In the visible wavelengths many common building materials are not good absorbers. The fraction of the black body radiation absorbed by a body at a given wavelength λ is given by the monochromatic absorptivity α_λ, the values of α_λ lie between zero and one. In the visible wavelengths, white paint, wood and plaster have α_λ values between about 0.1 and 0.5 see Table 3. In the infrared above 4 or 5µm wavelengths almost all common building materials except shiny metals have α_λ of 0.8 or higher. Plate glass which is transparent in the visible wavelength becomes a very good black body in the infrared wavelengths with α_λ greater than 0.9.

Table 3
Absorptivity of Materials to Radiation
from a Room Temperature Source and a Solar Source

	Absorptivity Solar Radiation	Absorptivity Source at 500°R
Polished Aluminum	0.3	0.06
Slate Roofing	0.9	0.9
Concrete	0.6	0.85
Red Brick	0.55	0.9
Paper, Wood, Cloth, Plaster	0.1 - 0.45	0.85 - 0.92

Sieber,W. ,Z. tech. Physik. 22:130 (1941)

Heat Transfer Between Black Bodies

Consider two flat plates, A_1 and A_2, parallel to each other with the spacing between the surfaces small compared to the width of the plates, see Figure 21. The air between the plates can be considered transparent.

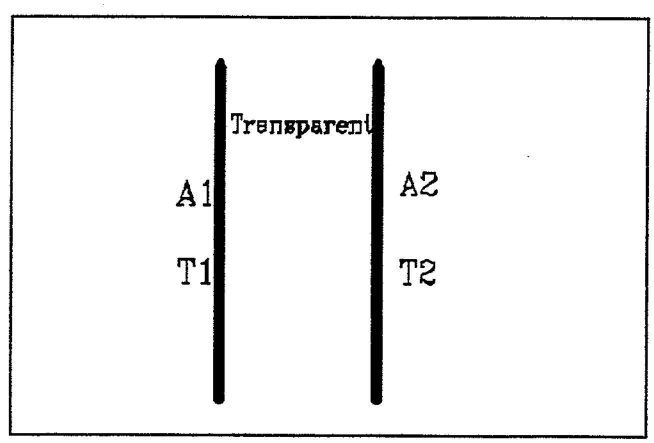

Figure 21
Parallel Black Plates

Each plate is a black body at a uniform temperature T_1 and T_2, respectively. The radiant energy emitted by the first plate is

$$q_{r1-2} = A_1 \sigma T_1^4 \tag{69}$$

All of the energy emitted by plate 1 is absorbed by plate 2. Similarly all of the energy emitted by plate 2, $A_2 \; \sigma \; T_2^4$ is absorbed by plate 1. The net radiative heat transfer is

$$q_r = A_1 \sigma \left(T_1^4 - T_2^4\right) \tag{70}$$

This heat transfer takes place independent of any conduction or convection heat transfer assuming the temperature T_1 and T_2 can be maintained. Thus the total heat transfer is the sum of the radiative heat transfer given by Eqn. 70 plus the convective heat transfer between the two plates.

In many cases it is useful to linearize the relationship given by Eqn. 70. Taking the mean temperature as $(T_1+T_2/2)$ and Δ as the temperature difference T_1-T_2 then $T_1=T_M+\Delta/2$ and $T_2=T_M+\Delta/2$ and

$$T_1^4 - T_2^4 \approx T_M^4 + 4\frac{\Delta}{2}T_M^3 + 6\left(\frac{\Delta}{2}\right)^2 T_M^2 - \left[T_M^4 - 4\frac{\Delta}{2}T_M^3 + 6\left(\frac{\Delta}{2}\right)^2 T_M^2 \tag{71}$$

When $\Delta << T_M$, where T_M is the absolute temperature,

$$\sigma\left(T_1^4 - T_2^4\right) \approx 4\sigma T_M^3 \left(T_1 - T_2\right) \tag{72}$$

An effective or radiative heat transfer coefficient can be defined so that

$$q_r = h_r \, A \left(T_1 - T_2\right) \tag{73}$$

and in this case,

$$h_r = 4\sigma T_M^3 \tag{74}$$

Table 4 gives values of h_r for different values of mean temperature. These values should be compared to the values given for convection in Table 2. At room temperatures heat transfer by radiation is comparable in magnitude to natural convection.

