PLASMA DYNAMICS

E. High-Temperature Toroidal Plasmas

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1. MODEL FOR THE TURBULENT ION HEATING IN TOKAMAK TM-3

Recently reported experiments on the Tokamak TM-3 machine have shown that the ion energy distribution contains a low-energy part at a temperature of approximately 50 eV and a high-energy tail.[1] The striking feature of this tail is that it continuously disappears as the applied magnetic field is increased from 10 kG to 26 kG. We propose that the enhanced heating of the tail is due to an electrostatic ion-cyclotron instability driven by the electron drift current parallel to the applied magnetic field. We summarize the pertinent results of linear and quasi-linear theories in support of this model.

Linear Instability Theory

The instability under consideration has been studied in the past by several authors.[2,3] The regime of plasma parameters pertinent to TM-3, where $T_e \gg T_i$ and, for $k_\perp a_i \sim 1$, $k\ell_{Di} \gtrsim T_i/T_e$ (a_i = ion Larmor radius, ℓ_{Di} = ion Debye length), was not covered by these previous studies, however, and the instability conditions must be rederived. The dispersion relation for electrostatic waves propagating almost across B_o ($k_\perp \gg k_\parallel$) and with phase velocities $v_i \ll (\omega/k_\parallel) \ll v_e$, where v_i and v_e are the ion and electron thermal velocities, has been solved. The growth rate is given by ($\gamma \equiv$ imaginary part of ω)

$$\gamma = \frac{\dfrac{T_i}{T_e}\dfrac{\sqrt{\pi}\,(k_\parallel v_e-\omega)}{k_\parallel v_e} - \dfrac{\sqrt{\pi}\,\omega}{k_\parallel v_i}\, e^{-k_\perp^2 a_i^2} \displaystyle\sum_{n=-\infty}^{\infty} I_n\left(k_\perp^2 a_i^2\right) \exp\left[-\left(\frac{\omega - n\omega_{ci}}{k_\parallel v_i}\right)^2\right]}{\dfrac{\partial f}{\partial \omega}}, \tag{1}$$

where $f(\omega)$ is given in Eq. 4. It is positive as long as

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$$\frac{u_e}{v_i} > \frac{(k_\perp/k_\parallel)(\omega/\omega_{ci})}{\sqrt{2}\, k_\perp a_i} \left[1 + \sqrt{\frac{m_i T_e^3}{m_e T_i^3}}\, e^{-k_\perp^2 a_i^2} \sum_{n=-\infty}^{\infty} I_n\left(k_\perp^2 a_i^2\right) e^{-\zeta_n^2}\right]. \tag{2}$$

where

$$\zeta_n = \frac{\omega - n\omega_{ci}}{k_\parallel v_i}, \tag{3}$$

and ω, $k_\perp$, and $k_\parallel$ satisfy the real part of the dispersion relation:

$$1 + \frac{T_i}{T_e} + (k\ell_{Di})^2 = e^{-k_\perp^2 a_i^2}\left[I_o\left(k_\perp^2 a_i^2\right) + \sum_{n=1}^{\infty} \frac{2\omega^2 I_n\left(k_\perp^2 a_i^2\right)}{\omega^2 - n^2\omega_{ci}^2}\right]$$

$$\equiv f\left(\frac{\omega}{\omega_{ci}},\ k_\perp a_i\right). \tag{4}$$

Equation 4 may be written

$$1 + \frac{T_i}{T_e} + (k_\perp a_i)^2 \left(\frac{\omega_{ci}}{\omega_{pi}}\right)^2 = f. \tag{5}$$

Consider first the limit $(k\ell_{Di})^2 \ll T_i/T_e$, which has been solved numerically.[3] In this limit we note that Eqs. 1, 2, and 4 depend only on ω/ω_{ci}, $k_\perp a_i$, and $k_\perp/k_\parallel$. Hence, for a constant ratio of T_i/T_e, as B_o is changed, both the growth rate γ/ω_{ci}, and the instability onset condition, Eq. 2, remain unchanged if $k_\perp$ and $k_\parallel$ are changed proportionately to B_o. Also, in this limit, the calculation by Lominadze and Stepanov[3] shows that for the onset of instability, $k_\perp a_i$ remains close to unity for a wide range of T_i/T_e (see Fig. VIII-1 and Table VIII-1). For the case of interest in TM-3, where $k_\perp \ell_{Di} \gtrsim T_i/T_e$, it can be seen from Fig. VIII-1 and Eq. 5 that increasing B_o moves ω closer to $n\omega_{ci}$ and thus cyclotron damping will decrease the growth rate γ/ω_{ci}.

Using TM-3 data[1] in the operating regime of strong anomalous resistance ($u_e \approx 60\ v_i \approx v_e/3$, $n_e \sim 10^{11} - 10^{12}/\text{cm}^3$, $T_e \sim 1$ keV, $T_i \sim 50$ eV), we observe the hot-ion tail at $B_o \approx 10$ kG, and it disappears at $B_o \approx 26$ kG. If the density is taken as 10^{11}, $(\omega_{ci}/\omega_{pi})^2$ varies from 1/25 to 1/4 for this variation in B_o. A rough scaling of the computations[3] to these parameters shows that at 10 kG the instability conditions are well satisfied and the growth rate should be large, while for 26 kG a strong

