Transient Heat Transfer

Conditions for Uniform Temperature

When the temperature of the interior changes, the building's structure may be sufficiently massive to provide significant thermal storage. However, if the structure is very thick it may require a substantial time interval before the temperature change is felt throughout the structure.

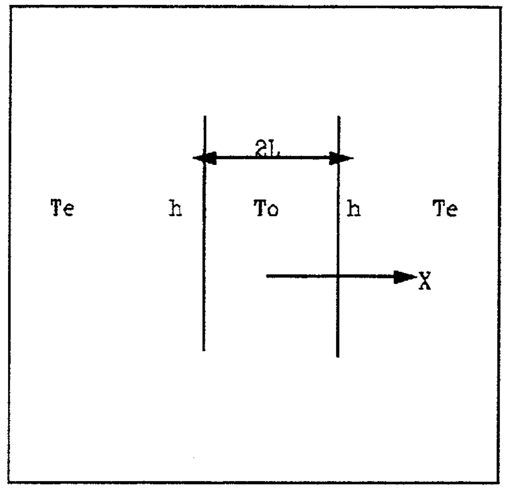

Figure 13

Consider the case of a plane homogenous slab of thickness 2L. The y and z dimensions are large compared to L so that the heat transfer can be assumed to be one dimensional, in the x direction only. Initially the slab is at a uniform temperature T_0 equal to the exterior air temperature. At time zero the air temperature suddenly increases to a new temperature level T_e. Both sides of the slab are in contact with the air and both sides have convective heat transfer coefficient h. At short times, the surface temperature of the slab will increase due to the heat transfer from the air. Some of the energy transferred to the slab will be used to raise the internal energy of the material near the surface. The balance of the heat transfer will be transferred to the next layer inside the slab where the same process occurs. The temperature distribution within he slab at some intermediate time is shown in figure 14. Right at the surface x=L where the slab temperature is T_s the heat transfer by convection must be equal to the heat transfer by conduction since a surface layer of zero thickness has a negligible energy change dE/dt because it has negligible mass. At the surface we can write,

$$Ak\left(\frac{dT}{dx}\right)_{x=L} = hA\left(T_e - T_s\right) \qquad (26)$$

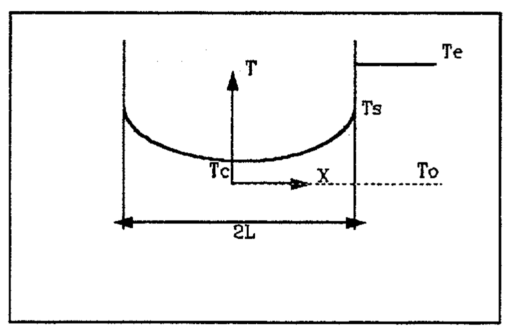

Figure 14

An upper estimate of the temperature difference between the surface and the centerline can be obtained as,

$$T_s - T_c > L\left(\frac{dT}{dx}\right)_{x=L} \qquad (27)$$

from equation 26,

The term hl/k which is dimensionless is known as the Biot number. When the Biot number is small, about 1/6 or less, the temperature

