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SINGLE ACTIVITY ACCIDENTS*

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Single Activity Accidents

Traditionally tort law has been viewed as having two functions--deterrence and compensation. Taxes and fines also serve the first purpose while private and social insurance join in the second (and naturally affect the first). Accidents fit neatly into the economists framework of externalities, and the roles of taxes and insurance have been examined in an equilibrium framework in this context. The purpose of this essay is to begin the extension of equilibrium models to include tort liability for accidents. Thus, the model building forming this analysis represents an attempt to examine some of the elements of the way tort law affects resource allocation, not an analysis of the development of tort law nor a serious putting together of all relevant questions in a way appropriate for policy analysis.

Obviously, any work in this area must be greatly influenced by the work of Guido Calabresi,[1] and I started this project as an attempt to set Calabresi to mathematics, although I've naturally wandered away from that definition of the task. While the paper concentrates on developing successively more complicated mathematical models, I have attempted to write this paper so that it will be readable for someone omitting mathematical details. The models considered analyze equilibrium behavior of all the participants in an activity, where participation exposes one to the possibility of accidents involving two (and only two) persons. For a world of risk neutral expected utility maximizers who do not engage in side transactions, equilibrium in the activity is related to the parameters (particularly the standard of due care) of a model describing the outcome of the tort law system. The focus is on the nature of equilibrium rather than the complexities of judicial decision making. Although much of the paper is devoted to simple cases where equilibrium is efficient, the main thrust of the paper is the development of models where full efficiency is not attainable at any level of the judicially controlled variables.

The models here are quite simple (at least measured by the degree of reality they capture) and hopefully have the potential for growth in manageable complexity.

1. Individual Decisions

We shall distinguish four types of decisions which affect expected accident costs. The distinctions are somewhat arbitrary in that some decisions can be modeled as several of these types, and many decisions involve components of several types at once. The typology, however, serves as an introduction to the formal modeling. The four types are choice of activity and three types of decisions affecting care. Activity choice is exemplified by the decision to walk, drive, or bicycle; or to run a railroad rather than an airline. The key elements of a decision of this type are that the choice be discrete and that the activity engaged in at the time of an accident be nearly indisputable. The decisions about care will be distinguished according to whether the decision variable is directly measured at the time of an accident, is stochastically related to a variable measured at accident time, or is a variable normally not measured at accident time. The variable measured at the time of an accident will be called care and will be the basis of negligence evaluations in negligence, contributory negligence or comparative negligence. Examples of care decisions are the quality of spark control devises on locomotives, the presence of radios on tugboats, the number of ropes tying a boat to a dock or the speed driven. If the actual decision is stochastically related to a care variable we shall call it a precaution variable. Examples are the general safety policy of a firm which affects the frequency with which its employees are negligent or the habit of turning around to talk to passengers which affects the likelihood of being in the wrong lane. The care variables here are not the control variables of the firm or automobile driver yet they are the elements contributing to the causation of particular accidents. The third type of variable, called level variables, affect expected accident numbers or costs but are not themselves viewed by courts as accident causes. Examples are the number of miles driven per year, the weight of the automobile, or the amount of planting near a railroad. A more basic analysis would attempt to analyze the reasons for and effects of treating these different variables differently. We shall take the distinctions as given to examine the role of each type of variable in the interplay of the legal system and accident related behavior. While the distinctions among the three care variables are fairly clear in terms of what individuals should reasonably be viewed as controlling and what courts can or choose to measure, the distinction of activities is less well based. In particular, I will argue that the legal system must have symmetric liability rules for two persons engaged in the same activity at the time of their accident. It is this restriction which distinguishes single activity accidents (analyzed in this paper) from two activity accidents (on which I am writing a separate paper). To pursue the basis of this restriction let us digress to analyze individual decisions in more detail.

An individual realizes that engaging in any activity in a particular way exposes him to a risk of accidents. The occurrence of an accident then involves him in the legal process which may require him to give funds to the other party in the accident (we assume that all accidents involve precisely two persons), or may enable him to collect from the other party. For this analysis we assume that the legal system is costless, so that payments and receipts are equal, and that there is no insurance. It does not seem appropriate to select each accident or potential accident as the basis for separate decisions. Rather we shall model individual behavior as the choice of care and activity levels that affect both the probabilities of accidents with everyone else and the costs of accidents which do occur. For example, the purchase of spongy bumpers, the decision to go on a long drive, and the habit of driving rapidly are decisions which potentially affect all other cars (and pedestrians) on the road. In choosing the value for each variable that affects the care and level of engagement in an activity the individual is trading off the pleasure or cost of marginal adjustments in these variables against their impact on his expected accident and liability costs. The decisions of others enter his decision calculus by affecting the probability of severity of accidents given his behavior in the activity. The legal system enters his decision calculus by relating expected liability payments to his accident involvement and the care decisions of himself and the parties with whom he may have accidents. The legal system predicates liability on behavior at the time of an accident. (We ignore the elements of damage measurement depending on mitigation before or after an accident.) The liability basis seems distinguishable into two types--what activity was engaged in at the time of the accident (e.g. blasting, crop dusting, driving, walking) and what care was shown in the activity at the time of (and proximately causing) the accident. The first aspect sometimes distinguishes potential plaintiffs from potential defendants while the second determines liability or the ability to collect (in a negligence/contributory negligence setting) or the amount of damages (under comparative negligence). The distinction is somewhat artificial in that we could define negligent driving as a different activity from nonnegligent driving (and reckless driving as a third activity), for a negligence system and ignore the distinction between activities for comparative negligence. (We might also try to distinguish activities by the care decision which was the proximate cause of a particular accident.) However, the distinction between activities and care shown in the activity seems useful for analysis and reflective of the fact that activity definition is probably rarely in dispute, while the level of care is often subject to dispute. Thus a legal system determining only activities is presumably less expensive and more accurate than one which measures care. For analysis, the distinction between activities permits asymmetic rules, like strict liability,and is essential for the concept of the cheapest cost avoider (Calabresi, p. 136). However, when the distinction between behaviors of the two parties to an accident is not easily drawn, the legal system is restricted to symmetric rules, which can still depend on the degree of care, and the analyst must use symmetric concepts. Thus the single activity accident (i.e. when both parties were engaged in the same activity at the time of an accident) is a simpler case with which to begin because of these restrictions.

Given the assumption that activity definition presents no problems,[2] we need next to model the measurement of care and the standard for due care for systems using this concept. (If there is no liability (each person bears his own costs) or if each person bears the costs of the other party, we have symmetric rules not requiring a measurement of care.) There are two starting places for modeling a due care standard, depending on whether quantities or shadow prices are viewed as the description of individual perception of judicial decisions. The approach which will be followed here is the presence of a distinct measurable physical definition of taking care, independent of the circumstances of the particular individual. Thus the standard is the speed driven at the time of the accident, the quality of the spark control device, or the depth of the water pipes. This approach seems particularly appropriate where legislated safety standards serve to define negligence. Obviously the quantity approach is only reasonable for an ongoing (long-run equilibrium) activity where people have learned the behavior patterns judged negligent by the courts. For a new activity or an extension of due care to a new dimension of behavior, it is necessary to view the problem as possibly requiring a benefit-cost analysis. For ongoing activities poeple are presumed to know what to do to avoid negligence, but not necessarily why.

2. Proximate Cause

Very many elements stretching back in time, can be viewed as contributing causes to any particular accident. The legal system uses the term proximate cause to label those causes which arise from potential defendents to a successful lawsuit as opposed to those elements which are ignored by the legal system. Obviously the definition and application of the proximate cause standard to particular accident situations is an important element in legal disputes. Following the approach in this essay I will not enquire into the difficulties of the edges of application of the standard. Rather I shall ask about the relationship between the structure of accident probabilities and a clear-cut proximate cause definition. I shall then proceed in the analysis under the assumption that proximate cause issues are not a problem for the analysis.

Let us assume that the only proximate causes of any accident are the decisions of the two parties to the accident. Let us also assume that the only determinants of the probability distributions of individual accidents are the decisions of all parties engaged in the activity. The assumption which will be made for the analysis is that the determinants of the expected number of accidents between any two particular persons are only their own decisions. Thus the only way that a decision by A, say, affects the expected utilities of others is by affecting their expectation about accidents with A.[3] This assumption permits the legal system to be in the position of examining all the possible consequences to others of a decision by any individual.

It seems readily apparent that this assumption is not strictly valid in many situations, such as auto accidents. It is an open empirical question how far from accurate it really is.[4] For example the bad driving of some occasionally results in accidents to two other parties who were responding to the bad driving. More generally, the presence on the highway of other drivers going at various speeds leads to decisions about lane selection, for example, which may affect the probability of accident, or the distribution of that probability across other drivers.[5] In the other direction having an accident with a bad driver (or more basically, making a care decision which increases the probability of an accident with a particular bad driver) may be saving some other good driver from an accident with the same bad driver.

Although these difficulties will be ignored for the analysis to follow, I want to argue here that the determinants of accident probabilities over some time period are likely to diverge in some degree from the simple structure which would be additive in probabilities relating to pairs of individuals. For example, when this condition is satisfied, increasing the number of persons engaged in the activity while holding accident determining behavior of the individuals constant increases the aggregate expected number of accidents roughly as the square of the number of people in the activity. This relationship does not appear to be a reasonable fundamental constant of nature. It is natural at this point to attempt to argue that the difficulty with the argument above is a failure to define decisions affecting accident probabilities correctly (i.e. that the behavior held constant when numbers increased was not the appropriate behavior). Perhaps (although it seems to me implausible), theory can argue its way out of this problem along these lines. It is unlikely that the legal system, with the limitations on the decision variables it can measure, could follow.

3. Equilibrium without Liability

If we assume that there is no legal system, no possibility of collecting for accident costs or paying that of others, we can view the analysis of this activity with its accident probabilities as a standard problem of economic equilibrium with externalities. After describing a model of equilibrium of this situation, we shall examine the change in equilibrium from the presence of a liability system, as well as the response of equilibrium to changes in the parameters describing the legal system. This parallels standard analysis of externalities which introduces additional markets or taxes to alter the equilibrium position. We shall consider only a legal system involving suits for damages and not the possible interaction of civil, criminal, and tax alterations in incentives.

We assume that there are a large number of people engaged in this activity, with accident possibilities present for any pair of persons. This large numbers assumption justifies the further assumption that no one attempts to directly alter the behavior of others. Conventionally the transactions costs involved to bribe, threaten, or reach agreement are viewed as prohibitively large so that the assumed impossibility is close enough to reality. A second assumption justified by the presence of large numbers is that each person assumes that his own decisions do not induce changes in the decisions of others in response.

Given these assumptions, an individual takes the behavior of others (as perceived by him) as given and selects his level of care to maximize expected utility, at the margin trading off the disutility of taking more care against the reduction in expected accident costs. We shall make the somewhat doubtful assumption that the perceptions of the behavior of others on which care decisions are based are correct. We shall then have a long run equilibrium when we have a simultaneous selection of care levels by each person, with each person's selection based on assumed behavior of others which correctly describes the selection of care levels simultaneously being made by the others. For the present we assume that everyone is the same.

Let us denote by x and y the levels of care chosen by the person being considered and perceived by him to be chosen by each other person. We assume that choosing care level x gives a utility V(x), apart from any accident involvement.[6] Since we shall assume that taking more care decreases expected accident costs, individuals will select a level of care in the range where V is decreasing.

Let us denote by C(x,y) the cost to a person taking care level x from being in an accident with someone who was taking care level y. Since these people are engaged in the same activity the cost of this accident to the other person is described by the same function, C(y,x). We shall denote the partial derivatives of C with respect to its first and second arguments as C_1 and C_2. Let us denote the expected number of accidents (per unit time) between two persons taking care levels x and y by $\pi(x,y)$. Naturally we assume that π is symmetric since any accident to the two of them occurs to each. C(x,y), however, will generally not be symmetric.

If there are n + 1 persons engaged in the activity (and so n others with whom he might have an accident) a person who believes the other n are choosing care level y will choose x to maximize utility of care less expected accident costs. We can express this as

$$\max_{x} \quad V(x) - n\pi(x,y)C(x,y) \tag{1}$$

The first order condition for expected utility maximization is obtained by differentiating expected utility with respect to care level x

$$V'(x) - n\pi_1(x,y)C(x,y) - n\pi(x,y)C_1(x,y) = 0 \tag{2}$$

This equation expresses implicitly the care level, x, chosen by an individual who believes that everyone else is choosing care level y. We will have a uniform equilibrium at care level $x°$ when belief that others are choosing $x°$ leads the individual also to choose $x°$. Thus $x°$, the no-liability equilibrium care level, must satisfy the equation

$$V'(x°) = n\pi_1(x°,x°)C(x°,x°) + n\pi(x°,x°)C_1(x°,x°) \tag{3}$$

We stated above that we assumed that care could be measured so that increased care decreased expected accident costs. Stated formally we assume for all x,y

$$\pi_1(x,y)C(x,y) + \pi(x,y)C_1(x,y) < 0 \tag{4}$$

To give structure to the analysis we shall assume that each person would like to see others take more care. Formally we are assuming for all x,y

$$\pi_2(x,y)C(x,y) + \pi(x,y)C_2(x,y) \leq 0 \tag{5}$$

Obviously there are many decisions which decrease costs per accident for the decision maker which increase costs per accident for others, like automobile weight. By assumption (5) we are ignoring this type of decision for now.

Even with these assumptions, an increased care level by all others does not necessarily decrease the taking of care. To see the relationship, let us examine (2) which defines x implicitly as a function of y. Differentiating implicitly we have

$$\frac{dx}{dy} = \frac{\pi_{12}(x,y)C(x,y) + \pi(x,y)C_{12}(x,y) + \pi_1(x,y)C_2(x,y) + \pi_2(x,y)C_1(x,y)}{n^{-1}V'' - \pi_{11}(x,y)C(x,y) - \pi(x,y)C_{11}(x,y) - 2\pi_1(x,y)C_1(x,y)} \quad (6)$$

For individual choice to be well behaved the denominator must be negative by the second order condition. The assumptions (4) and (5) above are not sufficient to guarantee that the numerator is positive, so we now make that assumption explicitly. Thus we are assuming that when others take more care an individual feels less need to take care. This relationship depends on the impact of the care of others on the expected cost savings from taking more care oneself. The assumption that care of others is liked is merely an assumption that expected costs are decreased by the care of others.

While ex post, different people may have borne different accident costs (due to the stochastic nature of accident occurrence), ex ante in uniform equilibrium everyone has the same expected utility, having the same disutility of care and the same expected accident costs. It is thus natural to ask what care decision, if taken by everyone, will maximize the expected utility position of each person in equilibrium. Let us call this the efficient level of care and denote it by x*. Thus x* is the level of care which maximizes expected utility

$$\max_x \quad V(x) - n\pi(x,x)C(x,x) \quad (7)$$

The first order condition for this maximization gives us the equation for the efficient level of care

$$V'(x^*) = n\pi(x^*,x^*)(C_1(x^*,x^*) + C_2(x^*,x^*)) +$$
$$nC(x^*,x^*)(\pi_1(x^*,x^*) + \pi_2(x^*,x^*)) \quad (8)$$

Under the sort of regularity assumptions we have been making the efficient level of care exceeds the equilibrium level of care in the absence of any liability system.[7]

$$x^* > x^\circ \tag{9}$$

The difference in the two positions can be characterized by the changed accident costs perceived when choosing a care level. When individuals choose a care level for themselves they examine the change in their own expected accident costs for a marginal change in care level. For efficiency, the decision should reflect the change in total expected accident costs, those of the person making the decision, and the sum of the changes in expected accident costs for everyone else with whom he might have an accident. If the cost function, C, were symmetric we could say that the no-liability equilibrium reflected attention to half the elements of social cost which appear in the condition for efficiency. We say half the elements rather than half the costs since in equilibrium the marginal expected individual costs are $\pi_1(x^\circ,x^\circ)C(x^\circ,x^\circ) + \pi(x^\circ,x^\circ)C_1(x^\circ,x^\circ)$ while for efficiency the marginal expected social costs are $2\pi_1(x^*,x^*)C(x^*,x^*) + 2\pi(x^*,x^*)C_1(x^*,x^*)$. Thus the same functions (elements of cost) are being evaluated at different levels of care.

The introduction of a tort liability system in this no-liability setting alters the incentives for taking care. The alteration comes from the possible liability implications of actually having an accident. Alternative general modes of affecting incentives are to check care taken during the activity (perhaps on a sampling basis) or to affect actions, like purchases, which are related to care decisions. Thus examples of these alternatives would be radar checks on speeding and subsidization of spongy bumper purchases (or regulations on automobile manufacture requiring them). With different sorts of care decisions, the different types of incentives may have very different administrative costs. Where purchases are the key element of care decisions, taxes and subsidies may be a very inexpensive way of altering incentives. However, many care decisions, like driving speed, are only tenuously connected with purchases. Checking performance in the activity, independent of accidents, will be inexpensive, relative to just monitoring accidents, when accidents are frequent relative to activity levels, when care is easy to monitor (speeding relative to driving when tired), when the location of the activity of different persons is concentrated and public, and when accidents themselves are difficult to monitor because of the incentives and opportunities for concealment. A more general theory of externality correction would explore information structures and the choice of incentive altering mechanism. We continue the discussion with the tort liability system as the only social alteration in the incentive to take care.

4. Individual Choice with a Negligence System

Let us assume that each person engaged in the activity chooses the level of a single variable which affects his expected accident costs and which is directly (and costlessly) measured at the time of any accident. If everyone is alike, the equilibrium in the absence of a liability system will occur at a care level (which we denote x°) which is below the efficient care level (which we denote x^*). A strict liability system in this context would result in each party to an accident bearing the costs of the other party. To the extent that actual costs of an accident are not symmetric, the equilibrium would diverge from x°. It does not seem worthwhile to seriously pursue this alternative. One might consider a negligence system without contributory negligence, but this would have the same unsatisfactory nature--where both are negligent each bears the costs of the other. Thus we shall consider a negligence-contributory negligence system. Both to avoid the solution where each bears the costs of the other and to reflect current legal theory,[8] we shall consider a system where the same standard of care is employed in the measurement of negligence and of contributory negligence.

The question we shall pursue is the relationship between the judicially set standard of due care and the level of care chosen by the individuals in equilibrium. Since we are considering only uniform equilibria, either everyone will be negligent (and contributorily negligent) or everyone will satisfy the due care standard. Thus there will never be successful lawsuits. Nevertheless, over some range, variations in the due care standard will lead to changes in the equilibrium level of care.

It is convenient for the general formulation to assume that there is a distribution $n(y)$ of other individuals choosing care level y. For convenience we assume that x and y lie in the unit interval. In the next section we will reduce this general function to the uniform case. Let us denote by U_1 the level of expected utility if an individual only bears his own accident costs when he has an accident with someone who is not negligent (and never bears anyone else's accident costs).

$$U_1(x) = V(x) - \int_d^1 \pi(x,y)C(x,y)n(y)dy \tag{10}$$

Let us denote by x_1 the level of care that maximizes[9] U_1. Similarly we shall denote by U_2 the level of expected utility for an individual who bears all of his own accident costs and also those of the other party when he has an accident with someone who is not negligent.

$$U_2(x) = V(x) - \int_0^1 \pi(x,y)C(x,y)n(y)dy - \int_d^1 \pi(x,y)C(y,x)n(y)dy \qquad (11)$$

(Note that C(x,y) appears in the first integral and C(y,x) in the second.) Let us denote by x_2 the level of care that maximizes[10] U_2.

