COMMUNICATION SCIENCES

AND

ENGINEERING

Academic and Research Staff

Prof. R. G. Gallager
Prof. F. C. Hennie III
Prof. E. V. Hoversten

Prof. R. S. Kennedy
Prof. C. E. Shannon

Prof. R. N. Spann
Prof. R. D. Yates
J. Max

Graduate Students

E. A. Bucher
J. N. Buckman
D. L. Cohn
R. L. Greenspan

S. J. Halme
L. Hatfield
H. M. Heggestad
Jane W-S. Liu

J. C. Moldon
P. S. Richter
J. H. Shapiro
M. A. Sirbu

A. EFFICIENT OPTICAL COMMUNICATION THROUGH A TURBULENT ATMOSPHERE

1. Introduction

Typical characteristics of optical communication through the atmosphere are (i) enormous available bandwidths for even small fractional bandwidths at carrier frequencies 10^{14}-10^{15} Hz; (ii) enormous antenna gains with physically small apertures; (iii) importance of quantum noise as a limiting factor; (iv) loss of spatial coherence, because of atmospheric turbulence; and (v) sensitivity to great path loss or fadeout at times, because of scattering from haze, rain, snow or fog on the atmospheric path. By using realistic atmospheric models, attempts have been made to evaluate the potential of the turbulent atmosphere as an optical communication channel.[1-3] In this report these analyses have been extended, with special emphasis put on the problems created by spatial incoherence in the received field. The optimum classical receiver, in the case of short signals as compared with the correlation time of the atmospheric turbulence, is found to consist of spatial and temporal filters followed by detection and diversity combining. The diversity combining is linear (in power) for Gaussian fields and nonlinear for log-normal fields. The optimum quantum receiver for short signals with unknown phase is seen to consist of a bank of photon counters (one for each signal mode) and a diversity combiner. The implementation is seen to lead to a lens-detector-diversity combiner array.

2. Field Models for Signals Distorted by a Turbulent Atmosphere

Using Kolmogorov's similarity theory of turbulence and Rytov's approximation, Tatarski[4] has shown that the received plane-wave field is log-normal, with following normalized amplitude distribution density of the field excitation u(r):

$$p(|u|) = \frac{1}{\sqrt{2\pi}\ \sigma_u} \exp - \frac{(\sigma^2 + \ln|u|)^2}{2\sigma^2}, \tag{1}$$

and with an almost uniform phase distribution.[1] The field covariance function can be found to be equal to

$$K(\vec{r}_1, \vec{r}_2) = \overline{u(\vec{r}_1)\ u^*(\vec{r}_2)} = \exp - \frac{1}{2}\ (D_A(|\vec{r}_1 - \vec{r}_2|) + D_\phi(|\vec{r}_1 - \vec{r}_2|)), \tag{2}$$

where D_A and D_ϕ are the structure functions for the normalized log-amplitude and phase, respectively. According to Tatarski,[5]

$$D_A(\rho) + D_\phi(\rho) = 3.44\,k^2 L C_n^2 l_o^{-1/3} \rho^2 \qquad (\rho \ll l_o)$$

$$= 2.91\,k^2 L C_n^2 \rho^{5/3} \qquad (\kappa_m \rho \gg 1) \tag{3}$$

where $k = 2\pi/\lambda$, L is the path length, C_n^2 is the structure constant of the refraction index, l_o the inner scale of turbulence, and $\kappa_m = 5.48/l_o$. To simplify analysis, the following covariance function will be adopted:

$$K(\vec{r}_1, \vec{r}_2) = \exp - \frac{|\vec{r}_1 - \vec{r}_2|^2}{2 r_c^2}, \tag{4}$$

where r_c is the coherence radius of the field.

By using the covariance function, the field can be expressed in terms of a generalized Karhunen-Loeve expansion with uncorrelated coefficients. For the purposes of analysis, these coefficients will be assumed to be independent and either normal or log-normal. The verification of these assumptions needs further research.

