PLASMA DYNAMICS

XIII. PLASMA PHYSICS

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A. INTERACTION OF A PLASMA WITH AN m = 1 RF MAGNETIC FIELD

This report is a description of advances made by the author on the problem of electrical coupling to the ions of a plasma. W. H. Glenn used a conventional Stix[1] coil to look at the line shape of the ion cyclotron resonance line in a helium plasma.[2] Using a twin T bridge developed by Graham and Badessa,[3] he was able to observe changes proportional to the magnitude of the plasma dielectric constant. He tried unsuccessfully to get to a region where ion Doppler broadening would be dominant.

By redesigning the conventional Stix coil, it has been possible to produce a field configuration which is much better for diagnostic and heating purposes of small moderate density plasmas than the conventional Stix coil. Whereas the Stix coil produces a purely symmetric azimuthal field that goes to zero at the axis, the new structure produces a field like that of a slow plane wave propagating along the axis of the plasma. This field is superior for measuring plasma density because it is almost uniform over the whole plasma cross section, rather than going to zero just at the most crucial spot as does that of the conventional coil. This fact makes loading larger so that detection is somewhat easier than with a conventional coil.

A new detection device was made by the author which could measure changes in the real and imaginary part of the loading, which Glenn's bridge was not able to do. With this device the real and imaginary parts of the plasma dielectric constant were measured for a hydrogen plasma at frequencies around the proton cyclotron frequency. A short coil wavelength was used and the neutral background pressure was kept low so that ion Doppler broadening predominates over collisional broadening.

Previous treatments usually ignored effects related to finite column size and finite electron conductivity. The author has considered these effects theoretically and found that they can be significant, particularly for small columns where they can sometimes cause the resonance to be washed out. The outline of this theoretical treatment is presented.

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1. Field Configuration

In the Stix coil the free space magnetic field B_z has the form $I_o(kr)\, e^{ikz-i\omega t}$, where I_o is the modified Bessel function of zero order. The new coil is designed to use the higher order $m = \pm 1$ fields in which the free space B_z goes like $I_1(kr)\, e^{ikz-i\omega t+im\theta}$. It can be shown that the transverse electric fields are of the form

$$E_r = if_1 Q I_1'(Qr) + \frac{m\omega}{Q} \frac{I_1(Qr)}{Qr}$$

$$E_\theta = -mf_1 Q \frac{I_1(Qr)}{Qr} - \frac{i\omega}{Q} I_1'(Qr),$$

where f_1 and Q are constants dependent on plasma parameters, and I' is the derivative of the modified Bessel function. Near r = 0,

$$\frac{I_1(Qr)}{Qr} \approx I_1'(Qr) \approx \frac{1}{2}.$$

On the axis we have simply a uniform right- or left-rotating field, depending on whether m = +1 or −1. For equal amounts of +1 and −1 we have a linearly polarized field, the experimental configuration.

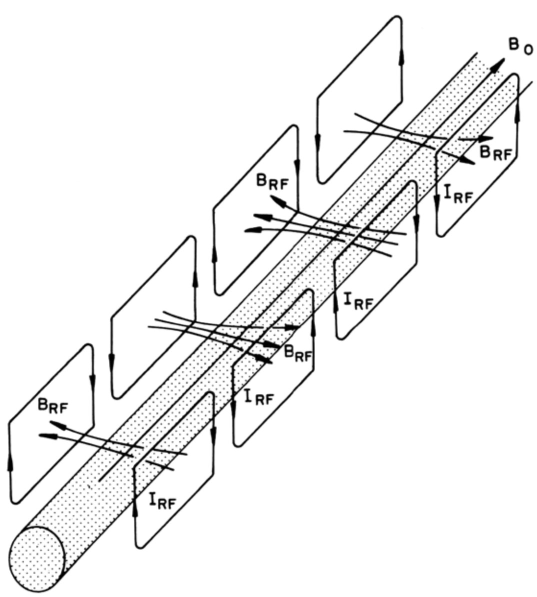

Fig. XIII-1. Coil arrangement for producing an m = ±1 field.