The relevant utility function for an individual is $U_1(x)$ when he chooses not to be negligent and $U_2(x)$ when he chooses a care level such that he is negligent.

$$U(x) = \begin{cases} U_1(x) & x \geq d \\ U_2(x) & x < d \end{cases} \qquad (12)$$

Since accidents result in nonnegative costs it is clear that $U_1(x) \geq U_2(x)$. From the assumptions we have made on the way care affects each person (4) and (5), it is also true that $x_1 \leq x_2$, i.e. if an individual bore more accident costs he would take more care. Thus, we are assuming that utilities are as shown in the diagram

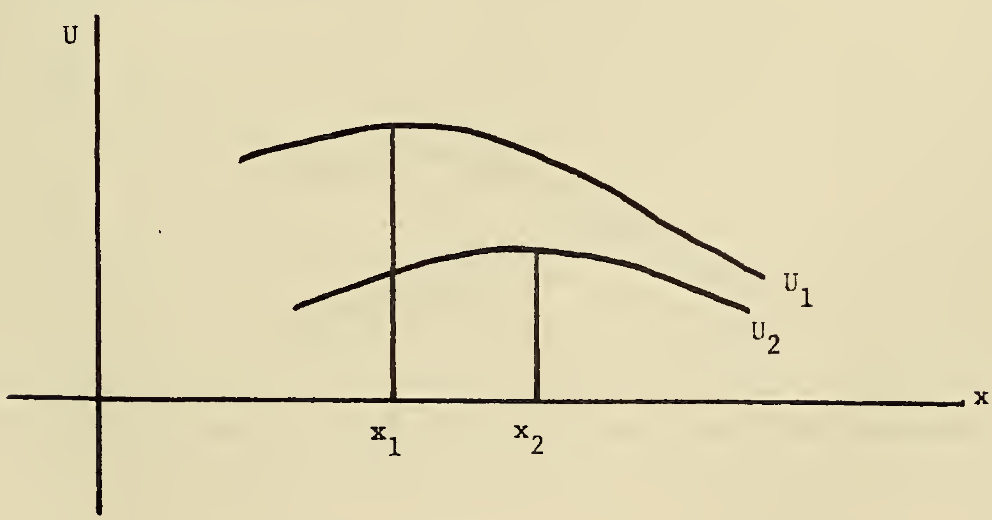


Figure 1

To describe individual choice, we must locate d relative to x_1 and x_2 since U_2 is the relevant utility function for $x < d$ and U_1 for $x \geq d$. Examining the diagram it is straightforward to confirm that a value of d less than x_1 leads to choice of x_1. The dotted curve represents the relevant portions of utility (i.e. represents U). Similarly it

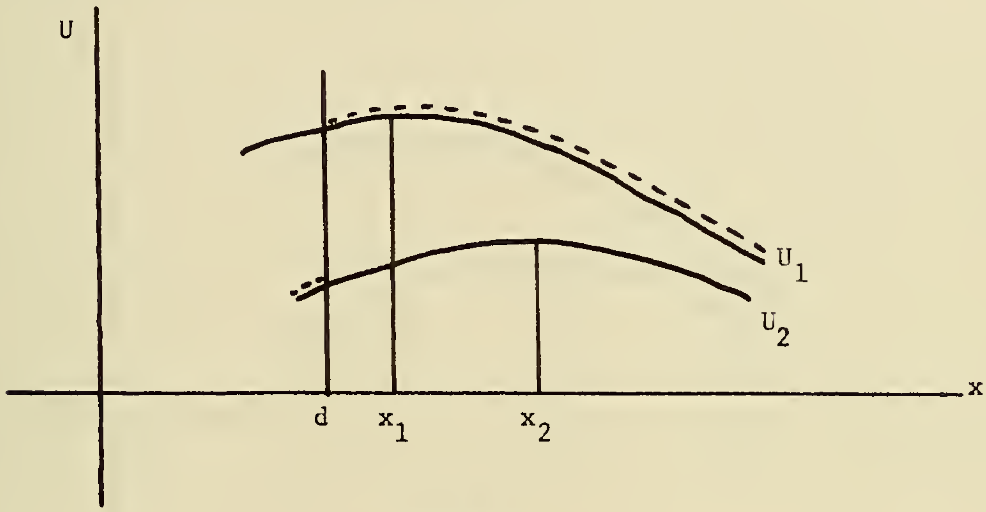


Figure 2

is clear that a level of due care between x_1 and x_2 leads to a choice of care level precisely equal to the due care level

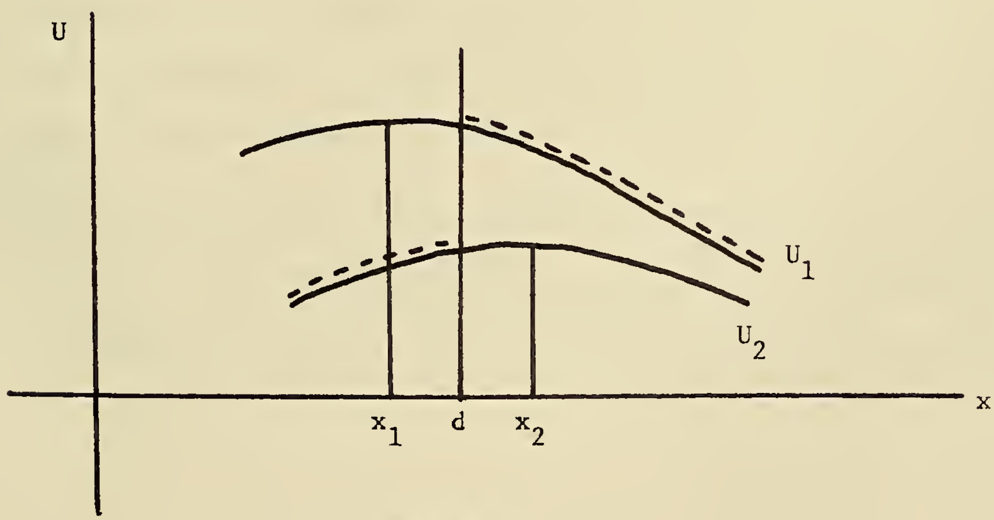


Figure 3

For due care levels in excess of x_2, we must compare the value of U_2 at its maximum, x_2, with the value of U_1 at d. Let us define $\hat{d}$ by the equality of these two levels

$$U_1(\hat{d}) = U_2(x_2) \tag{13}$$

The value of $\hat{d}$ is shown in Figure 4

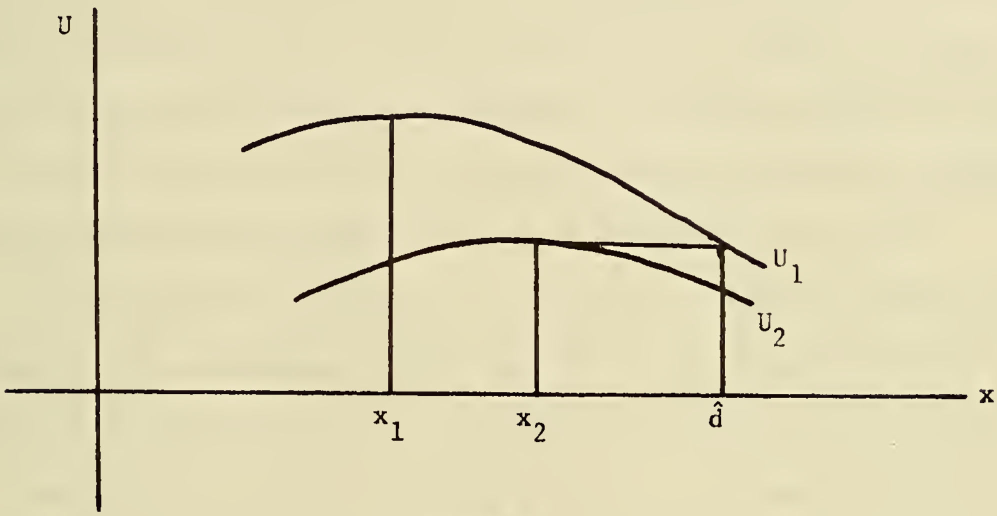


Figure 4

For due care levels above x_2, values below $\hat{d}$ lead to a choice of d but those above $\hat{d}$ lead to a choice of x_2, i.e. lead to a decision to be negligent. Summarizing these conclusions we have

level of due care	$d \leq x_1$	$x_1 \leq d \leq \hat{d}$	$\hat{d} \leq d$	(14)
chosen level of care	x_1	d	x_2	

Since expected costs depend on d (see (10) and (11)), all three of x_1, x_2, and $\hat{d}$ are functions of d for the individual. In addition, in determining equilibrium, the behavior of others (n(y)) will also vary with the due care level.

5. Uniform Equilibrium

We now wish to examine the possible uniform equilibria (i.e. with everyone selecting the same care level) that may arise with different judicially selected levels of due care. Since we are examining uniform equilibria, either everyone will be negligent or everyone will show at least due care. Thus there are never any damage awards. Nevertheless the level of due care can affect the equilibrium level of care. We shall conclude that there are only two candidates for equilibrium--the no-liability care level (x°) and the due care level (d) and shall examine the conditions which determine the equilibrium level as a function of due care.

If we have an equilibrium at x° without any liability system, the introduction of a negligence system with a due care standard below the equilibrium level (i.e. below x°) clearly has no effect on the system. No one has any reason to alter his behavior, since everyone is showing due care, although paying attention only to his personal costs. This corresponds to Figure 2 above with x° coinciding with x_1 and everyone being nonnegligent.

If the due care level is set slightly above the no-liability equilibrium, everyone will find it worthwhile to increase his care level precisely to the due care standard to avoid potential liability. This will be true despite the fact that the increased care taken by others reduces the incentive to take care for self-protection. Thus over some range the equilibrium care level rises with the due care standard. The next step is to determine the range for d such that equilibrium occurs at d. To ask when the due care level is a uniform equilibrium we must ask when each person chooses his care level at the due care level given that he assumes that everyone else is selecting precisely the due care level. We return to the structure with n + 1 persons. Since everyone else chooses the due care level (y = d) the utility function takes the form

$$U(x) = \begin{cases} U_1(x) = V(x) - n\pi(x,d)C(x,d) & x \geq d \\ U_2(x) = V(x) - n\pi(x,d)(C(x,d) + C(d,x)) & x < d \end{cases} \quad (15)$$

From the discussion above (14) we know that d will be chosen provided $x_1 \leq d \leq \hat{d}$ for this utility function. To make clear the functional relation we state this formally. A due care level d results in a uniform equilibrium at d provided

$$x_1(d) \leq d \leq \hat{d}(d) \quad (16)$$

where the limits are defined by

$x_1(d)$ maximizes $V(x) - n\pi(x,d)C(x,d)$

$$\hat{d}(d) \text{ satisfies } V(\hat{d}) - n\pi(\hat{d},d)C(\hat{d},d) = \underset{x}{\text{Max}}\ V(x) - n\pi(x,d)(C(x,d) + C(d,x)) \quad (17)$$

Recalling the definition of $x_2(d)$ as maximizing $U_2(x)$, $\hat{d}$ satisfies

$$U_1(\hat{d}) = U_2(x_2)$$

Let us consider the lower bound constraint. At $d = x°$, $x_1(d)$ is also equal to $x°$, so the constraint is satisfied. From the assumption made above (6), that care taken decreases with the care of others we know that $x_1(d)$ decreases with d and thus the lower bound constraint is satisfied for any d above $x°$.

Now let us consider the upper bound constraint. From Figure 4, $V(\hat{d}) - n\pi(\hat{d},d)C(\hat{d},d)$ is decreasing in $\hat{d}$. Thus from (17)

$$\hat{d}(d) \gtreqless d \text{ as } \phi(d) \equiv \underset{x}{\text{Max}}\ V(x) - n\pi(x,d)(C(x,d) + C(d,x)) - V(d) + n\pi(d,d)C(d,d) \lesseqgtr 0. \quad (18)$$

Note that $\phi(x^*)$ is negative (since the maximizing x is x^*). Since increased care decreases accident costs for both parties, (4) and (5),

$\pi(x,d)(C(x,d) + C(d,x))$ is decreasing in x. By one of the many concavity assumptions $V(d) - n\pi(d,d)C(d,d)$ is decreasing in d for $d > x^*$. Thus ϕ increases with d for $d > x^*$, giving a unique value, $\hat{d}_2$ as the upper bound for this range of solutions. To check the region between $x°$ and x^*, let us make the further assumption[11] that x_2 is decreasing with d, i.e. care decreases with the care of others for someone who is bearing all costs. As can be seen in Figure 4, $\hat{d}(d) > x_2(d)$. Thus when $x_2(d) > d$ the upper bound constraint is satisfied. If the due care level is set at the efficient care level, i.e., $d = x^*$, then $x_2(d)$ is equal to x^*. Hence the upper bound constraint is satisfied for all d below x^*. Thus we have shown that for all d between $x°$ and $\hat{d}_2$ (with $\hat{d}_2 > x^*$) there exists a uniform equilibrium with everyone just choosing the due care level.

For sufficiently high due care levels everyone will choose to be negligent and the legal system will effectively be equivalent to a no-liability system. Thus everyone will choose $x°$. We now wish to ask which values of d (above $x°$) will lead to a choice of $x°$ by someone who thinks that everyone else is choosing $x°$. Given that everyone else has chosen $x°$, expected utility is now

$$U(x) = \begin{cases} U_1(x) = V(x) & x \geq d \\ U_2(x) = V(x) - n\pi(x,x°)C(x,x°) & x < d \end{cases} \tag{19}$$

Thus U_2 coincides with utility in the absence of liability when everyone else chooses $x°$. Thus we know that x_2 coincides with $x°$. From (14) above we know that x_2 will be chosen for d above a critical value $\hat{d}_1$ defined by

$$V(\hat{d}_1) = V(x°) - n\pi(x°,x°)C(x°,x°) \tag{20}$$

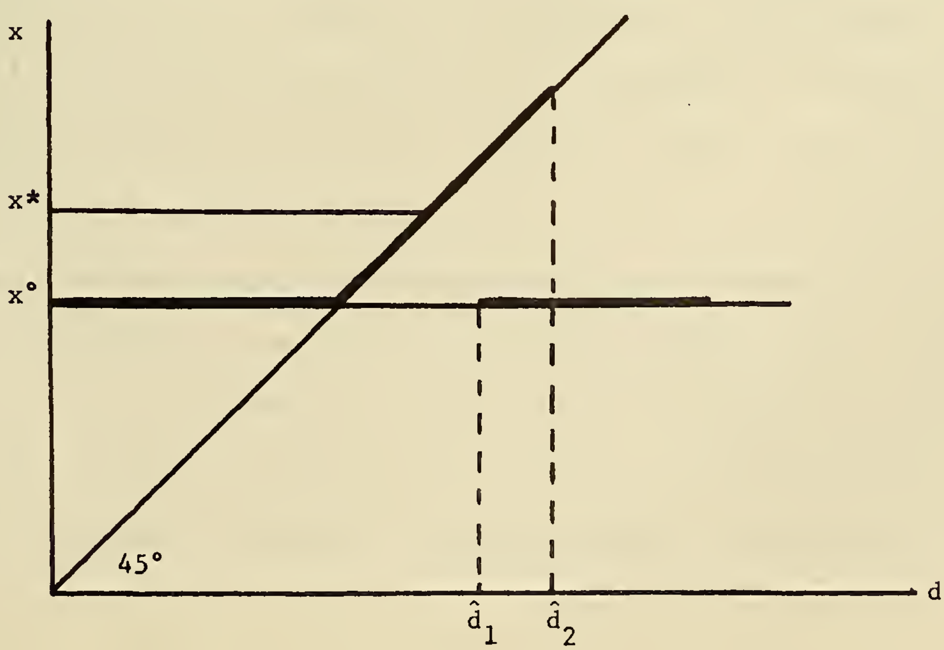
Let us note first that $\hat{d}_1$ is strictly greater than x^* since V is decreasing in x and

$$\begin{aligned} V(x^*) &> V(x^*) - n\pi(x^*,x^*)C(x^*,x^*) \\ &> V(x°) - n\pi(x°,x°)C(x°,x°) = V(\hat{d}_1) \end{aligned} \tag{21}$$

The remaining question is the relationship between[13] $\hat{d}_1$ and $\hat{d}_2$. That is whether there are ranges of d that give rise to no equilibria ($\hat{d}_1 > \hat{d}_2$) or two equilibria ($\hat{d}_1 < \hat{d}_2$). Both of these situations seem possible without further restrictions.[14] It also seems possible to have non-uniform equilibria in this range.[15]

We have examined the situations under which x° and d can be uniform equilibria. It remains to argue that these are the only uniform equilibria. A formal argument would check that possible demanded points, x_1, x_2, and d, coincide with either x° or d when they represent uniform equilibria. An informal argument will suffice for our purposes. In a uniform equilibrium there are no successful lawsuits. Thus either equilibrium is at d (where expected utility is discontinuous) or locally expected utility coincides with that of the no-liability equilibrium. For the latter situation x° is the only possible equilibrium.

Let us aummarize the situation described above. For $d \leq x°$ there exists an equilibrium at x° with no one negligent. For $x° \leq d \leq \hat{d}_2$ (with $\hat{d}_2 > x^*$) there exists an equilibrium at d with no one negligent. For $\hat{d}_1 \leq d$ there exists an equilibrium at x° with everyone negligent. Thus the possibilities are as shown in Figure 5.



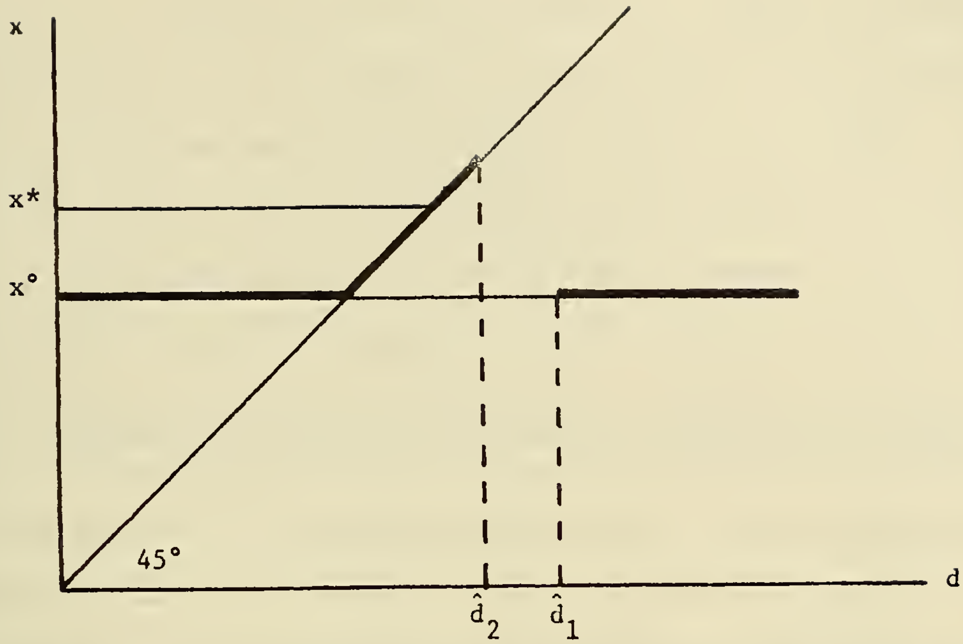


Figure 5. Uniform Equilibrium as a Function of the Due Care Standard

6. Stochastic Control of Care

Many accident situations arise in circumstances where it would not be reasonable to model individual decisions as deliberately selecting a negligent course of action. Rather the control variables of individuals (or firms) result in different distributions of actual behavior over time. Let us call the actual control variable precaution, and assume that a greater expenditure on precaution leads to a higher average level of care, but does not eliminate the variations. There are two aspects of the precaution-care pattern which complicate the discussion of the previous section and which seem worth discussing separately. First, people are often careful independent of any particular expenditures of money or effort on being careful. To model this phenomenon, we shall consider the case of care uniformly distributed between the precaution level and some upper bound, i.e., we shall assume that precaution is a lower bound for care.[16] Alternatively, in some circumstances, people will be careless some of the time no matter how much effort goes into trying to be careful. Greater efforts can only reduce the probability of being careless, not eliminate it. To model this aspect of behavior we shall consider precaution as an upper bound for care and assume that care is uniformly distributed between some lower bound and the precaution level. We shall also examine the case of a triangular distribution of care with its peak at the precaution level.

