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Multidisciplinary System Design Optimization (MSDO)

Multiobjective Optimization (II)

Lecture 17

April 5, 2004

by

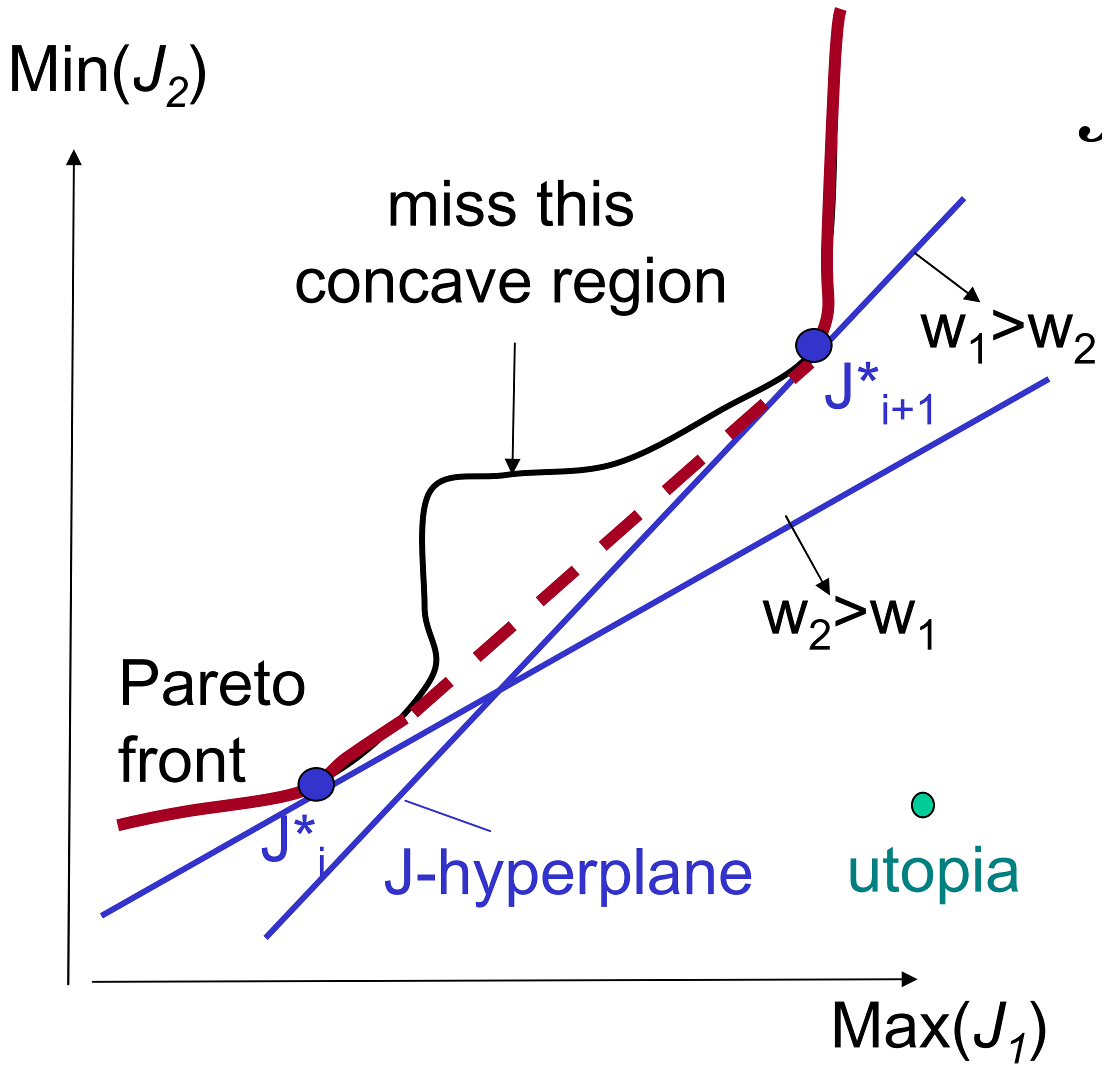
Prof. Olivier de Weck

MOO 2 Lecture Outline

Lecture 2 (today)

- Alternatives to Weighted Sum (WS) Approach
- Multiobjective Heuristic Programming
- Utility Function Optimization
- Physical Programming (Prof. Messac)
- Application to Space System Optimization
- Lab Preview (Friday 4-9-2003 – Section 1)

Weighted Sum (WS) Approach



$$J_{MO} = \sum_{i=1}^{z} \frac{w_i}{sf_i} J_i$$

- convert back to SOP
- LP in J-space
- easy to implement
- scaling important !
- weighting determines which point along PF is found
- misses concave PF

Weighted Square Sum Approach

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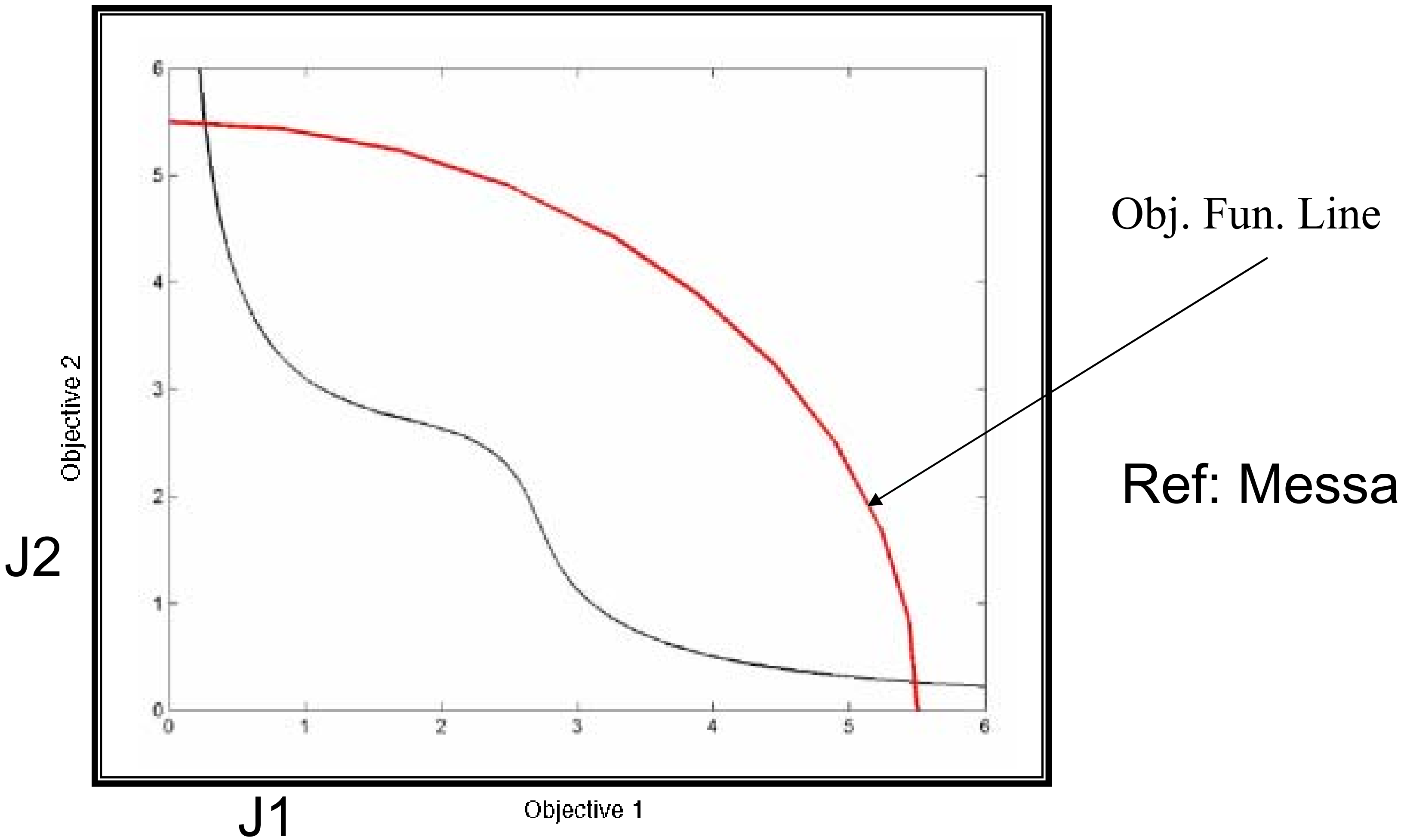
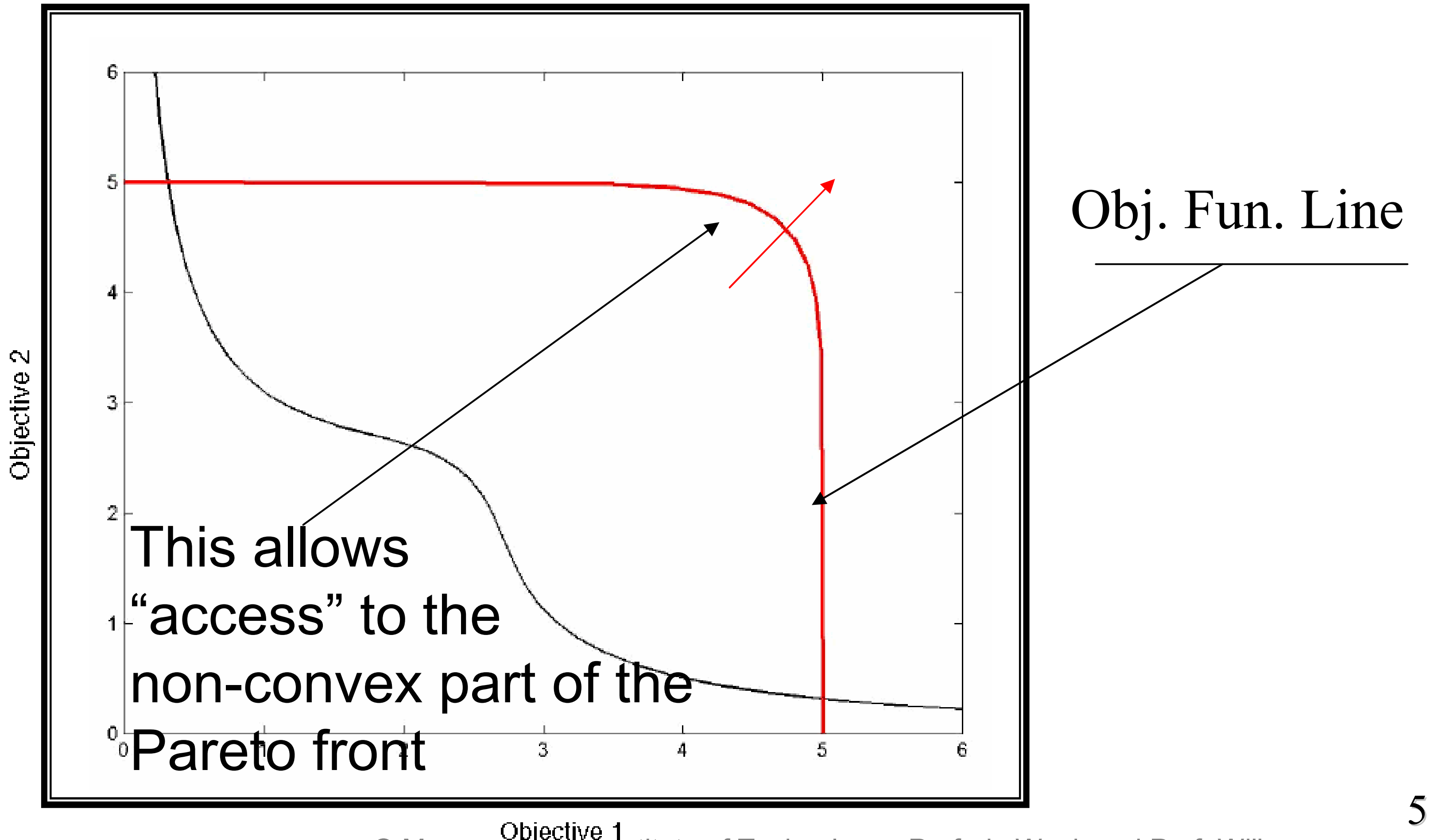
$$J = w_1 J_1^{\ 2} + w_2 J_2^{\ 2}$$