The field fluctuates so that its power spectrum range is 1-1000 Hz.[4] This fluctuation can also be interpreted as Doppler spread resulting from the motion of scattering inhomogeneities by wind. For clear weather conditions the time spread is negligible at present available signaling speeds.

The quantum models for spatially coherent fields[2,3] can be easily generalized to partially coherent fields, if the receiver is assumed to be large enough so that the eigenfunctions are approximately plane waves in both modal and Karhunen-Loeve representations. Then the modal functions are uncorrelated, indeed independent in the normal case. The strength (average number of photons) of each lateral electric field mode is equal to[6]

$$N_k = \frac{2\epsilon_o c \, T \, S(\vec{\kappa}_k)}{\hbar\omega},$$

(5)

where ϵ_o is the dielectric constant of the vacuum, $\hbar$ is Planck's constant, c is the velocity of light, T is the duration of the signal, and $S(\vec{\kappa}_k)$ is the two-dimensional angular spectrum of the electric field at the lateral wave number $\vec{\kappa}_k$. The field is pictured as consisting of a bunch of plane waves arriving from various directions.

The background noise will be assumed to be both spatially and temporally the white Gaussian field within the bandwidth and field of view of interest. Scattering and absorption losses will be neglected or assumed to be constant.

3. Optimum Optical Receivers and Their Performance

Based on the discussion above the complex envelope of the received signal, $y(\vec{r}, t)$, is taken to be equal to

$$y(\vec{r}, t) = S_k(t) \, z(\vec{r}) + n(\vec{r}, t).$$

(6)

Here $S_k(t)$ denotes a waveform corresponding to the message "k" from an orthogonal signal set with duration T and unit energy, and $z(\vec{r})$ denotes the complex multiplicative fading of the field. The signal is assumed to be short compared with the correlation time of the fading, so that $z(\vec{r})$ may be called a constant within a signal interval. If this assumption is made, we talk about <u>short</u> <u>signals</u>. The term $n(\vec{r}, t)$ denotes both spatially and temporally white Gaussian envelope noise with spectral density N_o.

First the fading z is assumed to be Gaussian. The treatment goes analogously to Van Trees'.[7] In the case of short signals the space-time covariance function factors as follows:

$$K_k(\vec{r}_1, t_1; \vec{r}_2, t_2) = Z^2 \, S_k(t_1) \, S_k(t_2) \, K_z(\vec{r}_1 - \vec{r}_2),$$

(7)

where $K_z(0)$ is normalized to unity, and Z is a constant. By using this as the kernel, the likelihood functional may readily be obtained.

$$l_k = \frac{1}{N_o} \sum_i \frac{|Y_{ik}|^2 \, Z^2 \lambda_i}{N_o + Z^2 \lambda_i},$$

(8)

$$Y_{ik} = \int \int_{A_r} \int_T y(\vec{r}, t) \, S_k^*(t) \, \psi_i^*(\vec{r}) \, d^2\vec{r} \, dt,$$

(9)

where λ_i and $\psi_i(r)$ are the ith eigenvalue and eigenfunction of the kernel $K_z(\vec{r})$, respectively. If only a finite number of the eigenvalues are nonzero, Eq. 8 can be realized
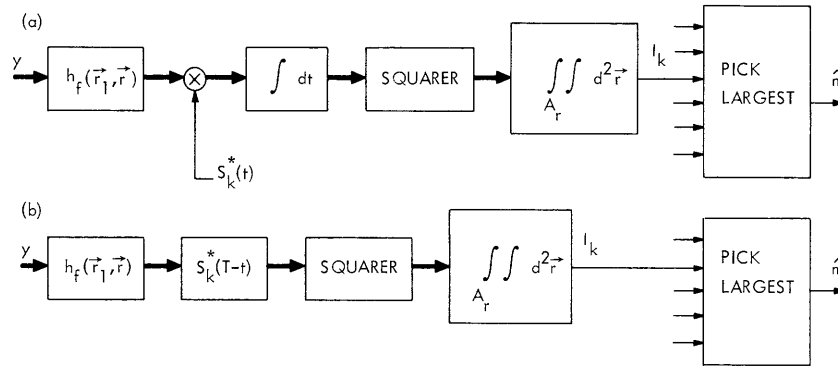
Fig. XIII-1. One-shot filter-squarer receiver for short signals. Fading is assumed to be Gaussian. Alternatives are (a) time correlator, and (b) matched filter. Field quantities are represented by heavy lines.

