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A. PRELIMINARY WORK ON A NEW MODEL OF BINAURAL HEARING

This is a report of preliminary work on a model of binaural hearing in which the firing patterns on the auditory nerve are regarded as inputs to a central processor. The model is an extension of work done by W. M. Siebert^{1,2} on monaural phenomena. The representation of the firing patterns is based upon data taken from the cat's auditory nerve by N. Y. S. Kiang and his collaborators.³ The major assumptions in the preliminary version of the model are: (i) the activity in the nerves is characterized by the firing times of the fibers, and the inputs to the central processor are sample functions from random-point processes^{1,2} in which the processes corresponding to the two ears are independent; (ii) the central processor is ideal, subject to the limitation that it only makes use of the times between firings, that is, it has no absolute time reference. By comparing the predictions of this version with psychoacoustic data, we can separate the limitations on performance imposed by the peripheral transformation from the limitations imposed by the central processor. These results can then be used to guide the development of a more realistic model of the processor.

We begin by presenting some general predictions that are independent of the random process selected to describe the firing patterns. We then choose a particular process and derive further predictions based on this process. Finally, we compare these predictions with available data. The predictions are made for situations in which the listener is required to discriminate interaural time delay or interaural amplitude ratio for tone bursts in a two-alternative-forced-choice experiment. We consider the just-noticeable difference (JND) in interaural time and interaural amplitude as a function of the amplitudes in the two ears, the interaural time delay, the frequency, and the duration of the tone. We assume that the task of the subject is to correctly identify the order of presentation of two binaural stimuli (a and b) that are identical except for incremental differences in interaural time or interaural amplitude, and that the JND is defined as the difference for which the probability of a correct response is 0.75. It can be shown that, under reasonable assumptions, the JND is equal to a constant factor times the variance of the minimum-variance estimate of the difference.

For interaural time,

$$\begin{aligned} \text{stimulus a: } s_1(t) &= A_1 \cos \omega \left(t - \tau - \frac{|\Delta|}{2} \right) & s_2(t) &= A_2 \cos \omega \left(t + \frac{|\Delta|}{2} \right) \\ \text{stimulus b: } s_1(t) &= A_1 \cos \omega \left(t - \tau + \frac{|\Delta|}{2} \right) & s_2(t) &= A_2 \cos \omega \left(t - \frac{|\Delta|}{2} \right). \end{aligned}$$

For interaural amplitude,

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$$\begin{aligned} \text{stimulus a: } s_1(t) &= A_1 \left(1 + \frac{|\delta|}{2}\right) \cos \omega(t-\tau) & s_2(t) &= A_2 \left(1 - \frac{|\delta|}{2}\right) \cos \omega t \\ \text{stimulus b: } s_1(t) &= A_1 \left(1 - \frac{|\delta|}{2}\right) \cos \omega(t-\tau) & s_2(t) &= A_2 \left(1 + \frac{|\delta|}{2}\right) \cos \omega t. \end{aligned}$$

The following notation is used throughout this report:

$s_j(t)$ = pressure signal to ear j ($j=1, 2$)

A_j = reference amplitude of signal to ear j

δ = incremental difference in interaural amplitude ratio to be detected

τ = reference interaural time difference

Δ = incremental difference in interaural time to be detected

\underline{t}_j = received data vector for ear j = firing times of all fibers of nerve
in ear j

$\underline{t} = (\underline{t}_1, \underline{t}_2)$

$p(\underline{t}_j/A, y)$ = probability density function for \underline{t}_j in response to the stimulus $s_j(t) = A \cos(t-y)$

$\delta_o(A_1, A_2, \tau)$ = value of δ such that the probability of being correct is 0.75 when amplitudes are A_1 and A_2 and the interaural delay is τ

$\Delta_o(A_1, A_2, \tau)$ = value of Δ such that the probability of being correct is 0.75 when amplitudes are A_1 and A_2 and the interaural delay is τ

\hat{a} = estimate of a (where a is any parameter).

We note here that, because of the assumption of ideal processing, $\delta_o(A_1, A_2, \tau)$ and $\Delta_o(A_1, A_2, \tau)$ are independent of τ . This is obvious when one considers that the ideal processing has perfect memory of interfering intervals. This is an artificial assumption and perfect memory will not be included in future versions of the model. In keeping with these statements, we shall restrict our attention to the case $\tau = 0$, and τ will be suppressed in our notation, since the predictions of the preliminary version are independent of τ .

In order to ensure that the central processor only makes use of the times between firings, we introduce an unknown time delay θ which is the same for both ears and different for each burst. (This has the effect of allowing the subjects to detect only interaural differences in time delay and, therefore, the interaural delays τ and Δ can be split up arbitrarily between the ears.) The reference stimuli ($\Delta = 0 = \delta$) during a

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particular burst can now be written

$$s_1(t) = A_1 \cos \omega(t-\theta) \quad s_2(t) = A_2 \cos \omega(t-\theta).$$

In the following discussion, we shall use Cramér's bound⁴ on the variance of an estimate \hat{a} , extended to the case of an unknown parameter.⁵ If $\text{Var}[\hat{a}]$ is the variance of \hat{a} , \underline{t} is a received data vector, and $f(\underline{t}/a, \theta)$ is the conditional probability of \underline{t} , given a and θ , the bound is

$$\text{Var}[\hat{a}] \geq \frac{E \left[\left(\frac{\partial \ln f(\underline{t}/a, \theta)}{\partial \theta} \right)^2 \right]}{E \left[\left(\frac{\partial \ln f(\underline{t}/a, \theta)}{\partial a} \right)^2 \right] E \left[\left(\frac{\partial \ln f(\underline{t}/a, \theta)}{\partial \theta} \right)^2 \right] - E^2 \left[\frac{\partial \ln f(\underline{t}/a, \theta)}{\partial a} \frac{\partial \ln f(\underline{t}/a, \theta)}{\partial \theta} \right]}.$$

Note that when

$$E \left[\frac{\partial \ln f(\underline{t}/a, \theta)}{\partial a} \frac{\partial \ln f(\underline{t}/a, \theta)}{\partial \theta} \right] = 0, \quad (1)$$

the bound can be written

$$\text{Var}[\hat{a}] \geq \frac{1}{E \left[\left(\frac{\partial \ln f(\underline{t}/a, \theta)}{\partial a} \right)^2 \right]}.$$

In all cases, there exists an estimate whose variance achieves the bound, at least asymptotically, and, therefore, we shall use the bound with equality. For simplicity, we assume that Eq. 1 is satisfied by $p(\underline{t}_j/A, y)$ and thus, that lack of knowledge of y does not affect the estimate of A . That random processes exist which describe the data reasonably well and satisfy Eq. 1 is demonstrated by the example discussed later in this report.