An m = 1 field is created by arranging coils around the plasma so that the axes of the coils are perpendicular to the axis of the plasma, causing the RF magnetic field to cross the plasma. By placing two or more coils along the axis with their windings wound in alternate directions, we cause this perpendicular magnetic field to reverse direction, giving a wavelength in the axial direction. This is shown schematically in Fig. XIII-1. Figure XIII-2 shows the relation of the magnetic field and the induced electric field on the axis, approximately a standing plane wave.

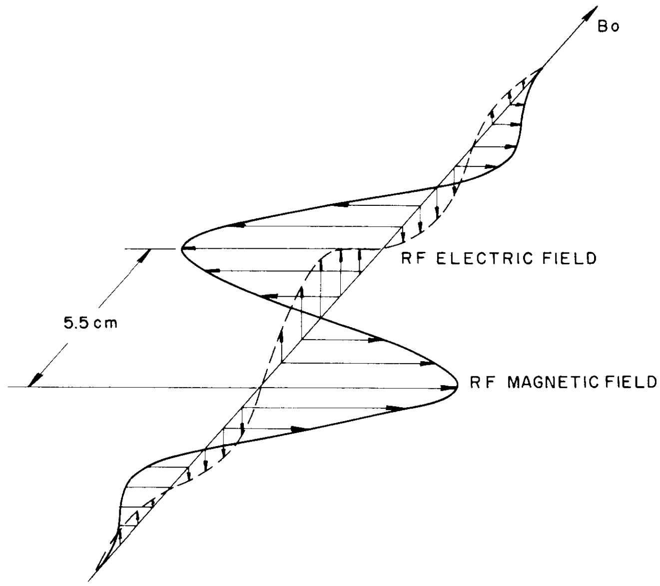

Fig. XIII-2. Relation of electric and magnetic fields produced by the coil arrangement of Fig. XIII-1.

2. Detection

Even with the relatively strong coupling of the m = 1 mode, it was necessary to use a highly sensitive detector, since the plasma used was small (1 cm diameter). A two-stage synchronous detector was designed for this purpose. The first stage extracts a sideband of the impressed RF signal caused by the plasma. The second stage measures the sideband. The first stage uses a Type 7360 tube having two deflection grids and two plates. A voltage across the deflection grids diverts the beam from one plate to the other. The current flowing between the plates is the product of the voltage across the deflection grids and that on the control grid. The 7360 multiplier circuit that was finally used was a slightly modified version of a circuit originally designed and built by Lyn D. Pleasance. Figure XIII-3 is a block diagram of the apparatus.

Because the plasma loads the coil slightly, there will be a small change in the coil inductance at 200 cycles, the plasma modulation frequency. The voltage across the coil is $-i\omega(L_o + L\cos 400\pi t)\, I\, e^{-i\omega t}$, where ω is of the order of megacycles, the ion cyclotron frequency, and I is kept constant by the 66-μH inductors in series with the voltage generator. In the multiplier this voltage is multiplied by a signal proportional to I, with a variable phase shift, ϕ. The signal out of the multiplier is

$$\mathrm{Re}\left(-i\omega(L_o + \Delta L \cos 400\pi t)|I|^2 e^{-i\phi}\right).$$

After this signal is processed through the 200-cycle filter and the synchronous detector, the DC output is proportional to Re $(-i\, e^{-i\phi}\Delta L)$. By varying the phase ϕ, we can look

