Multidisciplinary System Design Optimization (MSDO)

Structural Optimization & Design Space Optimization

Lecture 18

April 7, 2004

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I. Structural Optimization

II. Integrated Structural Optimization

III. Design Space Optimization

Structural Optimization

* Definition

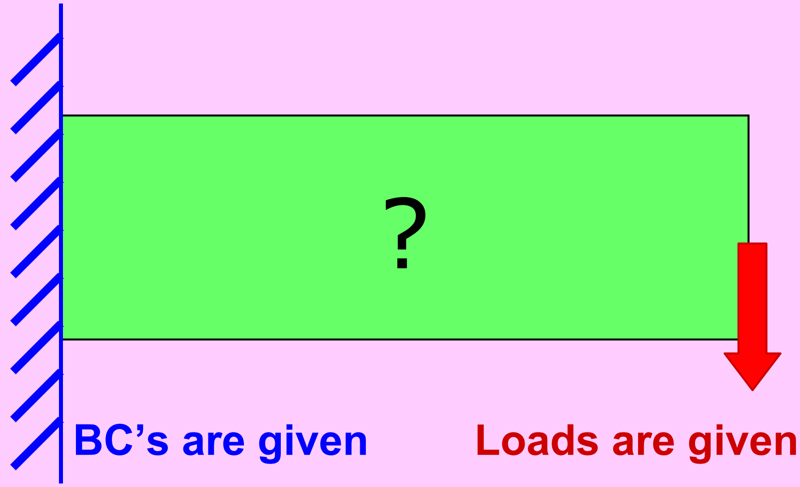
- An automated synthesis of a mechanical component based on structural properties.

- A method that automatically generates a mechanical component design that exhibits optimal structural performance.

Structural Optimization

$$\begin{aligned} &\text{minimize} \quad f(\mathbf{x}) \\ &\text{subject to} \quad g(\mathbf{x}) \le 0 \\ &\quad h(\mathbf{x}) = 0 \\ &\quad \mathbf{x} \in S \end{aligned}$$

Typically, FEM is used.



<Q> **How to represent the structure?** **or**
Which type of design variables to use?

min compliance
s.t. $m \le m_c$

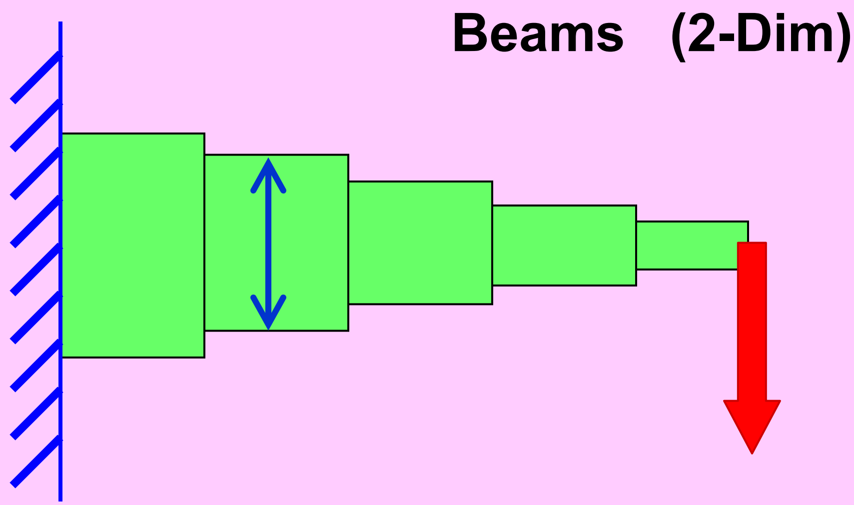
<A> **(1) Size Optimization**

(2) Shape Optimization

(3) Topology Optimization

Size Optimization Example

$$\begin{aligned} &\text{minimize} \quad f(\mathbf{x}) \\ &\text{subject to} \quad g(\mathbf{x}) \leq 0 \\ &\quad h(\mathbf{x}) = 0 \\ &\quad \mathbf{x} \in S \end{aligned}$$



- **Design variables (x)**

  x : thickness of each beam

- **Number of design variables (ndv)**

  ndv = 5

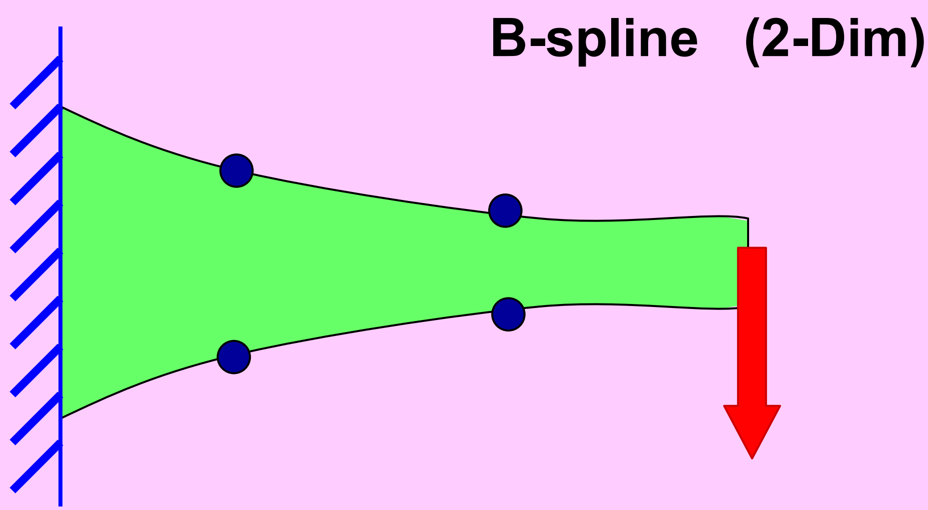
***f*(x) : compliance**

***g*(x) : mass**

***h*(x) : state equation**

Shape Optimization Example

$$\begin{aligned} &\text{minimize} \quad f(\mathbf{x}) \\ &\text{subject to} \quad g(\mathbf{x}) \leq 0 \\ &\quad h(\mathbf{x}) = 0 \\ &\quad \mathbf{x} \in S \end{aligned}$$



- **Design variables (x)**

  x : control points of the B-spline (position of control points)

- **Number of design variables (ndv)**

  ndv = 8

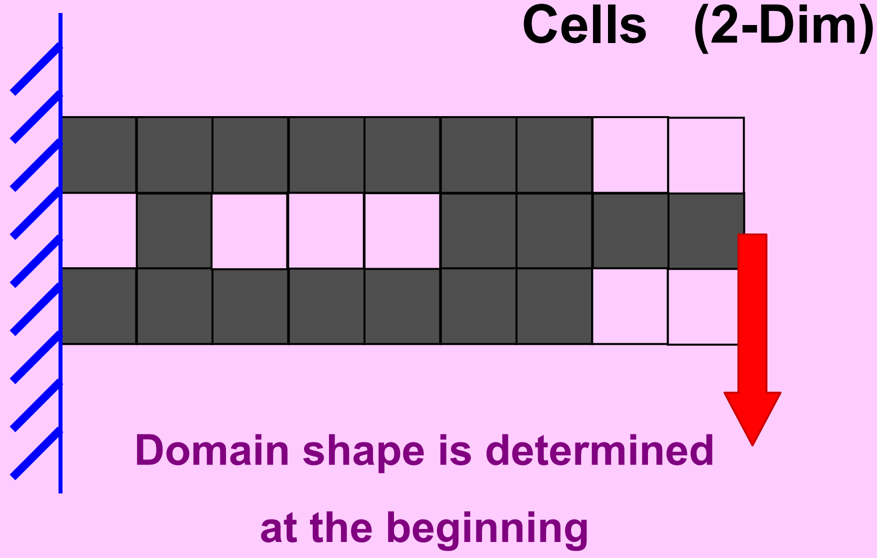
***f*(x) : compliance**

***g*(x) : mass**

***h*(x) : state equation**

Topology Optimization Example

$$\begin{aligned} &\text{minimize} \quad f(\mathbf{x}) \\ &\text{subject to} \quad g(\mathbf{x}) \le 0 \\ &\quad h(\mathbf{x}) = 0 \\ &\quad \mathbf{x} \in S \end{aligned}$$