Ref: Messac

Compromise Programming (CP)

$$J = w_1 J_1^{\,n} + w_2 J_2^{\,n}$$



Multiobjective Heuristics

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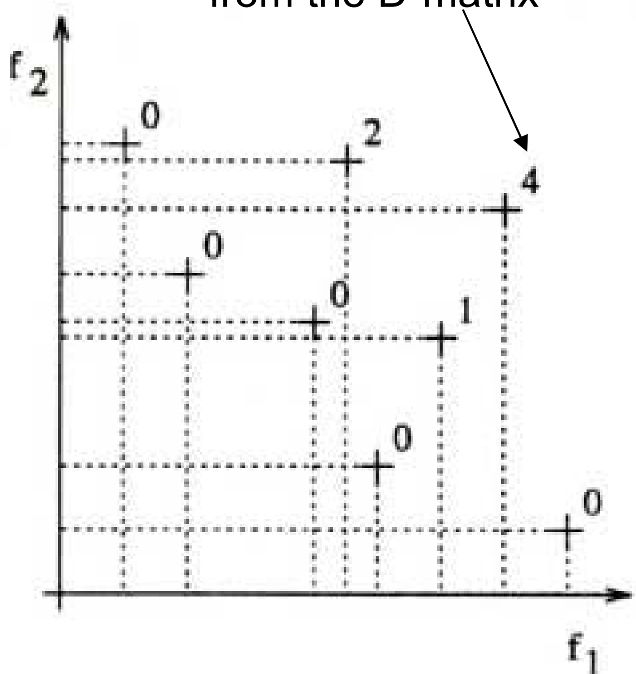
Pareto Fitness - Ranking

This number comes
from the D-matrix



Pareto ranking for
a minimization problem.

Recall: Multiobjective GA

- Pareto ranking scheme
- Allows ranking of population without assigning preferences or weights to individual objectives
- Successive ranking and removal scheme
- Deciding on fitness of dominated solutions is more difficult.

Example Multiobjective GA

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Minimization

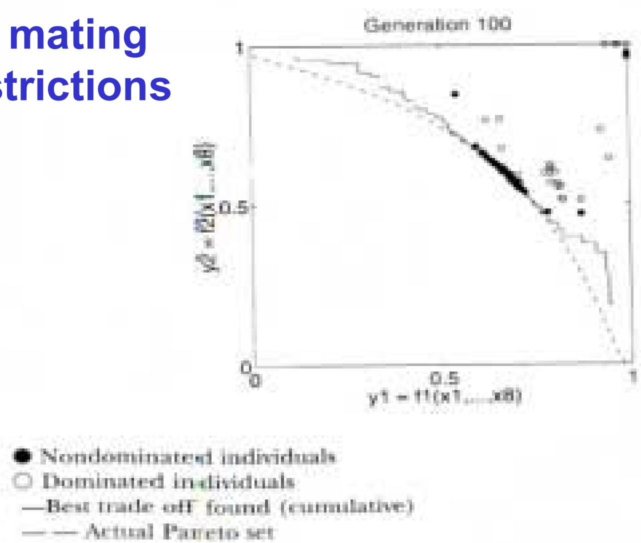
Objective 1

$$f_1\left(x_1,\ldots,x_n\right)=1-\exp\left[-\sum_{i=1}^{n}\left(x_i-\frac{1}{\sqrt{n}}\right)^2\right]$$

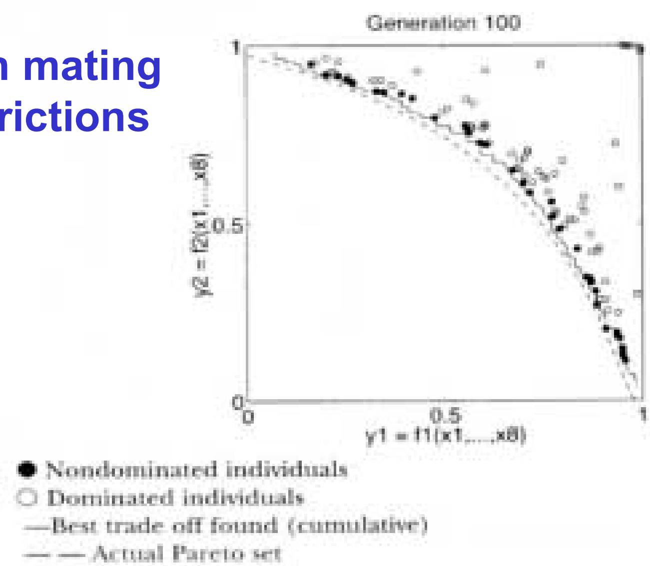
Objective 2

$$f_1\left(x_1,\ldots,x_n\right)=1-\exp\left[-\sum_{i=1}^{n}\left(x_i+\frac{1}{\sqrt{n}}\right)^2\right]$$