as a <u>diversity</u> <u>receiver</u>. The estimator-correlator and <u>filter-squarer</u> (Fig. XIII-1) realizations are also possible. The filter function h_f equals

$$h_f(\vec{r}_1, \vec{r}_2) = \sum_i Z \sqrt{\frac{\lambda_i}{N_o + Z^2 \lambda_i}} \, \psi_i(\vec{r}_1) \, \psi_i^*(\vec{r}_2).$$

(10)

For apertures large compared with the coherent areas the eigenfunctions approach plane waves, and the eigenvalues λ_i approach the spatial spectrum $S_z(\vec{k})$ of the field. Hence the two-dimensional Fourier transform of the <u>filter</u> <u>function</u> h_f is asymptotically

$$H_f(\vec{\kappa}) = Z \sqrt{\frac{S_z(\vec{\kappa})}{N_o + Z^2 S_z(\vec{\kappa})}} \, .$$

(11)

Roughly speaking, the spatial filter (Fig. XIII-2) restricts the field of view so that most of the signal plane-wave components are passed, while as much as possible of the background is excluded.

NOTE: After this report was submitted for publication, I became aware of R. O. Harger's results[8] that parallel Eqs. 8-11 and 23.

The performance of the optimum receiver for the Gaussian signal in Gaussian noise is known to be bounded by[9,7]

$$P(\epsilon) \lesssim \sqrt{\frac{1.83 N_o}{\bar{E}_r}} \, \exp - 0.149 \bar{E}_r / N_o$$

(12)

which is obtained by using an equal strength diversity system with the optimal diversity $D_o = \bar{E}_r/3.07N_o$. Here $\bar{E}_r/N_o$ is the envelope signal-to-noise ratio on a diversity path.
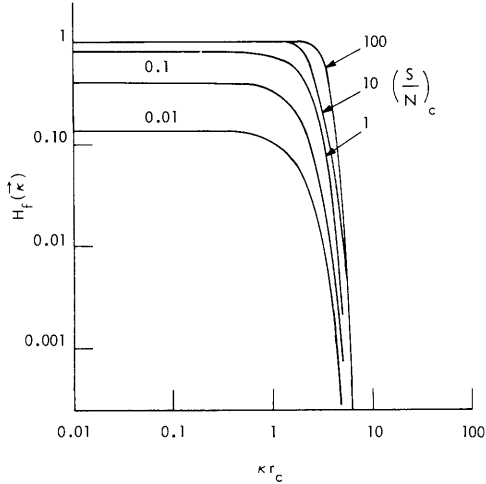

Fig. XIII-2.

Optimum spatial filter characteristics for different $(S/N)_c = Z^2 \pi r_c^2/N_o$, and Gaussian correlation function in Eq. 4.

The error probability of an optimum receiver for the correlation function of Eq. 4 can be bounded as follows:

$$\ln P(\epsilon) < \rho \ln M - \sum_i \left[\rho \ln\left(1 + \frac{2\lambda_i}{N_o}\right) - (1+\rho) \ln\left(1 + \frac{2\lambda_i \rho}{N_o(1+\rho)}\right) \right]. \tag{13}$$

This expression is evaluated for the case of asymptotically <u>large</u> receiver aperture, with the use of following notation: $(S/N)_c = A_c Z^2/N_o$, $D = A_r/A_c$, $A_c = \pi r_c^2 =$ coherence area, and $Li_2(\cdot)$ is the dilogarithm[10] (integral of $\ln (1-z)/z$).

$$\ln P(\epsilon) < \rho \ln M - \frac{\rho D}{2} Li_2(-2(S/N)_c) + \frac{(1+\rho)D}{2} Li_2\left(-\frac{2\rho}{1+\rho}(S/N)_c\right). \tag{14}$$

