XXII. NEUROPHYSIOLOGY*

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A. SUMMARY OF RESEARCH PROGRESS: THEORY OF THE RETICULAR FORMATION

Research on the functional organization of the reticular core of vertebrate control nervous systems has reached a significant landmark. Our problem is as described in Quarterly Progress Report No. 76 (page 313), but our progress has been achieved with a somewhat different model than that mentioned there.

The new model is enclosed within heavy lines in Fig. XXII-1, with everything outside only for generating an appropriately structured environment for computer simulation. The γ_{ij} in Fig. XXII-1 are all 3-variable symmetric switching functions of the binary σ_i. The typical module interconnection scheme is suggested by the M_5 hookup. Each module in Fig. XXII-1 is a hybrid probability computer, with schematic as shown in Fig. XXII-2. It receives 4-component probability vectors $P_{\delta i}$ from above and $P_{\alpha i}$ from below, as well as generating a corresponding $P_{\pi i}$ from its N_{ia} part. The j^{th} component in each case is the probability as computed by the module of origin that the over-all reticular formation model's present γ_{ij} input-signal configuration is properly a mode j one. The $P_{\alpha i}$, $P_{\delta i}$, and $P_{\pi i}$ vectors are passed componentwise through an f function as shown in Fig. XXII-3, and weighted in the subsequent 'Av' units according to formulas of the type

$$P = \frac{C_\pi P_\pi + C_\alpha P_\alpha + C_\delta P_\delta}{C_\pi + C_\alpha + C_\delta},$$

where $C_\pi = C_{\pi_1} C_{\pi_2} Q$, with all factors variable and determined according to two module decoupling principles and a potential command principle which demands that information

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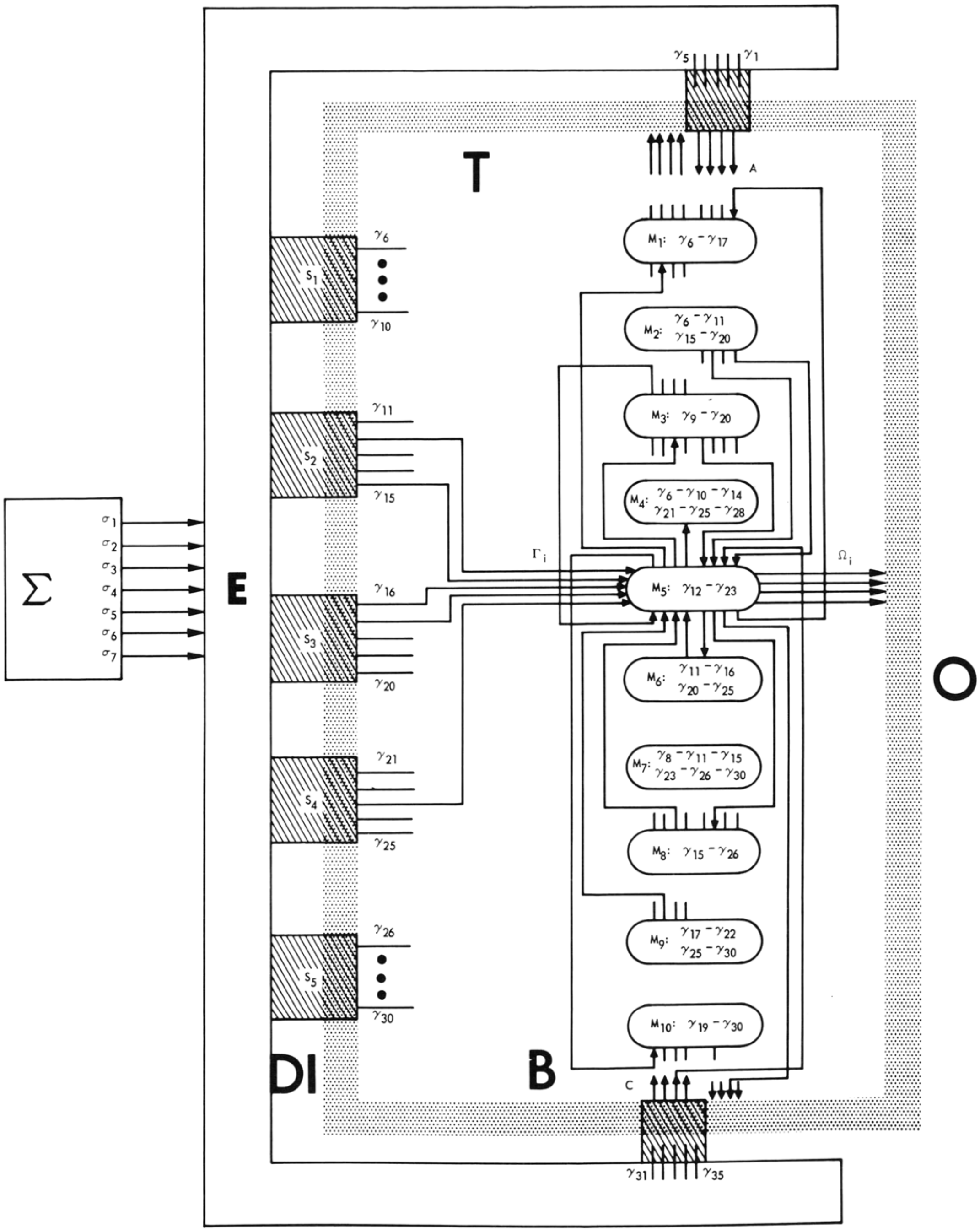