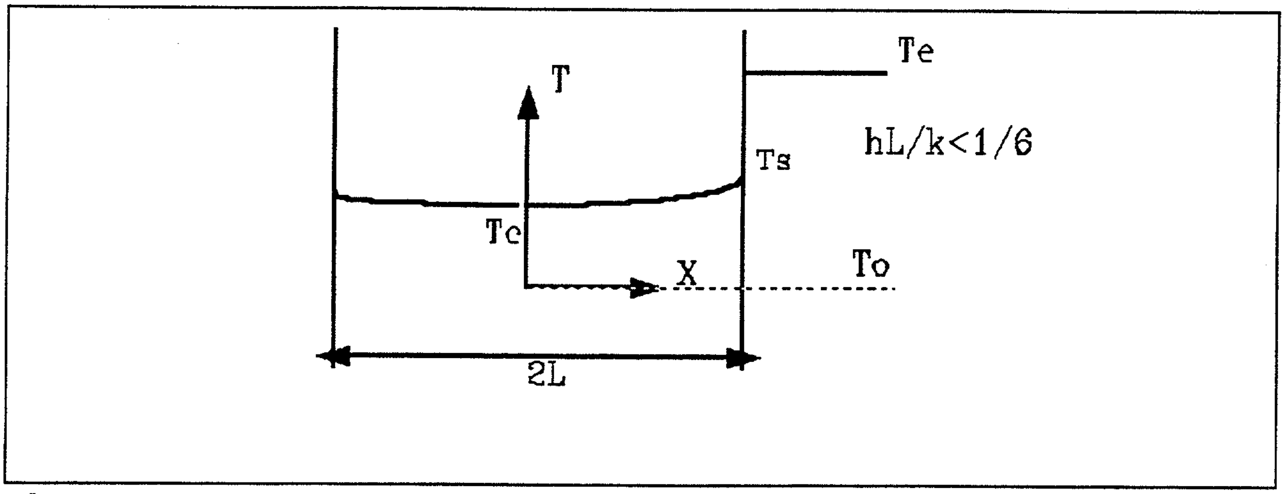

Figure 15

$$T_s - T_c > \left(\frac{hL}{K}\right)\left(T_e - T_s\right) \tag{28}$$

difference between the slab surface and the centerline is small compared to that between the slab surface, T_s, and the new air temperature, T_e. For this case, illustrated in figure 15, it is reasonable to assume that the temperature of the slab is uniform over its entire width. The slab temperature T only varies with time. The energy equation becomes,

$$\frac{dE}{dt} = q = -hA\left(T - T_e\right) \tag{29}$$

where A is the entire surface area of the slab which is in contact with air at T_e. When there isn't any change of phase and the energy change is proportional to $Mc\Delta T$, where M is the total mass of the slab, equation 29 becomes,

$$Mc\frac{dT}{dt} = -hA\left(T - T_e\right) \tag{30}$$

Since T_e is a constant this can be rearranged to read,

$$\frac{d\left(T-T_e\right)}{\left(T-T_e\right)} = -\frac{hA}{Mc}dt \tag{31}$$

with the initial condition for the slab that

$$at \quad t = 0 \qquad T = T_0 \tag{32}$$

The solution of equation 31 is

$$T-T_e = \left(T_0-T_e\right) \exp\left(-\frac{hAt}{Mc}\right) \tag{33}$$

or

$$T-T_e = \left(T_0-T_e\right) \exp\left(-\frac{t}{\tau}\right) \tag{34}$$

where τ is the thermal time constant of the slab with convective heat transfer coefficient h and

$$\tau = \frac{Mc}{hA} \tag{35}$$

<u>Example</u>

A two inch thick steel structural beam has natural convection heat transfer over one surface, the opposite side is insulated. Find its time constant.

<u>Solution</u>

First it must be determined if the steel can be assumed uniform in temperature across its width. Since only one side has convective heat transfer, referring to figure 13, L in this case should be the full width of the steel beam. The Biot number is

$$\frac{hL}{k} = \frac{2\left(\frac{2}{12}\right)}{20} = \frac{1}{60} \ll 1 \tag{36}$$

where h is estimated for natural convection from table 2. The assumption of uniform temperature is clearly justified. Note that if we were considering a two inch thick concrete section with a conductivity of about 1 BTU/hr ft °F the Biot number would be 1/3 and the concrete could not be assumed to be uniform in temperature.

The thermal time constant for the steel beam is,

$$\tau = \frac{Mc}{hA} = \frac{\rho cLA}{hA} = \frac{\rho cL}{h} = \frac{400(0.1)\frac{2}{12}}{2} = 3.5\ hr \tag{37}$$

The time constant is a function of the steel properties and the heat transfer coefficient at its surface.

Non-Uniform Temperatures

When the Biot number is much larger than 1/6 there is a substantial temperature difference across the width of a body as its temperature is changing with time. The solution of this case is more involved mathematically. Standard cases are given in any heat transfer textbook. The uniform temperature solution which was discussed previously can not be used in this case; such a solution may significantly overpredict the total heat transfer over a given period of time.

