{U_j}{\nu}$ and $\frac{V_j}{\nu}$ are small.

In the special case of simple shear flow, with $W(x_1, x_2) = Vx_1$, we have found the appropriate expression for the pressure for weakly divergence-free initial velocity fields, namely

$$p = \frac{2V}{4\pi}\int_{E^3} d^3y \frac{\partial \frac{1}{r}}{\partial x_3} u(\vec{y}, t) - \frac{2V^2t}{4\pi}\int_{E^3} d^3y \frac{\partial^3 r}{\partial x_1 \partial x_3^2} u(\vec{y}, t) - \frac{V^3t^2}{4\pi}\int_{E^3} d^3y \frac{\partial^3 r}{\partial x_3^3} u(\vec{y}, t), \tag{5a}$$

where $r = |\vec{x}-\vec{y}|$, and

$$u(\vec{x}, t) = \int_{E^3} d^3y\, K(\vec{x}, \vec{y}, t)\, u_1^{(0)}(\vec{y}). \tag{5b}$$

Substituting (5a) in our solution (3b) for u_3, we discover after some formal manipulation that the leading term in u_3 as t tends to infinity has the form

$$\begin{aligned} u_3(\vec{x}, t) &\sim 3\int_0^t dt' \int_{E^3} d^3y\, K(\vec{x}, \vec{y}, t-t')\, t'^2 u(\vec{y}, t') \\ &= 3\int_0^t dt'\, t'^2 u(\vec{x}, t) = t^3 u(\vec{x}, t) = t^3 \int_{E^3} d^3y\, K(\vec{x}, \vec{y}, t)\, u_1^{(0)}(\vec{y}). \end{aligned} \tag{6}$$

If $u_1^{(0)} \in L_p$ for $1 \leq p < 3$, that is, if

$$\left[\int_{E^3} |u_1^{(0)}|^p d^3y\right]^{1/p} < \infty,$$

we may estimate (6), using Hölder's inequality and our explicit representation for K. In the case of simple shear flow this expression reduces to the expression previously exhibited.[1] The result is that $u(\vec{x}, t) = 0\left(t^{-\frac{5}{2p}}\right)$, uniformly in the x_i, or

$$u_3 \sim 0\left(t^{3-\frac{5}{2p}}\right). \tag{7}$$

The order relation (7) fails to reveal conditions under which u_3 will remain bounded as t tends to infinity; it gives at best $u_3 \sim 0(\sqrt{t})$ when $u_1^{(0)}$ is integrable. On the other hand, u_1 will itself tend uniformly to zero as t tends to infinity, provided that $u_1^{(0)} \in L_p$ for $p < 5/2$.

It is easy to provide a counterexample to stability criteria involving only the integrability in some mean of the $u_i^{(0)}$. If we let $u_1^{(0)}$ be any bounded, integrable, nonnegative function that vanishes for $|\vec{x}| \geq 1$, then u_3 "uses up" the order relation (7) in the sense that

$$\sup_{x_3 \in E^3} |u_3| = 0(\sqrt{t}) \text{ but } \sup_{x_3 \in E^3} |u_3| \neq o(\sqrt{t}). \tag{8}$$

If the formal result (8) is substantiated in the more rigorous version that is now being worked on, we shall have demonstrated the instability of simple shear flow in the free-field case under perturbations $u_i^{(0)}$ belonging to the L_p classes.

The limiting case $\nu \to 0$ of the above-mentioned stability problem is of interest in elucidating the role of viscosity in stability theory. The inviscid initial-value problem has solutions (other than those among the Schwartz distributions) only under much more restrictive conditions than those of the viscid problem, owing to the absence of the regularizing action of viscosity. Solutions exist, provided that $\frac{\partial u_i^{(0)}}{\partial x_3}$ exists for each i, and belongs to an L_p class for

$p < 3/2$ free-field case

$p < 2$ flow between bounding planes

$p \leq \infty$ periodic boundary conditions.

The solutions then look like the viscid ones (Eqs. 3) except that the viscid propagated terms $f(\vec{x}, t) = \int_{E^3} K(\vec{x}, \vec{y}, t)\, f(\vec{y})\, d^3y$ are to be replaced by the corresponding inviscid transformations $f(x_1, x_2, x_3) \to f(x_1, x_2, x_3 - Vx_1t)$. [Formally, $K(x, y, t) \xrightarrow[\nu \to 0]{} \delta(x_1-y_1)\, \delta(x_2-y_2)\, \delta(x_3-y_3-Vtx_1)$.]

Specializing as before to the case of simple shear flow, and using the inviscid version of (5), we can show that the inviscid shear flow is not stable under any perturbation for which the solution corresponding to (3) and (5) exists. This generalizes the result previously given for the inviscid Couette flow,[2] in which the additional assumption was made that $u_1^{(0)}$ be twice differentiable with respect to x_1.

H. L. Willke, Jr.

References

1. H. L. Willke, Jr., Quarterly Progress Report No. 70, Research Laboratory of Electronics, M.I.T., July 15, 1963; see Eq. 3, p. 70.

2. Ibid., p. 73.