Fig. XXII-1. Simulation model (S-Retic).

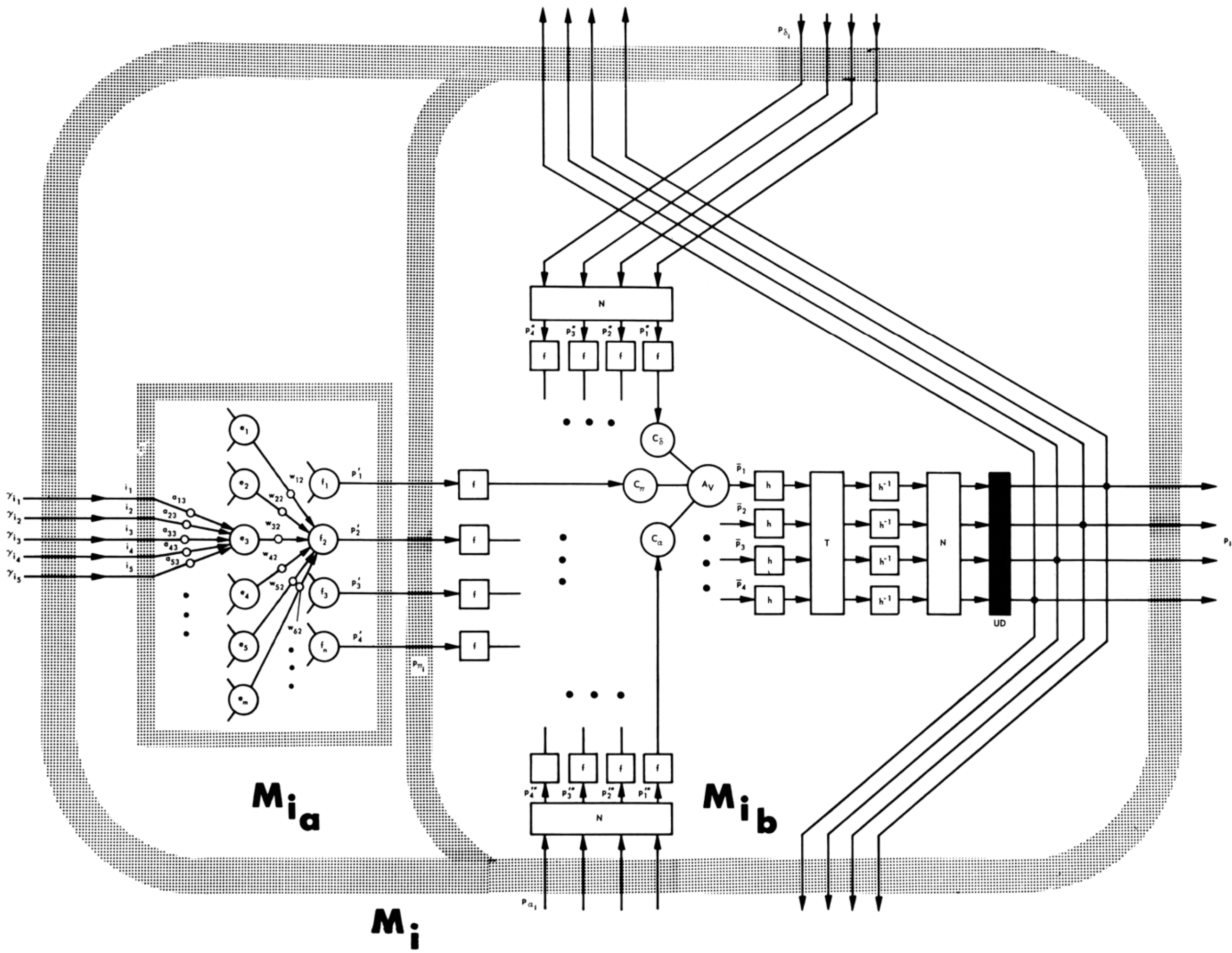

Fig. XXII-2. A typical M_i of Fig. XXII-1.