No mating restrictions



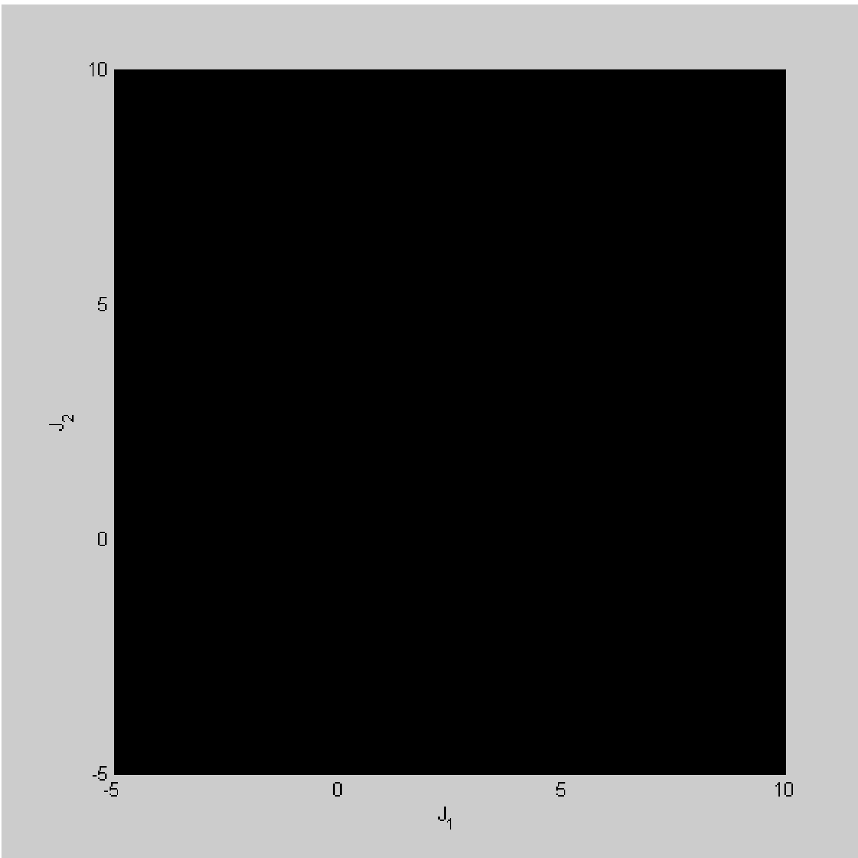
With mating restrictions



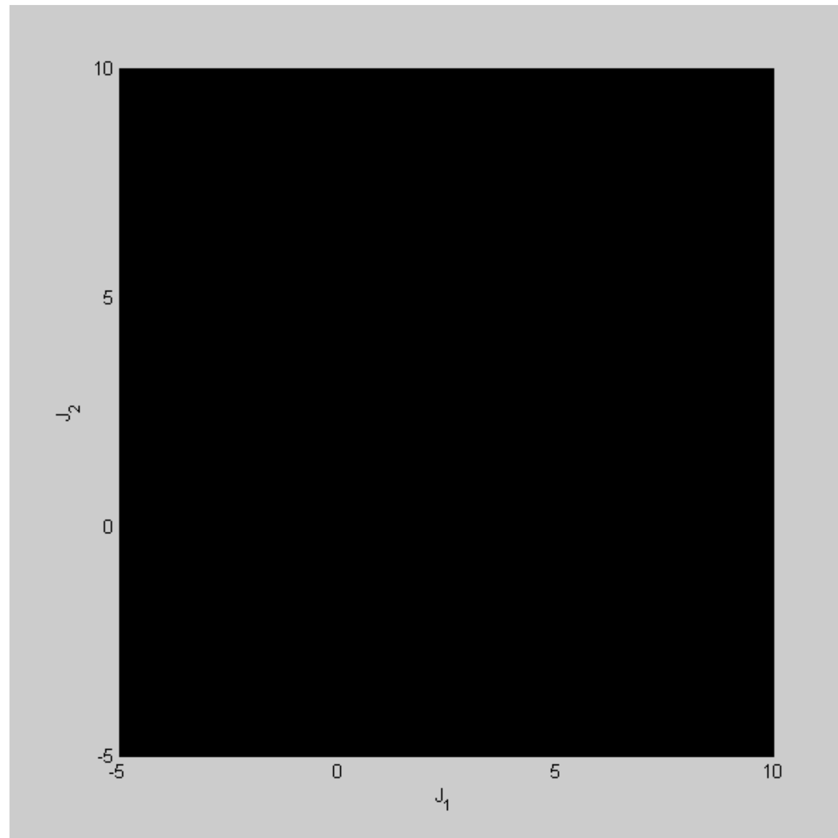
Double Peaks Example: MO-GA

Multiobjective Genetic Algorithm

Generation 1



Generation 10



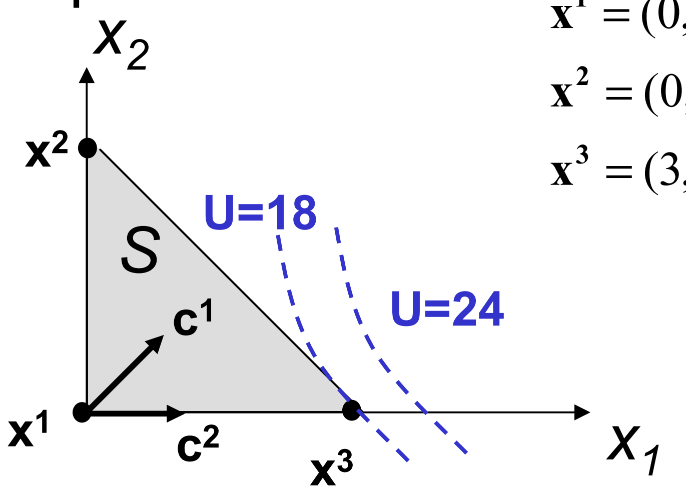
Utility Function Approach

Decision maker has utility function $U : \mathbb{R}^z \rightarrow \mathbb{R}$
This function might or might not be known mathematically
U maps objective vector to the real line

MOLP: $\max\{U(\mathbf{J}) | \mathbf{J} = \mathbf{Cx}, \mathbf{x} \in S\}$

MONLP: $\max\{U(\mathbf{J}) | \mathbf{J} = f(\mathbf{x}), \mathbf{x} \in S\}$

Example:



$\mathbf{x}^1 = (0,0)$

$\mathbf{x}^2 = (0,4)$

$\mathbf{x}^3 = (3,0)$

$\max\{J_1 = x_1 + x_2\}$

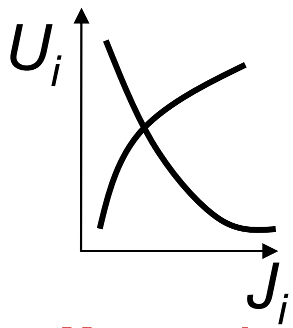
$\max\{J_2 = x_1\}$ s.t.

$4x_1 + 3x_2 \leq 12$

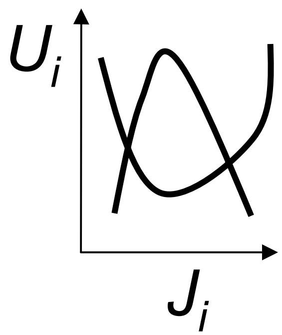
$x_1, x_2 \geq 0$ where

$U = 2J_1J_2$

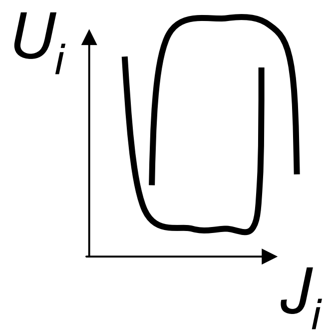
Utility Function Shapes



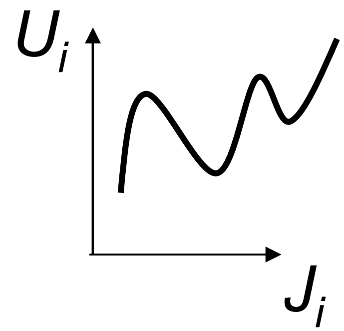
Monotonic increasing decreasing



Strictly Concave Convex



Concave Convex



Non-monotonic

Cook:

Smaller-is-better (SIB) **Larger-is-better (LIB)**	**Nominal-is -better (NIB)**	**Range -is-better (RIB)**	**-**

Messac:

Class 1S Class 2S	Class 3S	Class 4S

Example: Room Control Optimization

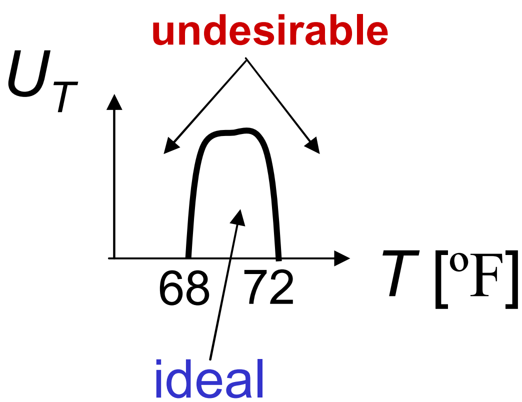
Want: - temperature in ideal range 68-72 ºF
- humidity above 56% is undesirable

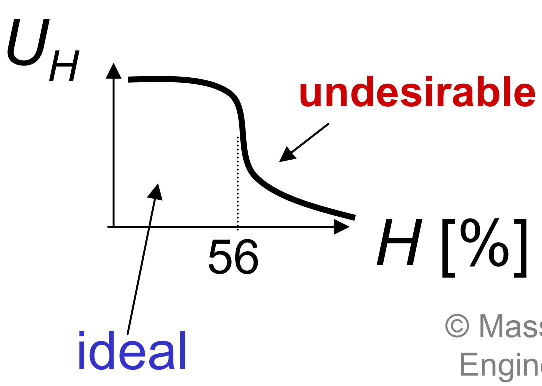
Assume:

$T = \mathbf{c}^1\mathbf{x}$ temperature

$H = \mathbf{c}^2\mathbf{x}$ humidity

Formulate as a MOLP





$$\min\left\{d_1^- + d_1^+\right\}$$

$$\min\left\{\quad d_2^+\right\} \quad \text{s.t.}$$

$$\mathbf{c}^1\mathbf{x} + d_1^- \geq 68$$

$$\mathbf{c}^1\mathbf{x} - d_1^+ \leq 72$$

$$\mathbf{c}^2\mathbf{x} - d_2^+ \leq 56$$

$$\mathbf{A}\mathbf{x} \leq \mathbf{b}, \quad \mathbf{x}, d_1^-, d_1^-, d_2^+ \geq 0$$

Using deviational variables

Aggregated Utility

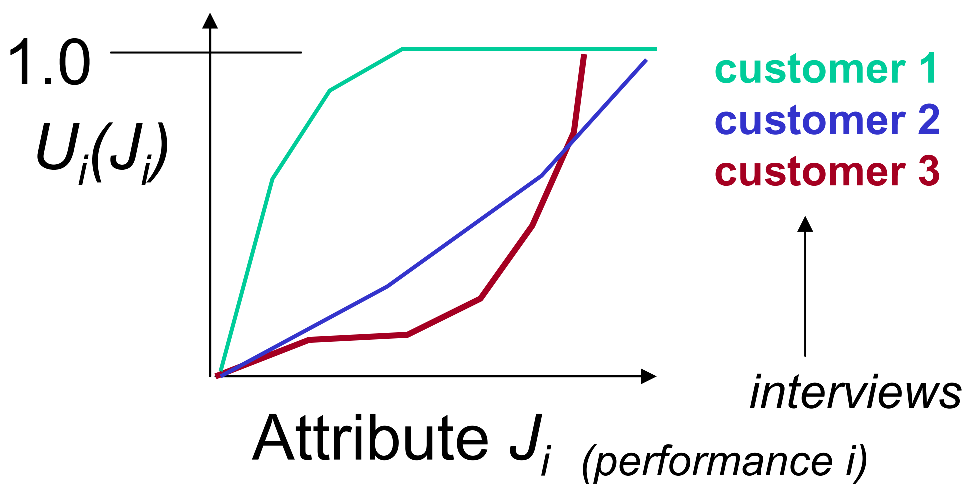
The total utility becomes the weighted sum of partial utilities:
… sometimes called multi-attribute utility analysis (MAUA)

E.g. two utilities combined: $U\left(J_1, J_2\right) = Kk_1k_2U(J_1)U(J_2) + k_1U(J_1) + k_2U(J_2)$

For 2 objectives: $K = (1 - k_1 - k_2) / k_1k_2$

Combine single utilities into overall utility function:

ki's determined during interviews
K is dependent scaling factor



Steps: MAUA
1. Identify Critical Objectives/Attrib.
2. Develop Interview Questionnaire
3. Administer Questionnaire
4. Develop Agg. Utility Function
5. Analyze Results

Caution: "Utility" is a surrogate for "value", but while "value" has units of [$], utility is unitless.