With precaution as a lower bound for care, there is no effect from the introduction of a negligence system with a due care level below the equilibrium level of precaution in the absence of liability. This situation is the same as in the determinate case. With increases in the due care standard over some range, the precaution levels rise exactly with the due care standard, so that each person is making sufficient effort to never be negligent. Thus over this range too the pattern from the determinate case is repeated. However, as the due care level rises out of this range, the equilibrium level of precaution decreases continuously with the due care level, approaching the no-liability equilibrium as it becomes less and less likely to be nonnegligent (i.e., as the due care standard approaches the upper bound in the distribution of care). Thus the situation fits the description in the diagram.

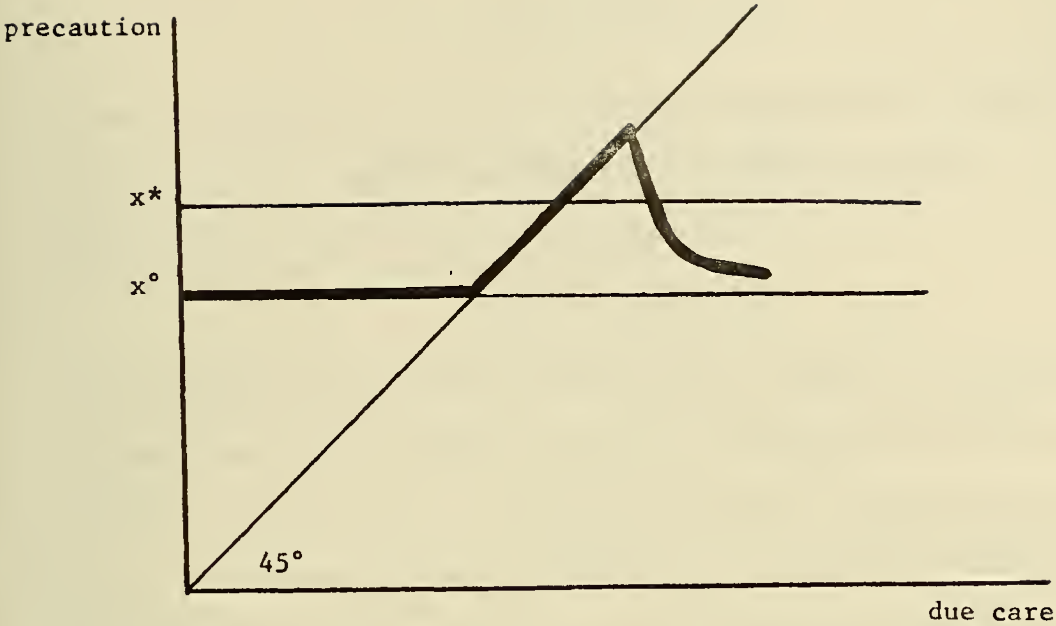


Figure 6. Equilibrium with Precaution as a Lower Bound for Care

Now let us derive this pattern of results explicitly. We continue to denote the control variable, now precaution, by x and the level of precaution chosen by everyone else by y. The choice of x results in a uniform distribution of actual care, a, between x and 1. The probability of an accident, given the level of care, depends only on the care being taken by the two parties a and b. For simplicity we take a particular form (1 - a)(1 - b). For simplicity we also assume that all accidents cost the same amount, C. In the absence of a liability system, expected utility can be expressed as

$$U(x) = V(x) - nC\int_x^1\int_y^1 \left(\frac{1-a}{1-x}\right)\left(\frac{1-b}{1-y}\right) db\,da$$

$$= V(x) - \frac{1}{4}nC(1-x)(1-y) \tag{22}$$

As above we can find the no-liability uniform equilibrium by maximizing U(x), given y, and finding a y so that the chosen x and y coincide.

$$V'(x^{\circ}) = -\frac{1}{4}\, nC(1 - x^{\circ}) \tag{23}$$

Also, as before, we can find the level of care which if chosen by everyone will lead to the highest uniform level of expected utility. For this we set x equal to y in (22) and maximize

$$V'(x^{*}) = -\frac{1}{2}\, nC(1 - x^{*}) \tag{24}$$

The particular forms chosen here show more clearly the distinction made above between the elements of marginal accident cost and the actual marginal accident cost. In functional terms, efficiency calls for examining twice the elements of cost examined for no-liability equilibrium. At their actual values, however, $\frac{1}{2}\, nC(1 - x^{*})$ is less than twice $\frac{1}{4}\, nC(1 - x^{\circ})$.

Let us start by examining equilibria where no one is ever negligent. Thus we begin be examining expected utility under the assumption that $y \geq d$ and look for those values of d which lead to a selection of x which coincides with y. As before we must distinguish expected utilities according to whether the person is never negligent, $x \geq d$, or may be negligent, $x < d$. Thus we write expected utility as

$$U(x) = \begin{cases} U_1(x) = V(x) - nC\displaystyle\int_x^1\int_y^1 \left(\frac{1-a}{1-x}\right)\left(\frac{1-b}{1-y}\right) dbda & : x \geq d \\[2ex] U_2(x) = V(x) - nC\displaystyle\int_x^1\int_y^1 \left(\frac{1-a}{1-x}\right)\left(\frac{1-b}{1-y}\right) dbda \\[2ex] \qquad - nC\displaystyle\int_x^d\int_y^1 \left(\frac{1-a}{1-x}\right)\left(\frac{1-b}{1-y}\right) dbda & : x < d \end{cases} \tag{25}$$

Thus from the utility of taking care we subtract expected accident costs when nonnegligent and when negligent the expected costs of others.

Performing the integrations we have

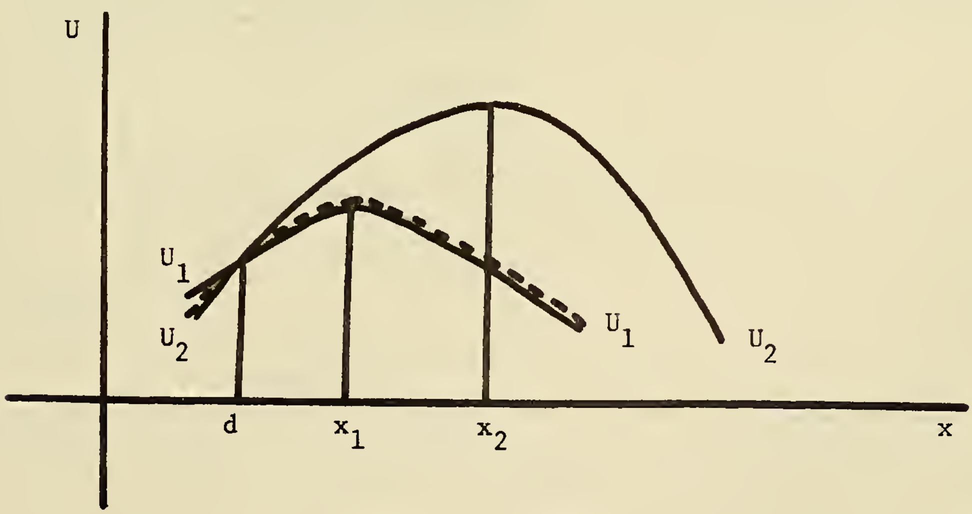
$$U_1(x) = V(x) - \frac{1}{4} nC(1-x)(1-y) \tag{26}$$

$$U_2(x) = V(x) - \frac{1}{4} nC(1-x)(1-y) - \frac{1}{2} nC(1-y) \frac{d-x-\frac{1}{2}(d^2-x^2)}{1-x} \tag{27}$$

Let us denote by $x_1(d)$ and $x_2(d)$ the maximizing arguments for these two functions (ignoring the values of x for which they are relevant). Inspecting the functions we see that there is no longer a discontinuity at d, rather U has a kink at that point. Examining the functions, we have $U_1(x) \gtreqless U_2(x)$ as $x \lesseqgtr d$. In addition, we can compare derivatives

$$U_2'(x) = U_1'(x) + \frac{1}{2} nC \frac{(1-y)}{(1-x)^2} (1 - d - x + \frac{1}{2} d^2 + \frac{1}{2} x^2) \tag{28}$$

Thus $U_2'(x) > U_1'(x)$ (and $x_1 < x_2$) and the utilities appear as in the diagram for different relative values of d, x_1, and x_2, with U as the dotted line -- U coincides with U_2 below d and with U_1 above d.



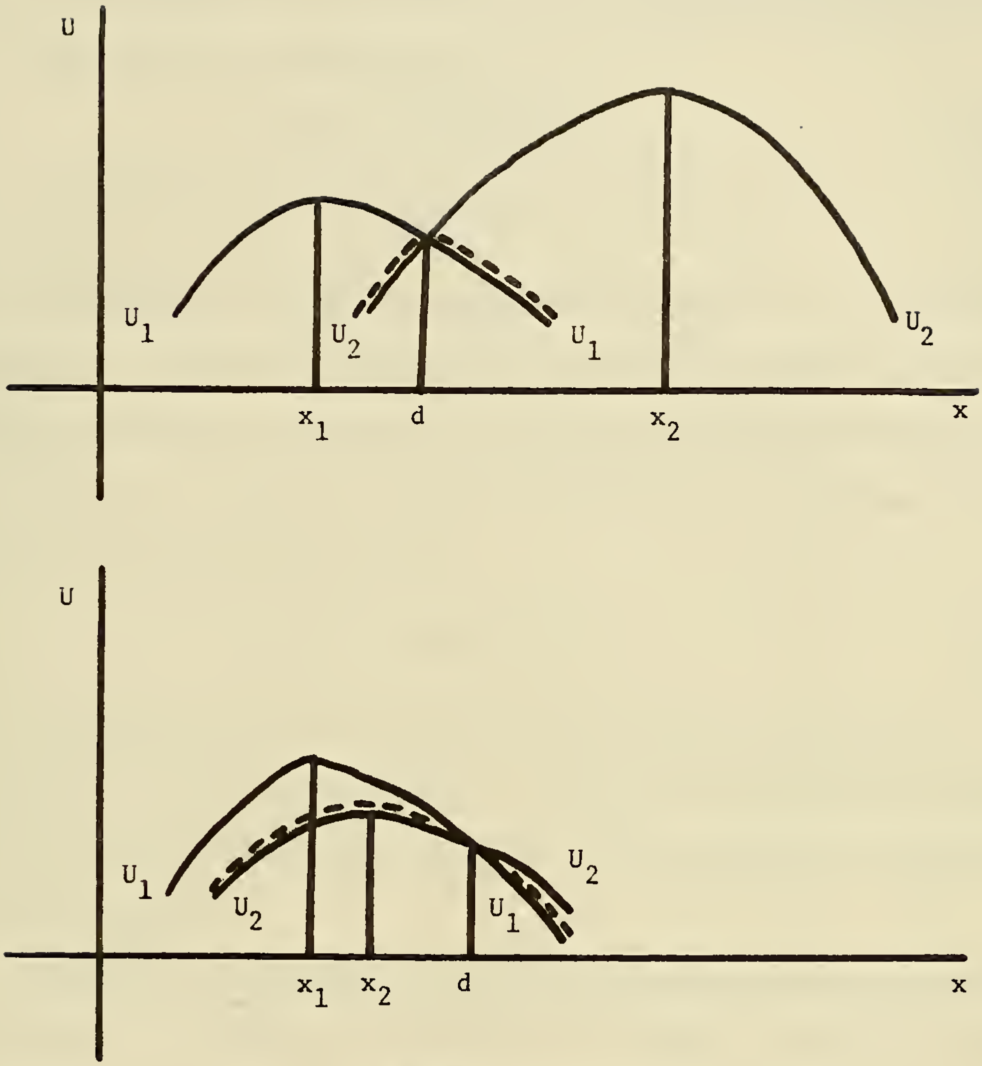


Figure 7

It is clear from the diagrams that the care level chosen satisfies

due care	$d \le x_1$	$x_1 \le d \le x_2$	$d \ge x_2$	
chosen care level /	x_1	d	x_2	(29)

Comparing this with the certainty / case, (14), we have the same pattern except that x_2 replaces $\hat{d}$. Since U_1 coincides with utility in the absence of liability (and so x_1 coincides with $x°$) we have two possible uniform equilibria with everyone nonnegligent at all times, $x°$ for $d \le x°$ and d for $x_1(d) \le d \le x_2(d)$. When $d > x_2(d)$, we do not have a uniform equilibrium since $y \geq d$. We have the definitions of $x_1(d)$ and $x_2(d)$/ for these limits by maximizing U_1 and U_2 in (26) and (27) evaluated at $y = d$.

$$V'(x_1) + \frac{1}{4} nC(1-d) = 0 \qquad (30)$$

$$V'(x_2) + \frac{1}{4} nC(1-d) + \frac{1}{2} nC \frac{1-d}{(1-x_2)^2} (1-d-x_2+ \frac{1}{2} d^2+ \frac{1}{2} x_2^2) = 0 \qquad (31)$$

Since x_1 is decreasing[17] in d, the lower bound constraint is not binding for d above x°. Since x_2 is decreasing[18] in d when x_2 equals d, there is a unique due care level $\hat{d}$ separating the values of d giving equilibria at d from / higher values of d that do not give equilibria at d. Equating x_2 and d in (31) we have the equation for $\hat{d}$:

$$V'(\hat{d}) + \frac{3}{4} nC(1-\hat{d}) = 0 \qquad (32)$$

Since V' is decreasing in x, comparing (24) and (32) we see that $\hat{d}$ is greater than x*. For greater values of d equilibria involve some negligence. Thus we have demonstrated the first two sections of the locus of equilibria in Figure 6.

To consider the case where in equilibrium everyone may have negligent care levels, we assume that everyone else chooses a value of y less than or equal to d. We can then express expected utility as

$$U(x) = \begin{cases} U_1(x) = V(x) - nC\displaystyle\int_x^1\int_d^1 \left(\frac{1-a}{1-x}\right)\left(\frac{1-b}{1-y}\right) dbda & : x \geq d \\ U_2(x) = V(x) - nC\displaystyle\int_x^1\int_d^1 \left(\frac{1-a}{1-x}\right)\left(\frac{1-b}{1-y}\right) dbda & \\ \qquad - nC\displaystyle\int_x^d\int_y^1 \left(\frac{1-a}{1-x}\right)\left(\frac{1-b}{1-y}\right) dbda & : x < d \end{cases} \qquad (33)$$

The integrals reflect bearing one's own cost when the other is nonnegligent plus an additional cost when one is negligent (one's own when the other person is negligent and his when he is not). Rewriting and integrating we have

$$U_1(x) = V(x) - \frac{1}{4} nC(1-x)(1-d)^2(1-y)^{-1} \quad (34)$$

$$U_2(x) = V(x) - \frac{1}{4} nC(1-x)(1-d)^2(1-y)^{-1}$$
$$- \frac{1}{2} nC(1-y)(d-x- \frac{1}{2}(d^2-x^2))(1-x)^{-1} \quad (35)$$

As above $U_1(x) \gtreqless U_2(x)$ as $x \lesseqgtr d$ and $U_2'(x) > U_1'(x)$. Thus we have the same possibilities depicted in Figure 7 and described in (29). Now, however, $y < d$ so we have a possible uniform equilibrium at $x_2(d)$. Differentiating $U_2(x)$ and equating y with x we have the equation for uniform equilibrium for $d > x_2(d)$

$$-V'(x) = \frac{1}{4} nC(1-d)^2(1-x)^{-1} + \frac{1}{2} nC(1-x) - \frac{1}{2} nC(d-x- \frac{1}{2}(d^2-x^2))(1-x)^{-1}$$
$$= \frac{1}{2} nC(1-x)^{-1}(1-d)^2 + \frac{1}{4} nC(1-x) \quad (36)$$

This is then the equation for the remaining section of the locus of equilibria in Figure 6. To check its shape, let us differentiate implicitly to get the effect of increased due care on the equilibrium care level

$$\frac{dx}{dd} = \frac{nC(1-d)(1-x)^{-1}}{V'' + \frac{1}{2} nC(1-d)^2(1-x)^{-2} - \frac{1}{4} nC} < 0 \quad (37)$$

The denominator is negative by the second order condition. Clearly, as d approaches one, the solution to (36) approaches $x°$.

To see how increases in the due care level can decrease the level of precaution, let us examine the structure of the individual decision. In choosing a precaution level, the individual examines, at the margin, the decreased probability of accident and increased likelihood of being nonnegligent relative to the cost of increasing the precaution level. An increase in the due care standard decreases the likelihood that the person will be nonnegligent at the time of an accident, and so increases the expected accident cost that must be borne by the individual. This serves as an incentive to increase precaution. However, an increased due care standard also decreases the likelihood that the other party to any accident will be nonnegligent. This decreases the incentive to take more precaution since the other person's costs are less likely to be borne. The balance of these two forces determines whether precaution rises or falls with the due care standard. At the efficient precaution level, precaution is still rising with the due care standard.

7. Stochastic Control of Care II

With precaution as an upper bound for care, there is some chance of being negligent at the time of an accident even with low due care standards. Thus the introduction of a negligence system with a due care standard below the equilibrium precaution level tends to increase the equilibrium level of precaution. This differs sharply from the determinate case where low due care standards had no effect at all. Since increased precaution decreases the probability of being negligent or contributorily negligent, the presence of a due care standard increases the benefits from increased precaution in this region. If the due care standard is set very high, however, individuals may choose a low level of precaution, realizing that they will be negligent in every accident they might have, but avoiding the high cost of a precaution level above the due care standard. This situation exactly parallels the determinate case and seems to allow the possibility of no or two uniform equilibria. It is no longer necessarily true that the efficient equilibrium is achievable, however. Thus the patterns of equilibria are shown in Figure 8.[19]

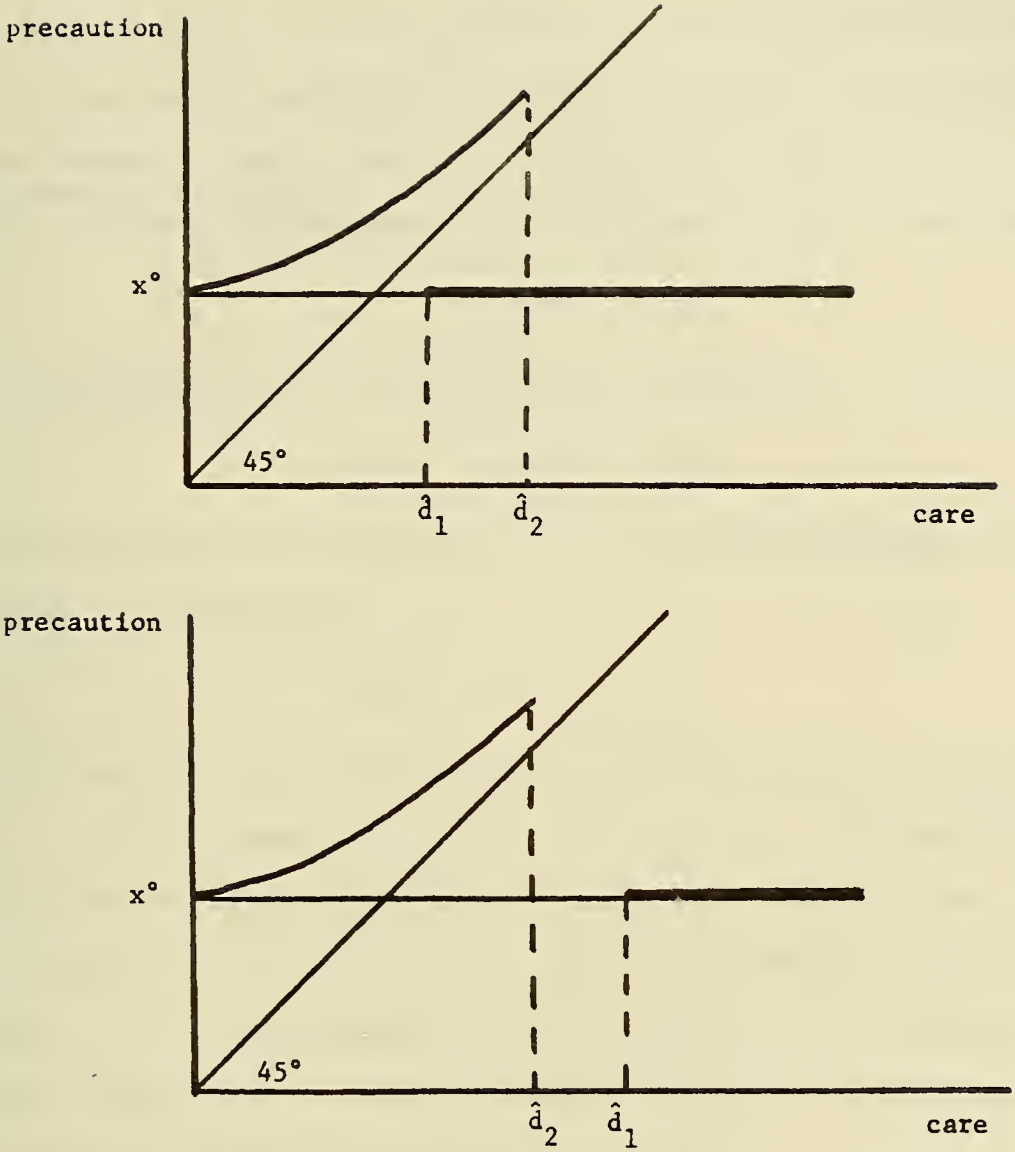


Figure 8. Equilibrium with Precaution as an Upper Bound for Care

Now let us derive this pattern of results. In the absence of a tort system, with everyone else taking the same precaution level y, expected utility of an individual taking precaution level x satisfies

$$U(x) = V(x) - nC\int_0^x\int_0^y \frac{1-a}{x}\,\frac{1-b}{y}\,dbda$$

$$= V(x) - nC(1 - \tfrac{1}{2}x)(1 - \tfrac{1}{2}y) \qquad (38)$$

where $(1 - a)(1 - b)$ is the probability of an accident when care levels a and b are present and $\frac{1}{x}$ and $\frac{1}{y}$ are the probabilities of these care levels for precaution levels x and y (when care is less than precaution). C is assumed to be a constant.