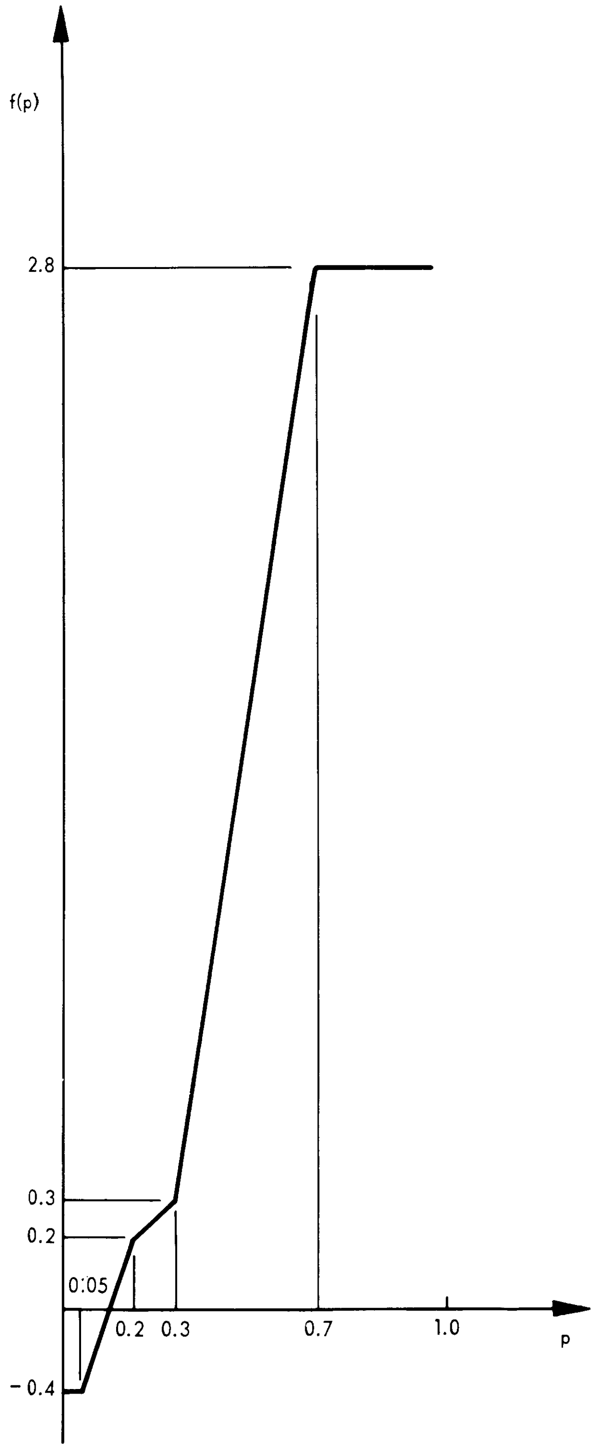

Fig. XXII-3. The f(p) function.