For convenience, we introduce the notation

$$\sigma_{\Delta}^2(A) = \frac{1}{E \left[\left(\frac{\partial \ln p(\underline{t}_j/A, y)}{\partial y} \right)^2 \right]}$$

$$\sigma_{\delta}^2(A) = \frac{1}{E \left[\left(\frac{\partial \ln p(\underline{t}_j/A, y)}{\partial A} \right)^2 \right]}.$$

This notation is reasonable, since $\sigma_{\Delta}^2(A)$ and $\sigma_{\delta}^2(A)$ are positive and are independent of y if edge effects are ignored.

In the interaural time cases, the probability function for the received data, $f(\underline{t}/\Delta, \theta)$,

is $p\left(\frac{t_1}{A_1}, \theta + \frac{\Delta}{2}\right) p\left(\frac{t_2}{A_2}, \theta - \frac{\Delta}{2}\right)$, where we have assumed for simplicity that the ears are statistically identical, as well as statistically independent. We can now prove that

$$\text{Var}[\hat{\Delta}] = \sigma_{\hat{\Delta}}^2(A_1) + \sigma_{\hat{\Delta}}^2(A_2) = 4\Delta_{\text{O}}^2(A_1, A_2)$$

and, therefore, that the interaural time JND on the midline is

$$\Delta_{\text{O}}(A, A) = \frac{1}{\sqrt{2}} \sigma_{\hat{\Delta}}(A), \quad (2)$$

and the JND's off the midline are related to those on the midline by

$$\Delta_{\text{O}}^2(A_1, A_2) = \frac{1}{2} \left(\Delta_{\text{O}}^2(A_1, A_1) + \Delta_{\text{O}}^2(A_2, A_2) \right). \quad (3)$$

Proceeding similarly for the interaural amplitude case and using the assumption that Eq. 1 is satisfied, we obtain

$$\text{Var}[\hat{\delta}] = \frac{4}{\frac{1}{\sigma_{\hat{\delta}}^2(A_1)} + \frac{1}{\sigma_{\hat{\delta}}^2(A_2)}} = 4\delta_{\text{O}}^2(A_1, A_2).$$

It follows directly that the interaural amplitude JND on the midline is

$$\delta_{\text{O}}(A, A) = \frac{1}{\sqrt{2}} \sigma_{\hat{\delta}}(A), \quad (4)$$

and the JND's off the midline are related to those on the midline by

$$\delta_{\text{O}}^2(A_1, A_2) = 2 \left(\frac{1}{\delta_{\text{O}}^2(A_1, A_1)} + \frac{1}{\delta_{\text{O}}^2(A_2, A_2)} \right)^{-1}. \quad (5)$$

If we assume that $\delta_{\text{O}}^2(0, 0) = \infty$, then

$$\delta_{\text{O}}^2(A, 0) = 2\delta_{\text{O}}^2(A, A) = \sigma_{\hat{\delta}}^2(A); \quad (6)$$

that is, the monaural amplitude JND is equal to $\sqrt{2}$ times the interaural amplitude JND on the midline.

Equations 2-6 summarize the general predictions; in the sequel we shall assume a particular form for $p\left(\frac{t_j}{A}, y\right)$, which is based on the neurophysiological data of N. Y. S. Kiang.³ We assume that the individual fibers are independent and that the firing times for the i^{th} fiber ($1 \leq i \leq M$) can be modeled as a sample function of a nonhomogeneous Poisson process,⁶ with intensity function $r_i(A, t)$. (For a discussion of the applicability of the Poisson model to auditory nerve firing patterns, see Siebert^{1, 2}.) With these assumptions and notation, we obtain

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$$\sigma_{\Delta}^2(A) = \left(\sum_{i=1}^M \int_0^T \frac{1}{r_i(A, u)} \left(\frac{\partial r_i(A, u)}{\partial u} \right)^2 du \right)^{-1} \quad (7)$$

and

$$\sigma_{\delta}^2(A) = \left(\sum_{i=1}^M \int_0^T \frac{A^2}{r_i(A, u)} \left(\frac{\partial r_i(A, u)}{\partial A} \right)^2 du \right)^{-1}. \quad (8)$$

Based again upon Kiang's data,³ a reasonable choice for low-frequency tones (and one for which Eq. 1 is satisfied) is

$$r_i(A, t) = a_i \exp[g_i(A, \omega) \cos \omega t]. \quad (9)$$

Here a_i represents the spontaneous activity of the fiber, and $g_i(A, \omega)$ is a function that flattens to a constant at large intensities and includes the frequency selectivity of the fiber. For higher frequencies $r_i(A, t)$ becomes progressively less time-dependent until, at frequencies greater than ~ 5 kHz, the time dependence is not present. We shall consider only the low-frequency case.

Using Eqs. 7, 8, and 9, we obtain approximately

$$\frac{1}{\sigma_{\Delta}^2(A)} = T\omega^2 \sum_{i=1}^M a_i g_i^2(A, \omega) F_0[g_i(A, \omega)] \quad (10)$$

$$\frac{1}{\sigma_{\delta}^2(A)} = TA^2 \sum_{i=1}^M a_i \left(\frac{\partial g_i(A, \omega)}{\partial A} \right)^2 \frac{d^2 I_0(x)}{dx^2} \Bigg|_{x=g_i(A, \omega)}, \quad (11)$$

where

$$F_0(x) = I_0(x) - \frac{d^2 I_0(x)}{dx^2},$$

and $I_0(x)$ is the modified Bessel function of the first kind of zero order. As indicated in Eq. 6, $\sigma_{\delta}^2(A)$ is equal to the monaural amplitude JND studied by Siebert.¹ He has derived and evaluated (approximately) a summation equivalent to Eq. 11. We refer to his paper for a justification of the following statements:

(i) For A near threshold, $\delta_0^2(A, A)$ decreases monotonically with A , perhaps as rapidly as A^{-2} .

