Heat Transfer

Leon R. Glicksman

Introduction

Heat transfer deals with the rate of heat transfer between different bodies. While thermodynamics deals with the magnitude of heat exchanged in a process. Heat transfer is necessary to determine the time required for a process or alternatively the size of a surface necessary to achieve a certain total rate of heat transfer.

Heat transfer analysis permits a calculation of the heat loss from a building surface to the surroundings for a given building size, window area and wall design, e.g. the level of insulation in the wall cavity. The comfort conditions for occupants in a room is determined by a balance of heat transfer from the person to the air surrounding him or her as well as the heat transfer to the walls of the interior. The size and cost of a heat exchanger is also determined by considering the heat transfer between the fluid streams in the exchanger.

In other fields, heat transfer plays a key role as well. The design of integrated microprocessors which contain very closely spaced elements, each with a finite amount of heat generation, is limited by the requirement for adequate cooling so that the operating temperature of the electronic components is not exceeded. Reentry of the space shuttle in the earth's atmosphere must be carefully programmed so that temperature extremes due to air friction is confined to the insulating tiles on the shuttle's surface.

Modes of Heat Transfer

Following thermodynamics, heat transfer is that energy transfer which takes place between two bodies by virtue of a temperature difference between the bodies. From the second law considerations it can be demonstrated that there is always a net positive energy transfer from the body at a high temperature to a second body at a lower temperature. Following the definition of heat, there are only two physical mechanisms for heat transfer: (1) electromagnetic waves produced by virtue of the temperature of a body, referred to as thermal radiation heat transfer and (2) atomic or molecular motion in a medium between the bodies exchanging energy, referred to as conduction heat transfer.

Sometimes conduction heat transfer takes place during the change of phase and is referred to as boiling or condensation heat transfer. Conduction heat transfer can also take place in the presence of fluid motion, which is called convection heat transfer.

The rate of heat transfer between two bodies is proportional to the temperature difference between the bodies and in some cases the temperature level of the bodies as well. In many instances the heat transfer process is analogous to the rate of transfer which appears in other fields. The analogy between heat transfer and electrical current flow will be used to illustrate some of the simpler heat transfer processes. Similarly, it can be shown that the rate of transfer of mass in an evaporation process follows a process very similar to that for heat transfer.

Conduction Heat Transfer

In a homogenous body which experiences a temperature gradient the rate of heat transfer due to microscopic motions is conduction heat transfer. In a gas the gas molecules in the higher temperature portion of the gas will have a higher kinetic energy. As the molecules of the gas randomly move through the gas volume there is a net energy transfer from the high temperature portion to the low temperature zones. In a solid, the energy transfer from high to low temperature may be due to the migration of electrons or the vibration of the molecular bonds.

Viewed as a macroscopic phenomena, the rate of heat transfer by conduction represented by the symbol q or Q is found to be directly proportional to the product of the local temperature gradient and the cross-sectional area available for heat transfer,

$$q \propto -A\nabla T \qquad (1)$$

In the case of one-dimensional heat transfer normal to a plane slab, figure 1, the conduction heat transfer can be given by Fourier's Equation,

$$q = -kA\frac{dT}{dx} \qquad (2)$$

The constant k is known as the thermal conductivity. q has the dimensions of BTU/hr or Watts and k has the dimensions of BTU/hr-ft °F or W/m °C.

The thermal conductivity defined by equation 2 is a thermophysical property of the material. If the composition and thermodynamic state is known then the thermal conductivity can be found. Table 1 lists the thermal conductivity of common

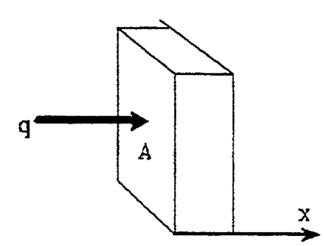


Figure 1 One Dimensional Conduction

Table 1
Thermal Conductivity of Common Materials

	k(BTU/hr ft °F)
Solids	
Copper	219
Aluminum	119
Steel	25
Brick, common	0.2 - 0.1
Concrete	0.5 - 0.8
Glass	0.5
Glass fiber insulation	.03
Ice	1.3
Plastic	0.1
Wood	0.1 - 0.2
Liquids	
Ammonia	0.3
Refrigerant-12	0.04
Light Oil	0.08
Water	0.34
Mercury	5
Gases	
Air, dry	0.015
Carbon Dioxide	0.009
Helium	0.09
Hydrogen	0.11
Water Vapor (Steam)	0.015 (at 212°F)
Refrigerant-11	0.005