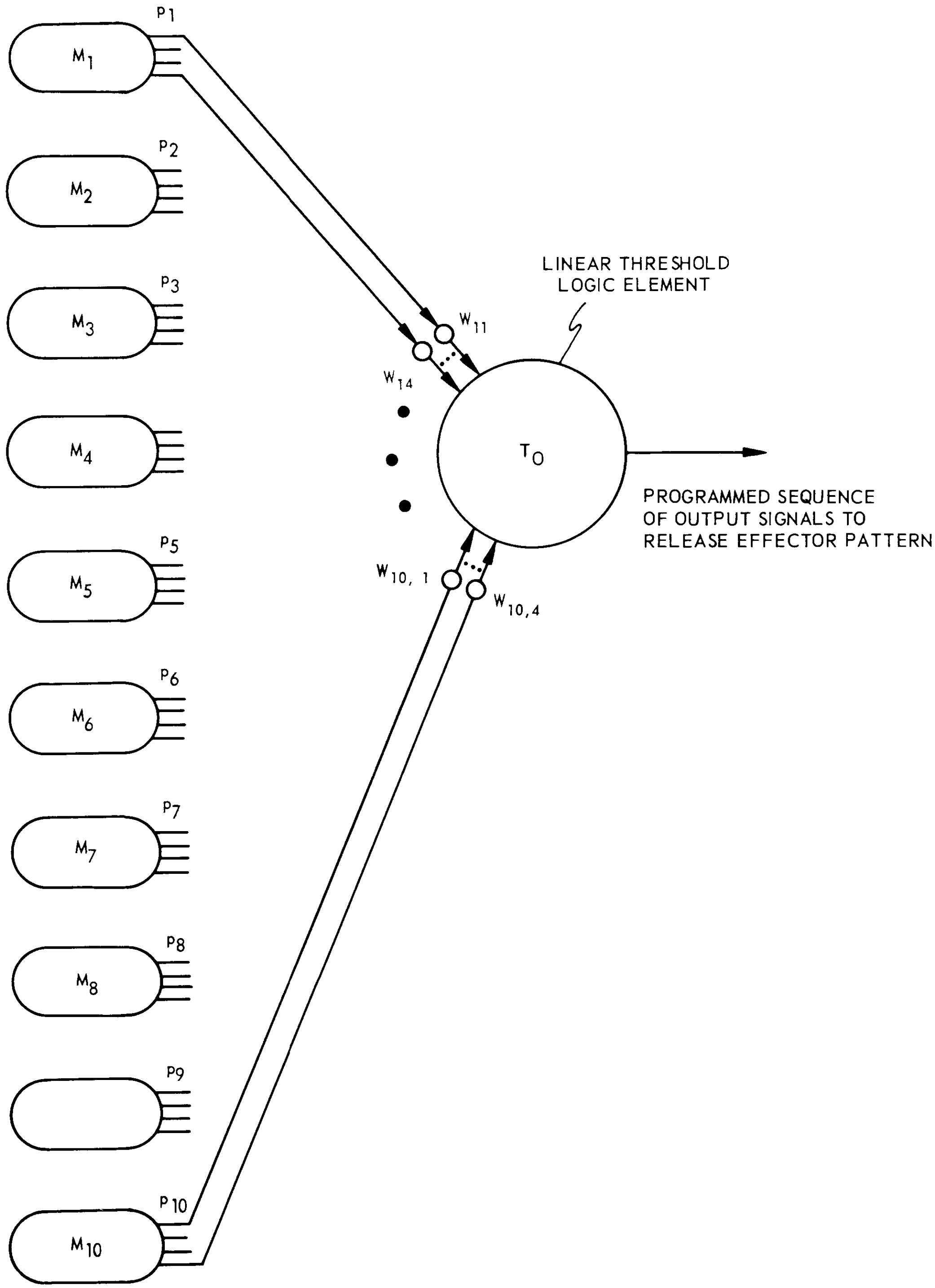

Fig. XXII-4. S-Retic output scheme.

constitute authority. The h, T, h^{-1}, N, and UD blocks in Fig. XXII-2 are to insure that every P_i is an appropriately normalized and delayed probability vector. Figure XXII-4 shows our Fig. XXII-1 model's output model detection scheme.

The model has been successfully simulated on the Honeywell Computer at Instrumentation Laboratory, M.I.T., in collaboration with J. Blum, W. L. Kilmer, E. Craighill, and D. Peterson. The model converged to the correct output model indication in each of approximately 50 test cases, and always in from 5 to 25 time steps. This is just what we had hoped for.

We are now concentrating on the functional design of a considerably enriched model that can handle conditioning and extinction in a satisfactory time-domain sense. The design will again be programmed for simulation on the Instrumentation Laboratory computer.

W. S. McCulloch, W. L. Kilmer

[Dr. W. L. Kilmer is now at Michigan State University, East Lansing, Michigan.]

References

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2. W. S. McCulloch, W. L. Kilmer, J. Blum, and E. Craighill, "A Cybernetic Theory of the Reticular Formation," a paper presented at the Bionics Symposium, Dayton, Ohio, May 2-5, 1966.