Convection Heat Transfer

Convective heat transfer is conduction though a gas or liquid in the presence of fluid motion. In these notes an introduction to the physics governing convection will be given along with some results for several different conditions.

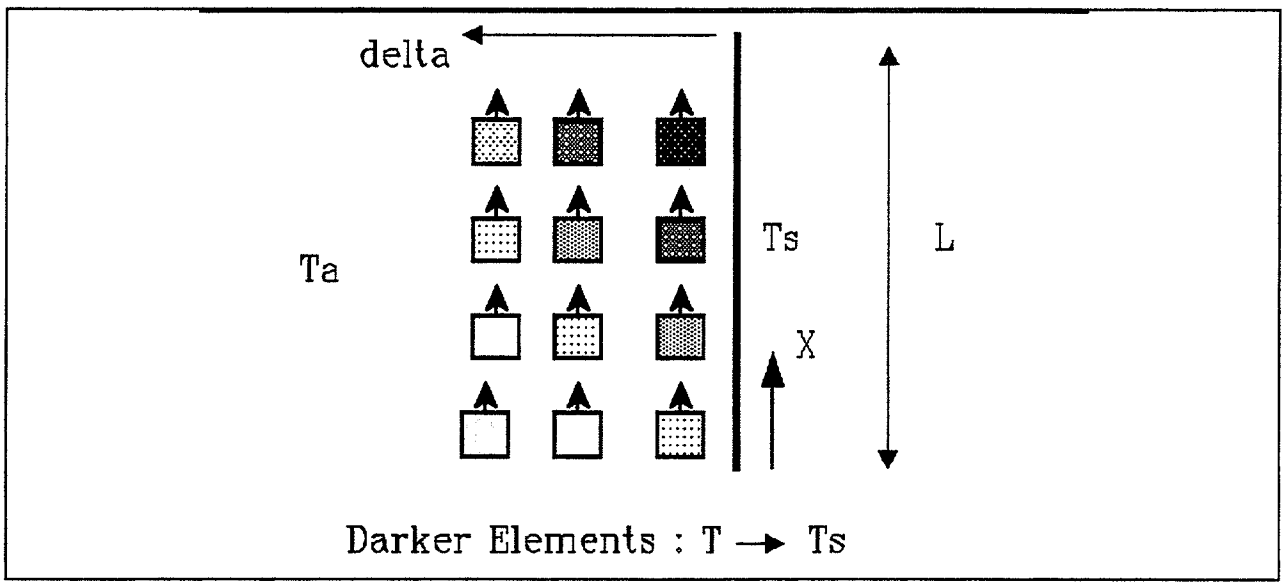

Figure 1
Fluid Flow Over a Heated Flat Plate

Consider the case of air at a uniform temperature T_a blown by a fan along a flat surface which is heated to a uniform temperature T_s, Figure 16. The element of air closest to the heated surface has a temperature increase as it starts to move up the plate. Elements further away, at a larger y coordinate still are at T_a. As the element becomes hotter it moves up and is replaced with another element at T_a and the process is repeated. At any location the temperature remains constant with time and there is a steady heat transfer for the surface to the air. As the air continues up the plate the elements further from the plate start to increase in temperature.

The maximum y distance at which the thermal effects are felt, at any location x, will be specified as δ. At any x location the conduction from the plate to the fluid can be calculated as

$$q = k_a \left(\frac{dT}{dy} \right)_{y=0} \tag{38}$$

This can be approximated in terms of δ as
Taking a control volume over the plates surface as shown in Figure 17, the energy equation gives,