1.73 k (W/m °C) = 1.0 (BTU/hr ft °F)

solids, liquids and gases at normal temperatures. Note that these values span many orders of magnitude with electrically conductors having the highest thermal conductivity and high molecular weight gases generally having the lowest thermal conductivity.

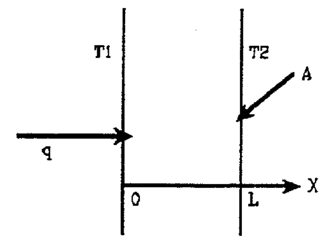


Figure 2 Conduction Through a Plane Wall

Consider a slab with a steady conduction heat transfer across it in the x direction, fig. 2, with the temperature equal to T_1 and T_2 at the surfaces corresponding to x equal to 0 and L, respectively. Then q is a constant and equation 2 can be integrated to give,

$$q = kA\frac{(T_1 - T_2)}{L} \tag{3}$$

For this case the temperature varies linearly across the width of the slab. One can consider an analogy between the solution for steady conduction and for steady D.C. electric current flow, Ohm's Law,

$$I = \frac{(V_2 - V_1)}{R} \tag{4}$$

The rate of heat transfer q is analogous to the current flow I, the potential difference ΔV is analogous to ΔT and the balance of equation 3 is analogous to the resistance. The term thermal resistance is used; for eqn. 3 the thermal resistance is L/kA.

Consider the case of steady heat transfer through a composite

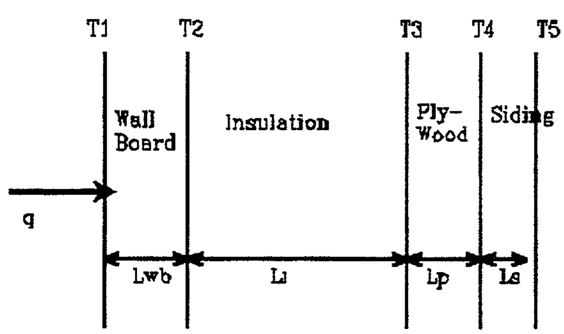


Figure 3 Steady State Heat Transfer Through a Composite Wall

wall as shown in figure 3. Each element of the wall has the same heat transfer rate q through it and for each an equation similar to equation 3 can be written. At steady state with no change in internal energy with time, no work, and no mass flows through each of the elements, the rate of heat transfer into and out of each wall element must be the same. For the wall board,

$$q = \frac{kA}{L_{wb}}\left(T_1 - T_2\right) \tag{5}$$

This can be rewritten as,

$$T_1 - T_2 = q\frac{L}{kA_{wb}} \tag{6}$$

For the insulation

$$T_2 - T_3 = q\frac{L}{kA_i} \tag{7}$$

Similar equations can be written for the plywood, T_3 - T_4 and and the siding, T_4 - T_5. When these equations are summed up the intermediate temperatures T_2, T_3 and T_4 cancel and the resulting equation becomes

$$T_1 - T_5 = q\left(\frac{L}{kA_{wb}} + \frac{L}{kA_i} + \frac{L}{kA_p} + \frac{L}{kA_s}\right) \tag{8}$$

or

$$q = \frac{T_1 - T_5}{\sum \frac{L}{kA}} \tag{9}$$

The electrical analogy for this case is resisters in series as shown in figure 4.

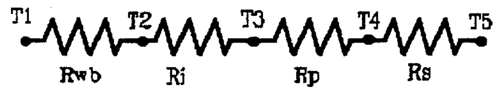


Figure 4 Electric Analogy to Steady State Heat Transfer

Then the overall solution can be easily written as,

$$q = \frac{\Delta T}{\sum R} \tag{10}$$

which is identical to equation 9.