B. REALIZABILITY OF A NEURAL NETWORK CAPABLE OF ALL POSSIBLE MODES OF OSCILLATION*

1. Introduction

Dr. McCulloch has called our attention to the need for investigating the modes of oscillation of neural nets with feedback and under constant input. The question "How many possible modes of oscillation are there for N neurons?" has already been answered by C. Schnabel.[1] There are $\sum_{K=2}^{K=2^N} (K-1)!\binom{2^N}{K}$ possible modes of oscillation. The next question is, "Are all of these modes realizable with a fixed anatomy?" The answer is affirmative, provided there is a minimum number of input lines to the network. The proof is presented here.

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2. $\mathscr{L}$-Networks

Consider N formal neurons with interactions of afferents (those described by McCulloch in "Agathe Tyche"[2]) forming a one-layer network with M binary inputs, and in which each neuron binary-output feeds back to the same neuron and to all the others. The number of inputs to each neuron is, then, M + N. At any time t the output (or state) – 0 or 1 – of any neuron is determined by the value of its inputs at time t - 1. Therefore, the state of any neuron at time t is determined by the states of all of the neurons and the inputs to the network, both at time t - 1. At any time t, the state of the network is defined as the array of N zeros and ones that indicate the state of each neuron at this time. Thus, we can say that the state of the network at time t is determined by the state of the network and the input configuration, both at time t - 1.

For a network of N neurons, there are 2^N possible states. We can imagine some particular networks in which some of these 2^N states are never reached. We are interested in those networks in which any of the 2^N states may be reached from any initial state by at least 1 input sequence. Such networks are here referred to as $\mathscr{L}$-networks. Thus, an $\mathscr{L}$-network is a network of N neurons forming one layer in which the output of each neuron is connected to itself and all of the others, and any of the 2^N states may be reached from any initial state by some input sequence.

Consider the set of all possible states $S = \left(S_1, S_2, \ldots, S_{2^N}\right)$ and the set of all possible configurations of the inputs $S = \left(W_1, W_2, \ldots, W_{2^M}\right)$ of an $\mathscr{L}$-network with M inputs. From these sets, we form all possible doubles

$$(S_i, W_k),$$

where $W_k \in W$ and $S_i \in S$. We have $2^M \cdot 2^N = 2^{M+N}$ doubles. We now generate 2^{M+N} successors of the form

$$(S_i, W_k) \rightarrow S_j$$

by arbitrarily assigning to each double (S_i, W_k) an element $S_j \in S$ and only one, and using any $S_j \in S$ at least once. The set of 2^{M+N} successors generated in this manner is referred to as a "set of successors, v(N, M)." We generate all possible sets v(N, M), and form from them a new ensemble $\mathscr{V}$. Thus, $\mathscr{V}$ is the ensemble of all possible sets of successors v(N, M).

Returning to any one $\mathscr{L}$-network, if S_i is the state at any time t - 1, W_k is the input configuration at time t - 1, and S_j is the state at time t, we can form a set, v, of 2^{M+N} successors of the form

$$[S_i(t-1), W_k(t-1)] \rightarrow S_j(t).$$

which describes completely the behavior of the net. Obviously $v \in \mathscr{V}$. We will prove that the inverse is also true, i.e., given any arbitrary v, there is an $\mathscr{L}$-network that is described by v.

LEMMA 1. Given an arbitrary set of successors $v(N,M) \in \mathscr{V}$, it is always possible to design an $\mathscr{L}$-network of N neurons and M inputs that verifies $v(N,M)$.

Proof. The proof consists in generating, from $v(N,M)$, N Venn diagrams of $N+M$ inputs each. The network can be designed from the N Venn diagrams (see Blum[3]).

We first note that each double (S_i, W_k) of each successor $(S_i, W_k) \rightarrow S_j$ of $v(N,M)$ determines one area in a Venn diagram of $N+M$ inputs.

Let us assume that we have drawn the Venn diagram for the first neuron. Then, we put a jot in those areas of the Venn represented by all (S_i, W_k) for which the S_j indicate that the state of that neuron is 1 (fires). We repeat the same for all of the N neurons. Thus, for each combination (S_i, W_k), the corresponding S_j is determined.

3. Modes of Oscillation

Consider a system of N formal neurons and M inputs. If we define the state of the system at time t as the array of N zeros and ones that indicate the state of each neuron at time t, there are 2^N possible states. A mode of oscillation of the system is defined as any sequence of states that is repeated under a constant input and involves more than one member state. A k-dimensional mode of oscillation is a mode that passes through k different states.