From this error bound the small rate error exponent and the channel capacity are evaluated:

$$E(1) = \frac{D}{2} \left(Li_2(-2(S/N)_c) - 2Li_2(-(S/N)_c)\right) \tag{15}$$

$$C = \frac{D}{2} \left(Li_2(-2(S/N)_c) + 2(S/N)_c\right). \tag{16}$$

Looking at Fig. XIII-3 one sees that the optimum diversity with energy constraint is attained at approximately $D = \bar{E}_r/4N_o$, while $\ln P(\epsilon) < -0.15\bar{E}_r/N_o$. These almost agree with the results in the equal strength diversity optimum case. Figure XIII-4 displays

the channel capacity.

If the fading z is assumed to be <u>log-normal</u>, the problem becomes very hard to deal with. Unlike as before,[1] we shall assume that the Karhunen-Loève coefficients of the
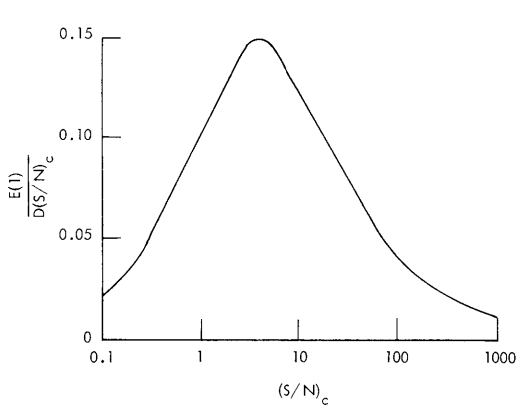

Fig. XIII-3.

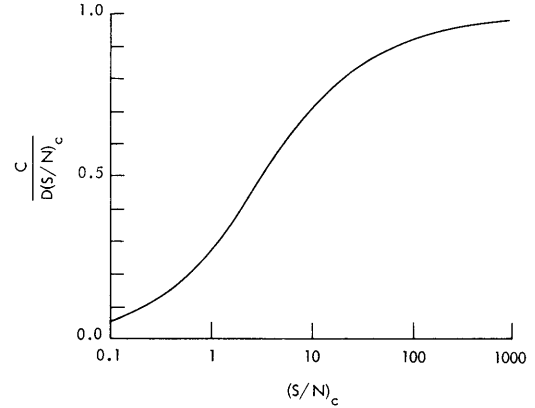Zero-rate error exponent for Gaussian channel with correlation function (4) and large receiver aperture.

Fig. XIII-4.

Channel capacity for Gaussian channel with correlation function (4) and large receiver aperture.

signal field are log-normal and independent. Each mode is assumed to have intensity λ_i and log-normal variance σ_i^2. Then the likelihood functional turns out to be equal (with (1), (6), and (9) used) to

$$l_k = \sum_i \ln \left\{ \int_0^\infty \frac{du}{\sqrt{2\pi}\,\sigma_i u} \, I_0\left(2u \, \frac{|Y_{ik}|}{N_o}\right) \exp -\left(\frac{u^2 \lambda_i}{N_o} + \frac{\left(\sigma_i^2 + \ln u\right)^2}{2\sigma_i^2}\right) \right\}. \tag{17}$$

Figure XIII-5 displays the optimum diversity receiver in this case. The nonlinear function $g_i(|Y_{ik}|)$ referred to in Fig. XIII-5 has been computed as a function of λ_i/N_o and σ_i and its curves are available.[11] The difference from the Gaussian case is in the nonlinear diversity combiner.