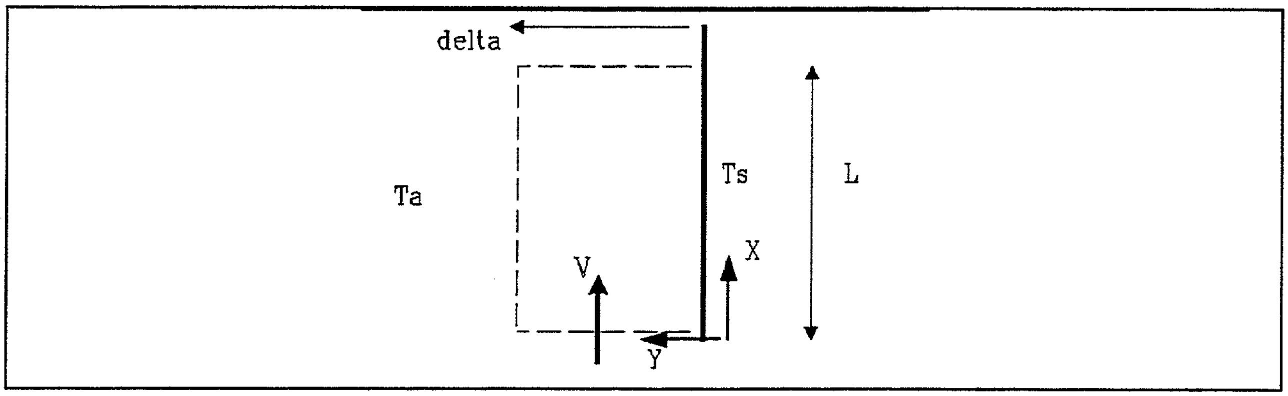

Figure 17 Control Volume for Energy Equation

$$q \approx \frac{k_a A \left(T_s - T_a\right)}{\delta} = \frac{L k_a z \left(T_s - T_a\right)}{\delta} \tag{39}$$

$$q = m \left(h_{out} - h_{in}\right) \approx \rho V \delta z c_p \left(\frac{T_s - T_a}{2} \right) \tag{40}$$

We can estimate the distance δ by equating q given by Eqn. 39 written over the area Lz[1] and q given by Eqn. 40:

$$k_a L z \frac{\left(T_s - T_a\right)}{\delta} \approx \rho V \delta z c_p \frac{\left(T_s - T_a\right)}{2} \tag{41}$$

Solving for δ,

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[1] This is only an approximation, to be more exact in equation 39, δ varies with the distance x.

$$\delta \approx \left(\frac{k_a L}{\rho V c_p} \right)^{1/2} \tag{42}$$

q can now be eliminated by substituting this expression for δ into
Eqn. 39,

$$q \approx k_a A \frac{\left(T_s - T_a\right)}{\delta} \approx \left(T_s - T_a\right) A k_a \left(\frac{\rho V c_p}{k_a L} \right)^{1/2} \tag{43}$$

From the definition of the heat transfer coefficient,

$$h = \frac{q}{A\left(T_s - T_a\right)} = \frac{k_a}{L} \left(\rho V L\right)^{1/2} \left(\frac{c_p}{k_a} \right)^{1/2} \tag{44}$$

Eqn. 44 indicates that the heat transfer coefficient increases with the fluid velocity V. As more fluid passes over the plate per unit time, there is a lower average temperature increase. Put another way, as the velocity increases the elements in Figure 16 further from the plate don't heat up and δ decreases. The conduction rate then increases because the temperature gradient $(T_s-T_a)/\delta$ increases. Similarly, when a fluid with a higher density and specific heat is used δ is smaller and q and h increase. Finally, as the fluid conductivity is increased, h increases.

To achieve an exact answer the viscosity of the fluid must also be considered. The velocity, μ, is the fluid property which governing the resistance force acting on a thin plate as it is pulled through a fluid at a constant velocity in a direction parallel to the plate length. The viscosity has units of lb_m/h-rft or kg/ms. Introducing the viscosity into Eqn. 44 and rearranging, there results,

$$\frac{hL}{k_a} = \left(\frac{\rho V L}{\mu} \right)^{1/2} \left(\frac{c_p \mu}{k} \right)^{1/2} \tag{45}$$

hL/k_a, a non-dimensional heat transfer coefficient is known as the Nusselt number. ρVL/μ is the Reynolds number, Re, and $c_p\mu/ka$ is the Prandtl number, Pr.