For a planar wall, the cross sectional area of each element is the same so that equation 9 can be rewritten as

$$q = \frac{\left(T_1 - T_5\right) A}{\frac{L}{kA_{wb}} + \frac{L}{kA_i} + \frac{L}{kA_p} + \frac{L}{kA_s}} \tag{11}$$

Each of the terms in the denominator is sometimes referred to as the R-value of that material. Note the R-value is independent of the surface area A while the thermal resistance R_T includes the surface area. For the plywood, a typical R-value for a one inch thickness is

$$\frac{L}{k_p} = \frac{1/12}{0.1} = 0.8 \frac{BTU}{hr\ ft^2\ °F}^{-1} \tag{12}$$

If the insulation is three and one half inches thick, a typical 2 by 4 construction,

$$\frac{L}{k_i} = \frac{3.5/12}{.028} = 11 \frac{BTU}{hr\ ft\ °F}^{-1} \tag{13}$$

For the composite wall in figure 3, the R-value of the insulation dominates all of the terms in equation 11.

<u>Convection Heat Transfer, Introduction</u>

In a typical wall construction, as shown in figure 3, the temperature on the inside wall surface, T_1, and the outside siding surface T_5 are not generally known. Rather the interior room air temperature, T_i, and the exterior air temperature, T_e, are the known quantities. Consider a wintertime condition, when the building is at a higher temperature then the exterior air. The temperature through the built-up wall continuously decreases from the inside wall at T_1 to the outside surface at T_5. This is shown in figure 5. The outside surface temperature T_5 is higher than the exterior air temperature T_e. In the air layer close to the building surface the air is in motion parallel to the surface. There is heat transfer by conduction from the building surface through this air layer. Because there is also energy transfer by the motion of the fluid the temperature through the air layer does not vary linearly. Rather, there is a large temperature gradient near the surface which decreases further from the surface until the temperature reaches the constant air temperature T_e. The layer over which the temperature change occurs is thin, typically one quarter of an inch or less.

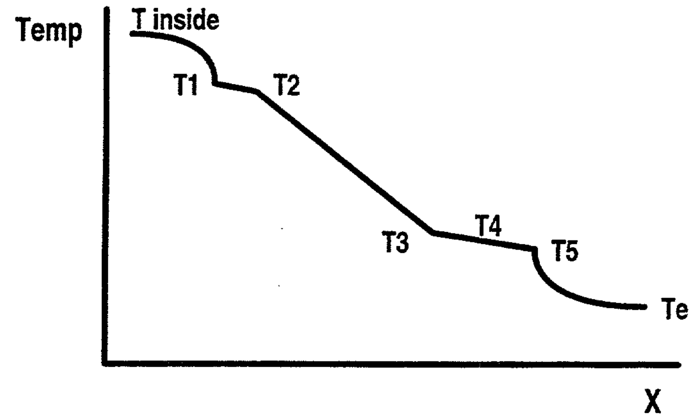


Figure 5 Temperature Distribution with Convection at the Surfaces

The process of conduction heat transfer through the air combined with energy transfer by fluid motion is called convection heat transfer. The rate of heat transfer is proportional to the surface area and the temperature difference between the surface and the uniform air temperature outside of the thin surface or boundary layer,

$$q \propto A\left(T_5 - T_e\right) \tag{14}$$

The expression is changed to an equality and in the process a new quantity, h, the heat transfer coefficient is defined,

$$q = hA\left(T_{surface} - T_{fluid}\right) \tag{15}$$

where h has the units of BTU/hr ft^2 °F or w/m^2 °C. Equation 15 is of no use until some way to calculate h is established.

Generally, the heat transfer coefficient, h, is a function of the fluid properties, the fluid velocity, the surface geometry and sometimes the temperature level. A more detailed discussion of convection will be given later. For now it is sufficient to observe that h increases as the air velocity increases and it increases with fluids of higher thermal conductivity.

