# *Network Coding and Its Implications on Optical Networking*

# Network Coding and Its Implications on Optical Networking

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**Abstract:** Network coding allows for minimum-cost multicast that is fault-tolerant against a link failure. However, it is often possible to achieve the calculated minimum cost without actually performing coding operations at interior nodes.



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## 1. Introduction

Network coding allows interior network nodes to perform arbitrary mathematical operations to combine the data received from different links, offering numerous advantages over traditional routing [1–3]. We investigate how network coding can be effectively utilized for fault-tolerant multicast in optical networks. Regarding the role of network coding in the context of protection or restoration, [4] presents an information-theoretic framework for network management for recovery from nonergodic link failures. Reference [5] presents the concept of 1+N protection that uses network coding over p-cycles, however, without considering the issue of the cost.

Among a variety of protection techniques that have been extensively researched in numerous studies, we particularly focus on the path protection scheme with live back-up that can provide extremely fast recovery. A conventional way to implement such a protection scheme is to transmit two live flows along link-disjoint paths so that upon link failure only receiver nodes can switch to the back-up flow [6]. However, it may require an excessive amount of redundant capacity as back-up capacity is not shared among connections.

Recently, [7] demonstrates that network coding can lead to significant savings in the back-up resources for the multicast scenario protected by live back-up. Notably, in contrast with other network coding solutions with a generic network models, [7] takes into account a unique and crucial characteristic of optical networks: converting photonic streams into electronic signals for data processing and then back to photonic streams for retransmission, which is called Optical/Electronic/Optical (O/E/O) conversion, may be expensive. Since arbitrary coding operations need to be performed in the electronic domain, [7] restricts the coding operation only to bitwise XOR, which can be done within the optical domain using a photonic bitwise XOR hardware. However, from the perspective of network coding, such bitwise XOR coding is too restrictive given that most existing code construction methods require a finite field much larger than binary.

In previous works [8, 9], we have pointed out that it is often possible to achieve the network coding advantage by coding only at a subset of nodes. In particular, [8] shows that while it is necessary to assume network coding at all possible nodes initially to calculate a minimum-cost subgraph for multicast (though without the consideration of fault-tolerance therein), there may be very few, if any, nodes in the resulting subgraph where network coding is actually required. In this paper, we examine if a similar argument holds when the fault-tolerance requirement is taken into account, and if so, discuss the implications of network coding in fault-tolerant optical networking.

## 2. Problem Formulation and Assumptions

The network is given by a directed graph $G = (V, E)$, where $V$ is the set of nodes and $E$ is the set of directed links. Each link has an arbitrary positive integer capacity representing the maximum number of wavelengths carried on the link. To represent a bidirectional link between two nodes, we allow a pair of unidirectional links with opposite directions, each assumed to have a fixed capacity. Given that the traffic is appropriately groomed a priori, we assume that a single wavelength channel is the unit granularity of the optical transport and thus a photonic stream with the data rate of a wavelength is not split into smaller streams. We consider the single multicast scenario in which a single source $s \in V$ wishes to transmit the data that amount to $R$, a positive integer, wavelengths to a set $T \subset V$ of receiver nodes, where $|T| = d$. In addition, we require the *fault-tolerance* condition which implies that the given multicast rate $R$ is still achievable in case of any single link failure by switching only at the affected receivers. Note that for a bidirectional link, which we represent by a pair of unidirectional links in each direction, a single link failure implies

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that the both unidirectional links fail. Though here we consider only a single link failure, it is not hard to generalize our method to the case of multiple link failures.

One of our objectives is to minimize the link cost, which more precisely is the total number of wavelengths used to achieve a fault-tolerant multicast of rate $R$. However, as mentioned earlier, network coding at interior nodes requires O/E/O conversions, which may incur significant costs that may cancel out some savings in the link cost. Therefore, we also need to minimize the coding cost, which is the number of wavelengths that require network coding. Having more than one objective, an optimal decision depends on the relative costs of wavelengths and O/E/O conversions. Below, we present two different strategies to tackle this problem.