In the <u>quantum</u> <u>Gaussian</u> case the likelihood functional can be shown to be equal to

$$l_k = \sum_i n_{ik} \ln\left(\frac{N + N_i}{N + N_i + 1} \, \frac{N + 1}{N}\right) = \sum_i c_i n_{ik}, \tag{18}$$

where N and N_i are the numbers of noise and signal photons in mode i. The weighting function tends toward the value in (11) squared in the classical limit. The
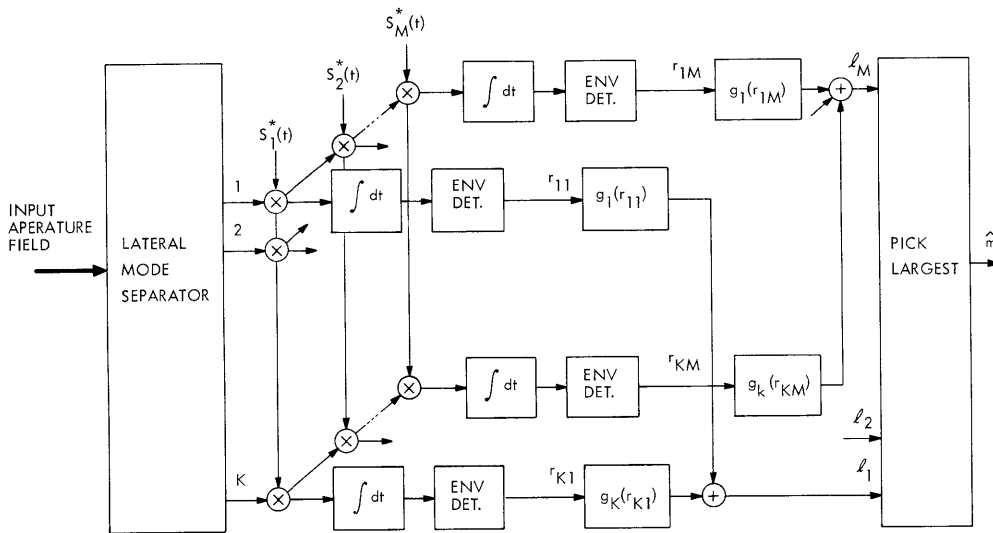
Fig. XIII-5. Optimum diversity receiver for a field with independent log-normal Karhunen-Loève coefficients in white Gaussian noise. The nonlinear eigenvalue processors are given within the wave brackets of Eq. 17.
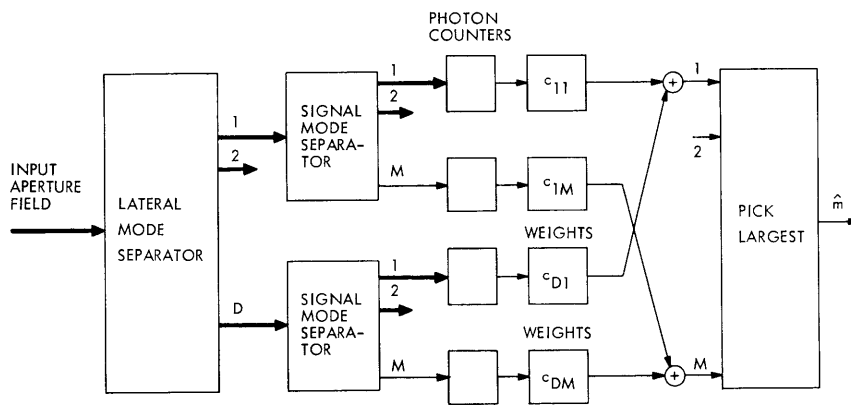

Fig. XIII-6. Optimum quantum receiver for Gaussian partially coherent thermal noise.
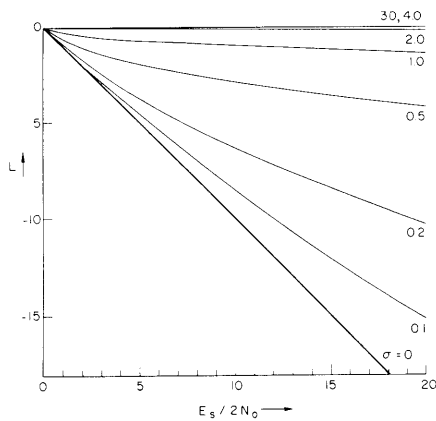

Fig. XIII-7.