There are two general forms of convection. When the air motion is set up by buoyancy effects due to the applied temperature difference between the surface and the fluid, e.g. the air flow over a hot 'radiator', the flow is natural or free convection. When the flow is due to an external source, e.g. the wind, a fan or by the motion of the surface, the flow is forced convection.

Rohsenow has presented a table which gives good estimates of the order of magnitude of h for convection heat transfer as well as boiling and condensation. It is reproduced in table 2.

Table 2
Convection Heat Transfer Coefficients
BTU/hr ft^2 °F

Gases Natural Convection	0.5 - 5
Gases Forced Convection	2 - 50
Liquids Forced Convection	30 - 1000
Liquid Metals	1,000 - 50,000
Boiling Liquids	200 - 50,000
Condensation	500 - 5,000

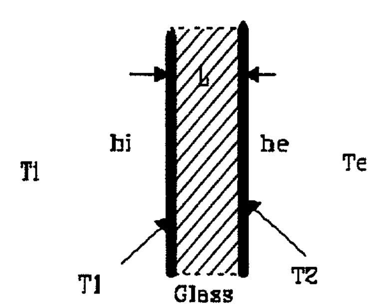
Rohsenow W.M. and Choi H., <u>Heat, Mass, and Momentum Transfer</u> Prentice-Hall, 1961.

1 BTU/hr ft^2 °F = 5.68 W/m^2 °C

Now returning to the concept of thermal resistance, from equation 15 the equivalent thermal resistance, R_T for convective heat transfer is (1/hA).

EXAMPLE

Consider a single glazed window. What is the increase in the energy efficiency if the glass is made of plastic with k = 0.1 BTU/hr ft °F instead of glass with a k = 0.5 BTU/hr ft °F? Assume that the radiation heat transfer remains the same.



SOLUTION

In this case convection heat transfer from the inside air at T_i to the glass surface acts in series with conduction through the glass and convection to the outside air. The equivalent electrical

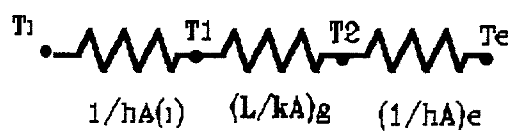


Figure 7 Electrical Analogy

circuit is shown on figure 7 and the steady state heat transfer, neglecting radiation, becomes

$$q = \frac{(T_i - T_e)}{\left(\frac{1}{hA}\right)_i + \left(\frac{L}{kA}\right)_g + \left(\frac{1}{hA}\right)_e} = \frac{(T_i - T_e)A}{\frac{1}{h_i} + \frac{L}{k_g} + \frac{1}{h_e}} \qquad (1)$$

Using table 2 the magnitudes of h_i and h_e are

$$\frac{1}{h_i} \approx \frac{1}{1} \approx 1 \left(\frac{BTU}{hr\ ft^2\ {}^\circ F}\right)^{-1}$$

$$\frac{1}{h_e} \approx \frac{1}{3} \approx \frac{1}{3} \left(\frac{BTU}{hr\ ft^2\ {}^\circ F}\right)^{-1}$$

For the glass, assuming it is 1/8 inch thick,

$$\frac{L}{k_g} \approx \frac{\left(\frac{1}{8}\right)\left(\frac{1}{12}\right)}{0.5} \approx \frac{1}{50} \left(\frac{BTU}{hr\ ft^2\ {}^\circ F}\right)^{-1}$$

Changing to plastic decreases k_g to 0.1 and increases L/kg to 1/10 but it will only change the overall value of q, given by equation 16 by less than 10 percent.

The overall heat transfer for composite systems such as figure 3 or figure 6, represented by equation 9 and 16, respectively is sometimes rewritten in terms of an overall heat transfer coefficient U defined as

$$q = UA\left(T_i - T_e\right) \qquad (17)$$

Although U has the same units as h, U can involve a combination of conduction and convection heat transfer and is not physically meaningful although it may be helpful for estimate purposes. A number of handbooks like the ASHRAE Handbook of Fundamentals list values of U for typical built up wall and roof construction. These values of U include convection heat transfer on the inside and outside for an assumed wind velocity and interior air circulation conditions.