Notes about Utility Maximization

- Utility maximization is very common and well accepted
- Usually U is a non-linear combination of objectives *J*
- Physical meaning of aggregate objective is lost (no units)
- Need to obtain a mathematical representation for $U(J_i)$ for all *I* to include all components of utility
- Utility function can vary drastically depending on decision maker …e.g. in U.S. Govt change every 3-4 years

Physical Programming

Classify Each Design Objective

SOFT

Class-1S	Smaller-Is-Better, i.e. minimization.
Class-2S	Larger-Is-Better, i.e. maximization.
Class-3S	Value-Is-Better.
Class-4S	Range-Is-Better.

HARD

Class-1H	Must be smaller.
Class-2H	Must be larger.
Class-3H	Must be equal.
Class-4H	Must be in range.

Ref: Prof. Achille Messac, RPI

Physical Programming

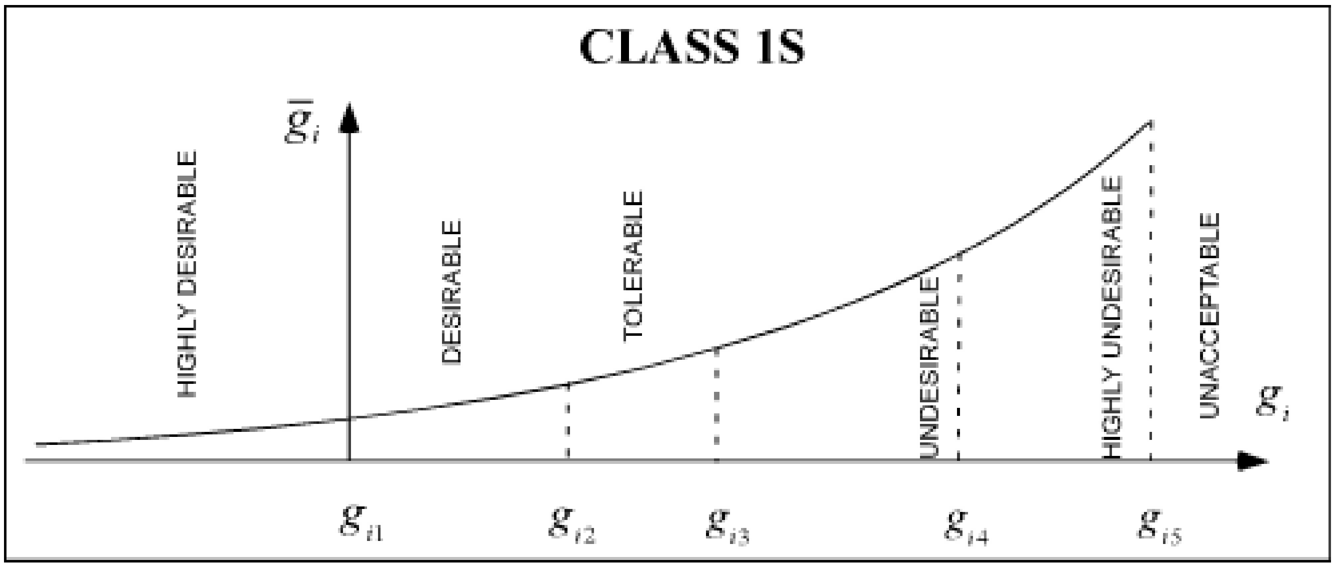
Quantify Preference for Each Design Metric

Ex: Mass of Beam

Highly Desirable	< 250 (kg)
Desirable	250 - 275
Tolerable	275 - 300
Undesirable	300 - 325
Highly Undesirable	325 - 350
Unacceptable	> 350

Physical Programming

Quantity Minimized Inside Code



Mass

Preference Function of Each Objective

- Cost (preference) is on the vertical axis, and will be minimized.
- The value of the design metric (obj) is on the horizontal axis.
- The designer chooses limits of several ranges for each design metric.
- Each range defines relative levels of desirability within a given design metric (obj).
- We then have a preference function for each design metric.
- These preference functions are added to form an aggregate preference function.

Physical Programming Problem Model

$$\min_x P(\mu) = \frac{1}{n_{sc}} \sum_{i=1}^{n_{sc}} P_i[\mu_i(x)]$$ (for *soft* classes)

Subject to

$\mu_i(x) \le v_{i5}$ (for class 1S metrics)

$\mu_i(x) \ge v_{i5}$ (for class 2S metrics)

$v_{i5L} \le \mu_i(x) \le v_{i5R}$ (for class 3S metrics)

$v_{i5L} \le \mu_i(x) \le v_{i5R}$ (for class 4S metrics)

$\mu_i(x) \le v_{i,\max}$ (for class 1H metrics)

$\mu_i(x) \ge v_{i,\min}$ (for class 2H metrics)

$\mu_i(x) = v_{i,val}$ (for class 3H metrics)

$v_{i,\min} \le \mu_i(x) \le v_{i,\max}$ (for class 4H metrics)

$x_{j,\min} \le x_j \le x_{j,\max}$ (for des. variable. constraints)

Nomenclature here μ is used similar to J in the class

Application to System Design

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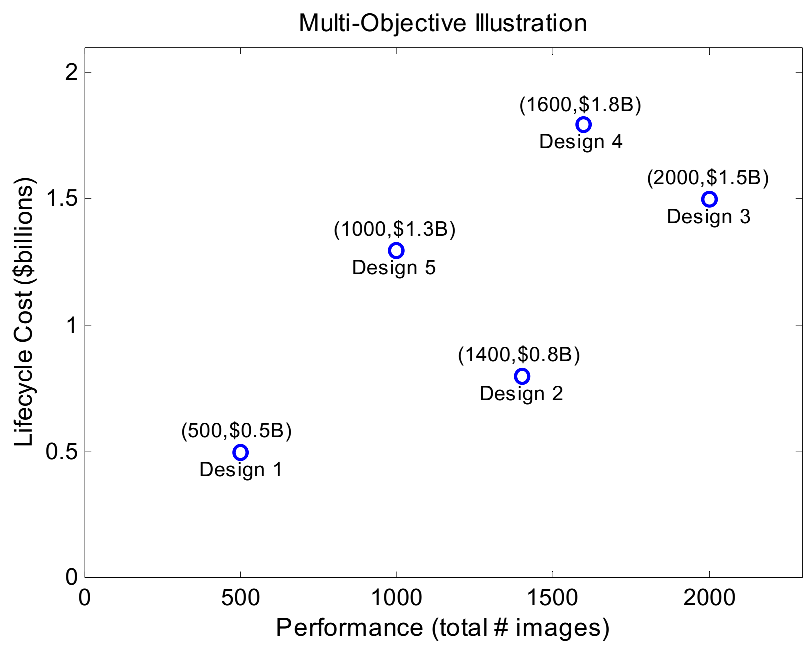
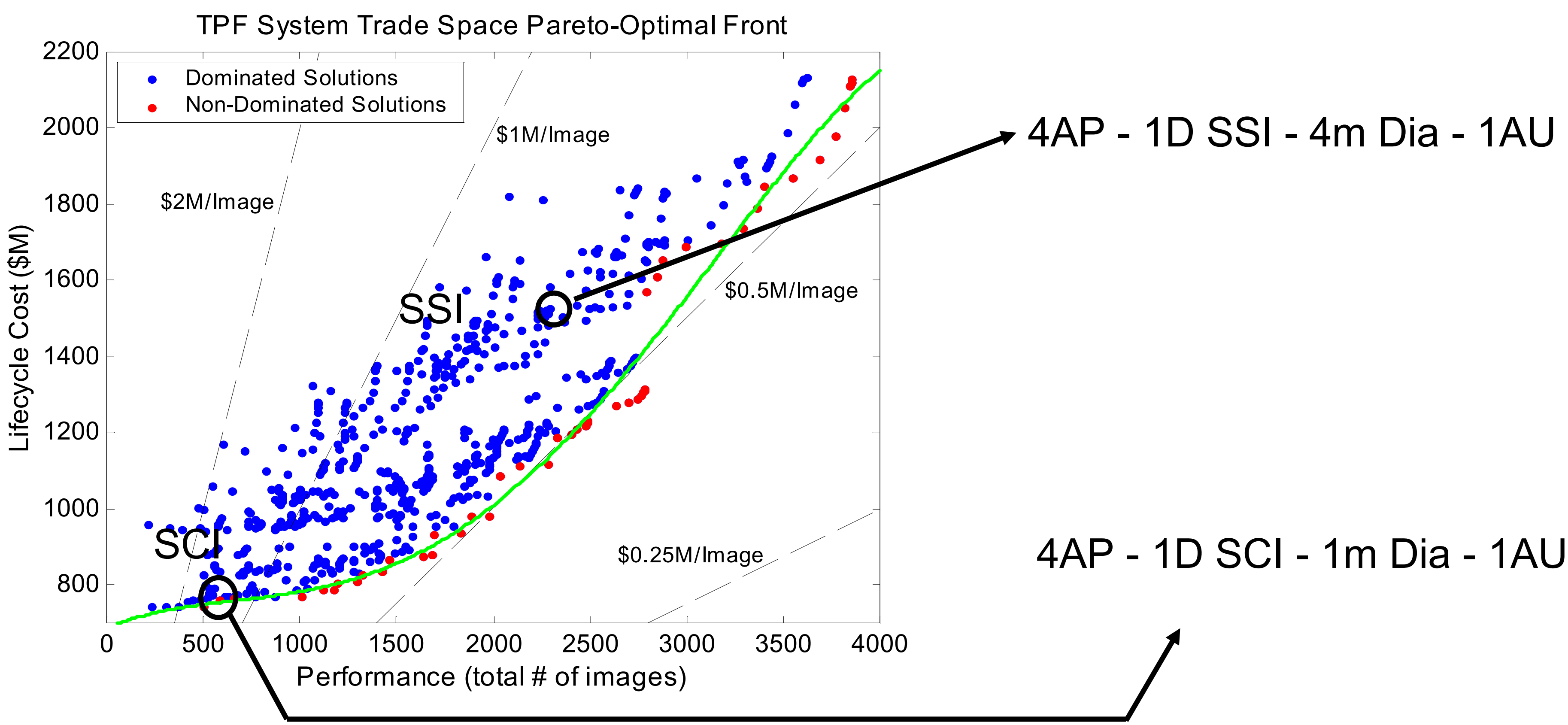
- **Multiobjective Problem:**
  - **Minimize Cost AND Maximize Performance Simultaneously**
- **Which design is best according to these decision criteria?**
- **Key Point:** Multi-Objective problems can have **more than one solution**! Single objective problems have only one true solution.