Maximization of expected utility gives the first order condition

$$V'(x) = -\frac{1}{2}nC(1 - \frac{1}{2}y) \tag{39}$$

For equilibrium x and y coincide giving the equation for equilibrium

$$V'(x^\circ) = -\frac{1}{2}nC(1 - \frac{1}{2}x^\circ) \tag{40}$$

For efficiency we set y equal to x in ( ) and then maximize. This gives the first order condition

$$V'(x^*) = -nC(1 - \frac{1}{2}x^*) \tag{41}$$

From the concavity of V we have $x^* > x^\circ$ so too little care is taken.

With a standard of due care, d, applicable to both negligence and contributory negligence and with uniform equilibrium either everyone is always negligent, $x < d$, or people may be nonnegligent some of the time. Let us start with the case $y < d$ where everyone else is always negligent, looking for a uniform equilibrium of this sort. We must distinguish utility depending on whether x is chosen above or below d

$$U(x) = \begin{cases} U_1(x) = V(x) - nC\int_0^d\int_0^y \left(\frac{1-a}{x}\right)\left(\frac{1-b}{y}\right)dbda & x \geq d \\ U_2(x) = V(x) - nC\int_0^x\int_0^y \left(\frac{1-a}{x}\right)\left(\frac{1-b}{y}\right)dbda & x < d \end{cases} \tag{42}$$

Unlike the determinate case there is no discontinuity in utility. For x below d everyone is negligent in all accidents and each one bears his own costs. For x above d, the individual only bears his own costs when his care level is below d. Performing the integration in the definition of utility, we can express utilities as

$$U_1(x) = V(x) - nCx^{-1}(d - \frac{1}{2} d^2)(1 - \frac{1}{2} y) \tag{43}$$

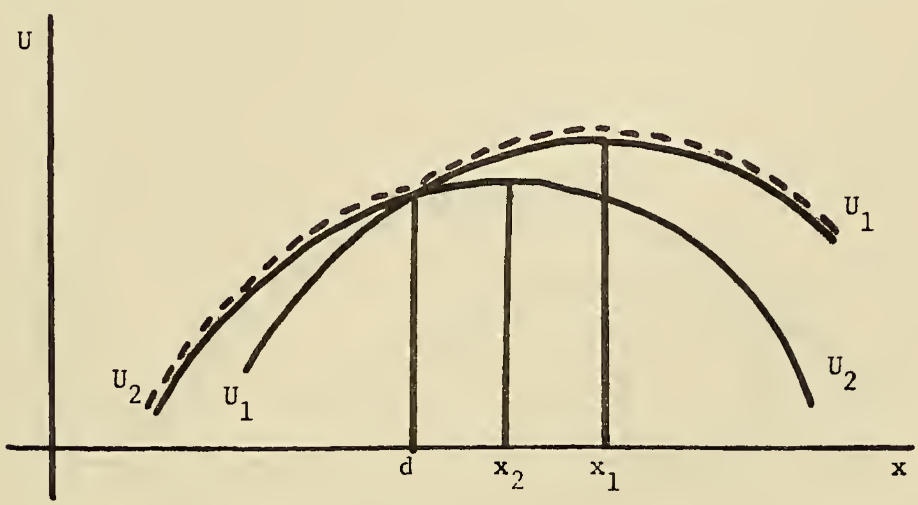
$$U_2(x) = V(x) - nC(1 - \frac{1}{2} x)(1 - \frac{1}{2} y) \tag{44}$$

Note that U_2 coincides with utility in the absence of liability and that $U_1(x) \gtreqless U_2(x)$ as $x \gtreqless d$. Also $U_1'(x) > U_2'(x)$ at $x = d$. Let us denote the utility maximizing levels of care, ignoring the constraints on the domain, by $x_1(d)$ and x_2. Note that x_2 is independent of d and coincides with $x°$ when it coincides with y. Then, the x_i are defined by setting the derivatives of U_i equal to zero:

$$V'(x_1) = - nCx_1^{-2}(d - \frac{1}{2} d^2)(1 - \frac{1}{2} y) \tag{45}$$

$$V'(x_2) = - \frac{1}{2}nC(1 - \frac{1}{2} y) \tag{46}$$

There are four possible configurations of utilities given the constraints above. These are shown in Figure 9 with U given as the dotted curve. The four possibilities depend on the relative positions of d, x_1, and x_2. (The remaining situation, $x_1 < d < x_2$, is ruled out by the conditions above.)



$d < x_1 \quad d < x_2$

$U_1(x_1) > U_2(x_2)$

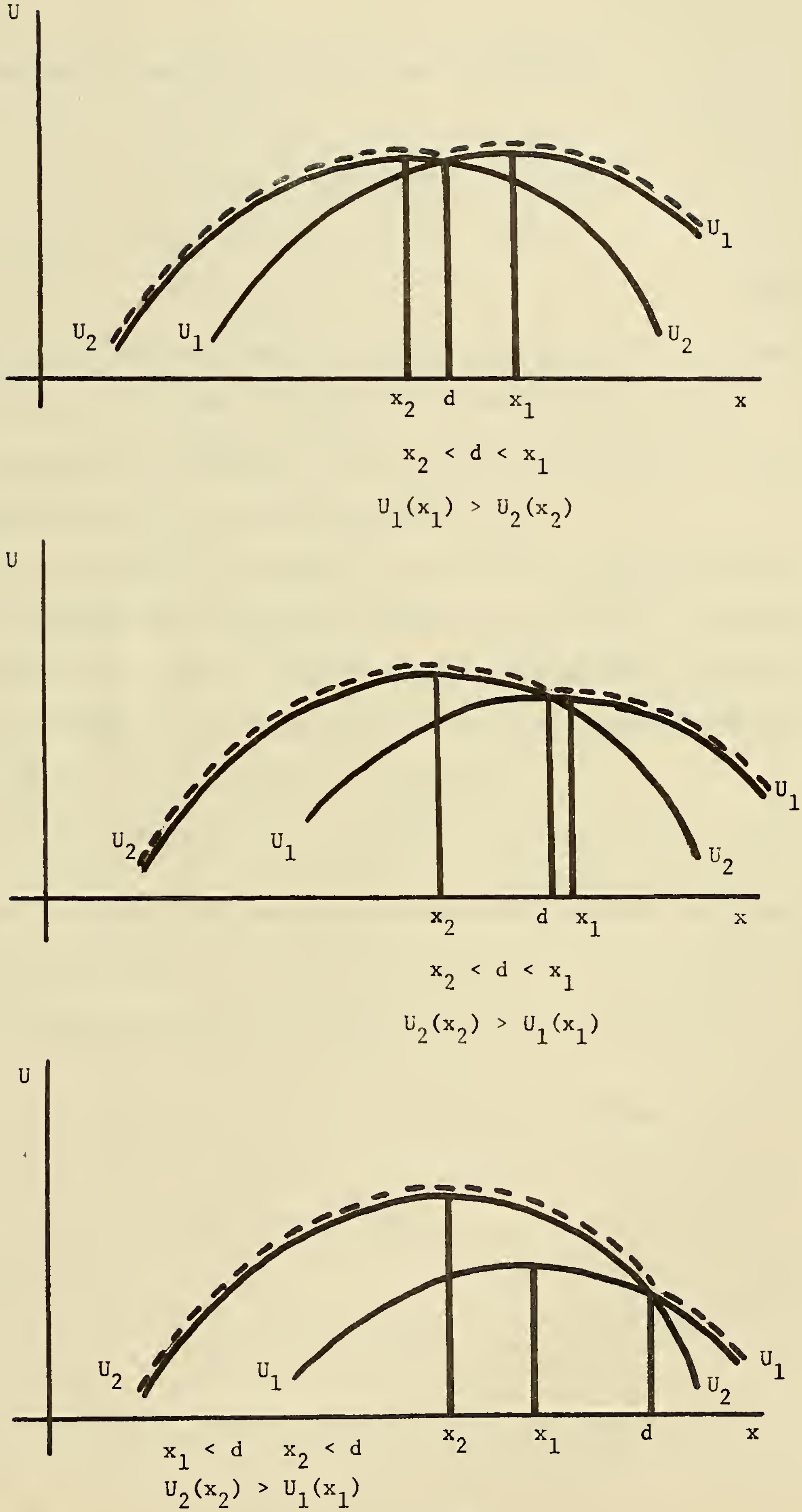

U
U₁
U₂
U₁
U₂
x₂
d
x₁
x
$x_2 < d < x_1$
$U_1(x_1) > U_2(x_2)$
U
U₂
U₁
U₁
U₂
x₂
d
x₁
x
$x_2 < d < x_1$
$U_2(x_2) > U_1(x_1)$
U
U₂
U₁
U₁
U₂
x₂
x₁
d
x
$x_1 < d \quad x_2 < d$
$U_2(x_2) > U_1(x_1)$

Figure 9

As the figures show we have the choice of care satisfying

$$\begin{aligned} x &= x_1 && \text{for } d < x_2 \\ x &= x_1 && \text{for } x_1 > d > x_2,\ U_1(x_1) \geq U_2(x_2) \\ x &= x_2 && \text{for } x_1 > d > x_2,\ U_1(x_1) \leq U_2(x_2) \\ x &= x_2 && \text{for } d > x_1 \end{aligned} \tag{47}$$

The only case which can give rise to a uniform distribution ($x = y$) must have $x < d$ (since $y < d$). This corresponds to the choice of x_2 under either of its possible circumstances. Note that d does not affect U_2 so that x_2 coincides with $x°$ at the uniform equilibrium.

Now let us consider the range of d resulting in this equilibrium. From the four diagrams we see that x_2 is chosen if and only if $U_1(x_1(d)) \leq U_2(x_2)$ evaluated at $y = x_2 = x°$. Since U_1 is decreasing in d, there will be a unique $\hat{d}_1$ serving as the lower bound for due care levels giving rise to this equilibrium. $\hat{d}_1$ is defined by the equation

$$V(x°) - nC(1 - \frac{1}{2} x°)^2 = \underset{x}{\text{Max}}\ V(x) - nCx^{-1}(1 - \frac{1}{2} x°)(\hat{d}_1 - \frac{1}{2} \hat{d}_1^{\,2}) \tag{48}$$

Now let us examine the case where others are not always negligent, $y > d$. To calculate utility, we must distinguish the cases where the individual is sometimes negligent and where he is always negligent.

$$U(x) = \begin{cases} \bar{U}_1(x) = V(x) - nCx^{-1}y^{-1}\Big[\int_0^d\int_0^d (1-a)(1-b)\,db\,da \\ \quad + 2\int_0^d\int_d^y (1-a)(1-b)\,db\,da \\ \quad + \int_d^x\int_d^y (1-a)(1-b)\,db\,da\Big] & : x \geq d \\ \bar{U}_2(x) = V(x) - nCx^{-1}y^{-1}\Big[\int_0^x\int_0^d (1-a)(1-b)\,db\,da \\ \quad + 2\int_0^x\int_d^y (1-a)(1-b)\,db\,da\Big] & : x < d \end{cases} \tag{49}$$

Performing the integrations we have

$$\bar{U}_1(x) = V(x) - nCx^{-1}y^{-1}[(d - \frac{1}{2} d^2)^2 + 2(d - \frac{1}{2} d^2)(y-d- \frac{1}{2}(y^2-d^2))$$
$$+ (x - d - \frac{1}{2} (x^2 - d^2))(y - d - \frac{1}{2} (y^2 - d^2))] \quad (50)$$

$$\bar{U}_2(x) = V(x) - nCx^{-1}y^{-1}[(x - \frac{1}{2} x^2)(d - \frac{1}{2} d^2)$$
$$+ 2(x - \frac{1}{2} x^2)(y - d - \frac{1}{2} (y^2 - d^2))] \quad (51)$$

Rearranging terms, we can write these as

$$\bar{U}_1(x) = V(x) - nCx^{-1}y^{-1}[(d - \frac{1}{2} d^2)(y - \frac{1}{2} y^2) \quad (52)$$
$$+ (x - \frac{1}{2} x^2)(y - d - \frac{1}{2} (y^2 - d^2))]$$

$$\bar{U}_2 = V(x) - nC \; y^{-1}[(1 - \frac{1}{2} x)(2y - d - \frac{1}{2} (2y^2 - d^2))] \quad (53)$$

As before, we have $\bar{U}_1(x) \gtreqless \bar{U}_2(x)$ as $x \gtreqless d$ and $\bar{U}_1'(x) > \bar{U}_2'(x)$ at $x = d$. We denote the maximizing levels of care for the two functions by $\bar{x}_1(d)$ and $\bar{x}_2(d)$. Thus the possible configurations are the same as in Figure 9 leading to the same description of choice, (47) with $\bar{U}_i$ replacing U_i. Now we will have a uniform equilibrium if and only if $\bar{U}_1(\bar{x}_1(d)) \geq \bar{U}_2(\bar{x}_2(d))$ both evaluated at $y = x_1(d)$.

For the uniform equilibrium in this situation, we have the first order condition for $\bar{x}_1$, obtained by differentiating (52),

$$V'(x) = -nCx^{-2}y^{-1}[(d - \frac{1}{2} d^2)(y - \frac{1}{2} y^2)$$
$$+ (x - \frac{1}{2} x^2)(y - d - \frac{1}{2} (y^2 - d^2))]$$
$$+ nCx^{-1}y^{-1}[(1 - x)(y - d - \frac{1}{2} (y^2 - d^2))] \quad (54)$$

This needs to be satisfied at $x = y$, giving the condition for equilibrium

$$-V'(x) = nCx^{-1}(1 - \frac{1}{2} x)^2 - nCx^{-1}(1 - x)(1 - \frac{1}{2} x)$$
$$+ nCx^{-2}(1 - x)(d - \frac{1}{2} d^2)$$
$$= \frac{1}{2} nC(1 - \frac{1}{2} x) + nCx^{-2}(1 - x)(d - \frac{1}{2} d^2) \quad (55)$$

Given this range of equilibria, it is natural to ask how x changes with d. Differentiating (55) we have

$$\frac{dx}{dd} = \frac{nC(1 - x)(1 - d)}{-x^2V'' + \frac{1}{4}nCx^2 + nC(d - \frac{1}{2}d^2)(2x^{-1} - 1)} > 0 \quad (56)$$

Thus the equilibrium level of precaution increases with the due care level throughout this range. Thus care rises with the due care level in the region where this equilibrium occurs. I have not shown that the set of equilibria is in fact an interval. We denote by $\hat{d}_2$ the maximal level of d giving rise to this type of equilibrium (i.e., maximal d satisfying $\bar{U}_1 = \bar{U}_2$)

Let us examine whether x* is a possible equilibrium. First let us find the due care level d* so that x* is the solution to (55), i.e.,

$$x^* = \bar{x}_1(d^*) \quad (57)$$

From the definitions of $\bar{x}_1$, (55), and of x*, (41), we see that $x^* = \bar{x}_1$ when

$$(1 - \frac{1}{2}x^*) = \frac{1}{2}(1 - \frac{1}{2}x^*) + x^{*-2}(1 - x^*)(d - \frac{1}{2}d^2) \quad (58)$$

Rearranging terms we have

$$(d^* - \frac{1}{2}d^{*2}) = (x^* - \frac{1}{2}x^{*2})\frac{x^*}{2(1-x^*)} \quad (59)$$

Now let us examine the utility comparison condition, making use of the definition of d*

$$\begin{aligned}\bar{U}_1(x^*) - \bar{U}_2(\bar{x}_2) &= V(x^*) - nC(1 - \frac{1}{2}x^*)^2 \\ &- \underset{x}{\text{Max}}\,[V(x) - nC(1 - \frac{1}{2}x)x^{*-1}(2x^* - d^* - \frac{1}{2}(2x^{*2} - d^{*2}))] \\ &= \underset{x}{\text{Min}}\,[V(x^*) - nC(1 - \frac{1}{2}x^*)^2 \\ &- V(x) + nC(1 - \frac{1}{2}x)(1 - \frac{1}{2}x^*)(2 - \frac{x^*}{2(1-x^*)})] \end{aligned} \quad (60)$$

This minimization is strictly less than the value achieved at x*. (since the minimizing x never coincides with x*).

$$\bar{U}_1(x^*) - \bar{U}_2(\bar{x}_2) < nC(1 - \frac{1}{2}x^*)^2(2 - \frac{x^*}{2(1-x^*)} - 1)$$

$$= nC(1 - \frac{1}{2}x^*)^2(\frac{2-3x^*}{2(1-x^*)}) \quad (61)$$

Since the right-hand side is nonpositive for $x^* \geq \frac{2}{3}$ the efficient equilibrium is not achievable for $x^* \geq \frac{2}{3}$.

8. Stochastic Control of Care[20] III

Some of the features of the two examples above can be combined into a single model if we assume that care has a triangular distribution with its peak at the point of precaution (we continue to assume that costs per accident are constant). This relationship is shown in Figure 10.

