MASSACHUSETTS INSTITUTE OF TECHNOLOGY
16.888/ESD.77J MSDO

16.888/ESD.77J Multidisciplinary System Design Optimization (MSDO) Spring 2004

Assignment 3

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Issued:	Monday March 8, 2004
Due:	Monday March 29, 2004
	(right after Spring Break)

You are expected to solve **Part (a)** individually and **Part (b)** in your project teams. Each person must submit their own complete write-up which includes Part (a) and Part (b). Please indicate the name(s) of your teammate(s) for Part (b). Submit a hardcopy

Topics: KKT, Sensitivity Analysis, Comparison of numerical and heuristic search algorithms, single objective optimization of your design projects.

Part (a)

Sensitivity Analysis

Recall the **dairy farm** example from Lecture 1. Remember, we were trying to optimize profit $J=P$, by choosing the number of cows N and the length L and radius R of the fence, subject to the cost constraint C< $100,000. Note that the design vector was $\mathbf{x} = [N\ L\ R]^T$.

(a1) Analytically calculate the solution, **x***, that will maximize J. For simplicity, assume that N is continuous, i.e. $N \in \mathbb{R}^+$. At the optimal point, **x***, compute the gradient vector ∇J with respect to the design vector **x**. Do a relative comparison (normalized sensitivity). Which design variable affects the optimal profit J^* the most? Show that the KKT conditions are met at the optimal point **x*.**

(a2) A number of parameters were fixed in the problem. What are they? Why are they not design variables? Compute the sensitivity of the optimal profit J^* with respect to these fixed parameters **p**? Compute the sensitivity of the optimal design point **x*** with respect to these fixed parameters **p**. What do you observe? What would you do if the price of milk dropped to 25% from its nominal level? Can you still make a profit?

(a3) Assume that we can increase the cost constraint C. What is the sensitivity of the optimal profit J^* with respect to this constraint? In other words: How many dollars of extra profit can we generate for every dollar the cost constraint is moved upward?

Comparison of Optimization Algorithms

(a6) Consider the following bivariate function J(x,y):

J **= 3*(1-x).^2.*exp(-(x.^2) - (y+1).^2) - 10*(x/5 - x.^3 - y.^5).*exp(-x.^2-y.^2) ...**
- 1/3*exp(-(x+1).^2 - y.^2)

Plot this function in the range (-5 < x,y <5). Note this is also known as the "peaks" function in MATLAB (>> help peaks). You may use any environment (Matlab, Mathematica, Excel… to solve this problem).

(a7) Find the global maximum of the function *J* , by using the following two optimization techniques:

a) Gradient search technique of your choice (e.g. Steepest Gradient, SQP…)
b) Heuristic technique of your choice (e.g. SA, GA, PSO)

(a8) Compare your techniques from above quantitatively and qualitatively as follows:

i. Dependence of answers on initial design vector (start point, initial population)
ii. Computational effort (CPU time [sec] or FLOPS)
iii. Convergence history
iv. Frequency at which the technique gets trapped in a local maximum

In order to answer this question you will need to implement your algorithms in some way (you may also use iSIGHT if you wish) and run a certain number of numerical experiments. Describe not just your conclusions, but also the process you followed. Do your conclusions still apply for larger, more complex design optimization problems?

Part (b)

In this assignment we want you to take the simulation code that you developed for your project in A2, refine it and couple it with an optimizer. First you should use a gradient-search technique. If you have non-continuous variables keep them at fixed values, or assume that they are continuous. Then use a heuristic technique and compare the answers and solution behavior.

(b1) Simulation completion
Complete the simulation code you started developing under part (b1) in assignment A2. Replace "placeholder" modules with actual code and rerun the entire analysis. Select interesting design points, based on what you learned in (b3) of assignment A2. What are remaining open issues in your project?

(b2) Gradient-based optimization

b2.1 Algorithm Selection
Select a gradient-based algorithm based on the characteristics of your project and the properties of the available algorithms. Rationalize in a few sentences, why your selection seems most appropriate for the problem at hand.

b2.2 Single objective optimization
Select a single (scalar) objective function for which to optimize your system. Describe why you selected this objective. Other potential objectives should be turned into equality or inequality constraints (for now). Using the gradient-based optimization technique identified in (b2.1), try to optimize your system with respect to the one objective function. Can you get the algorithm to converge? Do you obtain an improvement in the design? If not, please give some reasons. You may use iSight or any other optimizer, but please specify in your write-up. What is the optimal solution $\mathbf{x}^*$?

b2.3 Sensitivity analysis
Conduct a sensitivity analysis at the optimal point $\mathbf{x}^*$ with respect to $\mathbf{x}$, and a few of your fixed parameters, p? What design variables seem to be the drivers in your problem? Does this match the intuition you had beforehand? What are the active constraints at $\mathbf{x}^*$? How can you tell? Try moving the most important active constraint by some amount. Reoptimize and compare the new optimum with the previous optimum, what do you observe?

b2.4 Global Optimum
How confident are you that you have found the true global optimum? Explain.

Note: Keep all the results from this assignment handy for A4, where will extend the work on your project by also considering:

- scaling and post-optimality
- heuristic search techniques
- introducing multiple objectives