XX. NEUROPHYSIOLOGY*

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A. A MACHINE-INDEPENDENT THEORY OF THE COMPLEXITY OF RECURSIVE FUNCTIONS

This report summarizes a Ph.D. thesis with this title that was submitted to the Department of Mathematics, M.I.T., May 5, 1964.

The complexity of recursive functions is measured by associating with the list of all partial recursive functions $\phi_o, \phi_1, \phi_2, \ldots$ another list of partial recursive functions $\Phi_o, \Phi_1, \Phi_2, \ldots$ having the properties: (1) For all i and n, $\phi_i(n)$ converges (stops) if and only if $\Phi_i(n)$ converges. (2) There exists a total recursive function M such that

$$M(i, n, m) = \begin{cases} 1 & \text{if } \Phi_i(n) = m \\ 0 & \text{otherwise} \end{cases}$$

For instance, $\Phi_i(n)$ may be the number of steps required to compute $\phi_i(n)$, or the amount of tape used in the computation.

THEOREM: Let r be a total recursive function of 2 variables. Then there exists a 0-1 valued total recursive function f such that to every index i for f there corresponds an index j for f such that $\Phi_i(n) > r(n, \Phi_j(n))$ for almost all n; i.e., for all but a finite number of integers n.

THEOREM: There exists a total recursive function h for which the following is true: To each Φ_i there corresponds a 0-1 valued partial recursive function f with the same domain as Φ_i such that (1) If j is any index for f, then $\Phi_i(n) < \Phi_j(n)$ for almost all n. (2) There exists an index k for f such that $\Phi_k(n) < h(n, \Phi_i(n))$ for almost all n. (3) There exists a total recursive function τ which maps the index i for Φ_i into the index k above for f. For example, if $\Phi_i(n)$ is the number of steps required to compute $\phi_i(n)$ on a 1-tape T.M., then $h(n, m) = (n+m)^7$.

Both theorems are proved by using a double diagonalization argument.

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