Optimize Architecture of **Terrestrial Planet Finder (TPF)** Mission (expected Launch 2011)



The Pareto Boundary

- In a **two-dimensional trade space** (I.e. two decision criteria), the Pareto Optimal set represents the **boundary** of the most design efficient solutions.



TPF Pareto Optimal Set

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Mission Cost & Performance

Low

Medium

High

# "Images"	LCC ($B)	Orbit (AU)	# Apert.'s	Architecture	Apert. Diam. (m)
502	0.743	1.5	4	SCI-1D	1
577	0.762	2.0	4	SCI-1D	1
651	0.767	2.5	4	SCI-1D	1
1005	0.768	1.5	4	SCI-1D	2
1114	0.788	2.0	4	SCI-1D	2
1171	0.790	2.5	4	SCI-1D	2
1195	0.807	1.5	6	SCI-1D	2
1292	0.811	1.5	6	SCI-2D	2
1317	0.830	1.5	8	SCI-1D	2
1424	0.836	2.0	4	SCI-1D	3
1426	0.838	1.5	8	SCI-2D	2
1464	0.867	2.5	6	SCI-2D	2
1631	0.877	1.5	6	SCI-1D	3
1684	0.881	1.5	6	SCI-2D	3
1687	0.932	2.0	6	SCI-1D	3
1828	0.936	2.0	6	SCI-2D	3
1881	0.980	1.5	8	SCI-2D	3
1978	0.982	1.5	6	SCI-1D	4
2035	1.086	2.0	8	SCI-2D	3
2132	1.112	1.5	8	SCI-1D	4
2285	1.120	1.5	8	SCI-2D	4
2328	1.190	2.5	6	SCI-2D	4
2398	1197	3.0	6	SCI-2D	4
2433	1.212	4.0	6	SCI-2D	4
2472	1.221	4.5	6	SCI-2D	4
2482	1.227	5.0	6	SCI-2D	4
2487	1.232	5.5	6	SCI-2D	4
2634	1.273	2.5	8	SCI-2D	4
2700	1.280	3.0	8	SCI-2D	4
2739	1.288	3.5	8	SCI-2D	4
2759	1.296	4.0	8	SCI-2D	4
2772	1.305	4.5	8	SCI-2D	4
2779	1.312	5.0	8	SCI-2D	4
2783	1.317	5.5	8	SCI-2D	4
2788	1.569	3.0	6	SSI-2D	4
2844	1.609	3.5	6	SSI-2D	4
2872	1.655	4.0	6	SSI-2D	4
2988	1.691	2.0	8	SSI-1D	4
3177	1.698	2.5	8	SSI-1D	4
3289	1.739	3.0	8	SSI-1D	4
3360	1790	3.5	8	SSI-1D	4
3395	1.850	4.0	8	SSI-1D	4
3551	1.868	2.5	10	SSI-1D	4
3690	1.919	3.0	10	SSI-1D	4

Family
4 ap.
SCI-1D
2 m Diam.

Family
6 ap.
SCI-2D
4 m Diam.

Family
6 ap.
SSI-2D
4 m Diam.

Family
10 ap.
SSI-1D
4 m Diam.

Family
4 ap.
SCI-1D
1 m Diam.

Intersection of Multiple Families

Family
8 ap.
SCI-2D
4 m Diam.

Transition fron SCI to SSI Designs

Family
8 ap.
SSI-1D
4 m Diam.

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Multi-Objective Optimization Example: Broadband Communication Satellite Constellation

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- **Goal:** Determine with minimal computational effort a 4-dimensional Pareto optimal set.
- **Broadband Design Goals:** To simultaneously
  - Minimize Lifecycle Cost
  - Maximize Lifecycle Performance (# T1 minutes provided)
  - Maximize # Satellites in View Over Market Served
  - Maximize Coverage Over Populated Globe
- **Key Question**: Is it better to find and then combine a series of 2-dimensional P-optimal sets or attempt to simultaneously optimize all of the metrics of interest.
- **Pareto Optimality:** A set of design architectures in which the systems engineer cannot improve one metric of interest without adversely affecting at least one other metric of interest. This set quantitatively captures the trades between the design decision criteria.

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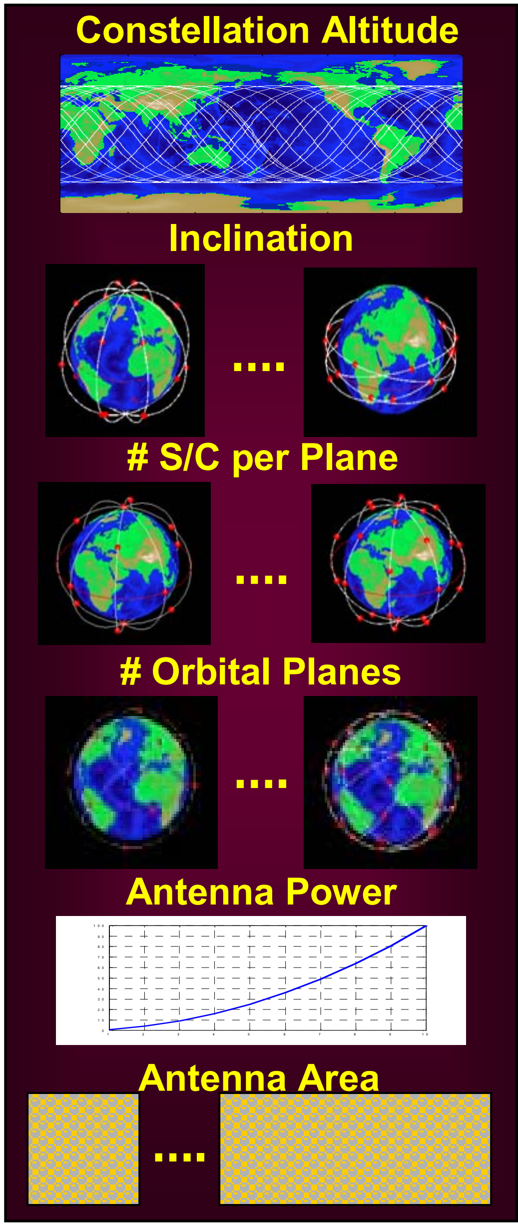
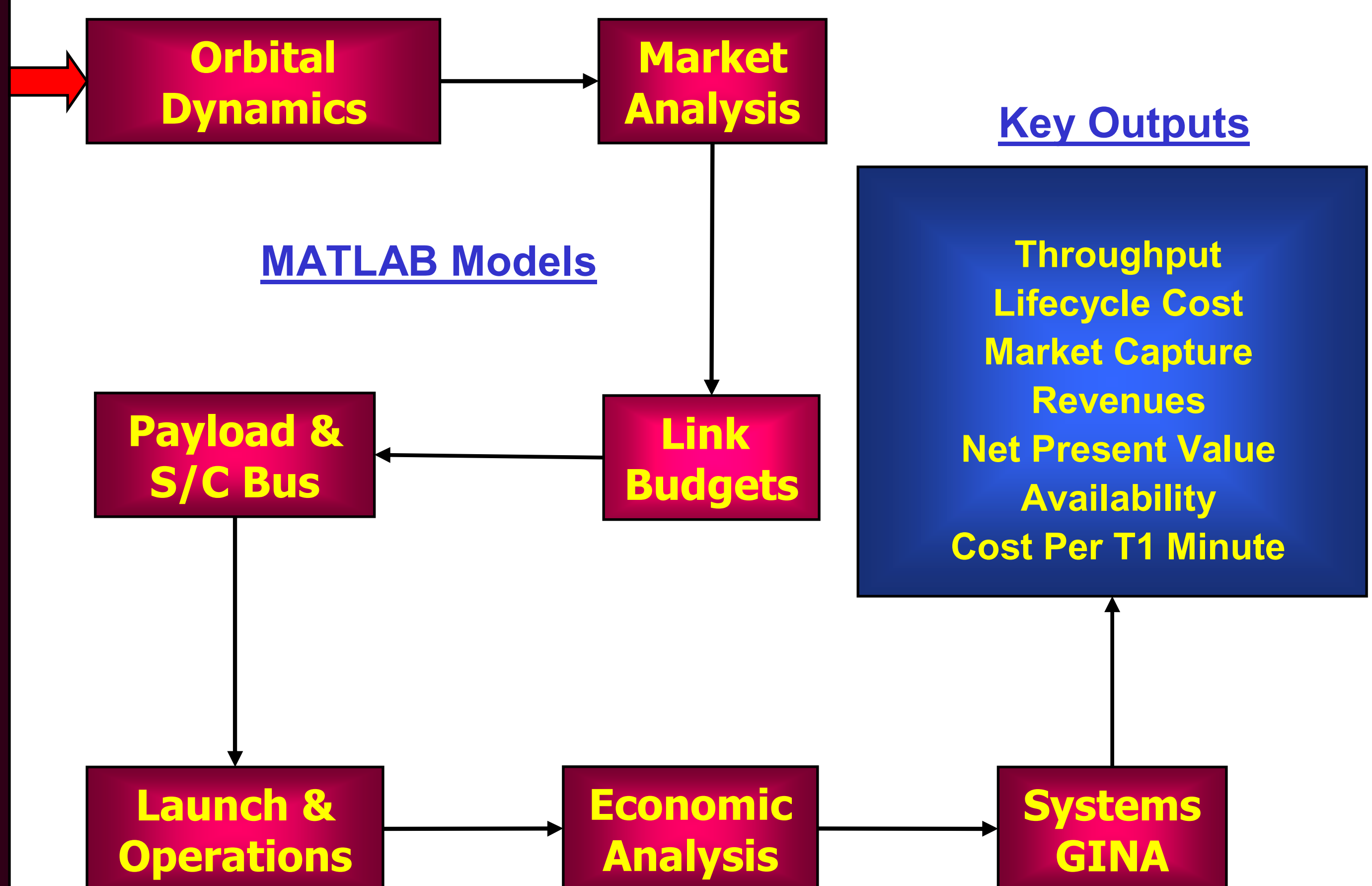
The Broadband GINA Model