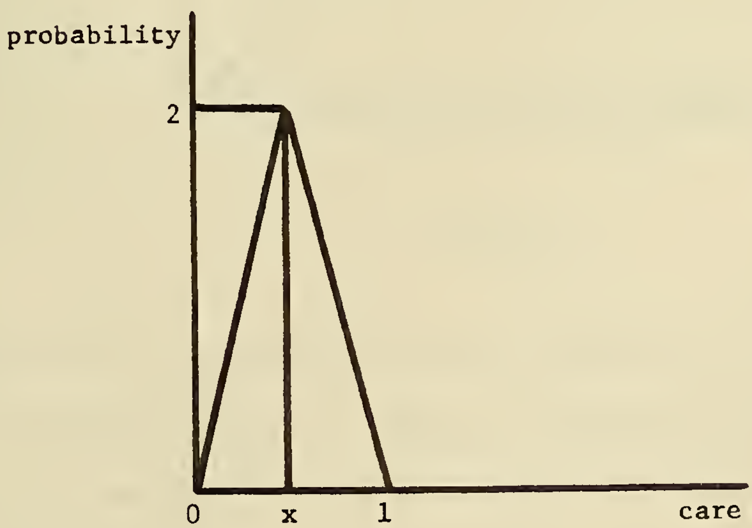


Figure 10

Now whatever the due care standard (within the unit interval) an individual will be negligent some of the time and will show more than due care some of the time. The level of due care now affects the equilibrium at all levels. The equilibrium level of precaution is now continuous in the due care level, rising up to a point and then declining with further increases in the due care standard (although there is a kink in the curve where it crosses the 45° line). With the due care standard at either extreme, we have the same equilibrium as in the absence of liability since either everyone is negligent or no one is. In Figure 11 is shown a typical locus of equilibria.

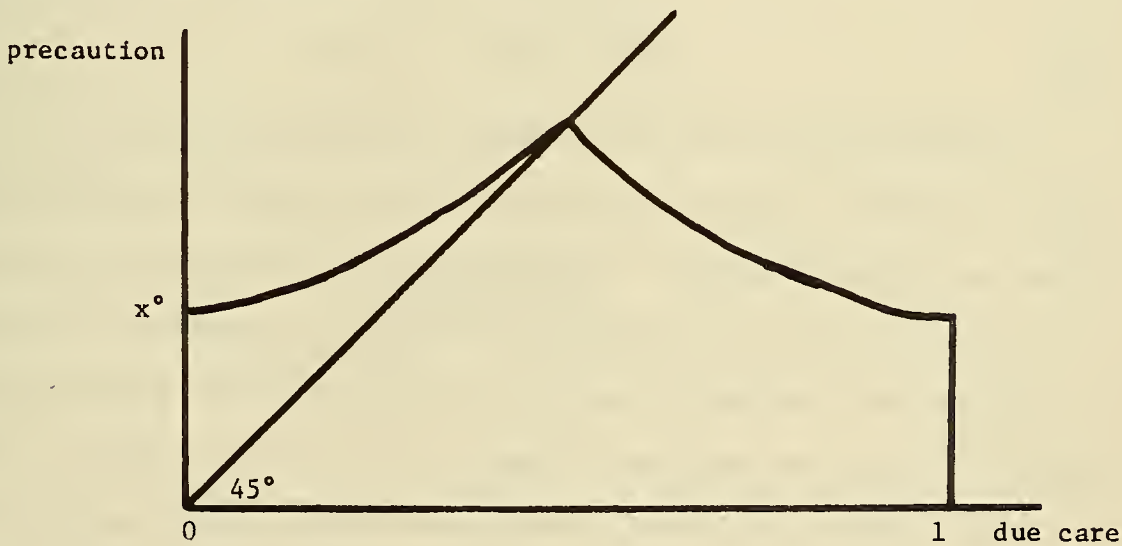


Figure 11

The ability to achieve the efficient equilibrium (i.e. whether x* is below the maximal level of x in Figure 11) depends on the parameters of the utility function. As with the previous section it is sometimes achievable and sometimes not.

Now let us derive these results. With a triangular distribution the density function f(a) for attention is $\frac{2a}{x}$ for $a \leq x$ and $\frac{2(1-a)}{(1-x)}$ for $a \geq x$. The expected value of (1-a) is thus $\frac{2}{3}(1 - \frac{1}{2}x)$. Thus expected utility in the absence of a liability system, assuming that everyone else has chosen care level y is

$$U(x) = V(x) - \frac{4nC}{9}(1 - \frac{1}{2}x)(1 - \frac{1}{2}y) \tag{62}$$

In the same way as above we can derive the no-liability equilibrium level as the solution to

$$V'(x^\circ) = -\frac{2nC}{9}(1 - \frac{1}{2}x^\circ) \tag{63}$$

and the efficient care level as the solution to

$$V'(x^*) = -\frac{4nC}{9}(1 - \frac{1}{2}x^*) \tag{64}$$

It is now the case that negligence and due care are possible for each individual for any level of the due care standard. However, we will consider the cases with $x \gtrless d$ separately for convenience since the expressions for the density of care when negligent differ in these two cases. Let us first write expected utility in terms of the density function covering both cases. One bears ones own costs whenever negligent and when nonnegligent and the other party is also nonnegligent. One bears the other's costs when negligent when he is not. Thus

$$U(x) = V(x) - nC[\int_0^d (1 - a)f(a,x)da \int_0^1 (1 - b)f(b,y)db$$

$$+ \int_d^1 (1 - a)f(a,x)da \int_d^1 (1 - b)f(b,y)db$$

$$+ \int_0^d (1 - a)f(a,x)da \int_d^1 (1 - b)f(b,y)db]$$

$$= V(x) - nC[\int_0^d (1 - a)f(a,x)da \int_0^1 (1 - b)f(b,y)db$$

$$+ \int_0^1 (1 - a)f(a,x)da \int_d^1 (1 - b)f(b,y)db] \qquad (65)$$

Now let us consider the case $y < d$. Performing the integrations we have

$$U(x) = \begin{cases} U_1(x) = V(x) - \frac{4nC}{3} [(1 - \frac{1}{2} y)x^{-1}(\frac{d^2}{2} - \frac{d^3}{3}) \\ \quad + (1 - \frac{1}{2} x) \frac{(1-d)^3}{3(1-y)}] & x \geq d \\ U_2(x) = V(x) - \frac{4nC}{9} [(1 - \frac{1}{2} y)(1 - \frac{1}{2} x - \frac{(1-d)^3}{1-x}) \\ \quad + (1 - \frac{1}{2} x) \frac{(1-d)^3}{(1-y)}] & x < d \end{cases} \qquad (66)$$

To see the relationship between the functions let us calculate the derivative of the difference between them

$$U_1(x) - U_2(x) = \frac{4nC}{9} (1 - \frac{1}{2} y)(1 - \frac{1}{2} x - \frac{(1-d)^3}{(1-x)} - \frac{3d^2}{2x} + \frac{d^3}{x}) \qquad (67)$$

$$= \frac{2nC}{9} (1 - \frac{1}{2} y)x^{-1}(1 - x^{-1})(-3x^2 + x^3 + 6xd - 3xd^2 - 3d^2 + 2d^3)$$

$$U_1'(x) - U_2'(x) = \frac{4nC}{9} (1 - \frac{1}{2} y)(- \frac{1}{2} - \frac{(1-d)^3}{(1-x)^2} + \frac{3d^2}{2x^2} - \frac{d^3}{x^2})$$

$$= \frac{2nC}{9} (1 - \frac{1}{2} y)(1 - x)^{-2}x^{-2}(-3x^2 + 2x^3 - x^4 + 6dx^2$$

$$- 3d^2x^2 - 6d^2x + 4d^3x + 3d^2 - 2d^3) \qquad (68)$$

Thus $U_1 - U_2$ and $U_1' - U_2'$ are both zero when $x = d$. To see the relationship over the rest of the range of x we note that

$$\text{sign } (U_1(x) - U_2(x)) = \text{sign } (-3x^2 + x^3 + 6xd - 3xd^2 - 3d^2 + 2d^3) \qquad (69)$$

The expression on the right is zero when $x = d$ and positive everywhere else in the unit interval since the sign of its derivative with respect to x is the same as that of (d - x).

Given these facts we see that U_2 is at least as large as U_1, with the two functions tangent at d. U coincides with U_2 below d and U_1 above it. Thus the situation coincides with one of the two positions in Figure 12, with U dotted in the figure.

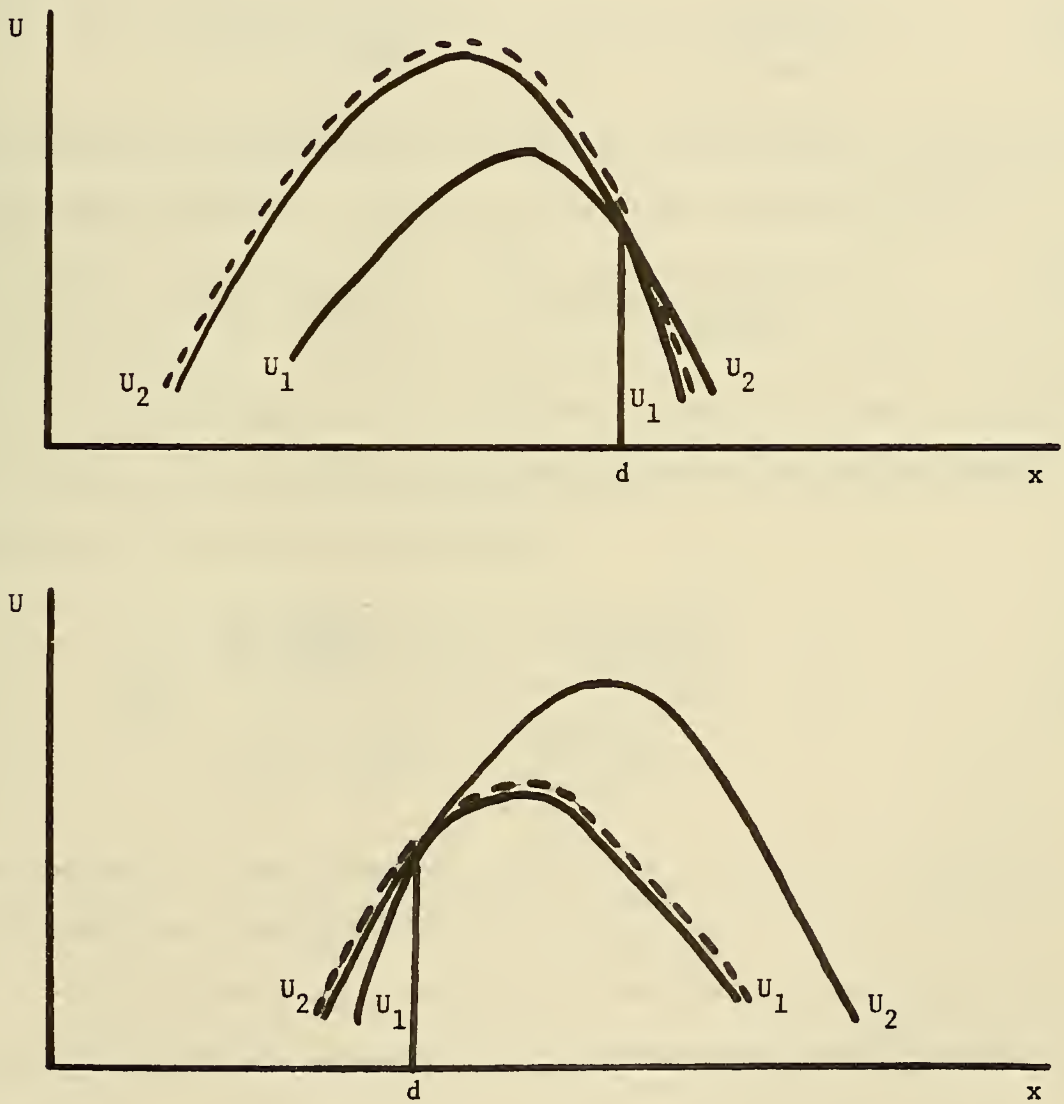


Figure 12

Thus the level of care chosen maximizes U_1 or U_2, whichever maximum is part of U. This depends on whether d is less than or greater than x_1 and x_2, the values which maximize U_1 and U_2. For $d \geq x_2$, x_2 is chosen, for $d < x_2$, x_1 is chosen. Since the calculation is done for $y < d$, we get a uniform equilibrium at x_2. Since x_2 maximizes U_2 we obtain its equation by maximizing U_2

$$V'(x) + \frac{4nC}{9}\left[\left(\frac{1}{2}\right)\frac{(1-d)^3}{(1-y)} + \left(1 - \frac{1}{2}y\right)\left(\frac{1}{2} + \frac{(1-d)^3}{(1-x)^2}\right)\right] = 0 \qquad (70)$$

The equation for equilibrium is obtained by substituting $y = x_2$ in the first order condition. Thus for $x_2 < d$ we have equilibrium where

$$V'(x_2) + \frac{2nC}{9}\left[\frac{(1-d)^3}{(1-x_2)} + \left(1 - \frac{1}{2}x_2\right)\left(1 + \frac{2(1-d)^3}{(1-x_2)^2}\right)\right] = 0 \qquad (71)$$

To examine this range of solutions we note first that x_2 coincides with $x°$ at $d = 1$. Differentiating (71) implicitly we get the slope of this part of the locus of equilibria

$$\frac{dx_2}{dd} = \frac{\frac{2nC}{3}\left\{\frac{(1-d^2)}{(1-x_2)} + 2\left(1 - \frac{1}{2}x_2\right)\frac{(1-d)^2}{(1-x_2)^2}\right\}}{V'' - \frac{nC}{9} + \frac{8nC}{9}\frac{\left(1 - \frac{1}{2}x_2\right)(1-d)^3}{(1-x_2)^3}} < 0 \qquad (72)$$

The numerator is clearly positive, while the denominator is less than the second order condition.[21] At $d = 1$, $\frac{dx_2}{dd} = 0$.

We can examine the other part of the locus of equilibria in the same fashion. We now assume $y \geq d$. Performing the integrations in (65) we have

$$U(x) = \begin{cases} \bar{U}_1(x) = V(x) - \frac{2nC}{9}\,[x^{-1}(1 - \frac{1}{2}\,y)(3d^2 - 2d^3) \\ \qquad + (1 - \frac{1}{2}\,x)(2 - y - y^{-1}(3d^2 - 2d^3)]\} \\ \bar{U}_2(x) = V(x) - \frac{2nC}{9}\,[(2 - x - \frac{2(1-d)^3}{(1-x)})(1 - \frac{1}{2}\,y) \end{cases}$$

$$+ (1 - \frac{1}{2}\,x)(2 - y - y^{-1}(3d^2 - 2d^3)]\} \tag{73}$$

Since

$$\bar{U}_2(x) - \bar{U}_1(x) = U_2(x) - U_1(x) \tag{74}$$

the discussion above on the relative positions of the curves carries over. Thus x_1 is chosen for $d < x_2$ and x_2 is chosen for $d \geq x_2$. For a uniform equilibrium we have $\bar{x}_1 = y \geq d$. The equation for this part of the locus is obtained by calculating the first order condition for the maximization of $\bar{U}_1$ and evaluating at $y = \bar{x}_1$

$$V'(x) + \frac{2nC}{9}\,[x^{-2}(1 - \frac{1}{2}\,y)(3d^2 - 2d^3)$$

$$+ \frac{1}{2}(2 - y - y^{-1}(3d^2 - 2d^3))] = 0 \tag{75}$$

$$V'(\bar{x}_1) + \frac{2nC}{9}\,[(\bar{x}_1^{-2} - \bar{x}_1^{-1})(3d^2 - 2d^3) + 1 - \frac{1}{2}\,\bar{x}_1] = 0 \tag{76}$$

Again, (76) can be differentiated to obtain the slope of this part of the locus of equilibria (also $\bar{x}_1 = x°$ and $\frac{d\bar{x}_1}{dd} = 0$ at $d = 0$).

$$\frac{d\bar{x}_1}{dd} = - \frac{\frac{2nC}{9}\,(\bar{x}_1^{-2} - \bar{x}_1^{-1})(6d - 6d^2)}{V'' - \frac{2nC}{9}\,[(2\bar{x}_1^{-3} - \bar{x}_1^{-2})(3d^2 - 2d^3) + \frac{1}{2}]} > 0 \tag{77}$$

Thus the part of the locus above the 45° line is rising, the part below is falling. From (71)/and (76) we confirm that each part hits the 45° line at the same value of d. From the slope conditions there is a kink in the locus at this point. This justifies the description of the shape of the curve given at the start of this section. It remains to check when x* is obtainable. The greatest obtainable equilibrium value of x occurs at x = d. From either (71) or (76) this value satisfies the equation

$$V'(x) = -\frac{2nC}{9}\left(4 - \frac{11x}{2} + 2x^2\right) \tag{78}$$

Since V is concave x* is achievable when the right-hand side of (64) is less than the right-hand side of (78); that is, when

$$\frac{2nC}{9}(2 - x) \leq \frac{2nC}{9}\left(4 - \frac{11x}{2} + 2x^2\right) \tag{79}$$

Solving this expression we see that the efficient solution is obtainable when

$$x^* \leq \frac{9 - \sqrt{17}}{8} \sim .6 \tag{80}$$

Thus the efficient solution is not always attainable.

The introduction of uncontrolled elements in individual behavior alters somewhat the description of equilibrium as a function of the level of the due care standard. It introduces the realistic element of the presence of successful law suits, at least for some range of judicial selection of the due care standard. It also tends to remove discontinuities in the response of equilibrium to the due care standard. However, the general picture of the impact of different due care standards on accidents is preserved--the presence of a negligence system tends to increase the level of care; with a higher due care standard increasing care in the low range, but decreasing it when the due care standard gets very high.

9. Stochastic Measurement of Care

Underlying the models discussed above was the assumption that the legal system accurately measured the level of care at the time of an accident. The stochastic models given above can also be interpreted as describing a situation where individual decisions are determinate but stochastic elements are present in the attempt by a court to measure what occurred at the time of the accident. To carry over the analysis we need to assume that the choice of any care level by the individual gives rise to a probability distribution of possible care measurements by a court. The first two cases described correspond to possible over and under estimation of care taken. To carry over the analysis we need two further assumptions--that the distribution of errors of measurement are independent of the due care standard, and that the errors of measurement relative to the two parties to an accident are also independent. It is not clear that these are good assumptions.

From the diagrams, we see that with the tendency to overestimate care taken, setting the due care level at the efficient care level results in an efficient equilibrium. With underestimation of care taken, the due care level must be set below the efficient care level to achieve the efficient equilibrium when it can be achieved at all.

10. Comparative Negligence[23]

There are two different forms of comparative negligence which have been employed--either total accident costs, or those of the less negligent, are divided between the two parties in fractions reflecting their relative contributions toward causing the accident.[24] By converting negligence issues to differences in degree rather than kind the legal process is considerably changed. We wish to ask, however, the impact of comparative negligence on care decisions. With differences across people the selection of comparative negligence (rather than negligence) will alter relative income distributions (loss-bearing) both ex ante and ex post. This selection also affects care choices, and so, efficiency.

Given the symmetry that comes from assuming that everyone is the same, there are two differences between comparative negligence (sharing total costs) and an absence of liability. The first is that one expects to bear some fraction of the costs of others to match the fraction of one's costs borne by others. Where there are decisions which affect the two parties to an accident differently, this alters incentives. Secondly the perception that increased care decreases the fraction of total costs borne serves to induce more care. The more sharply the cost sharing fractions change with the care decisions, the greater the inducement to increased care, and the greater the care taken in equilibrium (under suitable assumptions). As the shares become less responsive to care differences between the two parties, as in other words, the system tends toward simply dividing accident costs between the parties, the ex ante equilibrium tends toward a position that only differs from the no-liability equilibrium because of the different impact of care decisions on the two parties to the accident. When care affects accident probabilities but not costs, the equilibrium tends to the no-liability equilibrium as care differences decrease in importance in cost allocation. These points are brought out clearly in the determinate model used above where the ability to select the cost sharing fraction permits the choice of the efficient equilibrium. In the determinate case, however, in all accidents both parties are equally negligent so the key element is merely the change in the fraction of cost bearing with increased care at this point. When we consider the stochastic model used above, a wide range of possible cost allocations have positive probabilities bringing the shape of the entire curve into the determination of equilibrium. Rather than considering all possible cost divisions, we shall consider the class of divisions $\left(\frac{e-a}{2e-a-b}, \frac{e-b}{2e-a-b} \right)$ for different values of e. As e increases, this division converges to equal division and the equilibrium level of precaution decreases, converging to the no-liability equilibrium. The efficient allocation is only achievable in some cases with this class of division rules (specifically for $x^* \leq \frac{2}{3}$).