- **Design variables (x)**

  x : density of each cell

  ($0 \le \rho \le 1$)

- **Number of design variables (ndv)**

  ndv = 27

***f*(x) : compliance**

***g*(x) : mass**

***h*(x) : state equation**

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Structural Optimization

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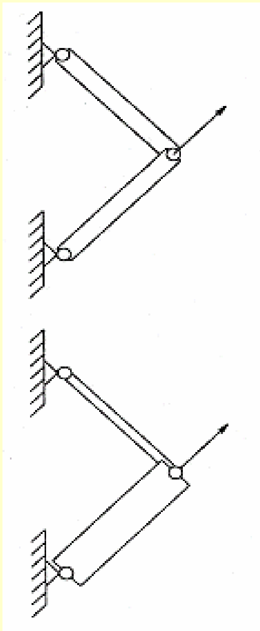
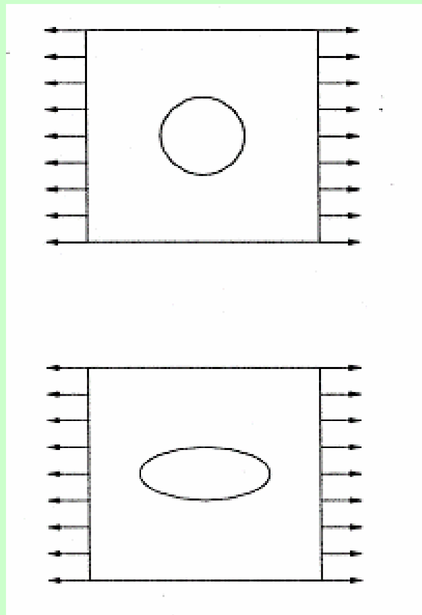
Size optimization



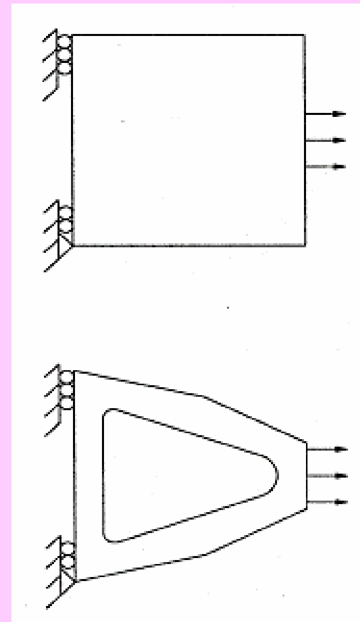
- Shape
  Topology } are given
- Optimize cross sections

Shape optimization



- Topology is given
- Optimize boundary shape

Topology optimization

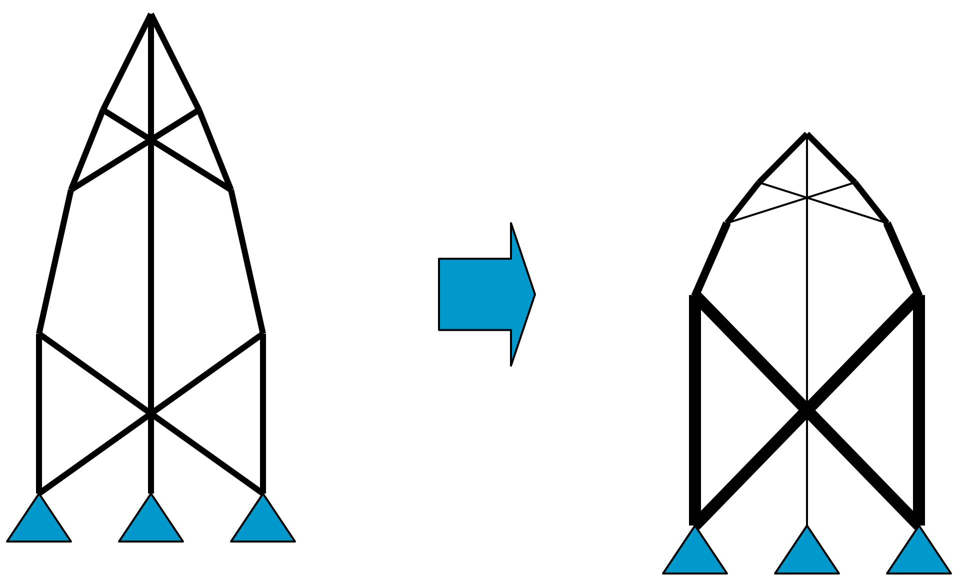


- Optimize topology

Size Optimization

- Simplest method
- Changes dimension of the component and cross sections
- Applied to the design of truss structures

Schmit (1960)
- General approach to structural optimization
- Coupling FEA & NL math. Programming



* Changed
  - Length of the members
  - Thickness of the members
* Unchanged
  - Layout of the structure

Ndv: 10~100

Shape Optimization

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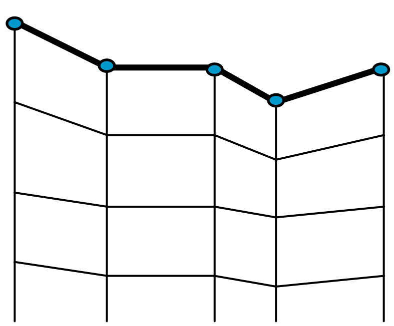
- Design variables control the shape
- Size optimization is a special case of shape optimization
- Various approaches to represent the shape

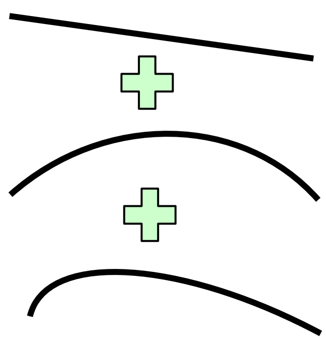
Zolesio (1981), Haug and Choi et al. (1986) – Univ. of Iowa

- A general method of shape sensitivity analysis using the material derivative method & adjoint variable method



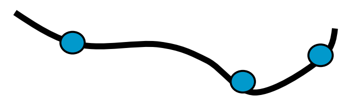
Nodal positions

(when the FEM is used)



Basis functions

$$\sum_{i=1}^{n} \alpha_i \phi_i(x, y, z)$$



B-spline
(control points)

Radius of a circle
Ellipsoid
Bezier curve
Etc…

Ndv: 10~100

Topology Optimization

(1) The evolutionary method

Xie and Steven (1993)

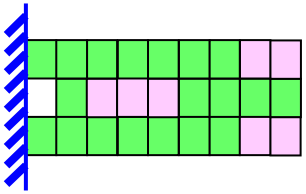
(2) The homogenization method

Bendsoe and Kikuchi (1988) – Univ. of Michigan

(3) Density approach

Yang and Chuang (1994)

*** cell-based approach**



Ndv > 1000

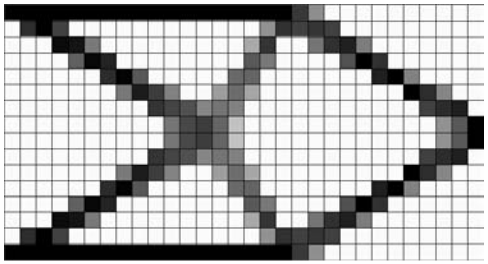
Topology optimization

❖ Homogenization method / Density approach

(1) Design variables: density of each cell

(2) The constitutive equation is expressed in terms of Young's modulus

$$\int_{\Omega} \sigma^{ij}(z)\varepsilon^{ij}(\bar{z})d\Omega = \int_{\Gamma} F^i \bar{z}^i d\Gamma \qquad \forall \bar{z} \in Z_{adm}$$