Table 4
h_r for Radiation at Mean Temperature T_m

Mean Temperature, T_m (°F)	h_r (BTU/hr ft^2 °F)
0	0.66
60	0.96
200	2.0
500	6.0
1500	51

1 BTU/{hr ft^2 °F} = 5.68 W/{m^2 °C}

For radiation between walls and ceiling or between a person and surrounding surfaces which are each black bodies at different temperatures the geometry of the configuration must be included.

The radiation emitted from surface A_1 which is absorbed by black body surface A_2 is

$$q_{r1-2} = A_1 F_{12} \sigma T_1^{\,4} \tag{75}$$

where F_{12}, the configuration factor, is the fraction of black body energy emitted from surface area A_1 which goes to surface area A_2. Similarly, the radiation emitted from surface A_2 absorbed by black body A_1 is,

$$q_{r2-1} = A_2 F_{21} \sigma T_2^{\,4} \tag{76}$$

where F_{21} is the fraction of the energy emitted by surface A_2 which is directed at surface A_1. When the two surface are at the same temperature the energy from A_1 to A_2 must equal the energy from A_2 to A_1. Thus the product of A_1F_{12} must equal A_2F_{21}. Since the configuration factors involve only geometry this equality must hold no matter what the temperatures T_1 and T_2 are. In general, the net radiation between the two constant temperature black bodies must be,

$$q_r = A_1 F_{12} \sigma \left(T_1^{\,4} - T_2^{\,4}\right) \tag{77}$$

For two parallel plates close together F_{12} is 1. For a person standing close to a large flat surface, F_{12} from the person to the flat surface is approximately one half. The configuration factor from a horizontal plate to the sun when directly overhead is the solid angle from an observer standing on the plate to sun, divided by π; this becomes the ratio of the square of the sun's radius to the distance from the earth to the sun. This is approximately 2×10^{-5}. The product of this configuration factor and $\sigma T_{solar}^{\,4}$ is approximately 400 BTU / hr ft^2. When the transmission loss through the earth's atmosphere is considered (a cloudless day with the sun directly overhead) the solar flux becomes approximately 200 BTU/hr ft^2.

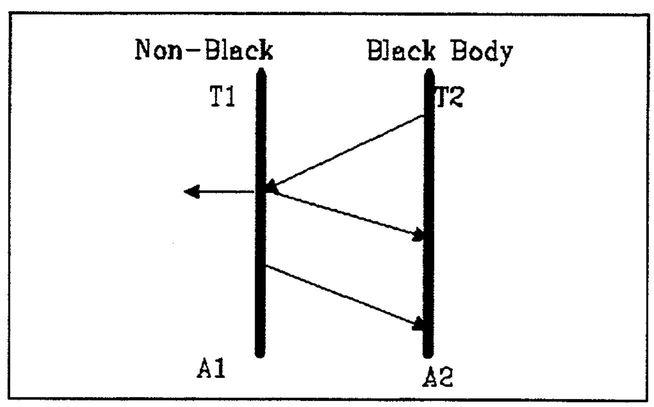

Figure 22
Non-Black Plates

Non-Black Bodies

Consider two large parallel flat plates A_1 and A_2. Surface A_2 is a black body and surface A_1 is a non-black body with absorptivity α_1. The amount of energy emitted by black body A_2 which is absorbed by body A_1 is $\alpha_1 \sigma T_1^4 A_1$, the balance $(1 - \alpha_1)$ is reflected back to A_2 where it is reabsorbed.

Since surface A_1 is not a black body, the amount of radiation energy it emits is less than σT_1^4. That amount is given at $\epsilon_1 \sigma T_1$ where ϵ_1 is the emissivity, defined as the ratio of the energy emitted by a non-black body to the energy emitted by a black body at the same temperature.

The net radiant energy exchanged between A_1 and A_2 becomes

$$q_r = \epsilon_1 A_1 \sigma T_1^4 - \alpha_1 A_1 \sigma T_2^4 \qquad (78)$$

If T_1 and T_2 are at the same temperature, the net heat transfer between the two bodies must be zero (If not, two bodies initially at the same temperature would have one body spontaneously drop in temperature while the second rose in temperature). If the net heat transfer is zero,

$$\epsilon_1 = \alpha_1 \qquad (79)$$

This hold when surface A_2 is a black body at a temperature near A_1. When the two surfaces are at temperatures far from each other or neither surface is a black body then a good engineering approximation is $\epsilon_{1\lambda} \approx \alpha_{1\lambda}$ at any given wavelength but α_1 does not necessarily equal ϵ_1.