Inputs (Design Vector)



Case 1 – Multi-Objective Optimization

Objective: Minimize LCC & Maximize Performance

[redacted]

Optimization Formulation & Pareto Plots

Objective: $Min \sum_{y=1}^{10} \phi_y(\Gamma)$ AND

$Max \sum_{y=1}^{10} \Psi_y(\Gamma)$

Constraints: *Subject to*

Isolation $MAE \geq 90\%$

$Eb/No \geq 4.4$ db

$Link\ Margin \geq 6.0$ db

Integrity $BER \leq 10^{-5}$

Rate $R \geq 1.54$ Mbps Per Link

Availability $\varepsilon_{min} \geq 10^{\circ}$

$P(Coverage) \geq 98\%$

Pareto Optimal Designs Found (60 Iterations)

LCC vs. Performance	LCC vs. Mean # Satellites in View	LCC vs. Global Population Coverage	4-Dimensional P-Opt.
12	4	6	12

Case 2 – Multi-Objective Optimization

Objective: Minimize LCC & Maximize Mean # Satellites in View

[redacted]

Optimization Formulation & Pareto Plots

Objective: $Min \sum_{y=1}^{10} \phi_y(\Gamma)$ AND

$$Max \frac{\sum_{i=1}^{480} \frac{\sum_{j=1}^{n} SIV_{ij}}{n}}{480}$$

Constraints: *Subject to*

Isolation: $MAE \geq 90\%$

$Eb / No \geq 4.4$ db

$Link\ Margin \geq 6.0$ db

Integrity: $BER \leq 10^{-5}$

Rate: $R \geq 1.54$ Mbps Per Link

Availability: $\varepsilon_{min} \geq 10^{\circ}$

$P(Coverage) \geq 98\%$

Pareto Optimal Designs Found (60 Iterations)

LCC vs. Performance	LCC vs. Mean # Satellites in View	LCC vs. Global Population Coverage	4-Dimensional P-Opt.
8	11	5	11

Case 3 – Multi-Objective Optimization

Objective: Minimize LCC & Maximize Global Population Coverage

[redacted]

Optimization Formulation & Pareto Plots

Objective: $Min \sum_{y=1}^{10} \phi_y(\Gamma)$ AND

$$Max \frac{\sum_{i=1}^{240} \frac{\sum_{j=1}^{480} COV_{ij}}{480}}{240}$$

Constraints: *Subject to*

- Isolation: $MAE \geq 90\%$
- $Eb / No \geq 4.4$ db
- $Link\ \ Margin \geq 6.0$ db
- Integrity: $BER \leq 10^{-5}$
- Rate: $R \geq 1.54$ Mbps Per Link
- Availability: $\varepsilon_{\min} \geq 10^{\circ}$
- $P(Coverage) \geq 98\%$

Pareto Optimal Designs Found (60 Iterations)

LCC vs. Performance	LCC vs. Mean # Satellites in View	LCC vs. Global Population Coverage	4-Dimensional P-Opt.
3	4	4	4

Case 4 – Multi-Objective Optimization

Objective: 4-Dimensional Simultaneous Optimization

[redacted]

Optimization
Formulation
& Pareto Plots

Objective:

$$Min \sum_{y=1}^{10} \phi_y(\Gamma) \text{ AND}$$

$$Max \sum_{y=1}^{10} \Psi_y(\Gamma) \text{ AND}$$

$$Max \frac{\sum_{i=1}^{480} \frac{\sum_{j=1}^{n} SIV_{ij}}{n}}{480} \text{ AND}$$

$$Max \frac{\sum_{i=1}^{240} \frac{\sum_{j=1}^{480} COV_{ij}}{480}}{240}$$

Constraints: $\vdots$

Pareto Optimal Designs Found (180 Iterations)

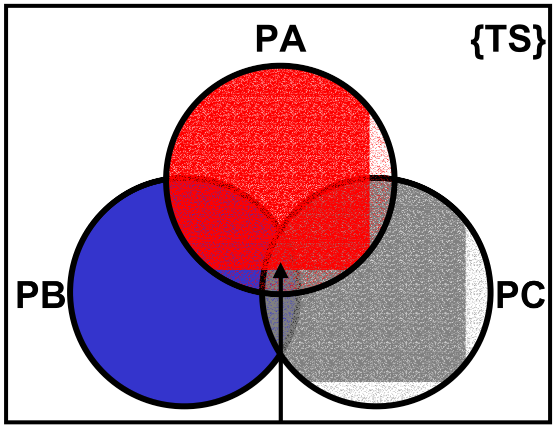
LCC vs. Performance	LCC vs. Mean # Satellites in View	LCC vs. Global Population Coverage	4-Dimensional P-Opt.
16	9	5	44

Multi-Objective Optimization Comparison

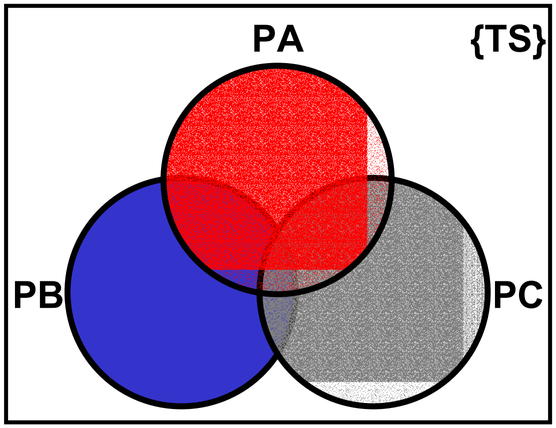
4-D Pareto Optimal Design Architectures Found

#	Approach	Mathematical Representation	Size of Pareto Optimal Set
1	Intersection of P-Opt. Sets	$(PA \cap PB) \cap PC$	1
2	Union of P-Opt. Sets	(PA U PB) U PC	21
3	Union of All Explored Designs	(A U B) U C	39
4	4-D Simultaneous Optimization	P-Opt.	44

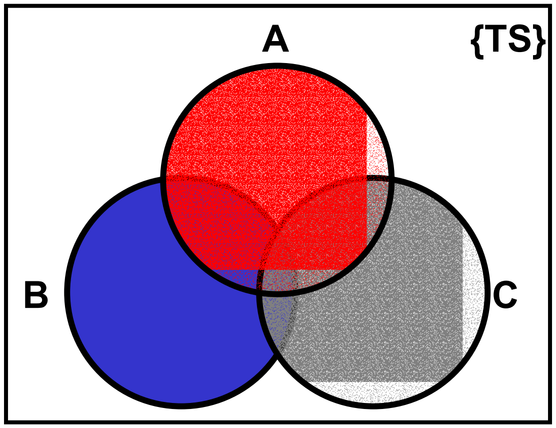
*Each case required the same amount of computational effort = 180 iterations.



$(PA \cap PB) \cap PC$



(PA U PB) U PC



(A U B) U C

Observations

- Combining a sequence of 2-D Pareto Optimal sets via {Set Theory} is a viable approach for finding n-dimensional P-optimal sets of design architectures.