Error exponent for a log-normal channel with no diversity and two orthogonal signals as a function of average signal-to-envelope-noise ratio $E_s/2N_o$, and the fading variance, σ.

binary error probability can be bounded as follows:

$$P(\epsilon) < \frac{1}{\sqrt{\frac{\pi}{2} \mu''\left(\frac{1}{2}\right)}} e^{\mu(1/2)},$$ (19)

where

$$\mu(1/2) = -\sum_i 2 \ln \left(\sqrt{(N+N_i+1)(N+1)} - \sqrt{(N+N_i)N}\right)$$ (20)

$$\approx \sum_i \left(\ln(1+N_i/N) - 2\ln(1+N_i/2N)\right) \quad (N_i, N \to \infty).$$

The lower line in (20) is the classical limit, and is seen to agree with the classical expression (cf. (13) for $\rho = 1$).

In the <u>quantum</u> <u>log-normal</u> case the optimum receiver is similar to that in Fig. XIII-6, except that the weighting is nonlinear. The error probability for zero background equals

$$P(\epsilon) = \prod_i \int_0^\infty \frac{1}{\sqrt{2\pi}\,\sigma_i} \exp -\left(x^2 N_i + \frac{\left(\sigma_i^2 + \ln x\right)^2}{2\sigma_i^2}\right) dx$$ (21)

$$= \exp\left(\sum_i L_i\right),$$

where L_i can be obtained from Fig. XIII-7 by setting "$E_s/2N_o$" equal to N_i.

4. Implementation of the Optimum Optical Receiver

For one-shot optimal receivers the basic operations according to Figs. XIII-1, XIII-5, and XIII-6 are the following.

1. Mode separation: Division of the field in the aperture into coherent components (spatial frequency sampling for large apertures).

2. Spatial filtering of background noise.

3. Correlation or matched filtering and detection of the individual modes.

4. Diversity combining of the individual mode outputs.

5. Decision making.

For large apertures the eigenvalues or mode amplitudes can be measured right at the focal plane of a lens on the aperture. The covariance function Γ of the field in the focal plane equals
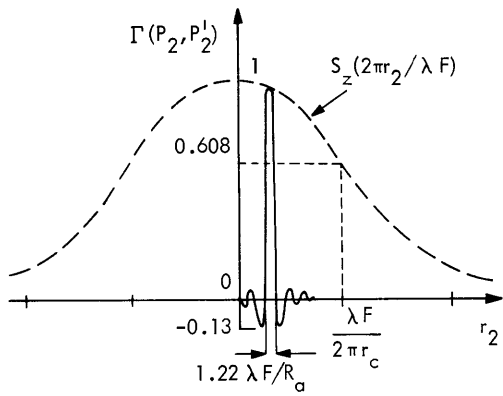
Fig. XIII-8. Field covariance function in the focal plane after Fraunhofer diffraction. λ = wavelength, and F = focal length.
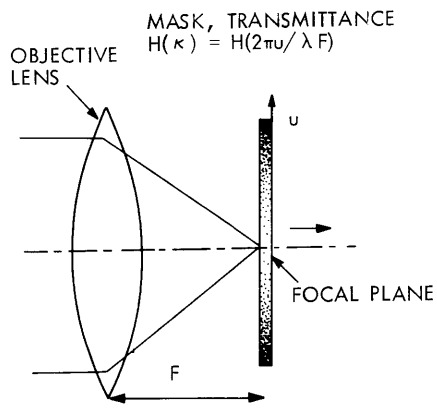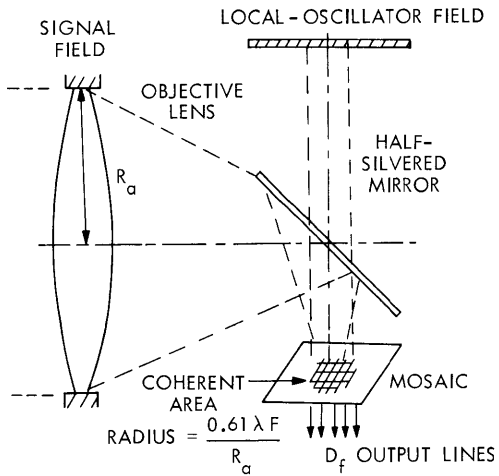

Fig. XIII-9. Spatial filtering with a mask.