In general, the results for forced convection are given as

$$\frac{hL}{k_a} = f\left(Re, Pr\right) \tag{46}$$

For flow over a flat plate at uniform temperature the exact solution is,

$$\frac{hL}{k_a} = 0.66\,(Re)^{1/2}\,(Pr)^{1/3} \tag{47}$$

Using air properties at room temperature h can be found as

$$h = 0.71\left(\frac{V}{L}\right)^{0.5} \tag{48}$$

while for water,

$$h = 12.7\left(\frac{V}{L}\right)^{0.5} \tag{49}$$

In this form V is in ft/sec and L is in ft. Note, water gives a much higher heat transfer coefficient than air because it has a much higher thermal conductivity as well as a higher density and specific heat.

Turbulent Flow

At low velocity, the fluid flows in very smooth paths about parallel to the plate surface. As the velocity is increased a point is reached where the fluid motion is much more chaotic characterized by eddies in the flow near the plate surface. This is termed turbulent flow. Turbulent flow over a flat plate is found to occur when the Reynolds number $\rho\ Vx/\mu$ exceeds 300,000. x is used to indicate the distance from the leading edge; the front of the flat plate can have laminar flow while the rear experiences turbulent flow. The distinction between laminar and turbulent flow is important because the eddies in the turbulent flow tend to bring fluid at the ambient temperature T_a much closer to the heated plate surface. In effect the eddies reduce the distance δ for conduction heat transfer and markedly increase the heat transfer coefficient.

The correlations for turbulent flow over a flat plate is of the same form as the laminar flow heat with different coefficients, for turbulent flow $Re = \rho VL/\mu \geq\ 320{,}000$

$$\frac{hL}{k_a} = 0.037\left(\frac{\rho VL}{\mu}\right)^{0.8}(Pr)^{1/3} \qquad (50)$$

For air at room temperature

$$\text{for } VL > 50\ ft^2/\text{sec} \qquad h = 0.55\ V^{0.8}/L^{0.2} \qquad (51)$$

For water

$$\text{for } VL > 3.9\ ft^2/\text{sec} \qquad h = 21.2\ V^{0.8}/L^{0.2} \qquad (52)$$

Flow Inside Tubes

The other important flow geometry is gas or liquid flow inside tubes. A similar development exists for the convective heat transfer with the exception that h is defined based on the mean temperature T_M of the fluid within the tube at the location in question. For a section within the tube of axial length between x and $x+\Delta x$,

$$h = \frac{q}{\pi D\,\Delta x\left(T_s - T_M\right)} \qquad (53)$$

where T_M is the mean fluid temperature at x.

Almost all practical cases of tube flow, the flow is turbulent. Exceptions are flows through very small tube diameters or the flow of viscous fluids such as oil. For turbulent tube flow, the heat transfer coefficient is given as

$$\frac{hD}{k} = 0.023\left(\frac{\rho VD}{\mu}\right)^{0.8}(Pr)^{0.4} \qquad (54)$$

For air at room temperature, the relationship becomes,

$$h = 0.34\frac{V^{0.8}}{D^{0.2}} \qquad (55)$$

while for water we get,

$$h = 13\frac{V^{0.8}}{D^{0.2}} \qquad (56)$$

where V is given in ft/sec and D is in feet.

Natural or Free Convection

In natural convection the fluid motion is solely due to buoyancy effects caused by the heating or cooling process of the fluid. Natural convection flow over vertical flat plates have similar physical considerations and analogous expressions to those used for forced convection, see Figure 16 and Eqn. 44 or 46. However, in this case the fluid velocity V is not set by external fans or by the motion of the heated body.