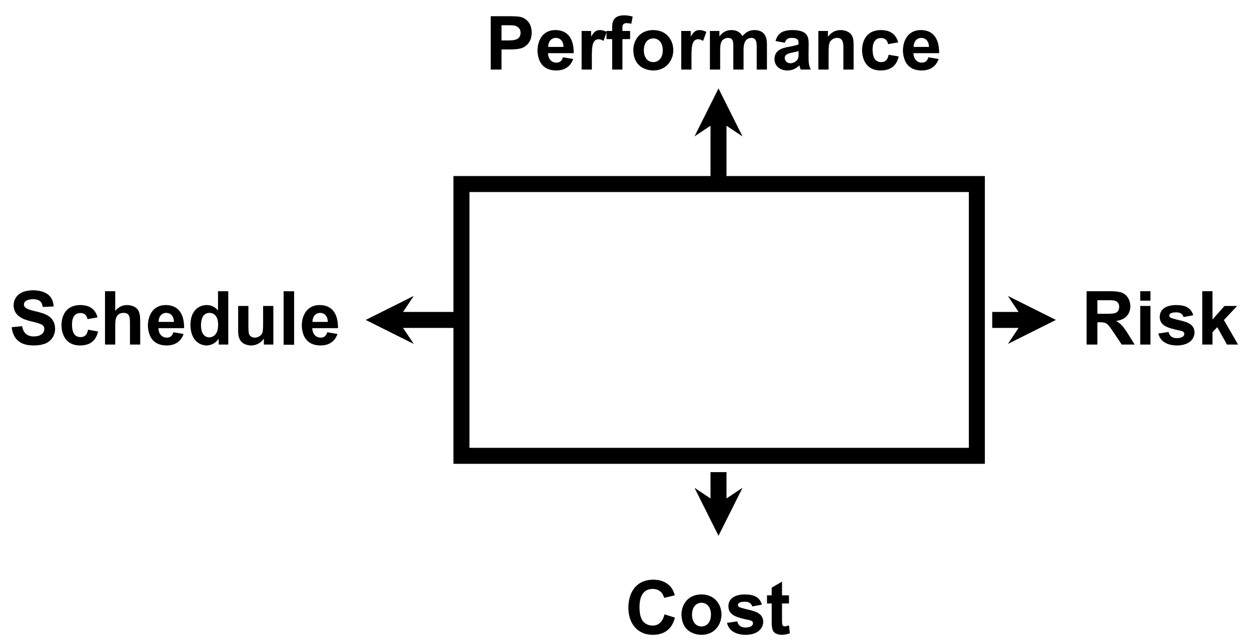
- However, it appears to be more computationally efficient to formulate a single n-dimensional multi-objective optimization problem, despite the difficulty in visualizing the solution (can't plot on orthogonal axes, can plot on "radar plot.")

N-Dimensional Problems

- The same principles of Pareto Optimality hold for a trade space with **any number *n* dimensions** (I.e. any number of decision criteria).
- 3 Criteria Example for Space-Based Radar
  - Minimize(Lifecycle Cost) AND
  - Minimize(Maximum Revisit Time) AND
  - Maximize(Target Probability of Detection)

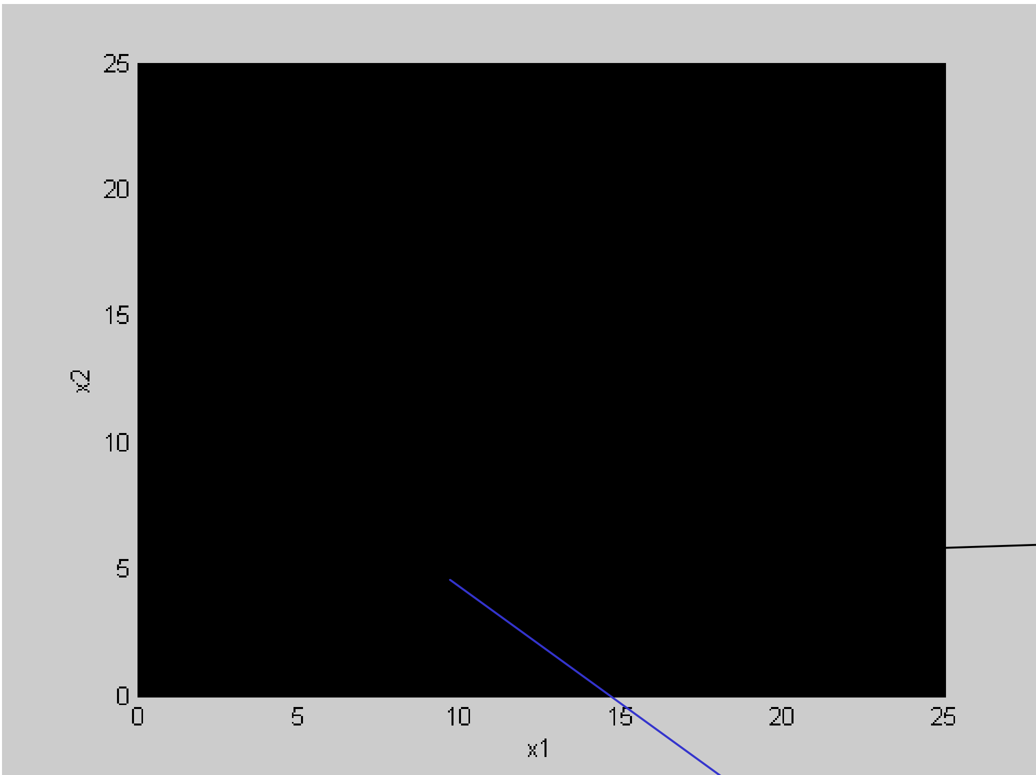
[redacted]

Four Basic Tensions (Trade-offs) in Product/System Development

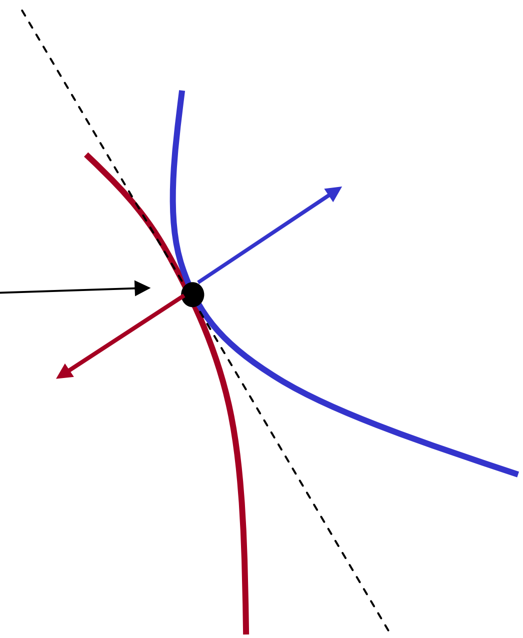


Ref: Maier and Rechtin,
"The Art of Systems Architecting", 2000

One of the main jobs of the system designer (together with the system architect) is to identify the principle tensions and resolve them



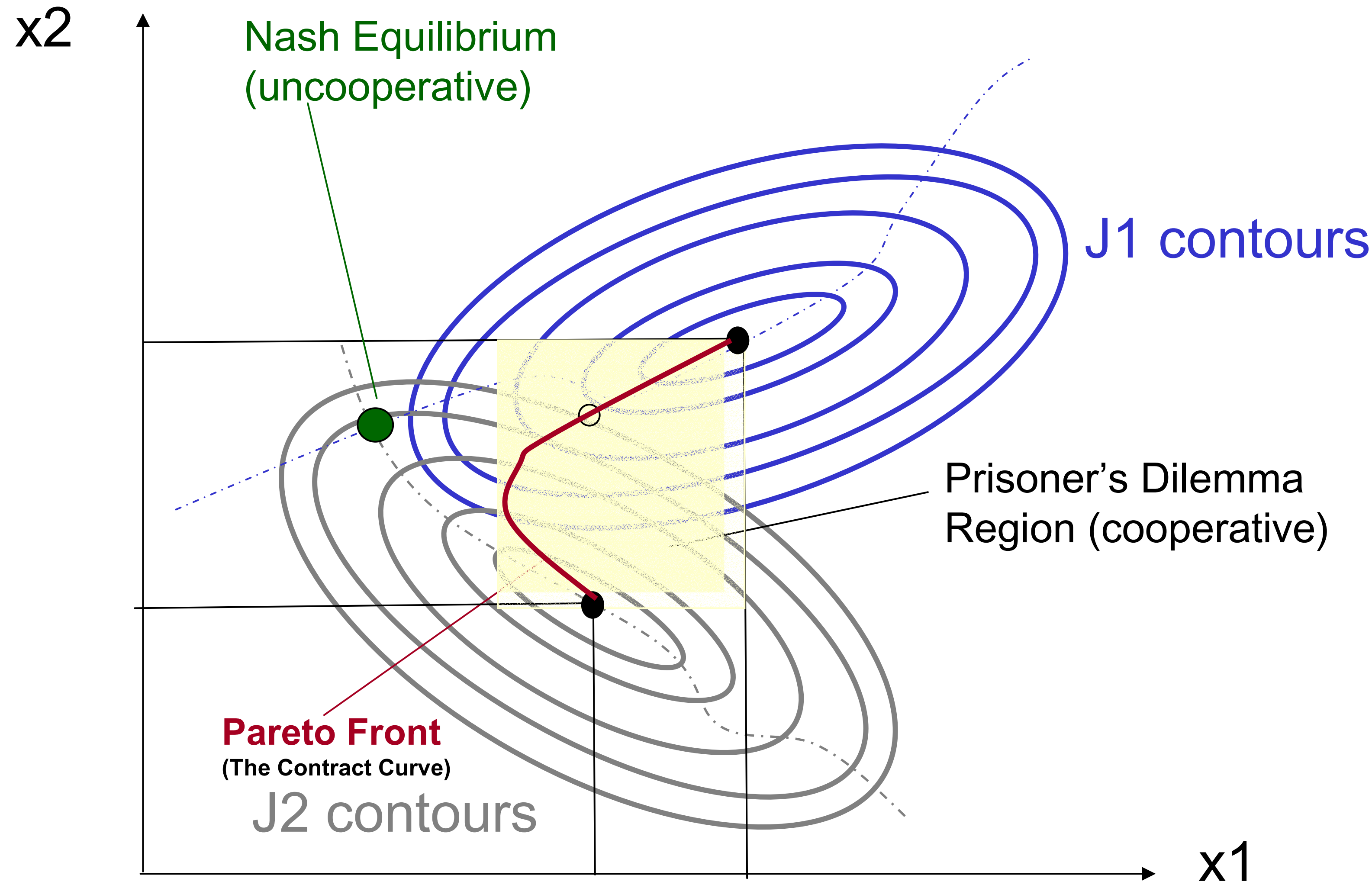
Non-dominated solutions occur, where Isoperformance curves are tangent to each other



Tensions in Engineering System Design can be quantified

Pareto-optimal Curve

Vector Optimization and Game Theory



In Practice

- Inefficient solutions are not candidates for optimality
- In practice a "near-optimal" solution is acceptable
- Solutions that satisfactorily terminate the decision process are called "final solutions"

Multiple Criteria
Decision Making (MCDM)

Multiattribute
Decision/Utility Analysis

- small # of alternatives
- environment of uncertainty
- resolving public policy problems
- e.g. nuclear power plant siting, airport runway extensions ...