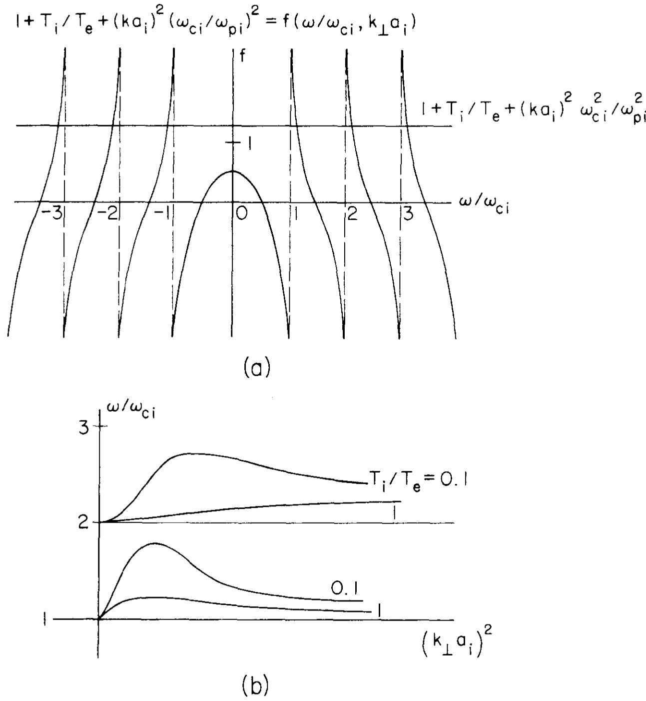

Fig. VIII-1. Dispersion characteristics for the electrostatic ion cyclotron wave (Lominadze and Stepanov[3]). (a) Eq. 4 for $k_\perp a_i$ constant. (b) $\omega(k_\perp)$ at $T_i/T_e = 0.1, 1$.

Table VIII-1. Conditions for onset of instability for $m_e/M_i = 1/3680$.

T_i/T_e	u_e/v_i	$k_\perp^2 a_i^2$	$\cos\theta$
1.00	23	1.1	.05
.33	15	.8	.12
.10	12	.8	.15

reduction in growth rate may be expected. Computer calculations for the growth rate have been initiated for obtaining quantitative estimates of the reduction in growth rate with increasing magnetic field.

Quasi-linear Theory

We shall now examine the way in which this instability heats the ions. The quasi-linear diffusion equation for ions in the presence of weak turbulence is well known,[4] and can be written

$$\frac{\partial f_i}{\partial t} = \nabla_v \cdot \overline{\overline{D}} \cdot \nabla_v f_i. \tag{6}$$

In order to estimate the ion heating rate, we shall assume that f_i is isotropic. In spherical coordinates in velocity space, integrating Eq. 6 over the spherical angles θ and ϕ, we obtain

$$\frac{\partial f_i}{\partial t} = \frac{1}{v^2} \frac{\partial}{\partial v} v^2 \tilde{D}_{vv} \frac{\partial f_i}{\partial v}, \tag{7}$$

where

$$\tilde{D}_{vv} = \frac{1}{4\pi} \int D_{vv} \sin\theta \, d\theta d\phi$$

$$= \begin{cases} \displaystyle\sum_{n,k} \frac{\pi}{2k_{\|} v} \left|\frac{eE(k)}{m}\right|^2 \left(\frac{\omega}{kv}\right)^2 J_n^2\left(\frac{k_\perp V_n}{\omega_{ci}}\right), & \text{for } v^2 > \left(\dfrac{\omega - n\omega_{ci}}{k_{\|}}\right)^2, \\ \text{where } V_n = \sqrt{v^2 - \left(\dfrac{\omega - n\omega_{ci}}{k_{\|}}\right)^2}, & \\ \text{and} & \\ 0 & \text{for } v^2 < \left(\dfrac{\omega - n\omega_{ci}}{k_{\|}}\right)^2. \end{cases} \tag{8}$$

For TM-3 parameters and the instability conditions discussed above, we find that all waves are linearly stable unless $(\omega - n\omega_{ci})/k_{\|} \gtrsim 2.5\, v_i$. This, together with Eq. 8, shows that only the tail ($\epsilon \gtrsim 6\, \epsilon_i$) of the ion energy distribution function will be heated, which is in agreement with experimental observations.[1] For the high-velocity ions ($v > 2.5\, v_i$) an order-of-magnitude estimate of $\tilde{D}_{vv}$ from Eq. 8 is

$$\tilde{D}_{vv} \approx \left|\frac{eE}{m}\right|^2 \frac{v_i^2}{k_{\|} v^3}, \tag{9}$$

where $|E|$ is an average field strength. A heating rate $\nu_h \equiv \tilde{D}_{vv}/v_i^2$ is then given approximately by

$$\nu_h \approx \omega_{pi} \left(\frac{\omega_{pi}}{k_{\|} v_i}\right) \left(\frac{v_i}{v}\right)^2 \left(\frac{\epsilon_F}{\epsilon_i}\right), \tag{10}$$

where ϵ_F is the average electric field energy, and ϵ_i the ion thermal energy. We can compare this heating rate with the electron-ion energy transfer rate,[5] $\nu_\epsilon \approx 2(m_e/m_i)\,\omega_{pe}/n\ell_{De}^3$. Assuming $v \approx 5\,v_i$ and $k_{\|} \approx 10\,\omega_{ci}/u_e$, we find $\nu_h/\nu_\epsilon \approx 10^6\,(\epsilon_F/\epsilon_i)$. Thus even if the total electric field energy is small compared with the ion thermal energy, turbulent heating may be dominant. Furthermore, if the nonlinear process that stabilizes the wave is such that the final electric field energy scales with the linear growth rate, then ϵ_F will decrease as the magnetic field increases. Therefore the heating rate of the tail (and hence its temperature) will also decrease with increasing magnetic field strength.

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