Fig. XIII-10. Mixer receiver for partially coherent light.

$$\Gamma(P_2, P_2') \cong C\ D(\rho)\ S_z(2\pi R/\lambda F),\tag{22}$$

where C is a constant, $D(\rho)$ is the diffraction pattern of the aperture (Airy disc), and S_z is the spatial power spectrum of the field. This result is accurate if the spectrum varies negligibly within an Airy disc (Fig. XIII-8). The numbers of practically independent coherent areas or samples in the focal plane and in the aperture plane can be defined as follows for circular apertures:

$$D_f = 3.68\ \frac{\int\int S_z(2\pi\vec{R}/\lambda F)\ d^2\vec{R}}{S_z(0)(0.61\ \lambda F/R_a)^2} = \frac{A_r}{2A_c}\tag{23}$$

$$D_a = \frac{K_z(0)\ A_r}{\int\int\limits_{A_r} K_z(\vec{r})\ d^2\vec{r}} = \frac{A_r}{2A_c},\tag{24}$$

where A_r is the aperture area, R_a is its radius, and $A_c = \pi r_c^2$ is the coherence area (Eq. 4). These numbers (equal by definition) may be called the degrees of freedom of the field in the aperture.

Spatial filtering may also be done in the focal plane (Fig. XIII-9). From the quantum point of view (Fig. XIII-6) weighting should be done after counting, but classically weighting can be done before detection.

The correlation and matched filtering are most efficiently done on radio frequencies, after heterodyning (Fig. XIII-10). The resulting receiver becomes rather complicated, especially if a large signal set is used. The complexity is described by the number $D \times M$ (D = diversity (23), (24), while M is the number of waveforms used).

It can be shown that by using tracking (perfect measurement) the performance of the unfaded channel is approached in the limit as $D \to \infty$ and the signal-to-noise ratio on each coherent area vanishes. By using the results above and the paper of Kennedy and Hoversten,[1] it can be seen that in a typical case the improvement available from tracking is 5 dB or more. In the optimal Gaussian case this improvement is 5.2 dB.

S. J. Halme

References

1. R. S. Kennedy and E. V. Hoversten, "On the Atmosphere as an Optical Communication Channel," IEEE Trans., Vol. IT-14, No. 5, September 1968.

2. Jane W-S. Liu, "Quantum Communication Systems," Ph.D. Thesis, Massachusetts Institute of Technology, 1968; Quarterly Progress Report No. 86, Research Laboratory of Electronics, M.I.T., July 15, 1967, pp. 233-239; Quarterly Progress Report No. 88, January 15, 1968, pp. 255-259.

3. C. W. Helstrom, "Detection Theory and Quantum Mechanics," Inform. Contr. 10, 254-291 (1967).

4. V. I. Tatarski, <u>Wave Propagation in a Turbulent Medium</u> (McGraw-Hill Book Company, New York, 1961).

5. <u>Ibid.</u>, see Equation 7.97.

6. C. W. Helstrom, <u>op. cit.</u> Equation 5.1 is similar to Eq. 5.

7. H. L. Van Trees, <u>Detection, Estimation, and Modulation Theory</u> (John Wiley and Sons, Inc., New York, 1968), Vols. I-II.

8. R. O. Harger, "Maximum Likelihood and Optimized Coherent Heterodyne Receivers for Strongly Scattered Gaussian Fields," Internal Memorandum, Institute of Science and Technology, University of Michigan, Ann Arbor, Michigan (unpublished, 1968).

9. R. S. Kennedy, <u>Fading Dispersive Communication Channels</u> (to be published by John Wiley and Sons, Inc.).

10. L. Lewin, <u>Dilogarithm and Associated Functions</u> (McDonald, Ltd., London, 1958).

11. S. J. Halme, "On Optimum Reception through a Turbulent Atmosphere," Quarterly Progress Report No. 88, Research Laboratory of Electronics, M.I.T., January 15, 1968, pp. 247-254.