For the case when ϵ_1 is equal to α_1 surface A_1 is termed a gray body. If A_2 is a black body then the net heat transfer between the two parallel plates becomes,

$$q_r = \epsilon_1 A \sigma \left(T_1 - T_2\right) \qquad (80)$$

If two parallel plates are both gray bodies it can be shown that the net heat transfer is

$$q_r = \frac{A\left(\sigma T_1^4 - \sigma T_2^4\right)}{\dfrac{1}{\epsilon_1} + \dfrac{1}{\epsilon_2} - 1} \qquad (81)$$

Note this reduces to equal equation 80 when ϵ_2 is unity. The net radiative heat transfer can be approximated as

$$q_r = A\frac{\epsilon}{2}\left(\sigma T_1^4 - \sigma T_2^4\right) \tag{82}$$

when $\epsilon_1 = \epsilon_2$ and $\epsilon_1 << 1$.

Example

An insulation is constructed from four parallel sheets of thin plastic sheeting which are 1/4 inch apart and are filled with air. Both sides of the sheets are coated with an aluminized material to give an emissivity of 0.1. Assume the emissivity is equal to the absorptivity. Find the R value of this insulation from the inside to the outside layer.

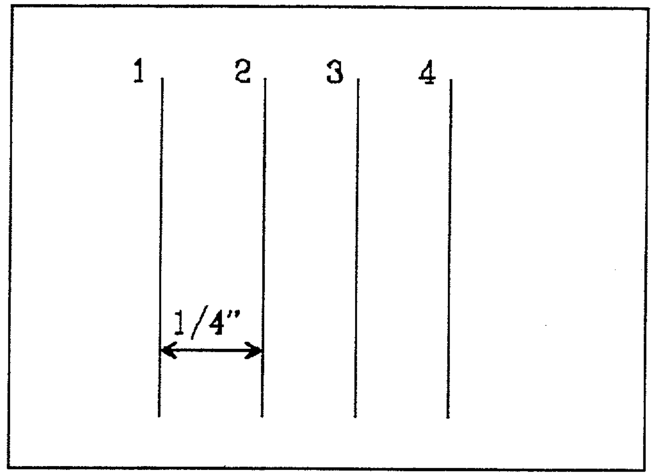

Figure 23
Parallel Layers

Solution

Numbering the successive sheets from 1 to 4, for each sheet the radiative heat transfer is given by equation 81,

$$\frac{q_r}{A} = \frac{\sigma T_i^4 - \sigma T_{i+1}^4}{\frac{1}{\epsilon_i} + \frac{1}{\epsilon_{i+1}} - 1} \tag{83}$$

$$\frac{q_r}{A} = \frac{\epsilon}{19}\left(\sigma T_i^4 - \sigma T_{i+1}^4\right) \tag{84}$$

This can be approximated as,

$$\frac{q_r}{A} = \sigma T_M^3 \frac{\epsilon}{19}\left(T_i - T_{i+1}\right) = h_r\left(T_i - T_{i+1}\right) \tag{85}$$

Note that in this case h_r includes $\epsilon/19$. The radiation heat transfer across the gap acts independently of the conduction heat transfer (for the gap width of 1/4 inch there isn't any convection, see figure 19). The conduction heat transfer is,

$$\frac{q_c}{A} = \frac{k_a}{L}\left(T_i - T_{i+1}\right) \tag{86}$$

The total heat transfer across each gap is the sum of the conduction and the radiation,

$$q_{total} = A\left(h_r + \frac{k_a}{L}\right)\left(T_i - T_{i+1}\right) \tag{87}$$

Using the concept of thermal resistances, each gap has an overall thermal resistance of

$$R_t = \frac{1}{\left[A\left(h_r + \frac{k_a}{L}\right)\right]} \tag{88}$$

If the temperature level from sheet 1 to 4 does not change appreciably than h_r and k_a can be assumed constant. The four parallel sheets form three gaps. The overall thermal resistance is three times that for a single sheet. The mean temperature will be evaluated at 60°F. The R value is the thermal resistance per unit area,

$$R\text{-}value = \frac{3}{h_r + \frac{k_a}{L}}$$

$$R\text{-}value = \frac{3}{\left[\frac{(4)(.17\times10^{-8})(520^3)(.1)}{19} + \frac{.016}{\left(\frac{.25}{12}\right)}\right]} \tag{89}$$

$$R\text{-}value = \frac{3}{(5\times10^{-3}+0.77)} = \frac{3}{0.775} = 3.9\left(\frac{BTU}{hr\ ft^2\ {}^\circ F}\right)^{-1}$$

Note that in this instance the heat transfer by conduction is much larger than that by radiation. The result would be about the same if only one surface in each gap had a low emissivity and the other was a black body.