Cylindrical Geometry

Besides plane walls, the other conduction geometry which is of interest is cylindrical geometry. In tubes carrying fluid in an air conditioner there is radial heat transfer from the interior fluid through the wall to the exterior fluid. In this case the heat transfer at any radius r within the tube material is a constant given by,

$$q = -k2\pi rz\frac{dT}{dr} \qquad (18)$$

where z is the axial length of the tube. This may be integrated to yield,

$$q = -\frac{k\left(T_1 - T_2\right)2\pi z}{\ln\left(\frac{r_2}{r_1}\right)} \qquad (19)$$

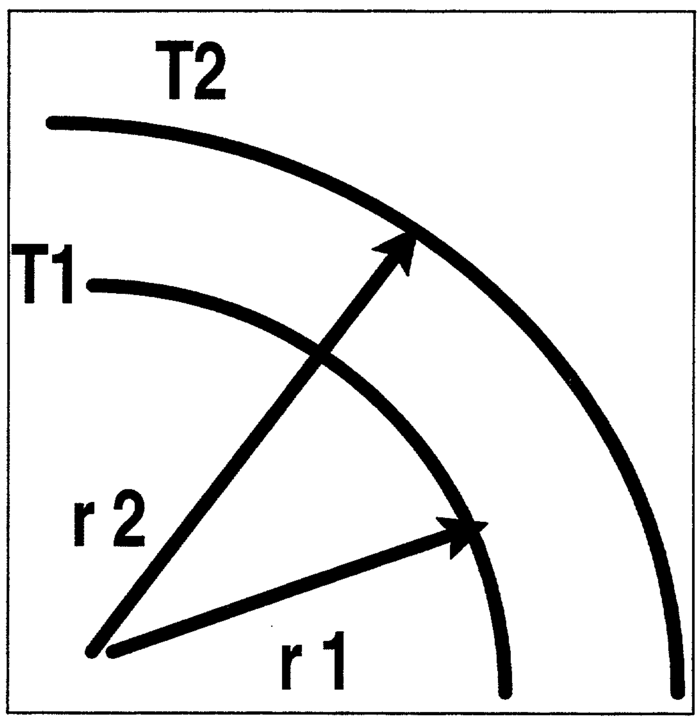


Figure 8 Cylindrical Geometry

and can be approximated by a planar form,

$$q = -\frac{k(T_1 - T_2)A}{t} \tag{20}$$

which is accurate within 5 percent for $r_2/r_1 < 2$ when A is evaluated at the midplane of the tube, i.e. $2\pi\, z\, (r_2 + r_1)\ /\ 2$ and t is $r_2 - r_1$.