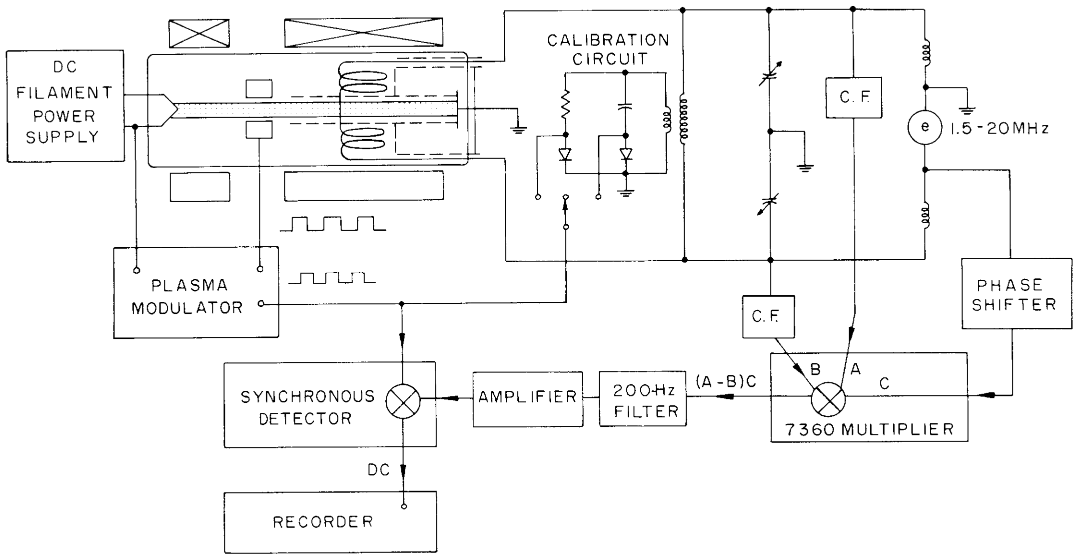

Fig. XIII-3. Detection system.

at the real or imaginary part of ΔL. A calibrating circuit with two dummy loads simulating plasma loading is used to adjust ϕ and to calibrate the over-all gain. This system is capable of detecting changes of inductance of one part in 10^6. The experimental signal, being of the order of one part in 10^4 or less, could not be detected on a conventional bridge.

It is shown in this report that when the electron conductance is high $\left(k^2a^2K_{\parallel} \gg K_{\perp} - iK_x\right)$, the coil impedance $\mathcal{Z}$ is given by

$$\mathcal{Z} = \sum_{m=\pm 1} -\frac{i\omega L}{2}\left(1 + k_o^2\eta \frac{a^2}{2s^2}(K_{\perp} + imK_x)\right),$$

where η is a numerical factor of the order of 1, and a and s are the plasma and coil radii, respectively. The fact that the loading is proportional to $a^2(K_{\perp} + imK_x)$ shows that the loading is proportional to the total number of ions within the coil. Evaluating the numerical factors and noting that resonance occurs for only the left-rotating field (m = -1), this expression is then

$$\mathcal{Z} \approx -i\omega L\left(1 + .0048\,\frac{\pi_i^2}{c^2}\,\frac{\omega}{kv_{\parallel}}\,Z(\xi_{-1})\right),$$

where $\xi_{-1} = \dfrac{\omega - \omega_c + i\nu}{kv}$, and Z is the plasma dispersion function tabulated by Fried and Conte.

A curve of plasma loading versus frequency is shown in Fig. XIII-4. The resonance