## 3. Our Strategies

### *3.1 First Strategy: Two-Stage Method*

Our first strategy is motivated by the experiments performed in [8], where we could separate two optimization problems without sacrificing optimality. In the first stage, we assume that network coding is allowed everywhere and calculate a subgraph incurring a minimum link cost to set up a multicast connection that achieves the target rate and satisfies the fault-tolerance requirement. From Theorem 1 in [1], the necessary and sufficient condition for a feasible fault-tolerant multicast is that the max-flow between the source and each receiver remains at least $R$ after any single link failure. We have formulated an integer linear optimization problem (which we omit here for space considerations but can be found in Section 6.2 in [10]) that finds the minimum link cost for a multicast of rate $R$ protected by live back-up against a single link failure, which is much more compact and tractable than the typical scenarios without network coding that involve Steiner trees [11]. Note that, after the first stage, it is guaranteed that we have at least the link cost minimized without a restriction on the coding cost.

In the second stage, based on the calculated subgraph, we apply our previously proposed evolutionary approach [8] to minimize the coding cost (for details, see [8]). More specifically, as there may be directed cycles in the subgraph after the first stage, we use the graph decomposition method described in Section III-B in [8] for fitness evaluation with slight changes to reflect the fault-tolerance requirement. If, similarly as in [8], network coding is found to be not required at all in the second stage, we end up reaching the ideal point where the found solution is optimal in the both objectives. However, if network coding is required for a nonzero number of wavelengths in the second stage, this two-stage method cannot provide further information on the possible tradeoff between the coding and link costs because the two optimization processes were performed separately. In such a case, our second strategy becomes much more useful, as will be described next.

### *3.2 Second Strategy: Multi-Objective Evolutionary Approach*

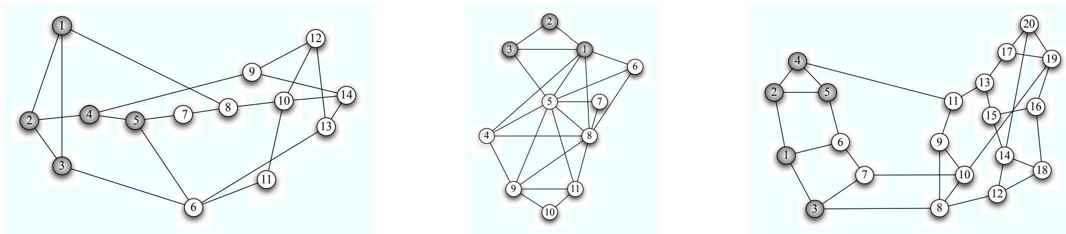
As a second strategy, we apply the multi-objective evolutionary approach we have proposed in [9], which utilizes evolutionary mechanisms to obtain a Pareto optimal front between the two objectives: the link and coding costs. Again, fitness evaluation must be done using the graph decomposition method as in the second stage of the two-stage method presented above. This multi-objective evolutionary approach can effectively reveal the possible tradeoff between the two costs, allowing for more informed decisions on whether or where to deploy network coding. Note, however, that the evolutionary approach may converge relatively slowly compared with the single-objective version used in the second stage of the two-stage method. Hence, we apply the multi-objective evolutionary approach only to those topologies that are found, by the two-stage method, to require at least one coded wavelength.

## 4. Experimental Results

We experimented the above two strategies based on the three topologies depicted in Fig. 1. We assume that all links are bidirectional having capacity (i.e., the maximum number of wavelengths that can be carried) 4 in *each* direction. For each topology, we generated 100 random multicast scenarios with a source chosen randomly out of gray-colored nodes and three receiver nodes out of the remaining nodes. In all scenarios, the target multicast rate is set to 4.

First, we applied the two-stage method to each of 100 multicast scenarios. Table 1 summarizes the calculated minimum number of coded wavelengths. Interestingly, we observe that, even with the fault-tolerance constraint, the minimum-cost multicast can mostly (in 99 out of 100 cases for each topology) be achieved without network coding at all, though, when calculating the minimum-cost subgraph, we have assumed network coding everywhere.

On each topology, for the single case where network coding is found to be required to achieve the minimum link cost, we applied our multi-objective evolutionary approach. Fig. 2 depicts the calculated Pareto front for the link cost and the number of coded wavelengths. The optimal decision depends on the relative costs of link usage and O/E/O



(a) NSFNET (b) NJ LATA (c) ARPANET

Fig. 1. Sample topologies used for simulations.