(ii) For larger values of A , $\delta_0^2(A, A)$ is approximately constant.

(iii) $\delta_0^2(A, A)$ is largely independent of frequency.

Equation 10 can be evaluated by methods essentially equivalent to those used by Siebert for Eq. 11. At low frequencies, the summation in (10) is approximately independent of frequency. Increasing A increases the number of fibers whose firings are synchronized with the variation of the stimulus, and more fibers are available from which information about the time structure can be obtained. At low intensities the increase is rapid; at higher intensities the number is dependent on the spread of stimulus-locked activity to fibers that are most sensitive to neighboring frequencies.

The resulting predictions for $\Delta_o^2(A, A)$ and $\delta_o^2(A, A)$ are the following.

For values of A well above threshold,

$$\Delta_o^2(A, A) = \frac{K_1}{T\omega^2(K_o + \ln A)} \quad (12)$$

$$\delta_o^2(A, A) = \frac{K_2}{T}, \quad (13)$$

where K_o , K_1 , and K_2 are constants.

For values of A near threshold, $\Delta_o^2(A, A)$ and $\delta_o^2(A, A)$ increase more rapidly with decreasing A .

We now compare the predictions expressed by Eqs. 2-6, which are independent of the random process chosen to model the firing patterns, with psychoacoustic data taken by R. M. Hershkowitz⁷ and reported in Section XIII-B. The experiment described there satisfies the assumptions implicit in our formulation. (Note that the incremental amplitude JND's in Sec. XIII-B are given in decibels, while, in the present formulation, δ_o is a fraction.) Generally speaking, the interaural amplitude JND's were found to be independent of interaural time difference and to be consistent with Eq. 5. Since the magnitudes of the predicted and observed changes were of the order of the variability of the data, however, this is not a sensitive test of the model. In the case of interaural time JND's, the results were strongly dependent on interaural time delay τ , and for values of $\tau \geq 600$ μ sec, conflicting cues and other little understood effects made it impossible to reliably measure JND's. It is not surprising that these results are not predicted by the model, since 1) we have assumed perfect memory of all interfiring times, and 2) we have not taken into account the ambiguities in the region near the antiphase point of a tone. (This was 1000 μ sec in the experiment above.) A comparison of the interaural time JND's on and off the midline with the relation predicted by Eq. 3 also revealed inconsistencies. As ear 2 was attenuated relative to ear 1, the JND increased much more rapidly than predicted. In fact, with ear 1 at 55 dB SPL and ear 2 at 25 dB SPL, the time JND was larger than when the ears were equal in amplitude at any level measured (measurements included values from 15 dB SPL to 80 dB SPL). This illustrates a significant and unexpected failure of the model which

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should be useful in future work.

Experimental results will now be compared with the predictions of the model when the Poisson process with the intensity function in Eq. 9 is chosen to describe the firing patterns. First, using Eq. 10 with reasonable values for a_1 , $g_1(A, \omega)$, and $F_O(g_1(A, \omega))$, the interaural time JND for 500 Hz and 55 dB SPL is predicted to be roughly 1 μ sec, as compared with values of approximately 10 μ sec in most empirical determinations.^{7,8} This discrepancy could be due to the approximations and guesses made in the evaluation of Eq. 10. It seems more likely, however, that the discrepancy results from the ideal processing assumption, which excludes significant time uncertainty in the higher centers. Second, for values of A near threshold, the data and the predictions show an increase in JND's with decreasing A. (This, of course, would be predicted by almost any model.) Finally, for higher values of A, the predictions expressed in Eq. 12 are generally consistent with the data. The frequency dependence of $\Delta_O(A, A)$ corresponds well with the data of Klumpp and Eady⁸ for frequencies below 500-1000 Hz. At higher frequencies, the interaural time JND increases and is essentially infinite at 1500 Hz. An increase at higher frequencies is expected, because of the loss of time dependence of $r_1(A, t)$; however, this is not yet quantified in the model and may not be a sufficient explanation. The dependence on duration has not been tested. The amplitude dependence in Eq. 12 predicts that, when A increases from 40 dB to 80 dB, the time JND decreases to a value between the value at 40 dB and $1/\sqrt{2}$ times the value at 40 dB. The data from Hershkowitz⁷ in Sec. XIII-B indicate that $\Delta_O(A, A)$ [$\tau_O(P)$ in Hershkowitz' notation] is constant or decreases slightly in this range, which is consistent with the model. Because of the magnitudes of the effects, however, this is not a sensitive test of the model. For a discussion of Eq. 13, we refer to Siebert.^{1,2}

The results of comparing the predictions of the preliminary version of the model with empirical data can be summarized as follows: in the case of interaural time, the model predicts performance that is better than the data; and in the case of interaural amplitude, the predictions are consistent with the data, although the effects are not large enough to provide a sensitive test of the model. Steps will now be taken to put constraints on the operations available to the central processor, in the hope of obtaining a model that better describes the behavioral data and of gaining insight into the processing that takes place in the physical system.

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B. DISCRIMINATION OF INTERAURAL TIME AND INTENSITY OFF THE MIDLINE

In most previous work on interaural just-noticeable differences (JND's) in time and amplitude, the JND's have been measured only on the midline.¹ Moreover, the time JND's and amplitude JND's have usually been measured in separate experiments (with different methods, subjects, and equipment), so that it is difficult to evaluate predictions on the relations of the two types of JND's.

In our research, we have performed a preliminary experiment on time and amplitude JND's both on and off the midline with a common experimental configuration and a common set of subjects. Specifically, we have made preliminary estimates of the following 6 functions: (i) interaural amplitude JND α_0 as a function of interaural amplitude difference α ; (ii) interaural time JND τ_0 as a function of interaural time difference τ ; (iii) interaural amplitude JND α_0 as a function of interaural time difference τ ; (iv) interaural time JND τ_0 as a function of interaural amplitude difference α ; (v) interaural amplitude JND α_0 on the midline as a function of signal level P; and (vi) interaural time JND τ_0 on the midline as a function of signal level P.