How to define the relation between the density and Young's modulus?

$$\rho \quad ? \quad E$$

Topology optimization

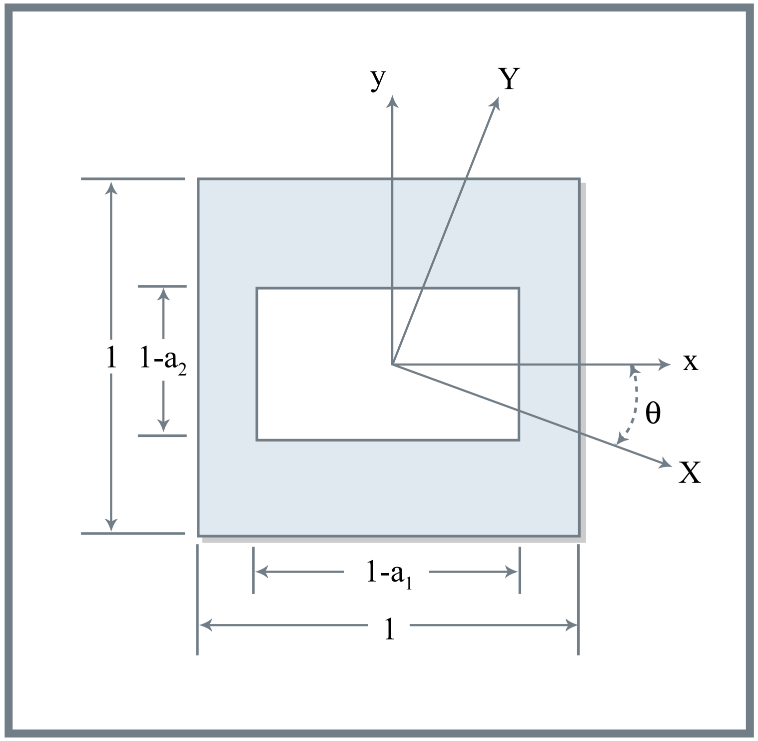
❖ Homogenization method

- Infinitely many micro cells with voids
- The porosity of this material is optimized using an optimality criterion procedure
- Each material may have different void size and orientation

Topology optimization

❖ Homogenization method

- Relationship between density and elastic modulus
- Design variables : a_1, a_2, θ



For 2-D elastic problem,

Solid part area : $\Omega_s = \int_\Omega (1 - a_1 a_2) d\Omega$

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{Bmatrix} = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{12} \end{Bmatrix}$$

$$D = D(a_1, a_2, \theta)$$

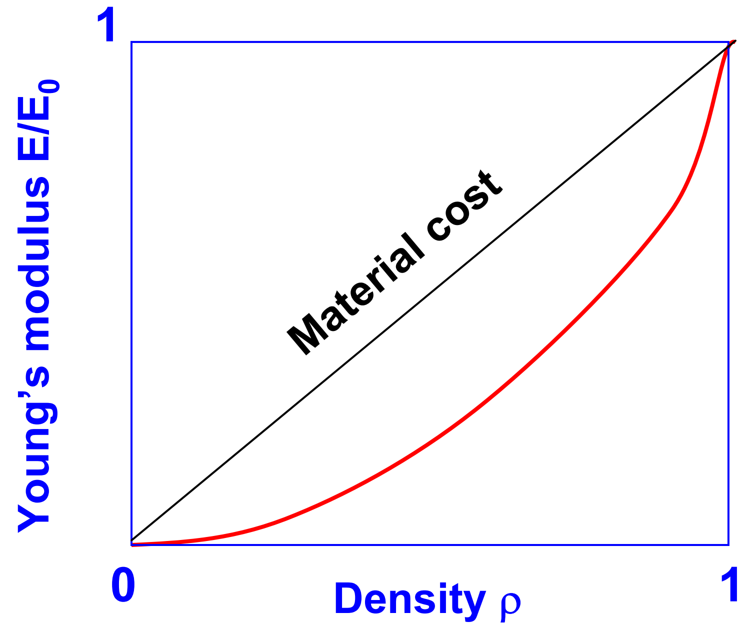
* Review papers : Hassani B and Hinton E (1998)

Topology optimization

❖ Density approach

Artificial material

- Design variable : density



Low computational cost

Simple in its idea

$$E = \rho^n E_o , \qquad 0 \le \rho \le 1$$

I. Structural Optimization

II. Integrated Structural Optimization

III. Design Space Optimization

Integrated Structural Optimization

❖ Motivation

1. Shape optimization

- Small number of design variables
- Smooth definite results
- Topology remains unchanged (Cannot make holes in the design domain)

2. Topology optimization

- Extremely large number of design variables
- Non smooth indefinite results
- Intermediate "densities" between void and full material

  $\Rightarrow$ unrealistic

→ Integrate shape optimization and topology optimization

Integrated Structural Optimization I

❖ On CAD-integrated structural topology and design optimization

- N. Olhoff, M. P. Bensoe and J. Rasmussen (1991)
- Interactive CAD-based structural system for 2-D
- Topology optimization $\Rightarrow$ CAD $\Rightarrow$ Shape optimization
- Topology optimization : Homogenization method (HOMOPT)
- Shape optimization : CAOS (Computer Aided Optimization of Shapes)
- CAD : Commercial CAD system AutoCAD

- The designer decides the initial shape for shape optimization interactively with the results of the topology optimization

Integrated Structural Optimization II

❖ Integrated Topology and Shape Optimization in Structural Design

-M. Bremicker, M. Chirehdast, N. Kikuch and P. Y. Papalambros, (1991)

- 3-phase design process

  Phase I : Generate information about the optimum topology

  Phase II : Process and interpret the topology information

  Phase III : Create a parametric model and apply standard optimization

- ISOS (Integrated Structural Optimization System)

-Image processing scheme instead of interactive scheme

Integrated Structural Optimization III

❖ Integrating Structural Topology, Shape and Sizing Optimization Methods

- E. Hinton, J. Sienz, S. Bulman, S. J. Lee and M. R. Ghasemi (1998)
- Interface : Interactive CAD data structure
  Automatic image processing
- Topology optimization : Evolutionary method
  Homogenization method
- Shape optimization : Mathematical programming
  Genetic Algorithm
- Shape optimization / Size optimization for 2-D elastic problems
- FIDO-TK

Integrated Structural Optimization

- Communications between SO and TO are not easy.

- The designer must provide many control parameters for optimization.

  ⇒ The optimal solutions highly depend on the user defined parameters

- Computationally very expensive.