One way to conserve energy in a building in the winter is to use the hot air exhausted from the building to heat the fresh cold outdoor air brought into the building for ventilation. The two air streams are kept separated by a metal surface and heat is transferred across this surface from the hot air stream to the cold air as shown on figure 24. The partition may be a flat surface or it may be a tube with one air stream flowing inside and the second flowing over the outside. It is necessary to calculate the size of the heat exchanger required to transfer a certain magnitude of heat between the two streams.

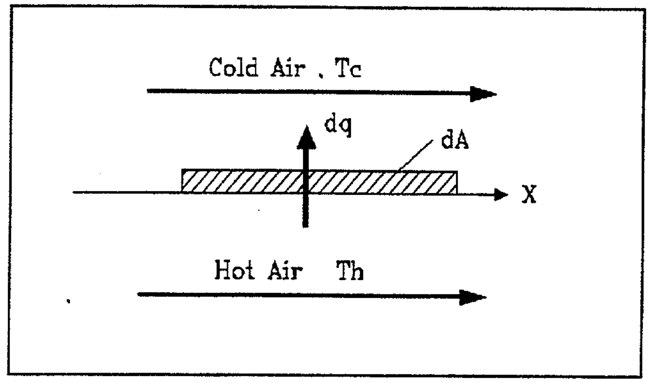

Figure 24 Heat Transfer Between two Air Streams

The overall coefficient of heat transfer for exchange between the two air streams is

$$U = \left(\frac{1}{h_H} + \frac{L}{k} + \frac{1}{h_C} \right)^{-1} \tag{90}$$

The heat transfer between the two streams for a differential area dA can be written as,

$$dq = U\, dA \left(T_H - T_C \right) \tag{91}$$

In this expression T_H and T_C represent the mean air temperature of the hot and cold stream, respectively, at the coordinate x where dA is located.

We can also write an energy balance for the hot and cold streams by taking a control volume over each of the streams from distance x to x+dx, see fig. 25. For the cold air stream with steady state conditions,

$$dq = m_C c_{pC} dT_C \tag{92}$$

and for the hot air stream,

$$dq = -m_H c_{pH} dT_H \tag{93}$$

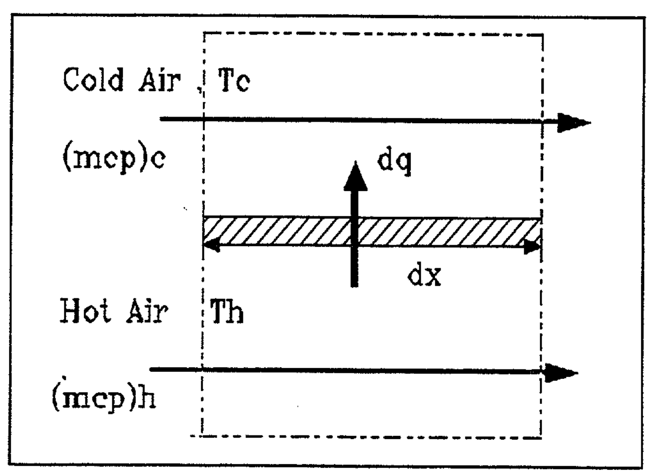

Figure 25

Proceeding in the x direction, the cold stream temperature rises while the hot stream temperature falls. For good energy efficiency it is useful to transfer as much energy from the hot to the cold stream as possible. Then T_C should approach the hot stream temperature, T_H. The question is how large must the surface area of the heat exchanger partitions be to achieve a given cold stream temperature rise?

The difficulty in dealing with this question is that T_H and T_C vary with x. In the general case equations 91, 92 and 93 must be solved simultaneously.