Let us start with the determinate case. We follow the notation of section 5 above. Let x and y be levels of care taken by the two parties to the accident; $\pi(x,y)C(x,y)$ and $\pi(x,y)C(y,x)$, the expected values to each party of costs from accidents between parties having these care levels (π symmetric); and $n + 1$ the number of identical individuals. Let $\phi(x,y)$ be the fraction of costs borne by person having care level x at the time of an accident ($\phi(x,y) + \phi(y,x) = 1$). As in section 5 efficiency is achieved by a uniform care level x^* satisfying

$$V'(x^*) = n\pi(x^*,x^*)C_1(x^*,x^*) + C_2(x^*,x^*))$$
$$+ nC(x^*,x^*)(\pi_1(x^*,x^*) + \pi_2(x^*,x^*)) \qquad (81)$$

and in the absence of any liability system equilibrium occurs at

$$V'(x^\circ) = n\pi(x^\circ,x^\circ)C_1(x^\circ,x^\circ) + nC(x^\circ,x^\circ)\pi_1(x^\circ,x^\circ) \qquad (82)$$

where $V(x)$ is the utility of taking care ($V'' < 0$). Under the comparative negligence system, utility net of costs satisfies (assuming everyone else chooses care level y)

$$U(x) = V(x) - n\pi(x,y)(C(x,y) + C(y,x))\phi(x,y) \qquad (83)$$

The first order condition for utility maximization is

$$V'(x) - n\pi(x,y)(C(x,y) + C(y,x))\phi_1(x,y) - n\pi(x,y)(C_1(x,y) + C_2(y,x))\phi(x,y) - n\pi_1(x,y)(C(x,y) + C(y,x))\phi(x,y) = 0 \quad (84)$$

We have equilibrium at x if (84) is satisfied for x = y.

Comparing the two allocation equations, (81) and (84), we have equilibrium at the efficient point if

$$\pi(x^*,x^*)(C_1(x^*,x^*) + C_2(x^*,x^*)) + C(x^*,x^*)(\pi_1(x^*,x^*) + \pi_2(x^*,x^*))$$
$$= 2\pi(x^*,x^*)C(x^*,x^*)\phi_1(x^*,x^*) + \pi(x^*,x^*)(C_1(x^*,x^*) + C_2(x^*,x^*))\phi(x^*,x^*) + 2\pi_1(x^*,x^*)C(x^*,x^*)\phi(x^*,x^*) \quad (85)$$

Since π is symmetric, $\pi_1(x^*,x^*) = \pi_2(x^*,x^*)$ and $\phi(x^*,x^*) = \frac{1}{2}$. Thus we can write this condition as

$$\pi(C_1 + C_2) + 2C\pi_1 = 2\pi C\phi_1 + \frac{1}{2}\pi(C_1 + C_2) + \pi_1 C \quad (86)$$

or, solving for ϕ_1

$$\phi_1 = \frac{\pi(C_1 + C_2) + 2C\pi_1}{4\pi C} \quad (87)$$

Comparing equilibrium (84) with that in the absence of liability, (82), an individual perceives different elements of gain from further care between the two equilibria of one-half the difference in expected marginal accident cost inflicted on himself and on the other party, $\frac{1}{2}n(\pi(C_2 - C_1) + \pi_1(C(y,x) - C(x,y)))$, plus the gain arising from the marginal decrease in the share of total costs for which he is liable, $\pi(C + C)\phi_1$. If the cost function is symmetric, $C(x,y) = C(y,x)$, the stimulus toward more care is precisely the value of the shift in the liability fraction, ϕ_1. (Of course this difference refers to the functional forms of the cost elements, since different levels of care give different actual values.)

Turning to the stochastic case, we follow the model used in section 7 above, that the choice of a precaution level x generates a uniform probability of care a between 0 and x (x < 1) and that total expected accident costs from accidents between two individuals having care levels a and b at the time of the accident are 2(1 - a)(1 - b)C, where C is a constant. From section 7 we know that efficiency requires a precaution level x* satisfying

$$V'(x^*) = -\, nC(1 - \frac{1}{2}\, x^*) \tag{88}$$

while in the absence of liability we have the equilibrium precaution level satisfying the equation

$$V'(x^\circ) = -\, \frac{1}{2}\, nC(1 - \frac{1}{2}\, x^\circ) \tag{89}$$

Let us examine the equilibrium level as a function of a parameter e which reflects how importantly differences in care are reflected in the division of costs. If costs are divided in proportions $\frac{e-a}{2e-a-b}$ and $\frac{e-b}{2e-a-b}$, (we assume e > x,y throughout) utility net of costs for someone choosing precaution level x when everyone else has chosen precaution level y is

$$U(x) = V(x) - 2nCx^{-1}y^{-1}\int_0^x\int_0^y (1-a)(1-b)\left(\frac{e-a}{2e-a-b}\right)dbda \tag{90}$$

The first order condition for utility maximization is

$$V'(x) + 2nCx^{-2}y^{-1}\int_0^x\int_0^y (1-a)(1-b)\left(\frac{e-a}{2e-a-b}\right)dbda$$

$$- 2nCx^{-1}y^{-1}\int_0^y \frac{(1-x)(1-b)(e-x)}{(2e-x-b)}\, db = 0 \tag{91}$$

We have equilibrium when this condition holds for x = y

$$V'(x) - 2nCx^{-3}\int_0^x\int_0^x\left[\frac{(1-x)(1-b)(e-x)}{(2e-b-x)} - \frac{(1-a)(1-b)(e-a)}{(2e-b-a)}\right]dbda = 0 \quad (92)$$

Let us write this as

$$V'(x) + nCF(x,e) = 0 \quad (93)$$

From this equilibrium condition we know that equilibrium changes with respect to the parameter e satisfy

$$\frac{dx}{de} = \frac{-\partial F/\partial e}{V''/nC + \partial F/\partial x} \quad (94)$$

We shall show that the numerator of this expression is positive and the denominator negative so that a decreased responsiveness of the fraction to care lowers the equilibrium level of precaution. Also we shall show that F tends to $\frac{1}{2}(1 - \frac{1}{2}x)$ as e increases without limit, confirming (from (89)) the convergence to the no-liability equilibrium as cost sharing converges to equal division. We shall see that F takes on the value $(1 - \frac{1}{2}x^*)$ for some value of e (above x^*) if and only if $x^* \leq \frac{2}{3}$. Thus from (88) the efficient equilibrium is only achievable in some situations. The trusting reader uninterested in tedious calculation can skip the remainder of this section where these propositions are derived.

Writing out the expression we have the definition of F

$$F = -2x^{-3}\int_0^x\int_0^x\left[\frac{(1-x)(1-b)(e-x)}{2e-b-x} - \frac{(1-a)(1-b)(e-a)}{2e-b-a}\right]dbda \quad (95)$$

Let us examine the integrand separately. Define

$$f(a,b,e) = \frac{(1-a)(1-b)(e-a)}{2e-b-a} \quad (96)$$

so that

$$F = -2x^{-3}\int_0^x\int_0^x (f(x,b,e) - f(a,b,e))dbda \tag{97}$$

First we note that by symmetry

$$\int_0^x\int_0^x (f(a,b,e) - f(b,a,e))dbda$$

$$= \int_0^x\int_0^x \frac{(1-a)(1-b)(b-a)}{(2e-b-a)} dbda = 0 \tag{98}$$

This implies that

$$\int_0^x\int_0^x \left(\frac{\partial f(a,b,e)}{\partial e} - \frac{\partial f(b,a,e)}{\partial e}\right) dbda = 0 \tag{99}$$

Since by direct calculation we also have

$$f(a,b,e) + f(b,a,e) = (1 - a)(1 - b) \tag{100}$$

we see that

$$\frac{\partial f(a,b,e)}{\partial e} + \frac{\partial f(b,a,e)}{\partial e} = 0 \tag{101}$$

Combining (99) and (101)we see that

$$\int_0^x\int_0^x \frac{\partial f(a,b,e)}{\partial e} dbda = 0 \tag{102}$$

Thus

$$\frac{\partial F}{\partial e} = -2x^{-3}\int_0^x\int_0^x \frac{(1-x)(1-b)(x-b)}{(2e-b-x)^2} dbda < 0 \tag{103}$$

where the inequality follows from b being less than or equal to x and both b and x being less than or equal to one.

Before turning to the derivative of F with respect to x let us calculate the second order condition by differentiating (91) and evaluating at y = x

$$V'' - 4nCx^{-4}\int_0^x\int_0^x f(a,b,e)dbda + 4nCx^{-3}\int_0^x f(x,b,e)db$$

$$+ 2nCx^{-2}\int_0^x \frac{(1-b)}{(2e-x-b)^2}\left[(e-x)^2 + (e-x)(e-b) + (1-x)(e-b)\right]db < 0 \quad (104)$$

Calculating the denominator of (94) (times nC) we have

$$V'' + nC\,\frac{\partial F}{\partial x} = V'' + 6x^{-4}nC\int_0^x\int_0^x (f(x,b,e) - f(a,b,e))dbda$$

$$- 2x^{-3}nC\int_0^x (f(x,x,e) - f(a,x,e))da$$

$$+ 2x^{-3}nC\int_0^x\int_0^x \frac{(1-b)}{(2e-x-b)^2}\left[(e-x)^2\right.$$

$$\left. + (e-x)(e-b) + (1-x)(e-b)\right]dbda \quad (105)$$

The denominator will be negative if (94) - (91) is negative. Calculating this difference we have

$$-2x^{-4}nC\int_0^x\int_0^x f(a,b,e)dbda + 2x^{-3}nC\int_0^x f(x,b,e)db - 2x^{-3}nC\int_0^x (f(x,x,e)-f(a,x,e))d.$$

$$= -2x^{-4}nC\int_0^x\int_0^x \left[f(a,b,e) - f(a,x,e) - f(x,b,e) + f(x,x,e)\right]dbda \quad (106)$$

Let us call the integrand g(a,b,x,e). Thus

$$-2x^{-4}nC\int_0^x\int_0^x g(a,b,x,e)dbda = -x^{-4}nC\int_0^x\int_0^x (g(a,b,x,e) + g(b,a,x,e))dbda$$

$$= -x^{-4}nC\int_0^x\int_0^x (a - x)(b - x)dbda < 0 \quad (107)$$

As e rises without limit F tends to

$$- 2x^{-3}\int_0^x\int_0^x \frac{1}{2}\left[(1-x)(1-b) - (1-a)(1-b)\right]dbda$$

$$= - x^{-3}\int_0^x (a-x)(x - \frac{x^2}{2})da$$

$$= - x^{-3}(\frac{x^2}{2} - x^2)(x - \frac{x^2}{2}) = \frac{1}{2}(1 - \frac{x}{2}) \tag{108}$$

Since x decreases with e, the maximal value of x achievable occurs at x = e. At this point

$$F = 2x^{-3}\int_0^x\int_0^x \frac{(1-a)(1-b)(x-a)}{(2x-b-a)} dbda$$

$$= x^{-3}\int_0^x\int_0^x (1-a)(1-b)dbda$$

$$= x^{-3}(x - \frac{x^2}{2}) = x^{-1}(1 - \frac{x}{2})^2 \tag{109}$$

where (98) and (100)were used to simplify the integration. Thus the maximal achievable precaution level satisfies

$$V'(x) = - nCx^{-1}(1 - \frac{x}{2})^2 \tag{110}$$

We know that the efficient level satisfies

$$V'(x^*) = - nC(1 - \frac{1}{2}x^*) \tag{111}$$

and that V' is decreasing in x. Equating the right-hand sides of (110) and (111)we see that the efficient point is the maximal achievable precaution level when $x^* = \frac{2}{3}$. Higher values of x* are not achievable (i.e. the solution to (110)is less that x*) while all lower values of x* are achievable.

11. Unmeasured Variables

Despite the complications of the analysis thus far, the models have been extremely restricted in that everyone was assumed to be the same and each person made but one decision. In the next section we will examine a difference across individuals in the utility of taking care. In this section we shall consider the interaction of the due care standard for care (with the distribution of care determined by a single precaution variable) with two variables which the court does not attempt to monitor. Before going to that analysis let us mention one element omitted in that discussion, namely the realistic possibility of a care variable the distribution of which is determined by several precaution decisions. Since we have considered precaution variables affecting the level of the distribution of care, with the form of the distribution given, it is natural to enquire into variables that affect the distribution function's shape.[25] In particular some decisions may compress the distribution, eliminating both high and low levels of care. While I have not examined any examples in detail, it seems that the presence of the due care standard gives great significance to shifts of probability between care levels above and below the standard relative to the significance given shifts strictly within either category. Thus the presence of a due care standard will affect this sort of decision in ways which will vary greatly with the details of the particular situation.

Above, in the first section we discussed the different types of individual decisions relative to the measurements which a court makes. To examine some of the interactions among these variables let us consider an activity where each individual makes three decisions which affect expected accident costs. We assume that the nature of the activity is such that given his other two decisions (and the decisions of others), the individual faces a constant rate of expected accident costs per hour engaged in the activity. (We denote the number of hours by x_2 for the individual and y_2 for all others.) We assume that the court, examining individual accidents, never asks about the total time devoted to the activity. Secondly, we assume that there is a safety decision which affects the costs of any given accident, but has no effect of accident probabilities. (We denote the level of the safety decision by x_3 and y_3.) Thus this decision is never viewed, by the court, as causing an accident, and so is never part of a judgment of negligence. (We ignore the possibility of attributing incremental accident costs to safety decisions which are too low.) The third decision is a precaution decision (denoted by x_1 and y_1), which generates a probability distribution of care which, in turn, affects accident probabilities but not costs per accident. We assume that accident probability given care levels a and b is (1 - a)(1 - b) while care is uniformly distributed between 0 and x_1.

Given this structure a person expects accident costs of $(1-a)(1-b)x_2y_2C(x_3,y_3)$ for accidents he has when taking care level a with a given person taking care level b. To determine total expected costs, we multiply by the number of other people, n, attach probabilities to the occurrence of care levels a and b, and add up across care levels.

Assuming everyone else is identical, the expected utility maximizing individual wishes to

$$\underset{x_1,x_2,x_3}{\text{Max}}\ V(x_1,x_2,x_3) - nx_1^{-1}x_2y_1^{-1}y_2C(x_3,y_3)\int_0^{x_1}\int_0^{y_1}(1-a)(1-b)dbda \qquad (112)$$

Integrating we can express consumer choice as

$$\max_{x_1,x_2,x_3} V(x_1,x_2,x_3) - n(1 - \frac{1}{2}x_1)(1 - \frac{1}{2}y_1)x_2y_2C(x_3,y_3) \quad (113)$$

From the maximization we have the first order conditions

$$V_1 = -\frac{1}{2}n(1 - \frac{1}{2}y_1)\ x_2y_2C(x_3,y_3) \quad (114)$$

$$V_2 = n(1 - \frac{1}{2}x_1)(1 - \frac{1}{2}y_1)\ y_2C(x_3,y_3) \quad (115)$$

$$V_3 = n(1 - \frac{1}{2}x_1)(1 - \frac{1}{2}y_1)\ x_2y_2C_1(x_3,y_3) \quad (116)$$

For uniform equilibrium these three equations are simultaneously satisfied at $x_i = y_i$.

Assuming for the moment that all three variables are centrally controlled, the maximization of utility of the representative individual is

$$\max_{x_1,x_2,x_3} V(x_1,x_2,x_3) - n(1 - \frac{1}{2}x_1)^2 x_2^{\ 2}C(x_3,x_3) \quad (117)$$

This given the first order conditions

$$V_1 = -n(1 - \frac{1}{2}x_1)\ x_2^{\ 2}C(x_3,x_3) \quad (118)$$

$$V_2 = 2n(1 - \frac{1}{2}x_1)^2\ x_2C(x_3,x_3) \quad (119)$$

$$V_3 = n(1 - \frac{1}{2}x_1)^2\ x_2^{\ 2}(C_1(x_3,x_3) + C_2(x_3,x_3)) \quad (120)$$

Thus as before, approximately half the elements of social cost are being examined by the individual in the absence of liability, since he ignores the impact of his decisions on the expected accident costs of others.

To examine equilibrium with a negligence standard let us assume that the due care level is not set so high that anyone chooses to be negligent all the time, i.e., we assume $y_1 > d$ and $x_1 > d$. Thus an individual bears his own accident costs when negligent or when neither party is negligent and bears the costs of the other person when negligent and the other party is not negligent. Thus we can write expected utility as

$$U(x_1,x_2,x_3) = V(x_1,x_2,x_3) - nx_1^{-1}y_1^{-1}x_2y_2[C(x_3,y_3)(\int_0^d\int_0^{y_1}(1-a)(1-b)dbda$$

$$+ \int_d^x\int_d^{y_1}(1-a)(1-b)dbda) + C(y_3,x_3)\int_0^d\int_d^{y_1}(1-a)(1-b)dbda] \qquad (121)$$

Performing the integration we have

$$U = V - nx_1^{-1}y_1^{-1}x_2y_2[C(x_3,y_3)((d - \tfrac{1}{2}d^2)(y_1 - \tfrac{1}{2}y_1^2)$$

$$+ (x_1 - d - \tfrac{1}{2}(x_1^2 - d^2))(y_1 - d - \tfrac{1}{2}(y_1^2 - d^2)))$$

$$+ C(y_3,x_3)(d - \tfrac{1}{2}d^2)(y_1 - d - \tfrac{1}{2}(y_1^2 - d^2))] \qquad (122)$$

For convenience let us refer to the term in brackets as B. Note that when $x_3 = y_3$ and $x_1 = y_1$, B is equal to $C(x_1 - \frac{1}{2}x_1^2)^2$. Calculating the first order conditions we have

$$V_1 = -nx_1^{-2}y_1^{-1}x_2y_2B + nx_1^{-1}y_1^{-1}x_2y_2$$

$$[C(x_3,y_3)(1 - x_1)(y_1 - d - \tfrac{1}{2}(y_1^2 - d^2))] \qquad (123)$$

$$V_2 = nx_1^{-1}y_1^{-1}y_2B \qquad (124)$$

$$V_3 = nx_1^{-1}y_1^{-1}x_2y_2[C_1(x_3,y_3)((d - \tfrac{1}{2}d^2)(y_1 - \tfrac{1}{2}y_1^2)$$

$$+ (x_1 - d - \tfrac{1}{2}(x_1^2 - d^2))(y_1 - d - \tfrac{1}{2}(y_1^2 - d^2)))$$

$$+ C_2(y_3,x_3)(d - \tfrac{1}{2}d^2)(y_1 - d - \tfrac{1}{2}(y_1^2 - d^2))] \qquad (125)$$

We have a uniform equilibrium when these three equations are solved simultaneously with x_i equal to y_i.