➔ Less expensive integrated scheme: design space optimization

I. Structural Optimization

II. Integrated Structural Optimization

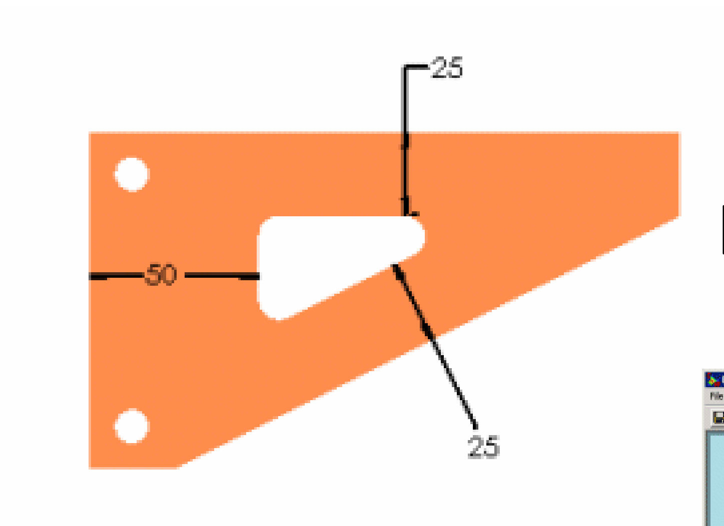
→ Structural Optimization Software

III. Design Space Optimization

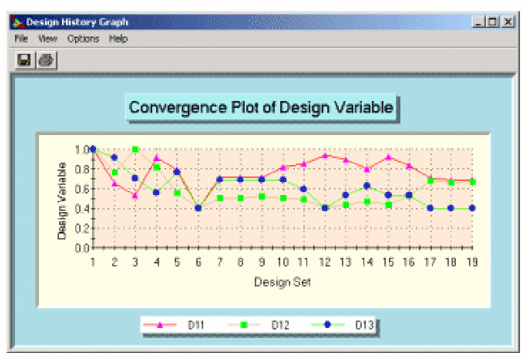
Shape Optimization Software

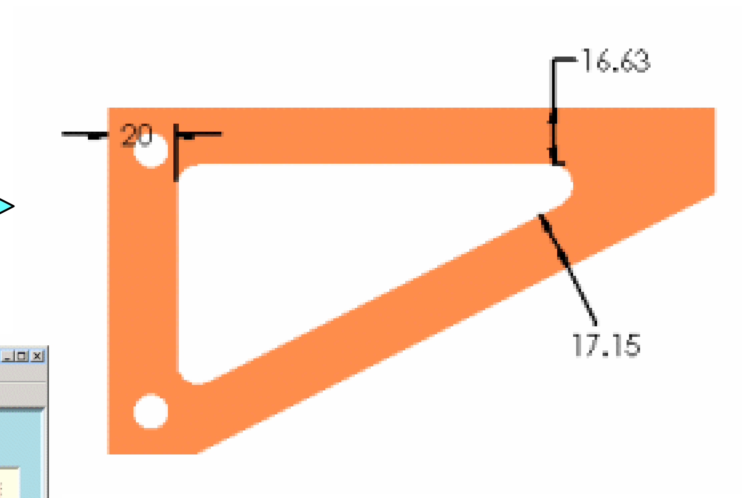
❖ Cosmosworks

Optimize parts and assemblies, whether for constraints such as static, thermal, frequency or buckling, or for objectives such as mass, volume or load factors.



Initial design





Optimal design

Topology Optimization Software

Altair OptiStruct

Topology, shape, and size optimization capabilities can be used to design and optimize structures to reduce weight and tune performance.

Topology Optimization Software

- **ANSYS**

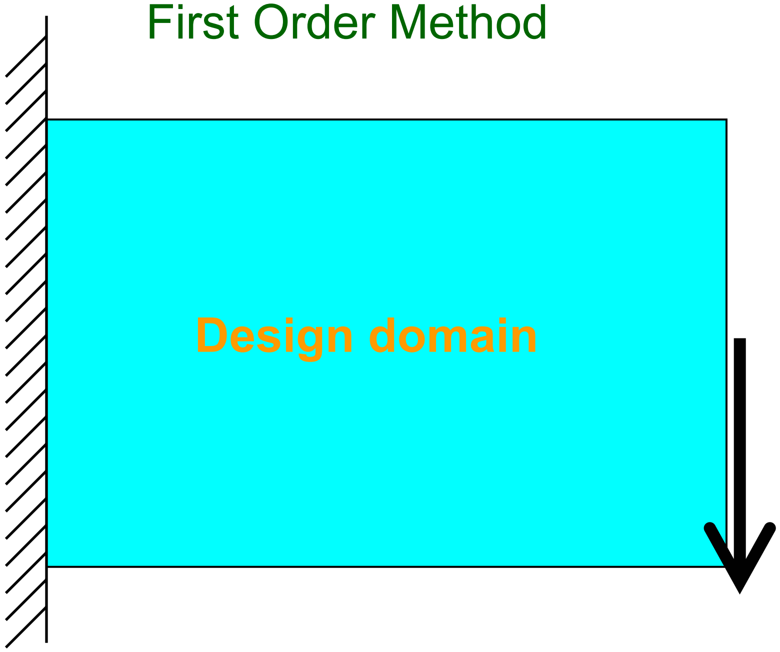
  Static Topology Optimization

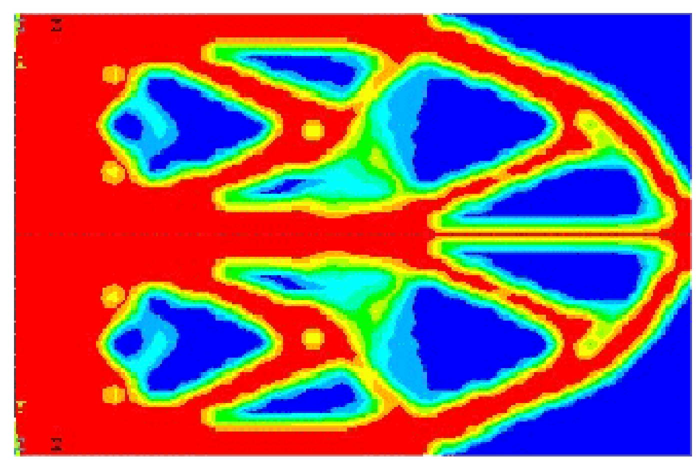
  Dynamic Topology Optimization

  Electromagnetic Topology Optimization

  Subproblem Approximation Method

  First Order Method





Topology Optimization Software

Design Space

Finite Element Analysis Software for engineering designers

CAE Templates – Input files for ANSYS, NASTRAN, ABAQUS are generated

Topology Optimization Software

MSC. Visual Nastran FEA

Elements of lowest stress are removed gradually.

Topology Optimization Software

❖ **Optishape**

- Mass/Rigid Element are available in Topology Optimization.
- Any type elements are available in Shape Optimization.

I. Structural Optimization

II. Integrated Structural Optimization

III. Design Space Optimization

Design Space Optimization

- The optimum solution depends on the optimization method used.

  e.g) gradient based search, GA, Simulated annealing, etc…

- But it also depends on the selection of the design variables.

  (objective functions and constraints given)

<Q> - What is the proper number of design variables for the given problem?

- What is the proper layout of the design variables?

What is the proper length of the chromosome ?

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1	0	1	1	1	0						n=6
1	1	1	0	1	0	1	0				n=8
0	0	1	0	1	1	0	1	0	1	1	n=11

1. **Which is the best length for a given design problem?**
2. **The longer, the better?**

Design space optimization

DSO formulation

$$\begin{aligned} \text{minimize} \quad & f(\mathbf{x}, n) \\ \text{subject to} \quad & g(\mathbf{x}, n) \le 0 \\ & h(\mathbf{x}, n) = 0 \end{aligned}$$

Dimension of the design vector **x** is to be determined.

or

The number of design variables is to be determined.

Applications

- **Topology optimization**
- **Plate optimization**
- **Eigenvalue problems**
- **MEMS (MicroElectroMechanical Systems) Design**

Design Space Optimization

- **Problem statement of design space optimization**

Conventional Optimization	Design Space Optimization
minimize $f(\mathbf{x})$ subject to $g(\mathbf{x}) \leq 0$ $h(\mathbf{x}) = 0$ $\mathbf{x} \in S$ (S is fixed)	minimize $f(\mathbf{x}, S)$ subject to $g(\mathbf{x}, S) \leq 0$ $h(\mathbf{x}, S) = 0$ $G(S) \leq 0$ $H(S) = 0$ $\mathbf{x} \in S$ (S is variable).

$S \equiv \{N, T_N, \{x_1, x_2, \ldots, x_N\}\}$

N : Number of design units

T_N : Topology of design units

$\{x_1, x_2, \ldots, x_N\}$: Remaining features of design units

Design space improvement is achieved by addition of new design variable.

What is the proper no. of design variables?