Two Dimensional Heat Transfer

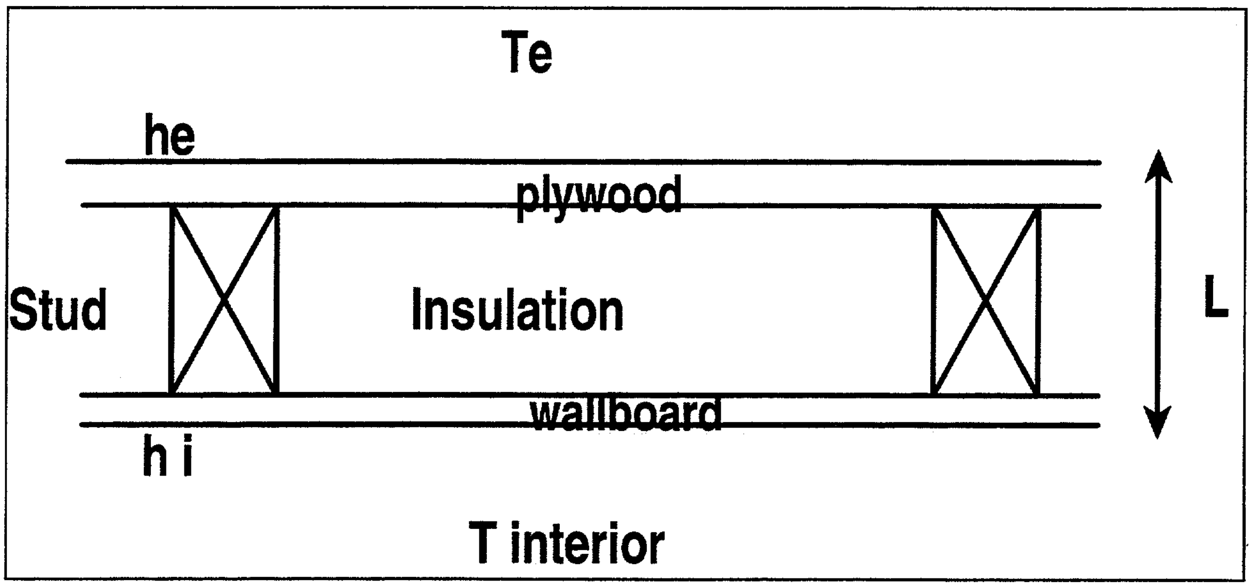


Figure 9

Most walls are not uniform across their entire surface area. Wood framing using 2 by 4's has studs spaced at regular intervals in the wall cavity, fig 9. Clearly the heat transfer through the studs is higher than the heat transfer through an equivalent cross-sectional area containing insulation.

The heat transfer through the wall cavity is due to two parallel conduction paths, one through the studs and the other through the insulation. The electrical analogy of this, in one limit, is shown in figure 10 with the insulation and the studs in parallel.

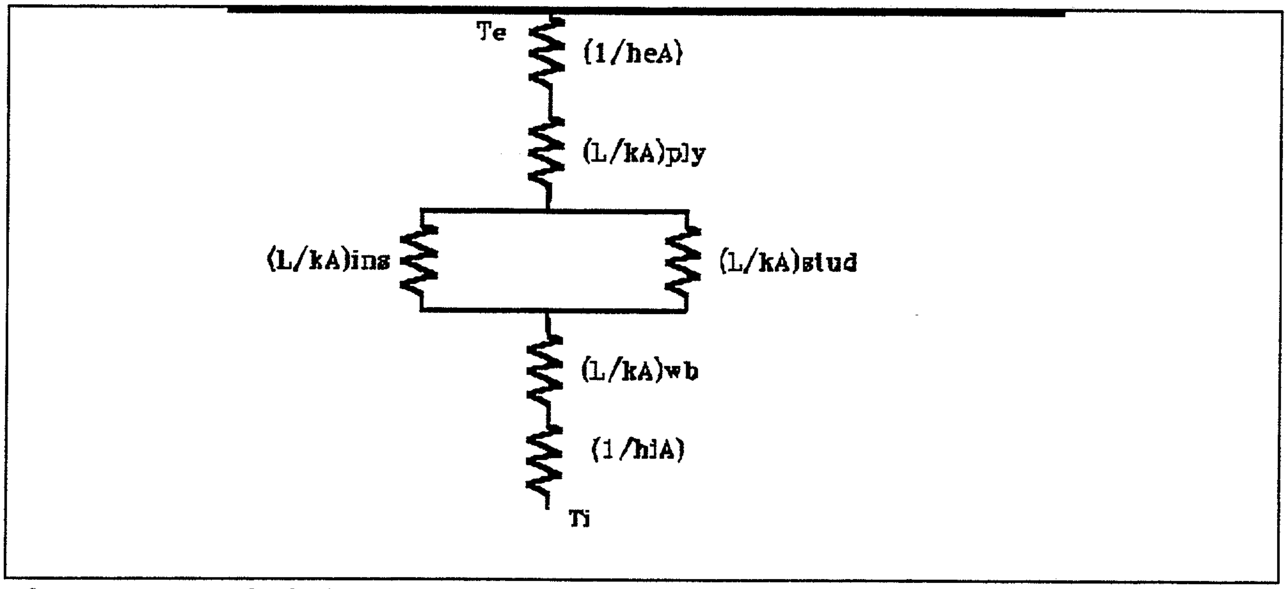


**Figure 10 Limiting Case for
Small Lateral Resistance to Heat Transfer in Sheathing**

Since the heat transfer through the two elements are summed,

$$q = \frac{(T_2 - T_3)}{\left(\frac{L}{kA}\right)_{ins}} + \frac{(T_2 - T_3)}{\left(\frac{L}{kA}\right)_{stud}} \tag{21}$$