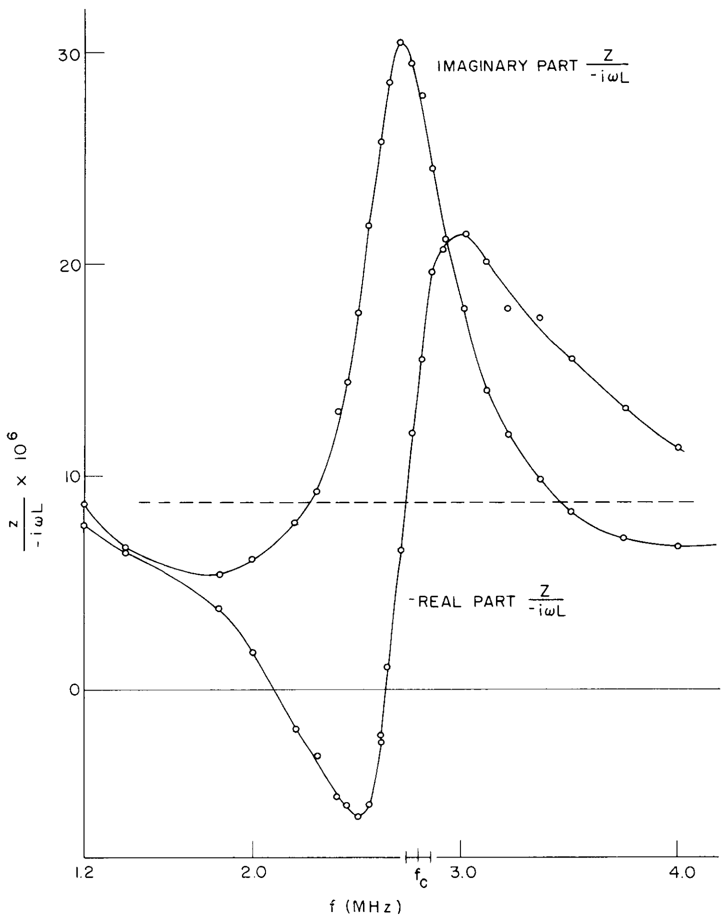

Fig. XIII-4. Plasma loading vs frequency.

curves have the form predicted by theory, and the resonance occurs at ω_c as predicted. The density of protons measured by the coil is 7×10^{10}/cc. No resonance occurs for H_2^+ because the electron conductance is too low. The width of the resonance curve is caused by a combination of ion Doppler broadening and electron Landau damping.

3. Theoretical Model

A theoretical model for this problem, including effects of finite electron conductivity, has been analyzed.[4] The method is to solve two coupled wave equations for E_z and B_z inside the plasma and to match these solutions with those in the empty space between the coil and plasma, and with that outside the coil. The solutions are obtained for the condition that the dominant wavelength of the coil is much shorter than the wavelength of ion cyclotron waves in the plasma. These solutions are matched at the coil with the driven current in the coil. One then derives an expression for the azimuthal electric field at the coil as a function of the driven current, which is

$$\frac{E_{\theta k}}{I_{\theta k}} = \frac{i\omega K_1'(ks)\, s\mu_o \left\{ I_1(ka)\, K_1'(ks) - I_1'(ks)\, K_1(ka) + \frac{\mathscr{I}}{\mathscr{K}} \left(I_1'(ks)\, K_1'(ka) - I_1'(ka)\, K_1'(ks)\right)\right\}}{K_1(ka) + \frac{\mathscr{I}}{\mathscr{K}} K_1'(ka)},$$

where s is the coil radius. Here $\frac{\mathscr{I}}{\mathscr{K}}$ is a function containing all of the plasma parameters. It is an expression in the form of an admittance that involves the two-wave solutions inside the plasma. For a small-diameter plasma and for a frequency near the resonance one can expand these wave solutions and arrive at the following result for the left-rotating component:

$$\frac{\mathscr{I}}{\mathscr{K}} \approx -\frac{I_1(ka)}{I_1'(ka)} \left\{ 1 + \frac{k_o^2}{k^2} \left[\frac{K_\ell \left(k^2 a^2 K_{\parallel}\right)/4}{K_\ell + \left(k^2 a^2 K_{\parallel}\right)/4} \right] \right\}^{-1}.$$

The experession in brackets represents simply a series combination of two conductances, K_ℓ and $\left(k^2a^2K_{\parallel}\right)/4$. This shows that in order to see the ion cyclotron resonance, $\left(k^2a^2K_{\parallel}\right)/4$ must be much bigger than K_ℓ, a result that can be derived from physical intuition. By evaluating $E_{\theta k}/I_{\theta k}$ for the dominant coil wavelength, we derive an expression for Z, the coil impedance. In the experiment the condition on the size of K is satisfied for only the proton resonance, not for the molecular ion.

P. W. Jameson

References

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2. W. H. Glenn, Ph.D. Thesis, Department of Physics, M.I.T., February 1966.

3. J. W. Graham and R. S. Badessa, Quarterly Progress Report, Research Laboratory of Electronics, M.I.T., April 15, 1958, p. 7.

4. P. W. Jameson (unpublished work).