The method used to obtain the JND's was two-alternative-forced-choice-plus-feedback. The signal was a 500-Hz tone burst of 300-msec duration. The rise and decay times were 50 msec, the interstimulus interval was 300 msec, and the subject had 1.6 sec in which to respond. Except when the signal level P was the independent variable (functions 5 and 6), the reference level in the left ear was maintained at 55 dB SPL. The incremental difference in amplitude for obtaining the JND in interaural amplitude was obtained by raising the signal in one ear and lowering the other, and the JND α_0 refers to the total difference. When an interaural amplitude difference α of non-unity was required as a reference, however, it was achieved by decreasing the amplitude in the right ear below 55 dB, with the amplitude in the left being held constant at 55 dB. Thus, when $\alpha = 55$ dB, the presentation was essentially monaural. In all cases, the

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JND was defined as the increment (in time or amplitude) for which the subject's responses were 75% correct. Two experienced subjects, LW and ND, were used throughout the experiment, and each of the 6 functions was measured at least twice on each subject. Because of the large number of JND's to be measured, it was impossible (within the given time constraints) to obtain complete psychometric functions in estimating the JND. Instead, a crude, informal, homing technique was used. In this technique, a few brief runs of 30 trials or less were presented first in order to locate the general region of the JND. Having located this region, 2-3 runs of 100 trials were then made to estimate the JND.

The results of the experiment are shown in Fig. XIII-1. For the on-the-midline JND's, these results can be compared with those of Mills² (amplitude JND) and those of Klumpp and Eady³ and Zwislocki and Feldman⁴ (time JND). Our average results at $P = 55$ dB SPL are roughly $\alpha_0 = 0.9$ dB and $\tau_0 = 11$ μ sec, and the average increase in

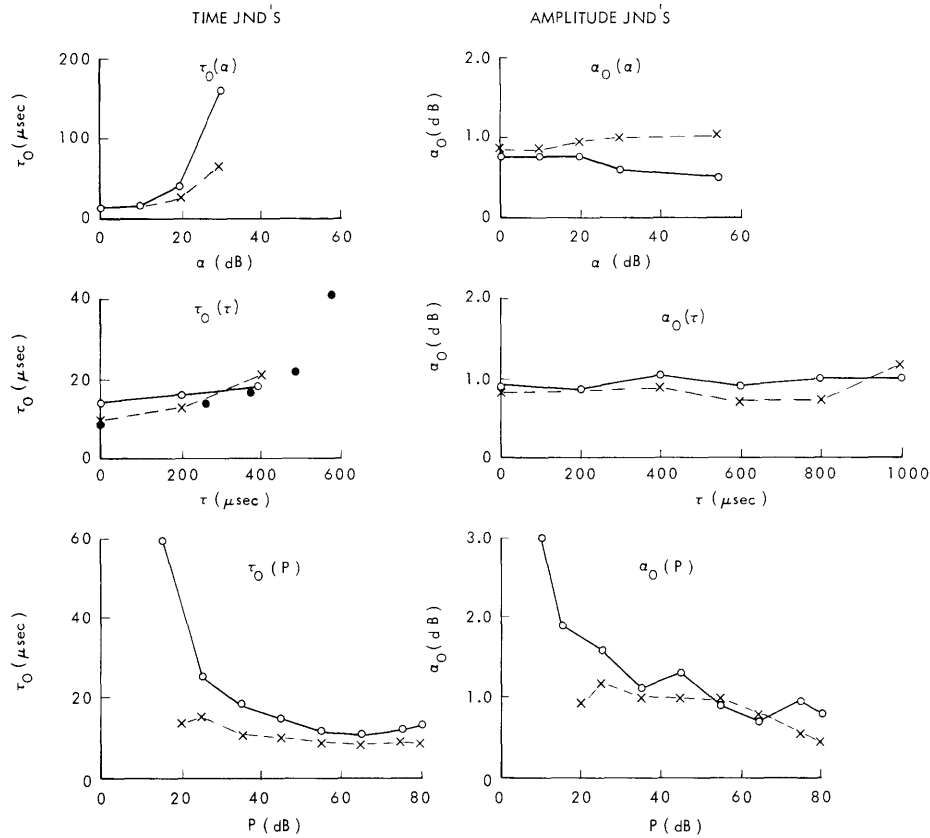


Fig. XIII-1. Just-noticeable differences in interaural time and interaural amplitude for a 500-Hz tone as functions of interaural time τ , interaural amplitude α , and power level P . O: Subject ND; X = Subject LW; ●: translation of minimum audible-angle data obtained by Mills.⁵

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τ_o as P is decreased from 80 dB to 30 dB is ~ 7 μsec . The comparable previous results are approximately $a_o = 0.7$ dB, $\tau_o = 18$ μsec , and an increase in τ_o of ~ 7 μsec . Assuming that changes in the azimuth of a tone source in a free-field environment are detected by observing variations in interaural time (a reasonable assumption at 500 Hz), one can translate the data of Mills on the minimum audible angle⁵ into data that can be compared with our data on $\tau_o(\tau)$. The results of this translation (under the assumption that the head is a sphere of 17-cm diameter) are shown along with our data on $\tau_o(\tau)$. The fact that our data on $a_o(\tau)$ are essentially independent of τ is inconsistent with the result of Mills⁶ which indicates that at 500 Hz the amplitude JND for antiphase tones (corresponding to $\tau = 1000$ μsec) is smaller than that for homophase tones ($\tau = 0$).

By taking the ratio τ_o/a_o , our results can also be compared with those of Moushegian and Jeffress⁷ on time-intensity trading. In their experiment, the listener was presented with a 500-Hz 75-dB SPL tone with interaural delays in the range 0-360 μsec and interaural amplitude ratios a in the range -9-+9 dB, and he was required to set the interaural time delay τ' of a noise signal to match the lateral position of the tone. The results for the 3 subjects tested in their experiment were roughly the following. For each subject, and each value of τ , the dependence of τ' on a was approximately linear (a measured in dB) with $\tau' = \tau$ at $a = 0$ dB. For subject 1, the slope of $\tau'(a)$ was 2.5 $\mu\text{sec}/\text{dB}$ for all τ . For subject 2, the slope was 18 $\mu\text{sec}/\text{dB}$ at $\tau = 0$ and increased monotonically to approximately 30 $\mu\text{sec}/\text{dB}$ at $\tau = 360$ μsec . For subject 3, the slope was 27 $\mu\text{sec}/\text{dB}$ at $\tau = 0$ and increased to ~ 40 $\mu\text{sec}/\text{dB}$ at $\tau = 360$ μsec .