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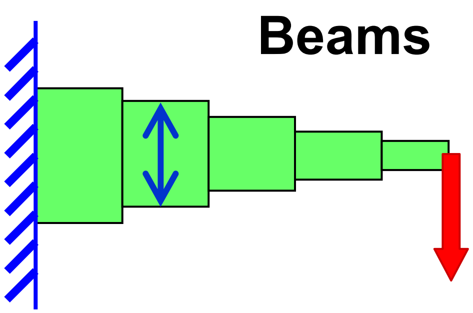
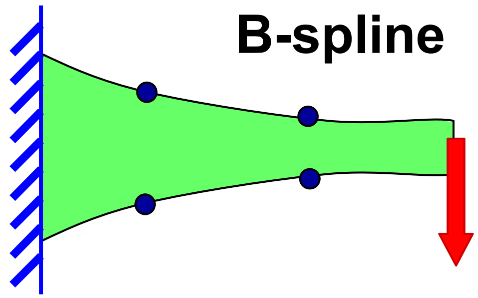
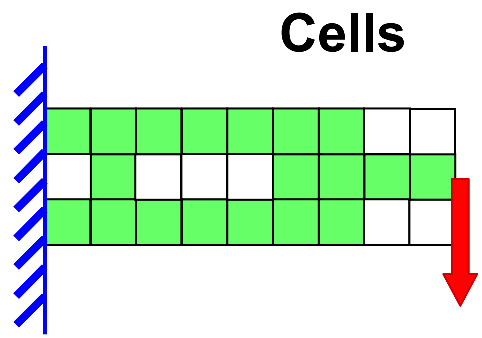






x : thickness of each beam

ndv = 5

x : control points of the B-spline

ndv = 8

x : density of each cell

ndv = 27

ndv = ?

ndv = ?

ndv = ?

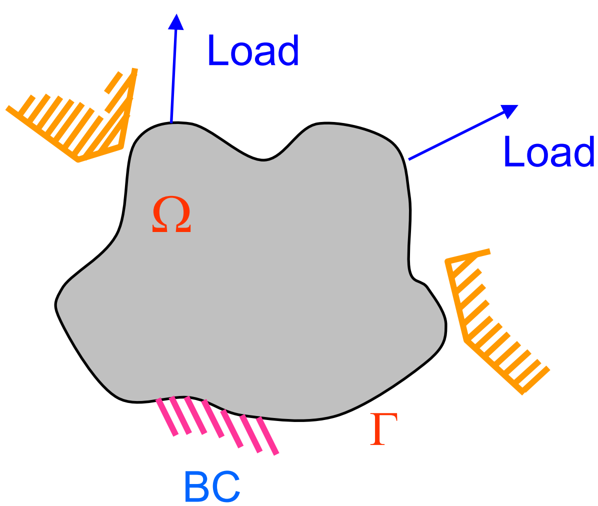
Design space optimization

Design Space Optimization

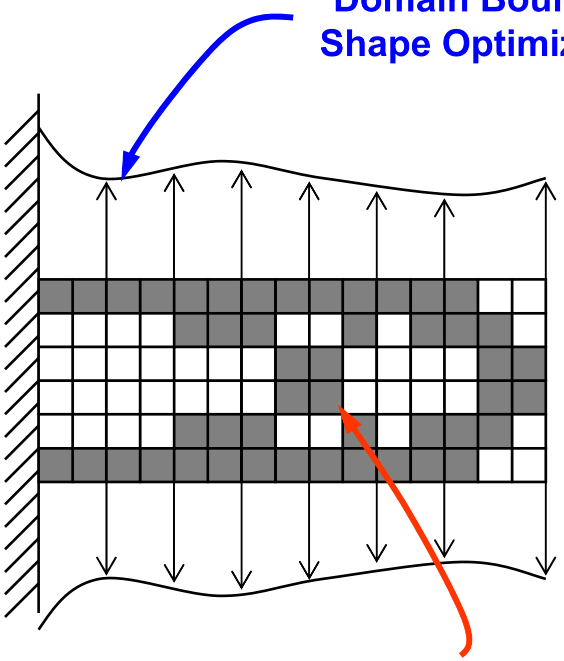
Topology Optimization
$\min \quad f(\rho)$
$\text{s.t.} \quad g(\rho) \le 0$
(Ω is given),

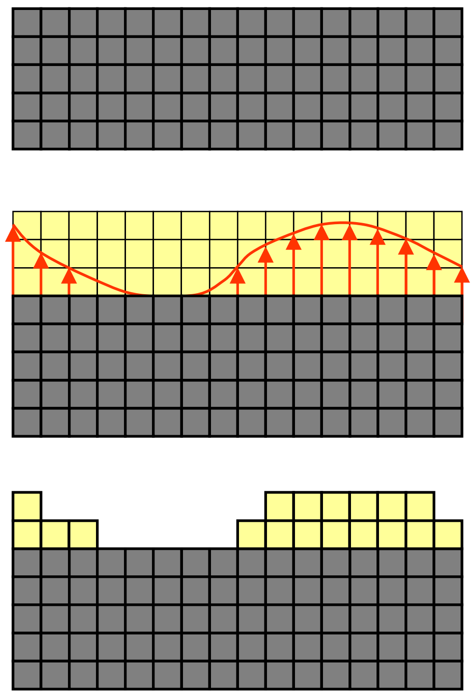
Design Space Topology Optimization
$\min \quad f(\rho; S(\Omega))$
$\text{s.t.} \quad g(\rho; S(\Omega)) \le 0,$