Multicriteria
Optimization*

- large # of feasible alternatives
- deterministic environment
- less controversial problems
- business and design problems

Ref: Keeney & Raiffa, 1976

Lecture Summary

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- Two fundamental approaches to MOO
  - Scalarization of multiple objectives to a single combined objective (e.g. Utility Theory)
  - Pareto Approach with a posteriori selection
- Methods for computing Pareto Front
  - Weighted Sum Approach (and variants)
  - Design Space Exploration + Pareto Filter
  - Normal Boundary Intersection (NBI)
  - Multiobjective Heuristic Algorithms
- Resolving Tradeoffs are an essential part of System Optimization

References

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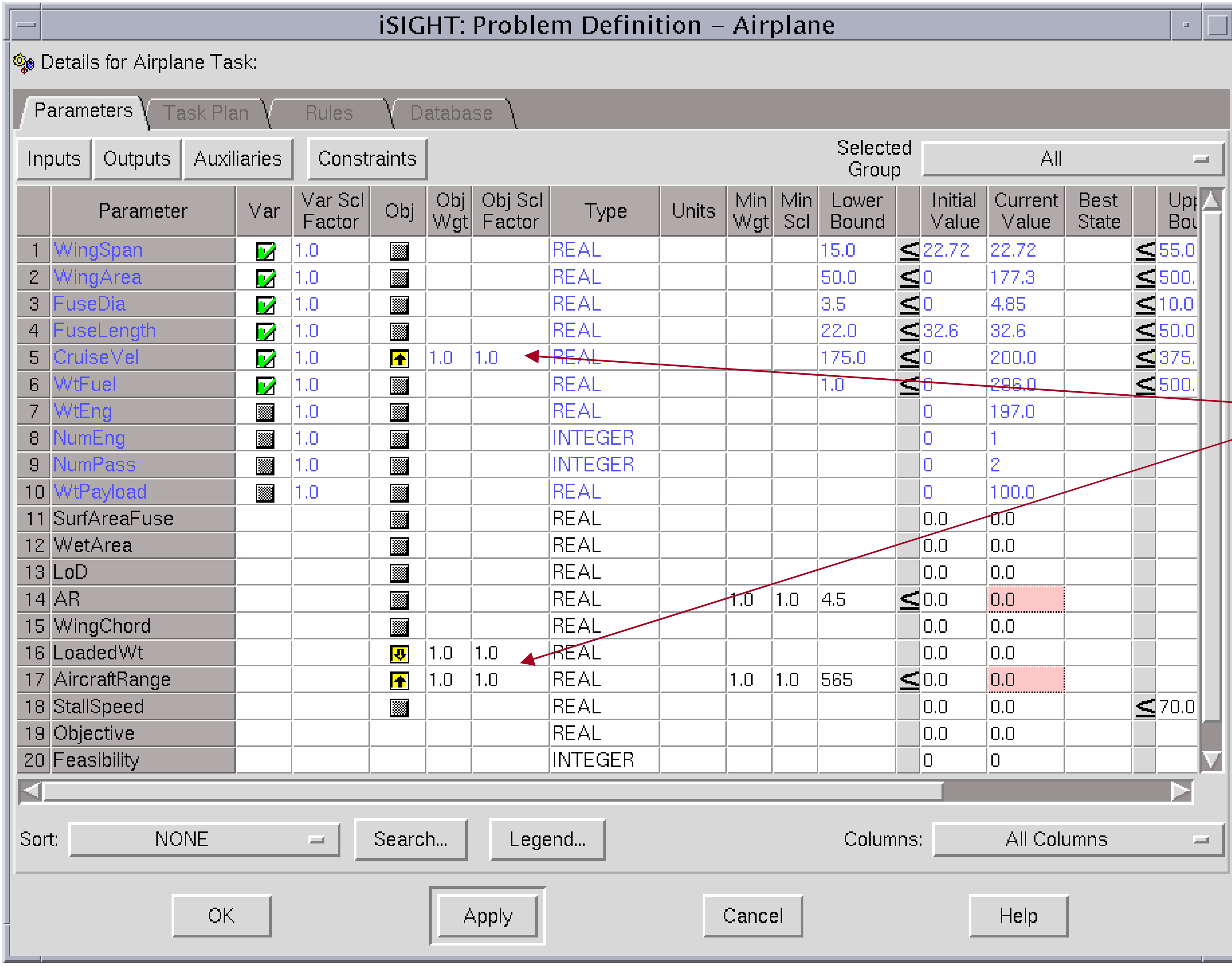
Lab#3: Friday - MO in iSIGHT



	Parameter	Var	Var Scl Factor	Obj	Obj Wgt	Obj Scl Factor	Type	Units	Min Wgt	Min Scl	Lower Bound		Initial Value	Current Value	Best State		Upp Bou
1	WingSpan		1.0				REAL				15.0	≤	22.72	22.72		≤	55.0
2	WingArea		1.0				REAL				50.0	≤	0	177.3		≤	500.
3	FuseDia		1.0				REAL				3.5	≤	0	4.85		≤	10.0
4	FuseLength		1.0				REAL				22.0	≤	32.6	32.6		≤	50.0
5	CruiseVel		1.0		1.0	1.0	REAL				175.0	≤	0	200.0		≤	375.
6	WtFuel		1.0				REAL				1.0	≤	0	296.0		≤	500.
7	WtEng		1.0				REAL						0	197.0			
8	NumEng		1.0				INTEGER						0	1			
9	NumPass		1.0				INTEGER						0	2			
10	WtPayload		1.0				REAL						0	100.0			
11	SurfAreaFuse						REAL						0.0	0.0			
12	WetArea						REAL						0.0	0.0			
13	LoD						REAL						0.0	0.0			
14	AR						REAL		1.0	1.0	4.5	≤	0.0	0.0			
15	WingChord						REAL						0.0	0.0			
16	LoadedWt				1.0	1.0	REAL						0.0	0.0			
17	AircraftRange				1.0	1.0	REAL		1.0	1.0	565	≤	0.0	0.0			
18	StallSpeed						REAL						0.0	0.0		≤	70.0
19	Objective						REAL						0.0	0.0			
20	Feasibility						INTEGER						0	0			

iSIGHT is set up to do Weighted Sum optimization

Note Weights and Scale Factors in Parameters Table

Lab #3: Multiobjective Optimization Game

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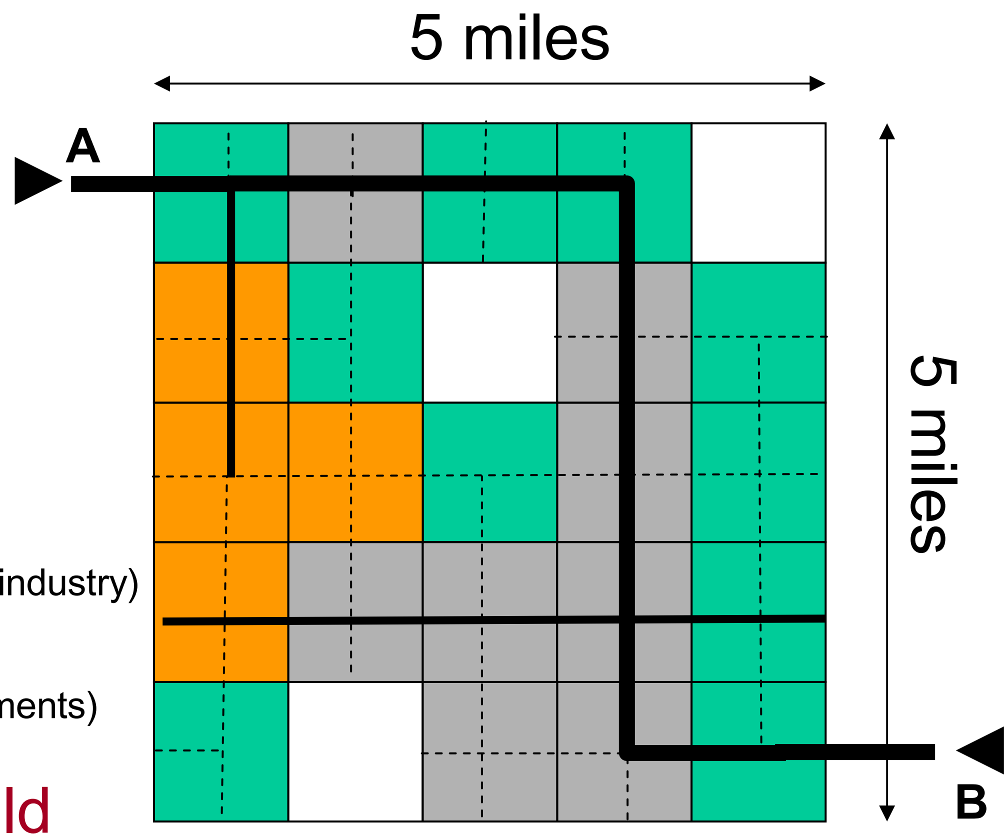
Task: Find an optimal layout for a new city, which comprises 5x5 sqm and 50’000 inhabitants that will satisfy multiple disparate stakeholders.

Stakeholder groups:

a) Local Greenpeace Chapter
b) Chamber of Commerce
c) City Council (Government)
d) Resident’s Association
e) State Highway Commission



- 0 Vacant Zone
- 1 Commercial Zone (shops, restaurants, industry)
- 2 Recreational Zone (parks, lakes, forest)
- 3 Residential Zone (private homes, apartments)

What layout should be chosen ?