Rather than starting with the most general case, consider a situation in which the hot stream has a much higher mass flow rate than the cold air stream. In this limit, comparing equations 92 and 93 it can be assumed that the hot stream temperature remains constant over the entire length x while the cold stream temperature continuously increases with x. Combining equations 91 and 92 since dq is the same,

$$U\,dA\left(T_H - T_C\right) = \left(m\ c_p\right)_C\ dT_C \tag{94}$$

This can be rewritten with T_H as a constant,

$$\frac{d\left(T_H - T_C\right)}{\left(T_H - T_C\right)} = -\frac{U\ dA}{\left(m\ c_p\right)_C} \tag{95}$$

Integrating equation 95 yields,

$$\frac{\left(T_H - T_C\right)_{x=L}}{\left(T_H - T_C\right)_{x=0}} = \exp - \frac{U\ A}{\left(m\ c_p\right)_C} \tag{96}$$

As the area is increased, the cold stream temperature approaches the hot stream temperature. Note the diminishing returns of increasing A as the magnitude of the coefficient on the right hand side of equation 96 gets larger.

The more usual representation of heat exchanger performance is in terms of the effectiveness defined as,

$$\epsilon = \frac{q\ (actual)}{q\ (A \to \infty)} = \frac{\left(m\ c_p\right)_C \left(T_{C,L} - T_{C,0}\right)}{\left(m\ c_p\right)_C \left(T_H - T_{C,0}\right)} \tag{97}$$

where the actual q is found from an energy balance on the cold stream. The maximum q is the limiting value of heat transfer for the same flow rate and inlet hot and cold stream temperatures while the area of the heat transfer surface gets very large. From the solution of equation 96,

$$\epsilon = \frac{T_H - T_{c,0} - (T_H - T_{c,L})}{(T_H - T_{c,0})} = 1 - \exp - \frac{U\,A}{(m\,c_p)_c} \tag{98}$$

The coefficient of the right hand term is written as NTU or number of transfer units. The final form of equation 92 is

$$\epsilon = 1 - \exp(-NTU) \tag{99}$$

In the general case when the mass flow rate of both streams are the same magnitude, the effectiveness, ϵ, is a function of the NTU, but it is also a function of the specific flow geometry and the ratio of product of the mass flow rate and specific heat of the hot stream to the cold stream, or the ratio of the mc_p product for the stream with the lower of the two values, $(mc_p)_{min}$ to the stream with the larger value, $(mc_p)_{max}$.

Consider the case of a hot and cold stream with equal value of mc_p with both streams flowing in the same direction, referred to as parallel flow, see figure 26. As the hot stream experiences a temperature drop, the cold stream experiences a corresponding temperature rise. In the limit both streams will approach the same temperature, the average of the inlet hot and