S(Ω) : Domain shape



Design Space Optimization

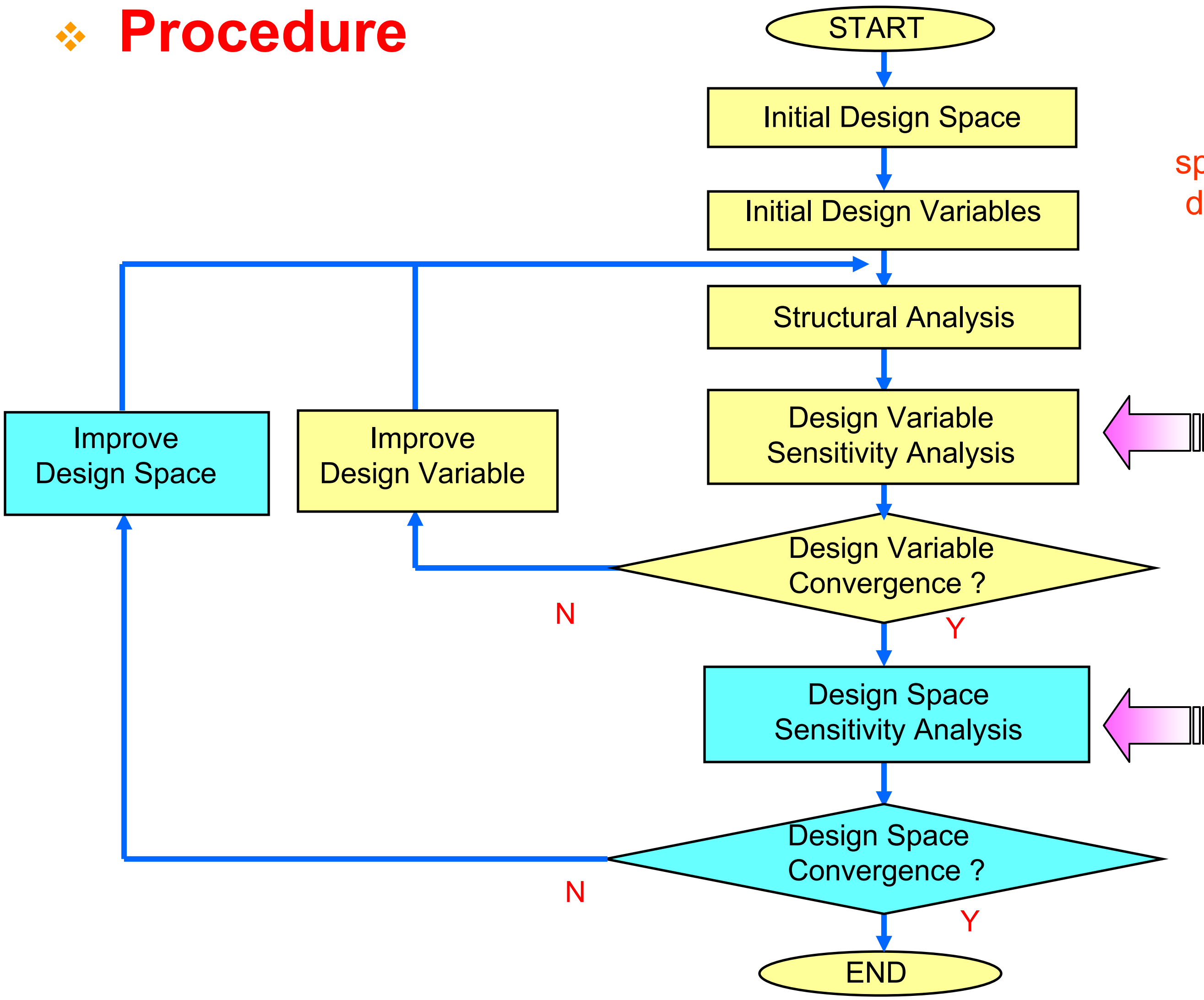




It is impossible to obtain sensitivities because addition of a design variable is a discreet process

Design Continuation Method

Procedure



In order to improve design space, increase the number of design variables is increased

$$\frac{d\Psi}{dx} = \frac{\partial\Psi}{\partial x} + \frac{\partial\Psi}{\partial x}\frac{dz}{dx}$$

$$\frac{d\Psi}{dn} = \frac{\partial\Psi}{\partial n} + \frac{\partial\Psi}{\partial z}\frac{dz}{dn}$$

$$\left(\frac{\Delta\Psi}{\Delta n} = \left.\frac{\Delta\Psi}{\Delta n}\right|_z + \frac{\partial\Psi}{\partial z}\frac{\Delta z}{\Delta n}\right)$$

Design Continuation Method

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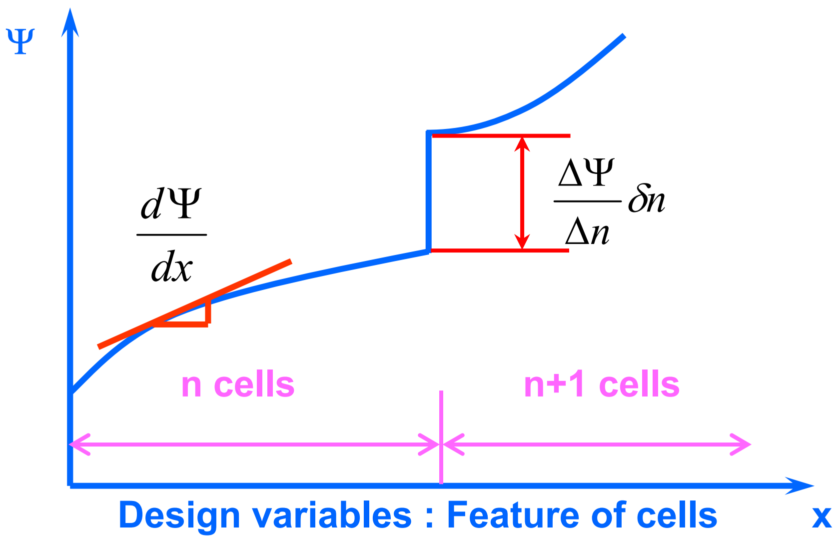
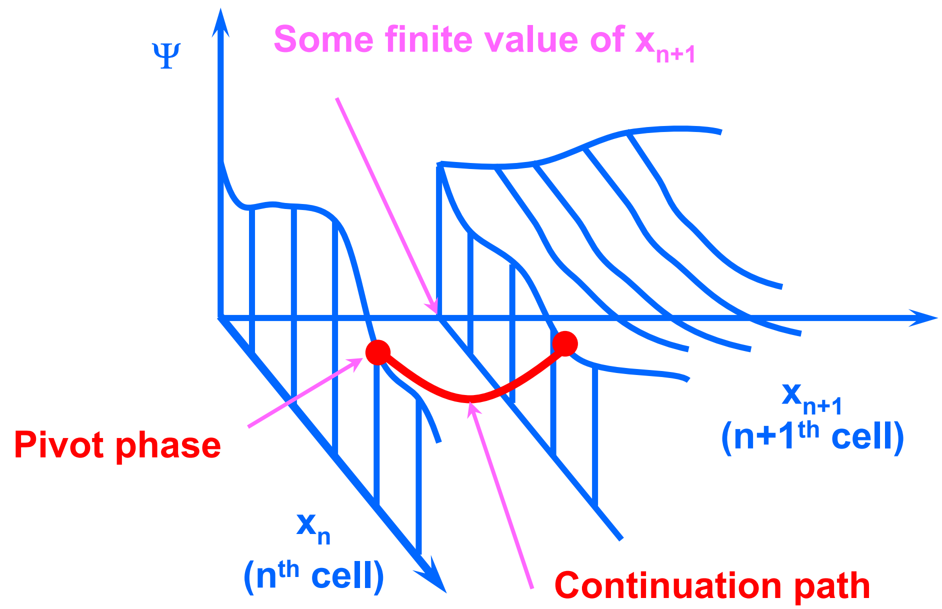
- **Design space change**