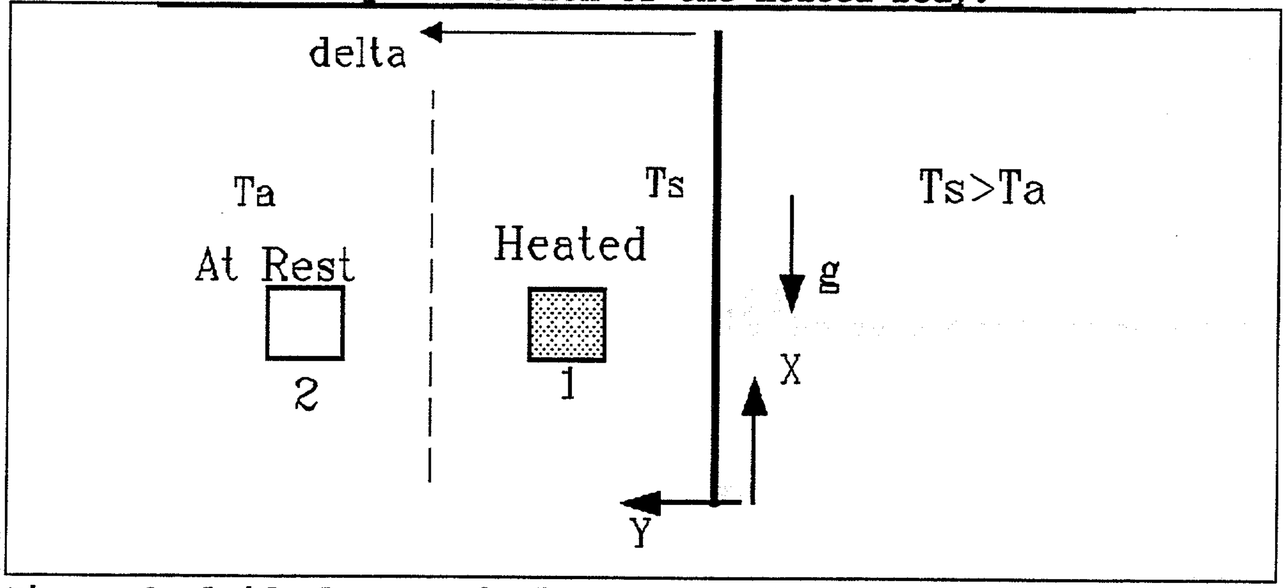

Figure 3 Fluid elements inside and outside the heated layer

We can make an estimate of V for free convection by considering a simplified case shown in Figure 18. A vertical heated plate at temperature T_s is surrounded by air at a temperature T_a. Air near the plate surface has a temperature close to T_s. The density of air decreases as its temperature increase. A fluid element of volume Vo far from the plate at T_a, position 2 in Figure 18 is at rest because it's weight $\rho_2 V_o g$ is balanced by the difference in the air pressure between its upper and lower surfaces. The fluid element of air at position 1 with the same volume V_o has a smaller weight, $\rho_1 V_o g$ because it is heated and has a lower density. But the element at position 1 has the same pressure acting on it as the element at position 2.
Thus, the difference between the weight at position 2 and 1 represent the net upward force exerted on the element 1 which causes it to accelerate. That net force acting over the distance L is the work done on the fluid elements which results in an increase in its kinetic energy, neglecting any viscous resistance to the fluid flow. At the bottom of the plate the velocity and kinetic energy are zero. V_B is the velocity due to the buoyancy efforts.

$$W = FL = \left(\rho_2 V_0 g - \rho_1 V_0 g\right) L = \frac{M V_B^{\ 2}}{2} = \frac{\rho_1 V_0 V_B^{\ 2}}{2} \tag{57}$$

Simplifying Eqn. 57,

$$\frac{\left(\rho_2 - \rho_1\right) g L}{\rho_1} = \frac{V_B^{\ 2}}{2} \tag{58}$$

The density change for air can be found from the ideal gas relationship

$$\rho = \frac{p}{RT} \tag{59}$$

and the change in the density becomes,

$$\frac{d\rho}{\rho} = -\frac{p\ dT}{RT^2}\frac{RT}{p} = -\frac{dT}{T} \tag{60}$$

and,

$$\frac{\Delta\rho}{\rho} = -\frac{\Delta T}{T} \tag{61}$$

V_B can be estimated from Eqn. 58 as

$$V_B = \left(\frac{2\left(T_s - T_a\right) g L}{T}\right)^{0.5} \tag{62}$$

Using V_B in the Reynolds number it can be seen that the Reynolds number is proportional to the temperature difference and the plate length,

$$\left(Re\right)^2 = \left(\frac{V_B \rho L}{\mu}\right)^2 \propto \left(T_s - T_a\right) L^3 \tag{63}$$

The heat transfer coefficient should follow the earlier form, Eqn. 46, with Re given by Eqn. 63.