Assume now that the effects of interaural time and interaural amplitude can be described purely in terms of a common, unidimensional, lateralization sensation (that is, that time and amplitude are completely tradeable) and the discrimination cue in our experiment for both the time JND and amplitude JND is derived solely from this sensation. Under this assumption (which, at best, can only be approximately valid in a region around the midline), we would expect our results on τ_o/a_o to be approximately equal to the ratios just cited. According to our data on $\tau_o(P)$ and $a_o(P)$, above approximately 30 dB SPL, the midline ratio τ_o/a_o lies in the range 9-21 $\mu\text{sec}/\text{dB}$ (LW) and 11-16 $\mu\text{sec}/\text{dB}$ (ND). The corresponding values in the Moushegian-Jeffress experiment are those for which $\tau = 0$ and are 2.5, 18, and 27 $\mu\text{sec}/\text{dB}$. According to our results on $\tau_o(\tau)$ and $a_o(\tau)$, as τ increases from 0 to 400 μsec , the ratio $\tau_o(\tau)/a_o(\tau)$ increases from ~ 12 to 24 $\mu\text{sec}/\text{dB}$ (LW) and 17 to 18 $\mu\text{sec}/\text{dB}$ (ND). The corresponding results from the trading experiment are 2.5 to 2.5 $\mu\text{sec}/\text{dB}$, 18 to 30 $\mu\text{sec}/\text{dB}$, and 27 to 40 $\mu\text{sec}/\text{dB}$. In general, the difference between the results obtained in the two experiments is no greater than the difference between the results obtained by different subjects in either of the two experiments alone.

In considering our data on $\tau_o(\tau)$, it is important to note that the graph terminates at $\tau = 400$ μsec only because we were unable to obtain reliable JND's for values

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of $\tau \geq 600 \mu\text{sec}$. At these larger values, the subjects found substantial changes in the discrimination cues and tended to become very confused. In particular, the subjects claimed that cues tended to reverse (in the sense that the stimulus that was leading in the left ear produced an image to the right of the stimulus that was leading in the right ear). In order to document this confusion, an additional short experiment was carried out. In this case, the incremental time disparity $\Delta\tau$ was held constant at $\Delta\tau = 100 \mu\text{sec}$, and the value of the reference delay τ was varied from $500 \mu\text{sec}$ to $1000 \mu\text{sec}$. In addition to ND, who participated in the main experiment, two additional subjects, RH and SC, were used. In order to keep track of cue reversals, the subjects were instructed to maintain a constant response code and reminded that getting all the answers wrong was just as good as getting them all right. The results of this experiment (in which each run consisted of only 50 trials) are shown in Table XIII-1. Note how the cues

Table XIII-1. Per cent correct as a function of τ for $\Delta\tau = 100 \mu\text{sec}$.

τ (μsec)	ND (run 1)	ND (run 2)	SC	RH
500	100%	98%	94%	82%
600	96	96	98	92
700	44	54	68	68
800	0	9	50	75
900	8	0	3	57
1000	0	6	14	25

tended to reverse and that near the point of reversal, the discrimination performance becomes very poor. Unfortunately, we did not have sufficient time to fully explore this phenomenon. It is our conjecture, however, that it is related to the "headwidth limitation" discussed in connection with binaural unmasking⁸ and to the lateralization ambiguity associated with the antiphase point at $\tau = 1000 \mu\text{sec}$. It is interesting to note in this connection that Mills reported a similar phenomenon at large angles in his minimum audible angle experiment.⁵

The primary purpose of our experiment was to provide data to help guide the development of Colburn's auditory-nerve model of binaural processing (see Sec. XIII-A). Based on the perfect-memory assumption (known to be inadequate), the model predicts that $\tau_0(\tau)$ and $a_0(\tau)$ should be independent of τ . This prediction is satisfied for $a_0(\tau)$, but not for $\tau_0(\tau)$. With regard to $a_0(a)$, the model predicts a small increase as we leave the midline (e. g., a maximum increase of 0.3 dB when the midline JND is in the range 0.8-0.9 dB). This prediction is consistent with the LW data, but not with the ND data. The data on $\tau_0(P)$ and $a_0(P)$ show that at very low levels, τ_0 and a_0 increase with a decrease in P , as Colburn's model (or any other model) predicts. At moderate-

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to-high levels, the data on $\tau_o(P)$ are relatively constant. According to the model, as P increases from 40 to 80 dB, $\tau_o(P)$ must decrease by a multiplicative factor q in the range $0.7 \leq q \leq 1.0$. For $a_o(P)$, the data show that a_o decreases slightly at moderate-to-high levels as P is increased, whereas the model predicts that a_o should be constant in this region. Aside from the case of $\tau_o(\tau)$, the most serious discrepancy occurs with $\tau_o(a)$. Making use of the data on $\tau_o(P)$ and Colburn's equation relating $\tau_o(P)$ to $\tau_o(a)$, we find that the prediction for $\tau_o(a)$ seriously underestimates the growth of τ_o with a . For example, at $a = 30$ dB, the predictions for ND and LW are 20 and 12 μ sec, respectively (as compared with the empirical values of 160 and 41 μ sec).

A second purpose of the experiment was to test the prediction of Durlach's equalization and cancellation (EC) model relating time JND's on the midline to amplitude JND's on the midline.⁹ According to this model, one should have $a_o(\text{dB})/\tau_o = 8.7 (2\pi f_o/\sqrt{k})$, where f_o is the frequency of the tone, and k is a frequency-dependent parameter of the model that includes the error variances associated with the "jitter" in the EC process. Assuming that $k = 1.185$ at 500 Hz (a value often used by Durlach in binaural unmasking applications and in the initial JND applications), we obtain $a_o(\text{dB})/\tau_o(\mu\text{sec}) = 0.025$. According to our results, however, the ratio is 0.08 (LW) and 0.07 (ND). Moreover, since the model requires that $k \geq 1$, there is no value of k for which the model predicts a ratio greater than 0.028.

Further details on this research are available in the author's Master's thesis.¹⁰

R. M. Hershkowitz

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C. EFFECT OF DURATION ON THE DISCRIMINATION OF PERIODICITY PITCH

It is well known that the frequency of a low-frequency tone is encoded in the neural firings of the auditory nerve in at least two ways: (i) by the set of fibers that respond to the stimulus (place encoding), and (ii) by the temporal patterns of the firings in a given fiber (time encoding). According to the model developed by W. M. Siebert,¹ if the higher centers make optimum use of the place encoding but ignore the time encoding, the just-noticeable difference (JND) Δf in frequency f should depend on the duration, T , of the stimulus according to the relation $\Delta f \propto T^{-1/2}$. If, on the other hand, the higher centers make optimum use of the time encoding but ignore the place encoding, the relation should be of the form $\Delta f \propto T^{-3/2}$.