and defining an equivalent resistance,

$$q = \frac{(T_2 - T_3)}{\left(\frac{L}{kA}\right)_{equiv}} \tag{22}$$

then the equivalent resistance becomes,

$$\left(\frac{L}{kA}\right)_{equiv} = \left[\left(\frac{L}{kA}\right)_{ins}^{-1} + \left(\frac{L}{kA}\right)_{stud}^{-1}\right]^{-1} \tag{23}$$

It is important to note that this approximation for the two-dimensional case represents one limiting case, the temperature T_2 on the inside of the wall board and the temperature T_3 on the inside of the plywood are uniform over the cross-section. In the actual case the heat flow may be uniform at the inside surface but it preferentially flows through the studs because they have a much higher conductivity than the insulation as shown on figure 11. There will be lateral heat flow in the wall board and the plywood in the y direction. To accommodate this heat flow by conduction

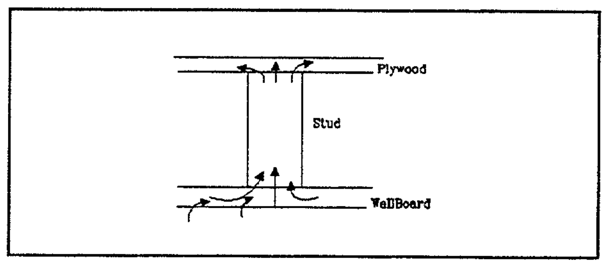


Figure 11
Heat Flow

there must be a temperature difference in the y direction in the wall board and the plywood. This acts as an additional resistance which will reduce the heat transfer through the stud.

The solution shown in figure 10 is only valid when the lateral resistance are small, i.e. the y direction conductivity of the wall board and the plywood approaches infinity. If on the other hand the lateral resistance are very large, i.e. the lateral conductivity of the wallboard and the plywood approaches zero, then a better model is two separate parallel heat flow paths from T_i to T_e, shown in fig. 12.

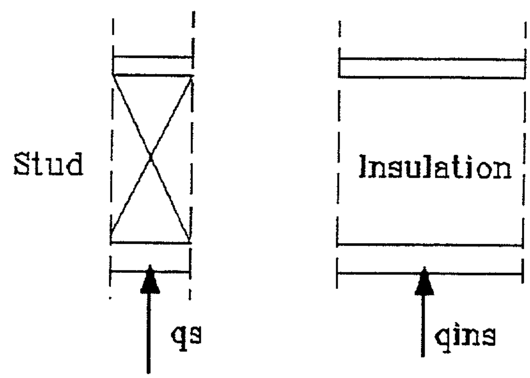


Figure 12 Limiting Case of Small Lateral Conductivity

The heat transfer through the studs is

$$q_s = \frac{(T_i - T_e)\,A_s}{\frac{1}{h_i} + \left(\frac{L}{k}\right)_{wb} + \left(\frac{L}{k}\right)_s + \left(\frac{L}{k}\right)_p + \frac{1}{h_e}} \qquad (24)$$

and through the insulation,

$$q_i = \frac{(T_i - T_e)\,A_i}{\frac{1}{h_i} + \left(\frac{L}{k}\right)_{wb} + \left(\frac{L}{k}\right)_i + \left(\frac{L}{k}\right)_p + \frac{1}{h_e}} \qquad (25)$$

The total heat transfer rate is the sum of equations 24 and 25. The true value of the two dimensional heat transfer lies between these two limiting cases of very small lateral conductivity and very large lateral conductivity.