By basing the determination of negligence on the level of care, which is (stochastically) affected by the precaution decision, the court is indirectly monitoring the precaution decision and directly affecting the equilibrium level of precaution by the choice of a due care standard.[27] The decision as to the amount of time to devote to the activity is not directly affected. However, by directly affecting the choices of precaution and safety, the legal system affects the benefits from engaging in the activity and so affects the time allocated to the activity.[28] In the normal case we would expect the legal system to reduce the accident rate per unit time (time held constant) and so to increase time devoted to the activity.[29] This response increases accidents per person in the activity by increasing time per person. Also, increased time in the activity by others increases the accident rate per hour, partially offsetting the improved precaution decision.[30]

The safety decision is also affected by the changed precaution and time decisions. In addition, it is affected directly by the court system, in that accident costs are sometimes borne by the other party to an accident. Given the symmetry of the ex ante positions of all individuals, this changed incentive takes a simple form. For the expected number of accidents where an individual is negligent and the other person is not, an individual pays attention to the impact of his marginal safety decision on the cost per accident of the other party to an accident $\partial C(y_3,x_3)/\partial x_3$ rather than its impact on his own cost per accident $\partial C(x_3,y_3)/\partial x_3$. (The simplicity of this change arises from the equal probabilities of being either party in an accident with one party negligent and the other party not.) Whether this improves resource allocation depends on the relative magnitudes of the impacts of one person's safety decisions on the two parties to an accident. For full efficiency, we would want individual decisions to reflect the marginal costs to both parties from a lower level of safety, so the larger impact is closer to the sum of impacts.

This system is rather complicated making it more difficult to trace out the efficiency implications of different due care standards. The legal system focuses on the precaution decision and so cannot attempt to achieve full efficiency.[31] In altering precaution decisions, it also has a direct impact on the safety decision, in that individuals sometimes must bear the accident costs of others, sometimes have others bear their accident costs. These two direct impacts also have indirect impacts on time and safety decisions due to the change in parameters describing the accident structure of the system and utility of engaging in the activity.

12. Different Individuals

While some differences in the abilities of individuals to be careful are recognized by the legal system (e.g. separate standards for children, blind, those with superior knowledge) there are many others which are not commonly considered. Some of these, such as wealth, are not recognized because of the philosophical stance of the legal system. Other differences are beyond the competence of the court to readily measure (e.g., general driving skill). In these circumstances a due care standard defined in physical terms is a blunt[32] instrument for altering behavior where different actions are desired from different people. To explore this issue let us consider a model where the accident structure is the same for everyone but people differ in the cost to them of taking care and the due care standard is the same for everyone. We shall follow the determinate model of section 5.

Given the assumptions on the nature of equilibrium made above, particularly that each person correctly perceives the behavior of others, we can follow the same pattern to determine equilibrium. A function giving the amount of care taken by people of different abilities to take care will represent an equilibrium when, for each utility level, it correctly describes the expected utility maximizing care level of a man who assumes that everyone else's decisions are correctly described by the function. Under the assumption that greater ability lowers the marginal cost of taking care, the equilibrium in the absence of liability will have a shape such as the one in the diagram.

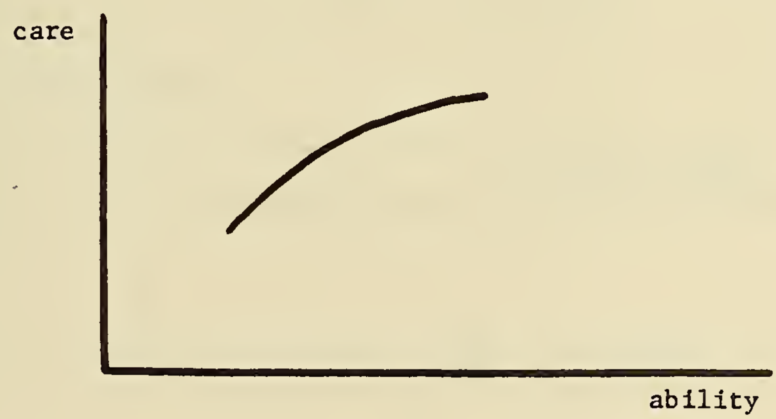


Figure 13

Let us derive this structure explicitly. We assume that the utility-of-taking-care level x for a man of type k satisfies[33]

$$V(x,k) = k^{-1}a^{-1}(1 - x)^a \qquad k > 0 \ , \ 0 < a < 1 \qquad (126)$$

We denote by n(h) the number of persons of type h and by y(h) the care perceived to be taken by a person of type h. The expected costs for accidents between two persons taking care levels x and y are (1-x)(1-y)C for each person, with C a constant. Expected utility for a person of type k can now be written as[34]

$$U(x,k) = k^{-1}a^{-1}(1-x)^{a} - C(1-x)\int (1-y(h))\, n(h)dh \qquad (127)$$

In the absence of liability an individual of type k maximizes U, taking the care of others as given. This gives a first order condition

$$k^{-1}(1-x(k))^{a-1} = C\int (1-y(h))\, n(h)dh \qquad (128)$$

For equilibrium x(k) and y(h) must be the same functions. Thus solving this for $x^{\circ}(h)$, we have the distribution of care in equilibrium

$$1 - x^{\circ}(h) = [hCK^{\circ}]^{\frac{1}{a-1}} \qquad (129)$$

where K° must satisfy

$$K^{\circ} = \int (1-x^{\circ}(h))\, n(h)dh = C^{\frac{1}{a-1}} K^{\circ\frac{1}{a-1}} \int h^{\frac{1}{a-1}}\, n(h)dh \qquad (130)$$

or

$$K^{\circ} = C^{\frac{1}{a-2}} \left[\int h^{\frac{1}{a-1}}\, n(h)dh\right]^{\frac{a-1}{a-2}} \qquad (131)$$

Thus expected accident costs for a person of type k are $C(1-x(k))K^{\circ}$.

Where everyone was the same, efficiency among uniform equilibria was suitably described by maximizing the common expected utility function. Where individuals differ, the problem of efficiency becomes more complicated. With utility defined in units commensurable with the resource units of accident costs, we shall define efficiency as maximizing the sum of expected utilities $\int U(x,h)\ n(h)dh$. Finding the efficient solution is simplified where the accident structure has the simple form we are taking here, of a single aggregate representing the state of the system which conveys to any individual the expected accident costs from his care decision. In this case efficiency calls for everyone to take more care than occurred in the no-liability equilibrium. (It is easy to construct hypothetical accident structures which do not have this property.) Thus the relationship between efficiency and no-liability equilibrium can be shown in a diagram

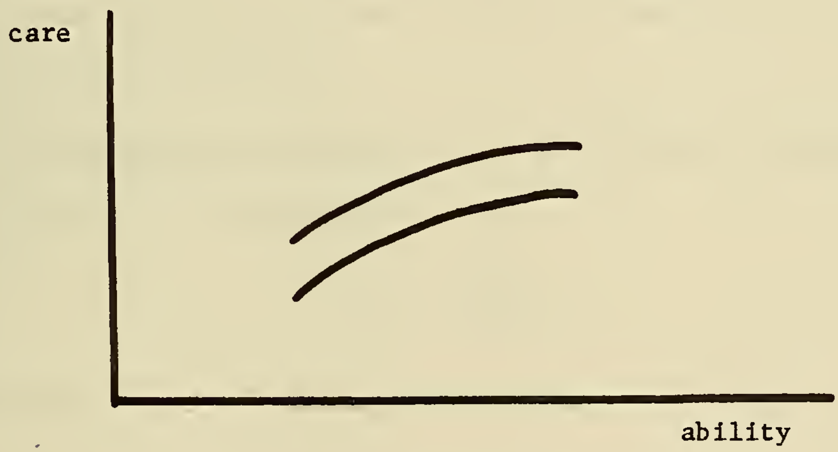


Figure 14

For efficiency we seek a function x*(h) to maximize

$$\int a^{-1}k^{-1}(1-x(k))^{a}n(k)dk - C\iint (1-x(h))(1-x(k))n(k)n(h)dhdk \qquad (132)$$

This yields the first order condition (also obtainable by maximizing U minus externality costs)

$$k^{-1}(1-x^*(k))^{a-1} = 2C\int (1-x^*(h))\ n(h)dh \qquad (133)$$

or, solving for x*(h)

$$1-x^*(h) = [2hCK^*]^{\frac{1}{a-1}} \quad (134)$$

where K* must satisfy

$$K^* = \int (1-x^*(h))\, n(h)dh = (2CK^*)^{\frac{1}{a-1}} \int h^{\frac{1}{a-1}}\, n(h)dh \quad (135)$$

or, solving for K*

$$K^* = (2C)^{\frac{1}{a-2}} \left[\int h^{\frac{1}{a-1}}\, n(h)dh \right]^{\frac{a-1}{a-2}} \quad (136)$$

Thus, in the efficient solution, expected accident costs for an individual of type k are C(1-x*(k))K*.

Comparing the no-liability equilibrium with the efficient solution, we see first (from (131)and(136)) that

$$K^\circ = 2^{\frac{1}{2-a}} K^* \quad (137)$$

Thus the comparison of care levels satisfies

$$1-x^\circ(k) = 2^{\frac{1}{2-a}} (1-x^*(k)) \quad (138)$$

Thus, for efficiency, every person is required to take more care. Expected accident costs for each person decrease both because he takes more care and because the other people take more care. Comparing expected accident costs, we have

$$C(1-x^\circ(k))K^\circ = 2^{\frac{2}{2-a}} C(1-x^*(k))K^* \quad (139)$$

Thus in the no-liability equilibrium expected accident costs for each person are $2^{2/(2-a)}$ times what they should be for an efficient solution.

Let us now consider a negligence system where the due care standard is selected within the range of care decisions. Some people may find the due care standard too difficult or expensive to maintain and will choose to be negligent. Others, to avoid the legal implications of negligence will choose precisely the due care level. Those with great ability (low marginal cost) to take care may choose a care level for self-protection (given the decisions of all others) which is above the due care level.[35] Thus, the equilibrium will have the following structure where we denote the range of abilities who select precisely the due care level by $[\bar{h},\bar{\bar{h}}]$.

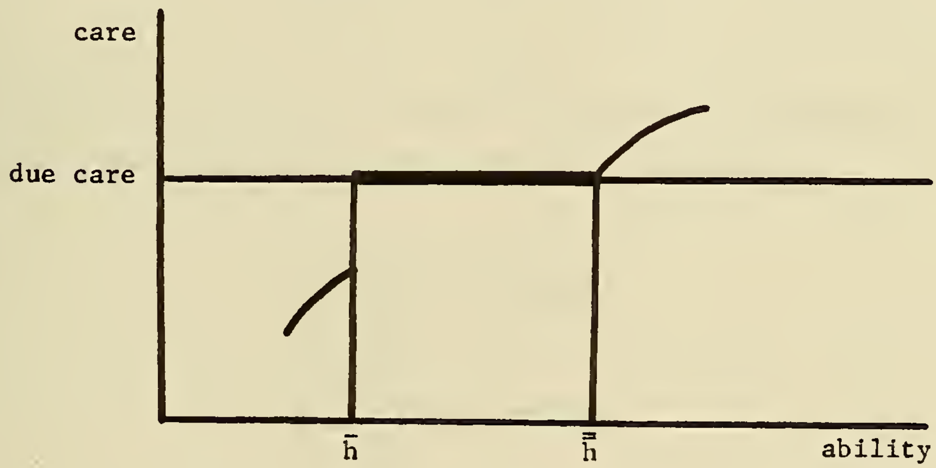


Figure 15

From the shape of this curve we can see that the efficient solution is not attainable.

Let us derive the formulas for this equilibrium structure. Given that the marginal utility cost of taking care decreases with h, the level of care in equilibrium is nondecreasing in h. Thus, there will be a unique index of type,[36] $\bar{h}$, such that for $h \geq \bar{h}$, type h is not negligent and for $h < \bar{h}$, type h is negligent. Expected utility for an individual of type k is thus the utility from taking care less his expected accident costs from accidents with the nonnegligent if he is nonnegligent. If he is negligent he bears his own costs in all accidents and the costs of others when he has an accident with someone who is not negligent.

$$U(x,k) = \begin{cases} U_1(x,k) = k^{-1}a^{-1}(1-x)^a - C\int_{\bar{h}}^{\infty}(1-x)(1-y(h))n(h)dh & : x \geq d \\ U_2(x,k) = k^{-1}a^{-1}(1-x)^a - 2C\int_{\bar{h}}^{\infty}(1-x)(1-y(h))n(h)dh \\ \qquad - C\int_{0}^{\bar{h}}(1-x)(1-y(h))n(h)dh & : x < d \end{cases} \tag{140}$$

Let us denote by K_1 and K_2 the contributions to the probability of accident by the nonnegligent and negligent respectively

$$K_1 = \int_{\bar{h}}^{\infty}(1-y(h))n(h)dh \tag{141}$$

$$K_2 = \int_{0}^{\bar{h}}(1-y(h))n(h)dh \tag{142}$$

Then we can rewrite expected utilities as

$$U_1(x,k) = k^{-1}a^{-1}(1-x)^a - (1-x)CK_1 \tag{143}$$

$$U_2(x,k) = k^{-1}a^{-1}(1-x)^a - 2(1-x)CK_1 - (1-x)CK_2 \tag{144}$$

From these expressions it is clear that $U_1(x) > U_2(x)$ and $U_2'(x) > U_1'(x)$ so that utilities appear as in the figure, where x_1 and x_2 are the maximizing levels for the two utility functions. U coincides with U_2 up to d and with U_1 for x greater than or equal to d. This is the same situation described above in section 4.

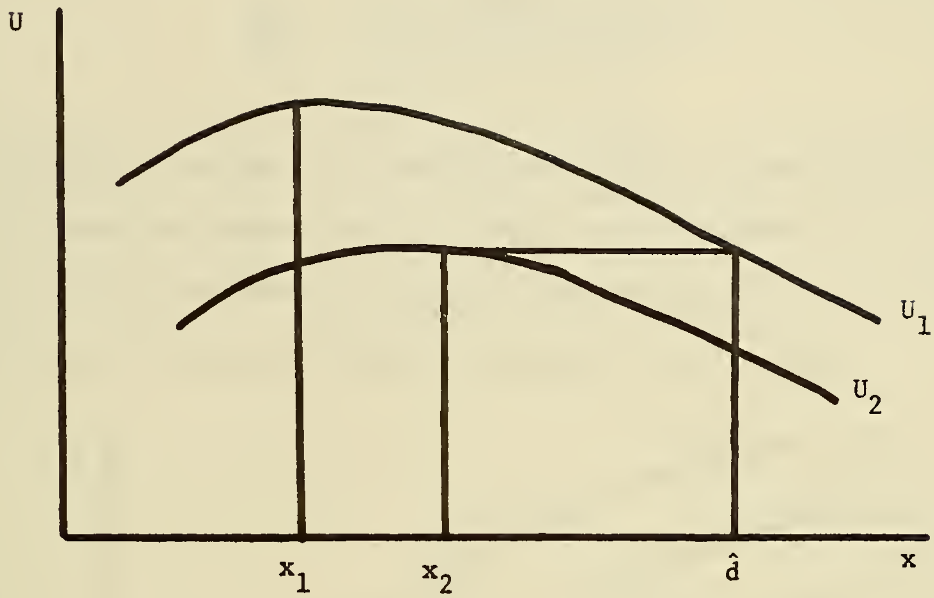


Figure 16

It is clear from the figure that the individuals choice satisfies

level of due care	$d \leq x_1(k)$	$x_1(k) \leq d \leq \hat{d}(k)$	$\hat{d}(k) \leq d$
chosen level of care	$x_1(k)$	d	$x_2(k)$ (145)

where $\hat{d}$ satisfies

$$U_1(\hat{d},k) = U_2(x_2,k) \tag{146}$$

and x_1 and x_2 are obtained by maximizing U_1 and U_2

$$k^{-1}(1-x_1)^{a-1} = CK_1 \tag{147}$$

$$k^{-1}(1-x_2)^{a-1} = 2CK_1 + CK_2 \tag{148}$$

It is clear that x_1 and x_2 increase with k. Implicitly differentiating the definition of $\hat{d}$, (146), we have

$$\frac{d\hat{d}}{dk} = - \frac{k^{-2}a^{-1}((1-\hat{d})^{a}-(1-x_2)^{a})}{-k^{-1}(1-\hat{d})^{a-1}+ CK_1} \qquad (149)$$

From the diagram we see that the denominator $\left(\frac{\partial U_1}{\partial d}\right)$ is negative while the numerator is positive (since $\hat{d} > x_2$). Thus $\hat{d}$ also increases in k. We can depict the division of the population according to ( ) in the following diagram, with the dotted line showing the choice of care level

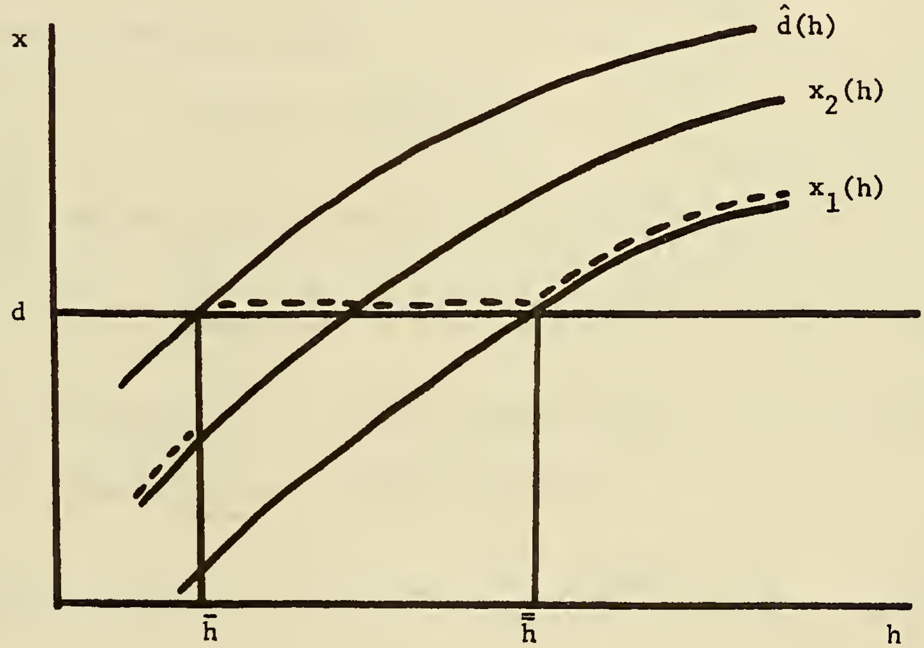


Figure 17

As the diagram shows we can partition the types of people by two values $\bar{h}$ and $\bar{\bar{h}}$ with choice satisfying

ability	$h < \bar{h}$	$\bar{h} \le h \le \bar{\bar{h}}$	$\bar{\bar{h}} < h$	
care	$x_2(h)$	d	$x_1(h)$	(150)

We can determine the levels of the partitions by the equations

$$\hat{d}(\bar{h}) = d \qquad (151)$$

$$x_1(\bar{\bar{h}}) = d \qquad (152)$$

From the expression for $x_1(h)$,(147), we can express $\bar{\bar{h}}$ implicitly in terms of K_1 by substituting in (152)

$$(1-d)^{a-1} = CK_1\bar{\bar{h}} \qquad (153)$$

Similarly from (146) and (151) we have

$$U_1(d,\bar{h}) = U_2(x_2(\bar{h}),\bar{h}) \qquad (154)$$

Substituting in the explicit expressions for U_1 and U_2 we have

$$\bar{h}^{-1}a^{-1}(1-d)^a - (1-d)CK_1 = \bar{h}^{\frac{1}{a-1}}(2CK_1 + CK_2)^{\frac{a}{a-1}}(a^{-1} - 1) \qquad (155)$$

Now let us examine the two constants which appear in the equations for x_1 and x_2, evaluated at the equilibrium levels.

$$K_1 = \int_{\bar{h}}^{\infty}(1-x(h))n(h)dh = (1-d)\int_{\bar{h}}^{\bar{\bar{h}}}n(h)dh + \int_{\bar{\bar{h}}}^{\infty}(CK_1h)^{\frac{1}{a-1}}n(h)dh \qquad (156)$$

$$K_2 = \int_0^{\bar{h}}(1-x(h))n(h)dh = \int_0^{\bar{h}}((2CK_1 + CK_2)h)^{\frac{1}{a-1}}n(h)dh \qquad (157)$$

We thus have four equations in four unknowns $\bar{h}$, $\bar{\bar{h}}$, K_1, and K_2. For further analysis we shall consider numerical solutions for particular values of a, C, and n(h).