There are reasons for suspecting that the higher centers do not make optimum use of the time encoding. In order to provide a direct test, however, an experiment was performed to determine the JND in frequency as a function of duration for a stimulus in which the frequency discrimination would have to be based purely on time information. The stimulus selected was chopped, white, Gaussian noise, and the aspect of the stimulus to be discriminated was the chopping rate. As pointed out previously by Miller and Taylor,² such a stimulus leads to a sensation of pitch (corresponding to the chopping rate), despite the fact that its spectrum is essentially the same as the unchopped noise, and it is believed to be very unlikely that the pitch arises from place encoding. According to the model, if the system makes optimum use of the timing information, the JND in chopping rate should vary with duration in the same manner as the JND for tones under the assumption that the place information is ignored, that is, $\Delta f \propto T^{-3/2}$.

Our method was that of two-alternative-forced-choice-plus-feedback (a priori probabilities of 0.5), and the subject's task on a given trial was to indicate which of the two signals in that trial had the higher chopping rate. The two base rates studied were 75 Hz and 300 Hz, the power level was 55 dB SPL, the duty cycle was 0.5, and the range of durations considered was 40-600 msec. Each run at a given set of conditions consisted of 100 trials, and the JND was defined to be the increment in chopping rate for which the subject's responses were 75% correct. The JND was estimated by varying the increment from run-to-run, obtaining psychometric functions, and interpolating. Four subjects were employed throughout the experiment.

In attempting to obtain psychometric functions, several difficulties were encountered. First, the training problem seemed to be unusually difficult for this task, in the sense that it took an exceptionally long time for performance to stabilize. Second, the psychometric functions were very often flat, that is, the %-correct responses varied very slowly with the increment in chopping rate. Thus, the results obtained for the JND's were less precise and less reliable than in many other experiments. In general, the experiment is regarded merely as a preliminary exploration, and it is clear that much

more work is required to obtain definitive results.

The JND's obtained for the two base frequencies and the various durations (averaged over subjects) are shown in Fig. XIII-2. The dip at 60 msec in the curve for

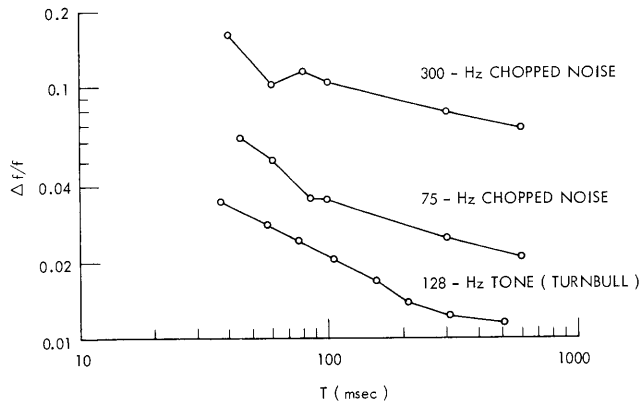


Fig. XIII-2. Weber fraction $\Delta f/f$ as a function of duration T .

300 Hz appeared in the data for each of the 4 subjects. The change in slope at the shorter durations for the 75-Hz case occurred for 3 of the 4 subjects. The slopes of these curves are approximately -0.3 (300 Hz), -0.3 (75 Hz, long durations), and -0.9 (75 Hz, short durations). Unless one assumes that the slope continues to decrease as the duration is decreased below 60 msec, there is no indication in these results of relation of the form $JND \propto T^{-3/2}$. Roughly speaking, the dependence of the JND on duration for the chopped noise appears to be approximately the same as the results obtained for tones (where there is place information, as well as time information). A comparison of the present results with those of Turnbull³ for a 128-Hz tone is included in Fig. XIII-2.

Further details about this study can be found in the author's Master's thesis.⁴

S. E. Portny

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D. SEQUENTIAL EFFECTS IN AUDITORY INTENSITY DISCRIMINATION

In psychophysical experiments, the response to a given stimulus depends not only on the stimulus, but also upon the "state" of the subject at the moment the stimulus is presented. Previous research¹ has shown that one important determinant of the subject's state is the sequence of previous events (previous stimuli, responses, and experimenter-subject feedbacks) in the experimental run. Since this sequence changes from trial to trial, the subject's state must also be regarded as changing from trial to trial. Although the dependence of state on the sequence of previous events is a generally recognized fact, the structure of this dependence is not well understood. Furthermore, in many of the models that are currently being used to interpret psychophysical data, this dependence is ignored. A common procedure is to assume that the various trials in a sequence are statistically identical and independent, and to interpret the fluctuations in the subject's responses to a given stimulus in terms of some unspecified "internal noise." In order to improve the quality of psychophysical research, it would be useful to devise experimental paradigms in which the fluctuations in state caused by variations in the sequence of previous events are minimized, or to understand this phenomenon well enough to take proper account of it when analyzing the data.

In a preliminary experiment that has just been completed, we have studied the effect of the just-previous trial on a subject's response in the context of a monaural, two-alternative-forced-choice, intensity-discrimination task. In each trial, the stimulus consisted of a pair of 350-msec, 400-Hz, tone bursts differing in intensity, and the subject was required to judge which of the two bursts was the more intense. With I denoting the mean intensity and ΔI the differential intensity (both in dB), the subject's task was to distinguish between the high-low pair ($I + \Delta I/2$, $I - \Delta I/2$) and the low-high pair ($I - \Delta I/2$, $I + \Delta I/2$). Each of these pairs was presented with an a priori probability of one-half.

This experiment consisted of four parts, denoted cases 1, 2, 3, and 4. In cases 1 and 2 (the "fixed-level" cases), the value of I was held constant at 75 dB SPL. In cases 3 and 4 (the "roving-level" cases), I varied randomly from trial to trial between 64 dB and 80 dB SPL (with a priori probabilities of one-half). In cases 1 and 3, the subject received no feedback from the experimenter. In cases 2 and 4, the subject was informed automatically after each trial whether or not his response was correct.