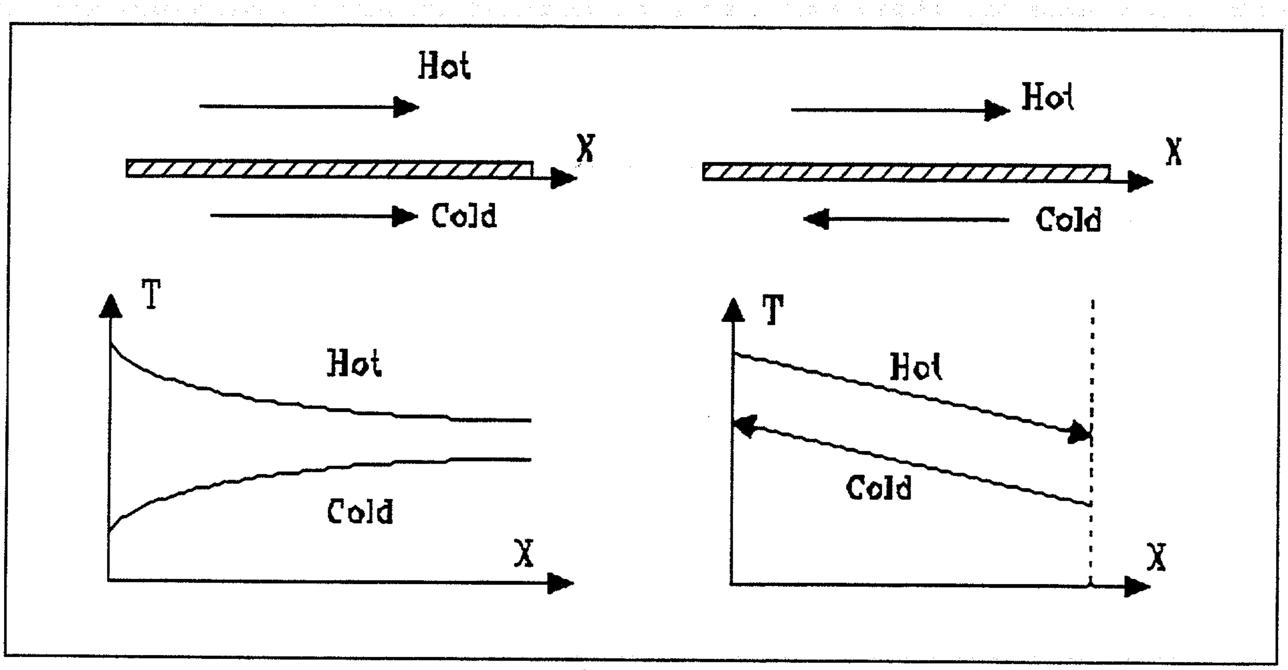

Figure 26
Parallel and Counter Flow
Heat Exchangers

cold temperatures.

When the two streams are flowing in opposite directions, termed counter flow, then the outlet cold stream is at the same x coordinate as the inlet to the hot stream. In counter flow when the area gets very large the cold stream temperature can approach the inlet hot stream temperature.

In the definition of effectiveness, the maximum q refers to the optimum flow geometry which is always the counterflow. Thus the limit of the effectiveness for counterflow is unity while the limit for the effectiveness in parallel flow is one- half. Figure 27 shows the effectiveness versus NTU for different flow geometries.

Example

An air-to-air heat exchanger is to be designed to pre-heat fresh air entering a house in winter. The house interior is 2000 ft^2 and is maintained at 70°F. The exterior air is at 40°F. One air change per hour is desired with the exhaust air preheating the incoming air. The incoming and exhaust air streams are separated by a series of thin parallel aluminum plates. The air velocity is 20 ft/sec and the spacing between plates is 1 inch. What is the total area of plates necessary if the outside air is to be heated to 60°F?

Solution

To find the surface area, first calculate the U values. The heat transfer coefficient for flow between parallel plates can be closely approximated by using the results for flow inside tubes with the tube diameter replaced by the hydraulic diameter, D_H, 4 × cross section area / wetted perimeter. In the parallel plate case, D_H is 2 × plate spacing. Note, eqn. 50 flow over a flat plate is only valid when the flow is exterior to a single plate and approaches a uniform temperature and velocity far from the plate surface.

Using equation 55,

$$h = 0.34 \frac{V^{0.8}}{D_H^{0.2}} = 0.34 \frac{(20)^{0.8}}{\left(\frac{1}{12}\right)^{0.2}} = 6.1 \frac{BTU}{hr\ ft^2\ °F} \tag{100}$$

Neglecting the thermal resistance of the metal plate,

$$U = \left[\frac{1}{h} + \frac{1}{h}\right]^{-1} = 3.05 \frac{BTU}{hr\ ft^2\ ^\circ F} \tag{101}$$

The heat exchanger effectiveness is

$$\epsilon = \frac{60-40}{70-40} = 0.67 \tag{102}$$

assuming counter flow with $C_{MIN}=C_{MAX}$, from figure 26 the number of transfer units, NTU, is approximately

$$NTU = \frac{UA}{C_{MIN}} \approx 2.0 \tag{103}$$

Then

$$A = \frac{2\ C_{MIN}}{U} \tag{104}$$

For one air change per hour

$$\begin{aligned} C_{MIN} &= (Volume\ flow\ rate)\ c_p \cdot \rho_{air} \\ C_{MIN} &= (2000)(10)(.24)(.076) \frac{ft^3}{hr} \frac{BTU}{lb_m\ ^\circ F} \frac{lb_m}{ft^3} \\ C_{MIN} &= 365 \frac{BTU}{hr\ ^\circ F} \end{aligned} \tag{105}$$

$$A = \frac{2(365)}{3.05} = 239\ ft^2 \tag{106}$$

If each plate is square, 2 ft × 2 ft, then 60 plates are necessary. With a one-inch plate spacing, the stack of plates are five feet wide. To make the heat exchanger more compact, narrow plate spacings would be used in combination with a different surface geometry--e.g., a roughened surface, which gives a higher heat transfer coefficient. Note in the calculation of NTU the area used is the area of one side of each plate.