The number N_o of possible modes of oscillation of N neurons is (see Schnabel[1])

$$N_o = \sum_{k=2}^{k=2^N} (k-1)! \binom{2^N}{k}.$$

LEMMA 2. All N_o possible modes of oscillation of N neurons can be described by a set of successors $v(N,M)$ such that

$$M \geq \log_2 \sum_{k=2}^{k=2^N} \frac{1}{\left(\frac{2^N}{k}\right)_E} (K-1)! \binom{2^N}{k},$$

where $\left(\frac{2^N}{k}\right)_E$ indicates the maximum whole number that is less than or equal to $2^N/k$.

Proof. Any k-dimensional mode of oscillation can be divided in k-steps. Each step can be expressed by a successor

$$(S_i, W_k) \rightarrow S_j,$$

where S_i and S_j are the states of the step, and W_k is the constant input word that produces the mode. If we require that two modes with at least one state in common cannot be specified by the same input configuration, we can insure that for each double (S_i, W_k) there is one and only one S_j.

Thus the number of k-dimensional modes of oscillation that may result from any one input configuration is the largest integer that is less than or equal to $2^N/k$. We denote this integer by $\left(\frac{2^N}{k}\right)_E$. The number of input configurations necessary to specify all k-dimensional modes, is

$$\frac{(k-1)!\binom{2^N}{k}}{\left(\frac{2^N}{k}\right)_E}$$

(number of k-dimensional modes divided by the number of k-dimensional modes specified by each input configuration). The minimum total number of input configurations then is

$$\sum_{k=2}^{k=2^N} \frac{(k-1)!}{\left(\frac{2^N}{k}\right)_E} \cdot \binom{2^N}{k}$$

and therefore the number of input lines, M, has to be such that

$$2^M \geq \sum_{k=2}^{k=2^N} \frac{(k-1)!}{\left(\frac{2^N}{k}\right)_E} \binom{2^N}{k}$$

or

$$M \geq \log_2 \sum_{k=2}^{k=2^N} \frac{(k-1)!}{\left(\frac{2^N}{k}\right)_E} \binom{2^N}{k}.$$

Since $\sum\limits_{k=2}^{k=2^N} k! \binom{2^N}{k}$ doubles have been used in describing all modes of oscillation, we can arbitrarily assign one and only one S_j to each of the remaining doubles; that is, to $2^{M+N} - \sum\limits_{k=2}^{k=2^N} k! \binom{2^N}{k}$ doubles. This could be, for example, the state $S_{000....00}$ to all of the doubles left.

THEOREM. Given N neurons and M input lines such that

$$M \geq \log_2 \sum_{k=2}^{k=2^N} \frac{(k-1)!}{\binom{2^N}{k}_E} \cdot \binom{2^N}{k},$$

it is always possible to design an $\mathscr{L}$-network that verifies all possible modes of oscillation.

Proof. All possible modes of oscillation of N neurons can be expressed by a set $v(N, M)$ of successors such that

$$M \geq \log_2 \sum_{k=2}^{k=2^N} \frac{(k-1)!}{\binom{2^N}{k}_E} \binom{2^N}{k} \qquad \text{(Lemma 2).}$$

According to Lemma 1, it is always possible to design an $\mathscr{L}$-network that verifies any $v(N, M)$, in particular, that which describes all possible modes of oscillation.

The minimum number of jots, $\mathscr{N}$, per Venn diagram for such an $\mathscr{L}$-network is the same for all of the neurons of the network. This number can be computed as follows. The number of modes of oscillation that pass through any one state is

$$\sum_{k=2}^{k=2^N} (k-1)! \binom{2^N-1}{k-1}.$$

This number gives the number of doubles that correspond to the same S_j in all successors $(S_i, W_k) \to S_j$. In describing the first neuron, for example, we put jots in the Venn areas for which S_j indicates that the neuron fires. There are 2^{N-1} of these states S_j. Therefore, the number of jots, $\mathscr{N}$, for the Venn of that neuron is at least

$$\mathscr{N} = 2^{N-1} \cdot \sum_{k=2}^{k=2^N} (k-1)! \binom{2^N-1}{k-1},$$

and $\mathscr{N}$ is the same for all neurons.

R. Moreno-Diaz

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2. W. S. McCulloch, Embodiments of Mind (The M.I.T. Press, Cambridge, Mass., 1965), pp. 203-215.

3. M. Blum, "Properties of a Neuron with Many Inputs," Principles of Self-Organization, H. Von Foerster and R. Zopf (eds.) (Pergamon Press, Oxford, London, New York and Paris, 1962), pp. 95-119.