Four values of ΔI were tested in each case, and at least 2000 trials were presented to each of 4 subjects at every value of ΔI . Thus, at least 32,000 trials were presented to each subject. Aside from the author (who served as Subject 3), none of the subjects knew that the primary purpose of the experiment was to study sequential effects. It had originally been intended to run some further tests in which the subjects were informed of their sequential dependencies and instructed to eliminate them, but the available

time was not sufficient for this purpose.

The data from this experiment were recorded on punched paper tape and processed on the PDP-4 computer. The data processing consisted of computing two matrices: a zero-order matrix, and a first-order matrix. The zero-order matrix summarizes the results without regard for the previous trial and presents information (for each value of ΔI) on the number of high-low trials, the number of low-high trials, the number of correct responses for each type of trial, the various relevant percentages, and the values of the parameters d' and β' of signal detection theory.² The first-order matrix presents the same information as the zero-order matrix, but subdivided according to the events of the previous trial. Additional processing consisted of estimating the amount by which a given psychometric function would have been increased if the previous trial had always been optimum (in the sense of leading to a maximum d' on the given trial) and if the subject's criterion had always been optimum (that is, $\beta' = 0$). A brief description of some of the results of this analysis is given below.

1. The effect of the just-previous trial depended strongly on the subject.

2. For those cases in which I was held constant, the difference between the optimum psychometric function (as described above) and the actual psychometric function was very small. For those cases in which I was varied, and for certain subjects, this difference was appreciable.

3. Subject 1 (whose over-all performance was exceptionally poor) showed a strong tendency to repeat consecutive responses in the no-feedback cases and a strong tendency to respond with the answer that was correct on the previous trial in the feedback cases.

4. Subjects 2, 3, and 4 showed a wide variety of small response dependencies. In addition to the dependency of β' on the previous trial, that is expected when feedback is introduced (that is, the adjustment of the criterion to improve performance), there were tendencies to (a) alternate responses when ΔI was small; (b) repeat responses when ΔI was large; and (c) respond oppositely to the previous presentation (interpreted as a microscopic adaptation effect, since it occurred when there was no feedback and increased with ΔI).

5. When the mean level I was varied from trial to trial, Subjects 1 and 2 showed a strong tendency to respond "low-high" when the given I was loud and "high-low" when the given I was soft. This effect (the usual macroscopic adaptation effect) was decreased, but not eliminated, by feedback. For Subject 2, most of this effect was attributable to the value of I on the previous trial. For Subject 1, the previous trial was not the dominant factor.

6. For most subjects and in most cases, the sensitivity d' was approximately a linear function of ΔI . When the mean level I was fixed, this function was found to be

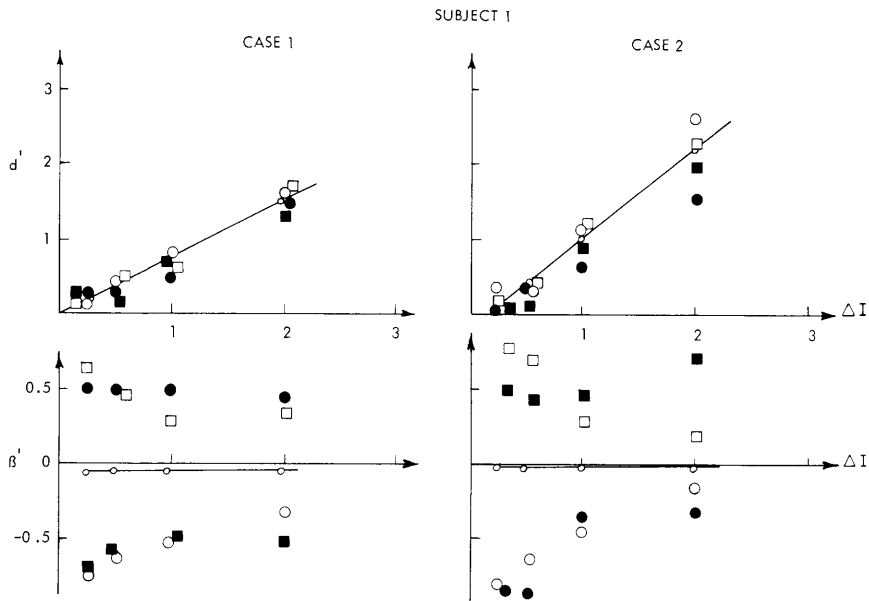


Fig. XIII-3. Effect of previous trial on d' and β' for Subject 1 in cases 1 and 2. Previous trial code:
 ○ = previous trial high-low, answered correctly
 ● = previous trial high-low, answered incorrectly
 □ = previous trial low-high, answered correctly
 ■ = previous trial low-high, answered incorrectly.
 Solid lines give d' and β' without regard to previous trial.

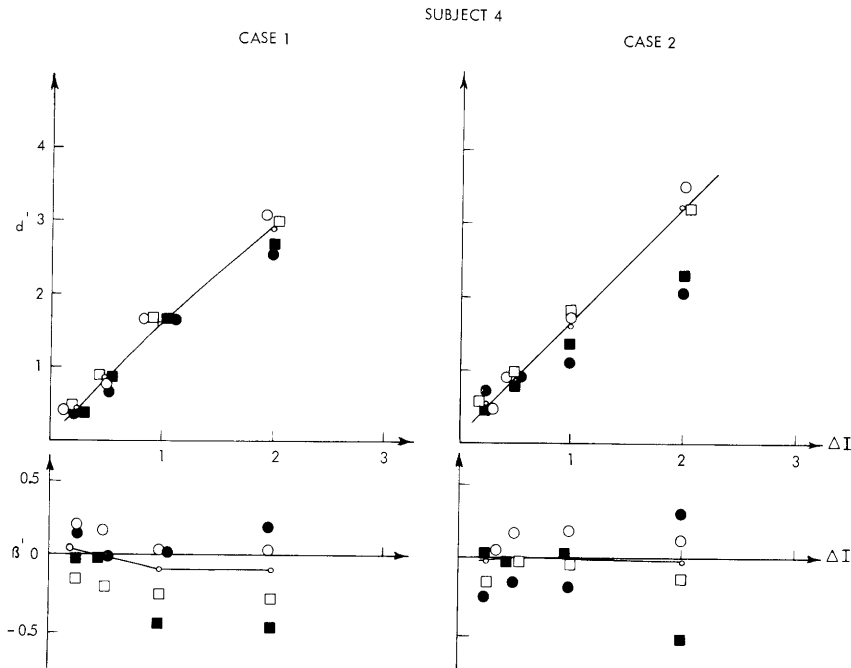


Fig. XIII-4. Effect of previous trial on d' and β' for Subject 4 in cases 1 and 2. Previous-trial code as in Fig. XIII-3.