  : Generating new design variable



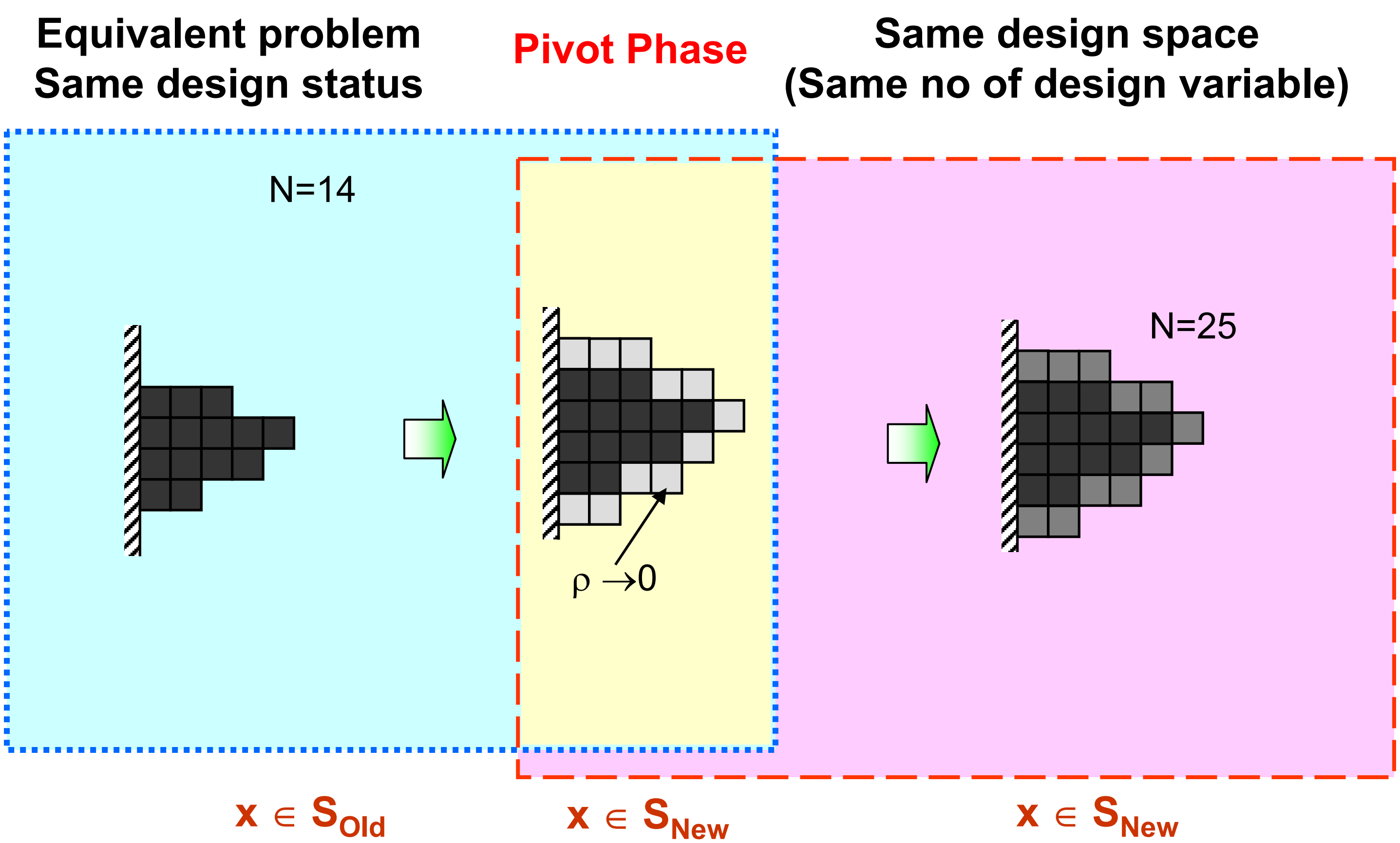


Functional is not continuous when a design variable is created.

⇒ It is impossible to obtain derivatives.

Design Continuation Method

Pivot phase



Sensitivity Analysis

❖ Topology Optimization

Problem statement

$$Minimize \quad \int_{\Gamma} F^i z^i d\Gamma,$$

$$Subject\ \ to \quad \int_{\Omega} \rho(x) d\Omega \le M_o,$$

$$0 \le \rho(x) \le 1$$

Relationship between E_i and ρ

$$\frac{E_i}{E_o} = \rho^n$$

E_i : Intermediate Young's modulus
E_o : Reference Young's modulus
n : exponent

Directional variation of the objective fn

$$\psi'_{+,\rho\to 0} = \int_{\Gamma} F^i z^{i\,\prime}_{\ +,\rho\to 0} d\Gamma \qquad \cdots\cdots ①$$

Sensitivity Analysis

❖ Topology Optimization (Cont'd)

State equation (variational form)

$$\int_{\Omega} \sigma^{ij}(z)\varepsilon^{ij}(\bar{z})d\Omega = \int_{\Gamma} F^{i}\bar{z}^{i}d\Gamma \qquad \forall \bar{z} \in Z_{adm}$$

$$\Rightarrow \int_{\Omega} [\sigma^{ij}(z'_{+,\rho\to0})\varepsilon^{ij}(\bar{z}) + \sigma^{ij}(z)\varepsilon^{ij}(\bar{z}'_{+,\rho\to0}) + \varepsilon^{ij}(z)D^{ijkl'}{}_{+,\rho\to0}\varepsilon^{kl}(\bar{z})]d\Omega$$

$$= \int_{\Gamma} F^{i}\bar{z}^{i'}{}_{+,\rho\to0}d\Gamma \qquad \forall \bar{z} \in Z_{adm}$$

Assume $\bar{z}'_{+,\rho\to0} = 0$

$$\Rightarrow \int_{\Omega} \sigma^{ij}(z'_{+,\rho\to0})\varepsilon^{ij}(\bar{z})d\Omega = -\int_{\Omega} \varepsilon^{ij}(z)D^{ijkl'}{}_{+,\rho\to0}\varepsilon^{kl}(\bar{z})d\Omega \qquad \cdots\cdots ②$$

Sensitivity Analysis

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Topology Optimization (Cont'd)

Adjoint equation

$$\int_{\Omega} \sigma^{ij}(\lambda)\varepsilon^{ij}(\bar{\lambda})d\Omega = \int_{\Gamma} F^i \bar{\lambda} d\Omega \qquad \forall \bar{\lambda} \in Z_{adm} \qquad \cdots\cdots③$$

Using the symmetry of the energy bilinear form and combining the eqn ①, ② , and ③

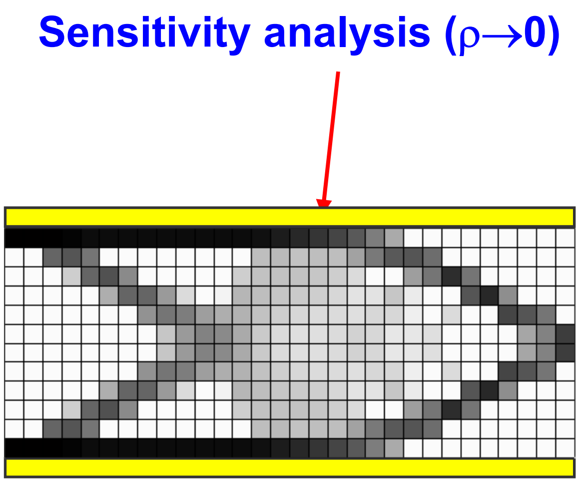
$$\psi'_{+,\rho\to 0} = \int_{\Gamma} F^i z^{i\,\prime}{}_{+,\rho\to 0} d\Gamma = -\int_{\Omega} \varepsilon^{ij}(z) D^{ijkl\,\prime}{}_{+,\rho\to 0} \varepsilon^{kl}(\lambda) d\Omega$$

Because z and λ are identical,

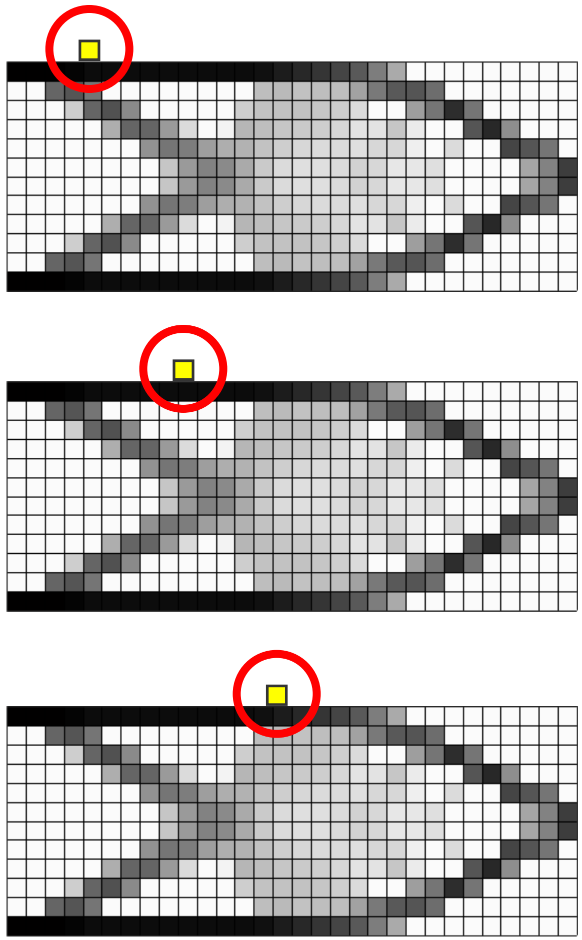
$$\psi'_{+,\rho\to 0} = -\int_{\Omega} \varepsilon^{ij}(z) D^{ijkl\,\prime}{}_{+,\rho\to 0} \varepsilon^{kl}(z) d\Omega$$