Table 1. Distribution of the calculated minimum number of coded wavelengths in 100 random multicast scenarios.

| # Coded Wavelengths | | 0 | 1 | 2 | ≥3 |
|---|---|---|---|---|---|
| # Scenarios | NSFNET | 99 | 1 | 0 | 0 |
| | NJ LATA | 99 | 0 | 1 | 0 |
| | ARPANET | 99 | 1 | 0 | 0 |

conversions required for network coding; i.e., we may want to employ an O/E/O conversion for network coding at a node to save a number of wavelengths, or if the cost of an O/E/O conversion exceeds that of those wavelengths, we can eliminate the coding requirement altogether, spending some extra wavelengths.

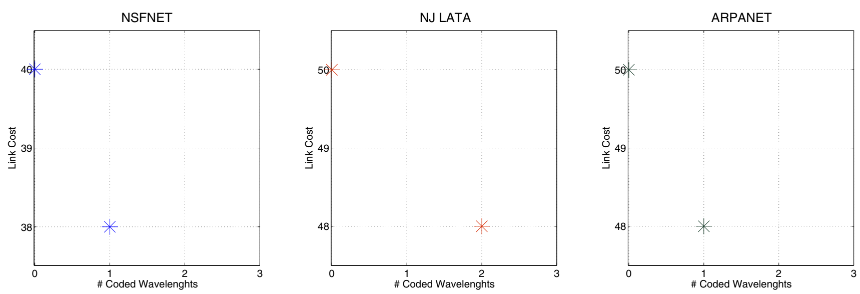



Fig. 2. Calculated Pareto front for the single scenario on each topology where network coding is found to be needed in Table 1.

## 5. Concluding Remarks

Our experiments show that the two-stage method often leads to the ideal point where we can achieve a minimum-cost fault-tolerant multicast without requiring network coding at all. In such cases, the benefit of network coding lies in that it allows for a more tractable optimization problem than the conventional methods involving Steiner trees merely by assuming coding operations during the calculation, rather than that it actually needs to be employed incurring O/E/O conversions. On the other hand, when network coding actually needs to be performed, our multi-objective evolutionary approach can lead to more informed decisions on whether or where to employ network coding.

## References


1. R. Ahlswede, N. Cai, S-Y. R. Li, and R. W. Yeung, "Network information flow," IEEE Trans. Inf. Theory, vol. 46, no. 4, pp. 1204-1216 (2000).
2. R. Koetter and M. Médard, "An algebraic approach to network coding," IEEE/ACM Trans. Netw., vol. 11, no. 5, pp. 782-795 (2003).
3. D. S. Lun, M. Médard, T. Ho, and R. Koetter, "Network coding with a cost criterion," Technical Report P-2584, MIT-LIDS (2004).
4. T. Ho, M. Médard, and R. Koetter, "An information-theoretic view of network management," IEEE Trans. Inf. Theory, vol. 51, no. 4, pp. 1295-1312 (2005).
5. A. E. Kamal, "1+N protection in optical mesh networks using network coding on p-cycles," in Proc. IEEE Globecom (2006).
6. M. Médard, R. A. Barry, S. G. Finn, W. He, and S. S. Lumetta, "Generalized loop-back recovery in optical mesh networks," IEEE/ACM Trans. Netw., vol. 10, no. 1, pp. 153-164 (2002).
7. R. C. Menendez and J. W. Gannett, "Efficient, fault-tolerant all-optical multicast networks via network coding," in Proc. OFC/NFOEC (2008).
8. M. Kim, M. Médard, V. Aggarwal, U.-M. O'Reilly, W. Kim, C. W. Ahn, and M. Effros, "Evolutionary approaches to minimizing network coding resources," in Proc. IEEE Infocom, 2007.
9. M. Kim, M. Médard, V. Aggarwal, and U.-M. O'Reilly, "On the coding-link cost tradeoff in multicast network coding," in Proc. MILCOM, 2007.
10. M. Kim, "Evolutionary approaches toward practical network coding," Ph.D. dissertation, MIT, 2008, (available at http://web.mit.edu/minkyu/www/doc/PhDthesis.pdf).
11. A. K. Somani, Survivability and Traffic Grooming in WDM Optical Networks, Cambridge University Press (2006).