The exact expression for natural convection on a vertical plate in air at room temperature is

$$\text{for } \Delta T\, L^3 < 1000^\circ F\, ft^3, \quad h = 0.29 \left(\frac{\Delta T}{L}\right)^{1/4} \tag{64}$$

$$\text{for } \Delta T\, L^3 > 1000^\circ F\, ft^3, \quad h = 0.21 \left(\Delta T\right)^{1/3} \tag{65}$$

Natural Convection in Enclosed Spaces

In an open wall cavity between interior and exterior walls air circulation will take place from the hot to the cold wall. The air will rise along the hot wall move horizontally to the cold wall at the upper end of the wall cavity (and at other vertical locations as well). The warm air will then flow down the cold wall. This process will result in energy transfer from the hot to the cold wall. The overall heat transfer can be represented by a heat transfer coefficient defined in terms of the two wall temperatures,

$$q = h_c A \left(T_{hot\ wall} - T_{cold\ wall}\right) \tag{66}$$

Figure 19 shows measured results for eight foot high walls. h_c is a function of the spacing between the walls and Θ, the temperature difference from the hot to the cold wall. At a small spacing and/or a small temperature difference the buoyancy effects are minimal and h_c is simply the ratio of air conductivity to wall spacing. As the spacing is increased, h_c reaches a constant, typically a spacing between 1/2 and 3/4 inch is optimum. Radiation heat transfer across the cavity must be added to the convection. If there is infiltration of outside air into the cavity the energy transfer may be increased considerably.

Radiation Heat Transfer

Thermal radiation along with conduction are the two fundamental mechanisms of heat transfer. Bodies, such as the sun, emit electromagnetic energy by virtue of their temperature level. The electromagnetic energy is exchanged between bodies at different temperatures giving rise to a net heat transfer.

For heat transfer by radiation the electromagnetic energy generally falls in the visible range, with wavelengths from 0.4 to 0.7 µm, the near infrared, from 0.7 to 25µm in wavelength, or the far infrared from 25 to 1000 µm. Electromagnetic radiation in other wavelengths such as X-Rays, and Radio waves is not thermally induced and will be excluded from consideration.

A black body is a body which absorbs all of the radiation incident on its surface over all wavelengths of importance for heat transfer. The absorbed energy represents an energy transfer to the black body which can contribute to the internal energy increase or it may be transferred through the body by one or more forms of heat transfer.

For a black body at a uniform temperature T, the rate of radiation heat transfer emitted by the body and leaving the surface, over all wavelengths, is given as

$$q_{r\ emitted} = A\sigma T^4 \tag{67}$$

where T is the absolute temperature and σ is the Stefan-Boltzmann constant, which has the value 0.17×10^{-8} BTU/hr $ft^2 \cdot R^4$ or 5.7×10^{-8} $W/m^2\ {}^\circ K^4$. Because of the non-linear nature of the expression, radiation becomes progressively more important at higher temperatures although there is considerable radiative heat transfer between bodies at room temperature.

The black body radiation is emitted over a range of wavelengths. The emitted energy at a single wavelength λ, $q_{re\lambda}$, can be defined so that

$$q_{r\ emitted} = \int_0^\infty q_{re\lambda}\, d\lambda \tag{68}$$

Figure 20 shows the distribution of $q_{re\lambda}$ as a function of wavelength for black bodies at three temperature levels. For solar radiation, the sun has an effective black body temperature near 10,400 °R, a majority of the emitted energy is in the visible wavelengths and the balance of the energy is in the near infrared below 3μm. In contrast, the energy emitted by a black body at room temperature falls in the infrared wavelength range from 8μm to 40μm. For any black body at temperature T eighty percent of the total black body emissions occurs when the product of the wavelength and the absolute temperature, λT, is between 4000 (μm)(°R) and 17,000 (μm)(°R). One half of the total radiation is emitted at wavelengths equal to or below the value of λT equal to 7400 (μm)(°R).