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independent of the stimulus on the previous trial, but to be steeper when the previous response was correct than when it was incorrect. This dependency occurred both with and without feedback, although it was stronger with feedback.

7. The slope of $d'(\Delta I)$ was increased by the introduction of feedback for Subjects 1 and 3, but not for Subjects 2 and 4 (whose performance without feedback was relatively good).

8. When the mean level I was varied from trial to trial, the slope of $d'(\Delta I)$ was appreciably smaller for the soft I than for the loud I. Also, the slope for the loud I in the variable-I cases was appreciably smaller than the slope for the fixed I cases.

Some sample data from these experiments are shown in Figs. XIII-3, XIII-4, and XIII-5.

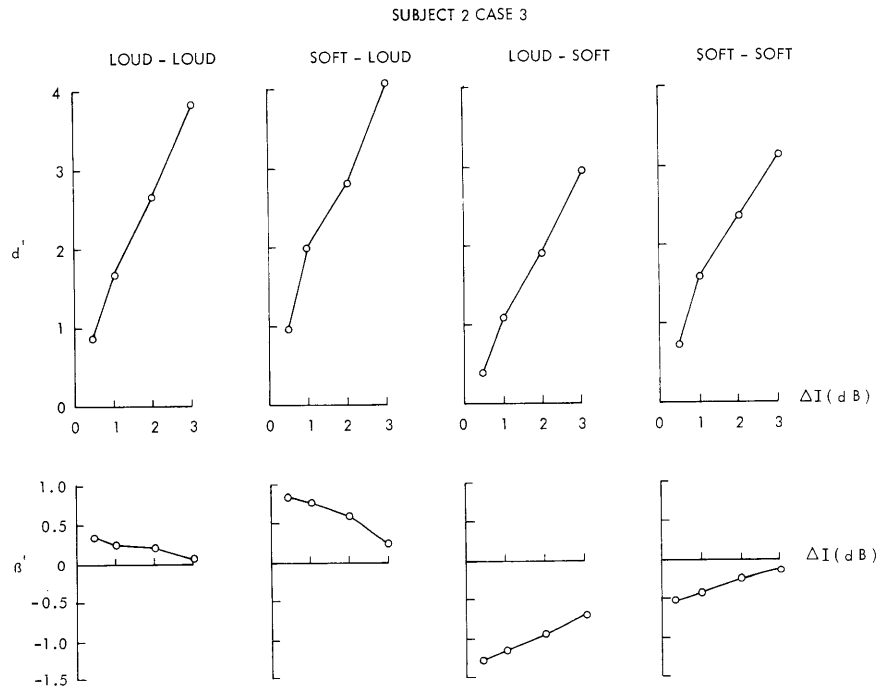


Fig. XIII-5. Effect of mean signal level on d' and β' for Subject 2 in case 3. SOFT-LOUD means that the previous trial was soft and the given trial was loud.

Further details on this work (as well as a comparison with earlier work) can be found in the author's Master's thesis.

H. S. Hsiao

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References and Footnotes

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2. In the version of signal detection theory that was used to analyze our data, it was assumed that the two density functions corresponding to the two stimuli $I + \Delta I/2$ and $I - \Delta I/2$ were Gaussian with means M_H and M_L and equal variance σ^2 , and that the listener responded "high-low" rather than "low-high" if and only if $X_1 - X_2 \geq \beta$, where (X_1, X_2) is the pair of samples arising from the two-alternative-forced-choice presentation on the given trial, and β is the criterion. The parameters d' and β' were then defined by the equations $d' = 2(M_H - M_L)/\sigma$ and $\beta' = \beta/\sigma$. To the extent that d' and β' depend on the events in the previous trial, the usual assumption that the probability densities and the criterion are constant from trial to trial is obviously a false one.
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E. SUBJECTIVE ESTIMATES OF UNCERTAINTY IN ABSOLUTE IDENTIFICATION

It has been shown in a variety of psychophysical tasks that the use of confidence ratings or second-choice paradigms can facilitate measurement without substantially decreasing performance level. The purpose of this research was to explore the extent to which similar benefits might be gained in a task of absolute identification.

The set of stimuli to be identified consisted of ten 1000-Hz, 500-msec tone bursts varying in level between 66 dB and 30 dB (SPL) in 4-dB steps. The stimuli were assigned numbers 1 through 10 in order-preserving fashion. The subject was not given

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feedback, but before each run he was exposed to the whole set 3 times in decreasing order.

Two types of responses were employed. In the straight identification task (S), the subject responded with a single integer between 1 and 10 that represented his best guess as to the stimulus presented. In the weighting task (W), the subject was instructed to distribute numerical weightings over the set of stimuli according to the certainty with which he thought each stimulus was presented on the given trial. In each format, each stimulus was presented 10 times in a given run, and the subject was given 10 runs with each format (S and W runs being alternated).

The data were processed to obtain three different confusion matrices. The S matrix was obtained from the S data in the usual fashion. The WP matrix was obtained from the W data by choosing the stimulus with the largest weighting as the response. The WA matrix was obtained from the W data by normalizing the weightings on a given trial and averaging the weightings obtained for a given stimulus over different trials. It was found that the information transfer, as well as the subject's biases, were roughly the same for all three matrices. It was also found that the widths of the distributions of weightings on a single trial in the W task was less than half of the widths determined from the WA and S matrices, and that the difference between the peak response and the average response in the W task was uncorrelated with the difference between the peak response and the correct response.

In general, we concluded that the additional burden in the W task of having to assign weightings did not degrade performance, but that the subject was unable to make use of the weightings to improve performance.

In future work, we intend to examine the extent to which the difference between the widths found on single trials in the W case and the widths found in the S and WA matrices (caused primarily by trial-to-trial variations in the peaks for the W case) can be ascribed to sequential effects.

Further details on this work can be found in the author's Bachelor's thesis.¹

S. J. Bayer

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