In the tables below we consider a situation where h varies between 1.0 and 60.00, having a uniform distribution. We have selected values for the other parameters of a = .5 and C = .01. (The example differs from the discussion in that the nonnegativity constraint on x is sometimes binding.) Table one gives values of the four parameters describing the system for different levels of the due care standard. We note that $\bar{h}$ and $\bar{\bar{h}}$ increase monotonically with the due care standard in the example. Table two gives the level of care selected by different individuals in equilibria with different due care levels. When an individual is above the due care level, his care decreases with the due care standard. This general property follows because the set of people at the due care level are taking more care (and thus giving less incentive for care) and the number of negligent people also increases (again giving less incentive for care since people above the due care standard bear no costs from accidents with those below the standard). Similarly the expected utility of those above the due care level increases with the due care level. The example shows that those below the due care level also decrease care with the due care standard. An incentive for decreased care comes from the increased care of those at the due care level and the increased number of negligent people. Offsetting this somewhat, the decreased care of those above the due care level is an incentive for more care. Table three gives the levels of expected utility for these individuals for different due care levels.

Table I

D	K_1	K_2	$\bar{h}$	$\bar{\bar{h}}$
.1	12.853	.001	1.00	8.201
.2	12.311	.390	1.40	9.082
.3	11.685	1.010	2.020	10.229
.4	11.060	1.630	2.640	11.672
.5	10.408	2.270	3.280	13.587
.6	9.688	2.970	3.980	16.320
.7	8.826	3.805	4.820	20.686
.8	7.624	4.942	6.200	29.329
.9	5.003	7.224	9.970	60.000
.95	2.188	9.956	16.250	60.000

Table II

D	Individual Type		
	5.9	10	40
.1	.1	.400	.962
.2	.2	.330	.958
.3	.3	.300	.954
.4	.4	.400	.951
.5	.5	.500	.943
.6	.6	.600	.933
.7	.7	.700	.923
.8	.297	.800	.894
.9	.0356	.900	.900
.95	0	.583	.950

Table III

D	Individual Type		
	5.9	10	40
.1	.210	.080	.0050
.2	.210	.080	.0050
.3	.196	.083	.0053
.4	.190	.088	.0055
.5	.18	.087	.0059
.6	.169	.086	.0064
.7	.158	.083	.0067
.8	.142	.072	.0080
.9	.166	.058	.0110
.95	.195	.060	.0080

With the due care standard set at 0 we have the no-liability equilibrium. If getting the care level precisely to one is prohibitively expensive we again have the no-liability equilibrium with a due care level of one. Thus the response of individuals to rises in the due care standard must reverse direction at some point. An example of equilibria for two due care levels is shown in Figure 18.

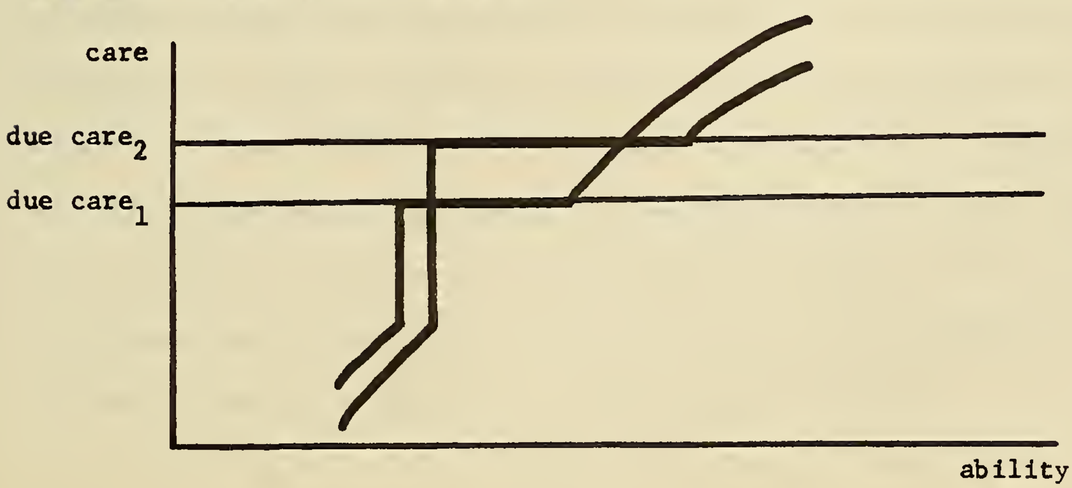


Figure 18

13. Equity and Efficiency

We have focused on the determination of the equilibrium levels of ex ante control variables which affect expected utilities and the distributions of accident costs. Since efficiency in this setting is an ex ante concept, it was natural to talk of the efficient solution relative to the different equilibrium positions occurring with different parameters for the legal system. There are a number of equity concepts which seem potentially relevant for a model in this stage of development.[37] There are two standard ex ante equity calculations. One is the standard tax incidence question of asking who gains and loses (in terms of expected utility) with any change in the legal system. For the bulk of the models, considering only uniform equilibria for a population of identical individuals, this question is not interesting. For the model with varying abilities to take care, we saw that those naturally above the due care standard were benefited by a higher standard, while those at or below the standard sometimes were hurt. The evaluation of a changed level of the due care standard is a calculation of utility differences between the position chosen and a projection of the historically given position.[38] The second standard ex ante approach is to evaluate the actual position in terms of some criterion (welfare function) which is not dependent on hypothetical or real alternatives. With the usual (individualistic) assumptions that welfare can be expressed as an increasing function of individual expected utilities, we are reduced to the efficiency concept where everyone is identical (and equilibrium is uniform). Given differing individuals and alternative welfare functions one could evaluate different due care standards in terms of a social welfare function.

In ex post terms, there are again several equity notions that seem potentially interesting. Economists have considered welfare functions of actual ex post positions and considered ex ante expectations of these welfare functions. In the absence of additivity of the welfare function this notion will not coincide with that of a welfare function of expected utilities. Evaluation with such a procedure would depend on the calculation of the distribution of accident costs across the population. (A similar calculation arises using expected utilities if individuals are risk averse and insurance is not available.) In parallel fashion to the distinction in ex ante concepts, we have ex post concepts which depend on differences between actual positions and hypothetical alternatives. The obvious alternative to use is the absence of some particular accident. The basic question is then whether accidents have moral significance. If not (if they are viewed morally as random events) it seems unfair to have people bearing accident costs above the average. If accidents have moral significance, and particularly if there are differences in the care of the parties at the time of an accident (either because of different care decisions or a random component coming from identical precaution decisions) then the fairness notion will focus precisely on behavior in the individual accident. With uniform decisions in a model where the stochastic structure is so explicitly present it is difficult to attribute moral significance to chance outcomes. This may be a shortcoming in the model rather than in moral intuitions from actual settings. The difficulty in pursuing moral intuitions in such an abstract setting lies behind my ending the discussion of equity with a mere listing of some possible approaches.

14. Omissions

When I read Calabresi's Costs of Accidents, I was struck by the number of complicated issues which he kept simultaneously before the reader. To be able to cope mathematically (or even at all) I have followed the opposite tack of selecting a single issue and assuming away the complications from all others. It seems appropriate, then, to review some of the omissions. Before this, however, let me draw out some of the other distinctions between my analysis and that of Calabresi. While I have ignored many elements affecting resource allocation, the ones remaining were considered in the context of equilibrium. That is, the decisions of all the participants in an activity were simultaneously considered to examine the indirect impacts of legal parameters--the effects of behavior modified to adjust to legal standards on the behavior of others and on choice in areas not covered by legal standards.[39] Thus we saw that an increased due care standard tended to lower the care taken by those previously above the standard. Also a standard for care affected safety decisions not covered by the standard and the time devoted to the activity. In addition, any decision affected accident possibilities with everyone else, which was taken to be a large number of people.[40]

In his analysis Calabresi sought the appropriate person on whom to place liability. Apart from placing liability on someone not involved in the accident this requires a two activity setting. Presumably, one of the purposes of a strict liability approach is to avoid the complicated factual issues arising in the determination of negligence. If this purpose is to be served, identification of the person strictly liable must not itself be complicated. This suggests that it should be simple

to identify which of two parties to an accident is strictly liable and that the answer should not depend on the details of the particular accident relative to other accidents which the same pair might have had. When it is desired to induce particular changes in behavior by both potential parties to some sort of accident (e.g., slow driving and careful walking) one can't usefully consider strict liability rules relative to both types of behavior and avoid fact determination which is complicated and thus similar to the situation with negligence. Thus we have focused on the legal rule (and its implications) rather than starting with the
in change
behavior/which/is desirable.[41, 42]

The model of individual behavior I have employed is highly simplified and excessively rational. The presence of simpli-
simplified form
fications is clear in the / of the expected utility function, the small numbers of decisions (often just one), the absence of other (alternative or related) activities, and the absence of a decision to engage in the activity at all. This latter decision, often involving a capital outlay (such as automobile purchase) may be one which is highly responsive to incentives. Decisions about care may be particularly subject to non-market forces. Knowledge of the consequences to others and of appropriate accident preventing modes of behavior may be significant determinants of behavior (with or without direct social pressure). Thus the assumption of concern solely with one's own expected utility is not fully accurate and the quality of the approximation will vary with the type of decision being considered. In addition, the imperfect nature of compensation (most clearly evident in case of death) limits the assumed blunting of incentives by the liability of others.[43] It is particularly

in the realm of decisions involving low probability events and involving one's own health and safety that the accuracy and rationality of individual choice is questionable.[44, 45]

To an even greater extent, the model of the legal system is greatly simplified, almost to the point of extinction. The legal system was assumed to be costless, prompt, and fully determinative of legal outcomes. There were no settlements out of court, failures to correctly initiate legal proceedings, legal fees and personal costs for litigants, or expenses to the public. Legal rules were fully worked out, so there were no problems from ignorance of the rules and no efforts to obtain determination of new cases.

The insurance industry also appeared as a costless, perfectly functioning institution. That is, individuals were assumed to base decisions (and society to evaluate outcomes) on the correctly calculated expected value of costs. In practice, there are administrative costs for insurance and complications and disputes over collection. Price setting by private companies, depending on industry structure, may well diverge from marginal cost pricing. There are also the distortions arising from imperfect perceptions by insurance companies which parallel the inabilities of courts to evaluate some variables. Thus individuals with different expected costs may be charged the same because of the cost or inability of telling them apart. Thus prices are quoted which may reflect costs on average rather than person by person. Inability (or expense) to measure behavior of the insured which affects expected costs leads to charges that do not vary with such behavior and so a blunting of incentives relative to these variables. Similar measurement problems after accidents occur can lead to inefficient repair or treatment of damage and excessive payments, both of which affect costs and so premia (leading again to incorrect price incentives). Attempts by insurance companies to classify people by past accident history create their own set of incentives as anticipated future payments for insurance affect current decisions. Drawing policy conclusions directly from models with so many omissions seems foolhardy. Understanding these models may help in drawing policy conclusions from less formal consideration of a more detailed view of the workings of accident law.

Footnotes

[1]Particularly Costs of Accidents (Yale 1970) and more recently Calabresi and John T. Hirschoff, "Towards a Test for Strict Liability in Torts," Yale Law Journal, 1972.

[2]The definition of new activities is a serious complication which we will not consider, since the analysis will be confined to long run equilibria.

[3]We are assuming that expected accident costs are a sum across all other people of the expected costs from accidents with them, with each of the latter expectations depending solely on the behavior of the two parties to the accident. This makes sense for very short time periods or moderately long ones. Otherwise we might need to consider separately behavior shortly after an accident.

[4]Recognizing these additional externalities not identified by the legal system, limits the ability of the legal system to affect efficiency in some contexts. This is more clearly seen in two activity accidents.

[5]If the presence of additional cars is viewed as 'causing' the lane switch rather than a change in accident probabilities, there is an accident unrelated externality from the additional car to the lane switcher, so the problem is merely shifted rather than eliminated.

[6]V is assumed to be twice continuously differentiable, strictly concave, and in units comparable to income. We make the stronger assumption that expected utility, V minus expected costs, is strictly concave for each of the cost allocation configurations we considered.

[7]To pursue efficiency further one would want to evaluate the cost

of inefficient private decisions relative to the cost of setting up any mechanism to achieve the efficient point.

[8]See W. Prosser, Law of Torts, p. 418.

[9]x_1 satisfies $V'(x_1) = \int_d^1 (\pi_1(x_1,y)C(x_1,y) + \pi(x_1,y)C_1(x_1,y))n(y)dy$.

[10]x_2 satisfies $V'(x_2) = \int_0^1 (\pi_1(x_2,y)C(x_2,y) + \pi(x_2,y)C_1(x_2,y))n(y)dy$

$+ \int_d^1 (\pi_1(x_2,y)C(y,x_2) + \pi(x_2,y)C_2(y,x_2))n(y)dy$.

[11]That is, we assume that $\pi_{12}(x_2,d)(C(x_2,d) + C(d,x_2)) + \pi_1(x_2,d)(C_2(x_2,d) + C_1(d,x_2)) + \pi_2(x_2,d)(C_1(x_2,d) + C_2(d,x_2)) + \pi(x_2,d)(C_{12}(x_2,d) + C_{21}(d,x_2)) \geq 0$.

[13]In determining $\hat{d}_1$, we look at demand assuming everyone else at x°. In determining $\hat{d}_2$ we look at demand assuming everyone else at d. Thus when the care of others affects demand we do not have identical equations determining these values.

[14] $$\phi(\hat{d}_1) = \max_x V(x) - n\pi(x,d)(C(x,\hat{d}_1) + C(\hat{d}_1,x)) - V(x^\circ) + n\pi(x^\circ,x^\circ)C(x^\circ,x^\circ) + n\pi(\hat{d}_1,\hat{d}_1)C(\hat{d}_1,\hat{d}_1)$$

$$\geq n\pi(x^\circ,x^\circ)C(x^\circ,x^\circ) - n\pi(x^\circ,\hat{d}_1)(C(x^\circ,\hat{d}_1) + C(\hat{d}_1,x^\circ)) + n\pi(\hat{d}_1,\hat{d}_1)C(\hat{d}_1,\hat{d}_1)$$

Thus if $C(x,y)$ and $\pi(x,y)$ take the form $f(x)f(y)$, this expression is positive and there is a range without equilibria.

[15]A second possible equilibrium configuration has some of the people, say n_1, at a nonnegligent point and $n+1-n_1$ people at a negligent point. For this to be an equilibrium, utilities must be the same for both types. This results in a picture as in figure 4 with some of the people at x_2, the others at $\hat{d}$, with d coinciding with $\hat{d}$. From the discussion above we know that this can only occur for due care levels above the efficient point. Spelling this out precisely we have the equal utility condition

$$V(x_2) - (n-n_1)\pi(x_2,x_2)C(x_2,x_2) - n_1\pi(x_2,d)(C(x_2,d) + C(d,x_2))$$
$$= V(d) - (n_1-1)\pi(d,d)C(d,d)$$

where x_2 maximizes $V(x) - (n-n_1)\pi(x,x_2)C(x,x_2) - n_1\pi(x,d)(C(x,d) + C(d,x))$. I have not explored the problem of the existence of such equilibria.

[16]We assume that precaution and care are measured in the same units.

[17]By implicit differentiation $\frac{dx_1}{dd} = \frac{1}{4}\, nC/V''$.

[18]Differentiating, the denominator of $\frac{dx_2}{dd}$ is the second order condition and is negative. The numerator is $\frac{1}{4}\, nC(1 + 2(1 - d - x_2 + \frac{1}{2}\, d^2 + \frac{1}{2}\, x_2^2)/(1 - x_2)^2 + 2(1 - d)^2/(1 - x_2)^2)$

[19]The upper end of the curve must be above the 45° line, as shown.

[20]The analysis of this section was done by Roger Gordon.

[21]The second order condition is $V'' + \frac{8nC}{9}\,(1 - \frac{1}{2}\, y)\frac{(1-d)^3}{(1-x)^3} < 0$.

[22]

[23]Without loss of continuity the section on comparative negligence may be omitted by the reader who wants to proceed to the discussion of negligence in more complicated settings.

[24]In addition a negligence standard may be imposed as a preliminary hurdle to the application of comparative negligence.

[25]Having several decisions which would merely combine to determine

the level of the distribution does not appear to be an interesting direction for extension.

[26]We assume that the cost per accident $C(x_3,y_3)$ is decreasing in x_3.

[27]This exactly follows the analysis in section 7, compare ( ) and ( ).

[28]In form ( ) and ( ) are the same, although the values solving them will be different since the first order conditions from differentiation relative to x_1 and x_3 are different.

[29]That is, if V is additive, time devoted to the activity will increase if $(1 - \frac{1}{2} x_1)^2 C(x_3,x_3)$ is smaller, or (provided provision for one's own safety doesn't raise the costs of others by too much) if x_1 and x_3 are larger.

[30]Expected accident costs per hour are $n(1 - \frac{1}{2} x_1)(1 - \frac{1}{2} y_1)y_2$ $C(x_3,y_3)$.

[31]I have not examined whether the negligence system can induce the best level of precaution given that time and safety are freely chosen, and so depend on the equilibrium level of precaution.

[32]A legislated safety standard, like a speed limit, will generally be uniformly applicable, or at least uniform over wide classes (e.g. auto, truck, trailer). No attempt is made to tailor the speed limit to the reflexes or sense of the driver.

[33]We are assuming that the utility of taking care decreases with k ($\frac{\partial V}{\partial k}$ < 0) but that the marginal disutility of taking care decreases ($\frac{\partial^2 V}{\partial x \partial k}$ > 0).

[34]Subject to the constraint $x \leq 1$.

[35]There may not be any parts of the population in either of the two rising parts of this curve, depending on the shape of n(h) and the level of due care.

[36]Individuals of type h will be indifferent between precisely due care and the appropriate level of negligence. We assume that they choose the due care level.

[37]Since the model is essentially static (there is a before and after as accident probability distributions become actual accidents, but no development of individual knowledge or positions) the equity concepts discussed are static. Thus no mention is made of the ongoing nature of social decisions.

[38]One could calculate differences relative to any hypothetical alternative, for example differences from the best equal utility allocation. See J. Rawls, Theory of Justice for a discussion of equal utility and of stability in terms of gains and losses relative to hypothetical alternatives, although the relevant hypothetical alternative to use for a stability discussion is not seriously analyzed.

[39]I did not consider general equilibrium interactions among different activities.

[40]The analysis of John P. Brown, "An Economic Theory of Liability," Journal of Legal Studies, forthcoming, is similar to mine in being an equilibrium analysis. It differs in considering only two persons and in considering the category of two activity accidents.

[41]In a book as meaty as Costs of Accidents, I am obviously commenting only on the part closely paralleling the analysis I have done.

[42]The consideration of accidents between strangers by Richard A. Posner, "A Theory of Negligence," Journal of Legal Studies (197) is similar in approach to that here in that particular standards of due care are considered relative to alternatives.

[43]In part the shape of V can correct for these misspecifications.

[44]See, e.g., J. D. Tumerin and H. L. D. Resik, "Risk Taking by Individual Option--Case Study--Cigarette Smoking," in Perspectives on Benefit Risk Decision Making, National Academy of Engineering, 1972; P. Slovic, "From Shakespeare to Simon: Speculations and Some Evidence--on Man's Ability to Process Information," unpublished; or A. Tversky and D. Kahneman, "Judgment under Uncertainty: Heuristics and Biases," unpublished.

[45]Incorrect perceptions of accident probabilities could probably be fitted into the models in a straightforward manner, as could incorrect perception of the utility of taking care.