Numerical Results - Sensitivities

- Topology
  Sensitivity analysis



- Finite Differencing



Numerical Results - Sensitivities

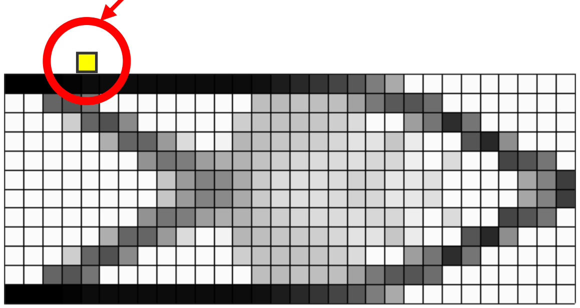
Topology Sensitivity analysis

$E = 210\times10^9$ N/m^2

$\nu = 0.3$

$\Delta b = 0.01$

Isoparametric 8-node plane element



New Design Variable No.	FDM $\Delta\Psi_i/\delta b$	Analytic $\Psi_i'/\delta b$	$(\Psi_i' /\Delta\Psi_i \times100)\%$
1	-116.6826	-116.6470	99.97
2	-8.5387	-8.5367	99.98
3	-1.6238	-1.6235	99.98
4	-2.3714	-2.3706	99.97
5	-3.6479	-3.6474	99.99
6	-3.9573	-3.9564	99.98
7	-3.4781	-3.4776	99.99
8	-3.2490	-3.2481	99.97
9	-3.0011	-3.0005	99.98
10	-2.7633	-2.7626	99.97
11	-2.5634	-2.5628	99.98
12	-2.2540	-2.2534	99.97
13	-2.2088	-2.2081	99.97
14	-3.5688	-3.5680	99.98
15	-5.8834	-5.8819	99.97
16	-6.7057	-6.7040	99.97
17	-5.8702	-5.8683	99.97
18	-3.7185	-3.7169	99.96
19	-2.0841	-2.0830	99.95
20	-1.1828	-1.1819	99.92
21	-1.6386	-1.6361	99.85

Sensitivity analysis vs. Finite differencing

16.888
ESD.77

Number of function evaluation per step

Analytical sensitivity analysis (SA): 1 + 1 = 2

Finite differencing (FD): 1 + ndv

Ex) Topology optimization with ndv = 500

(1) Number of function evaluations per step

SA: 2

FD: 501

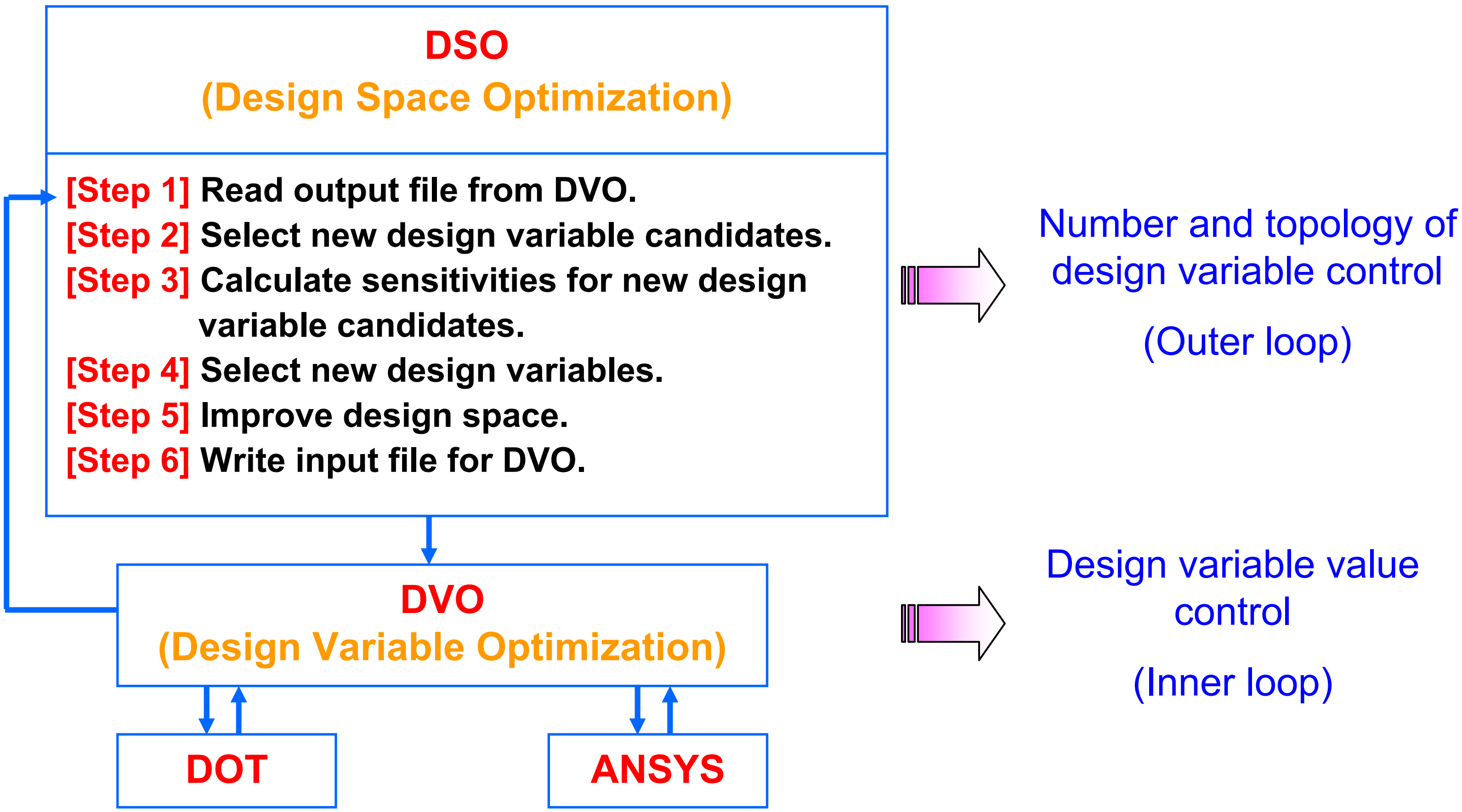
(2) Computing time (1 minutes per function evaluation, 10 steps)

SA: $2 \times 1 \times 10$ = 20 minutes

FD: $501 \times 1 \times 10$ = 5010 minutes = 3 days 11.5 hours

Optimization Scheme

- Computer program structure



Problem Statement

$$\text{Minimize} \quad \int_{\Gamma} F^i z^i d\Gamma$$ → **compliance**

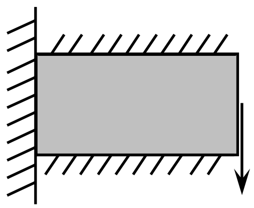
$$\text{Subject to} \quad \int_{\Omega} \rho(x) d\Omega \leq M_o$$ → **mass**

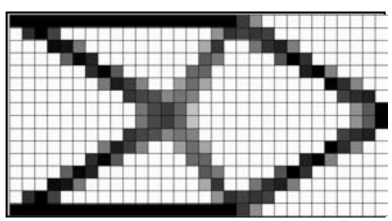
$$0 \leq \rho(x) \leq 1$$ → **density of cells**

Topology Optimization Results

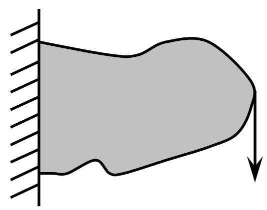
Traditional optimization

Number of
design variables
(pixel)
: 30×16 = 480

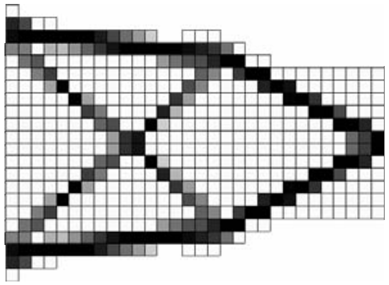




Design Space Optimization

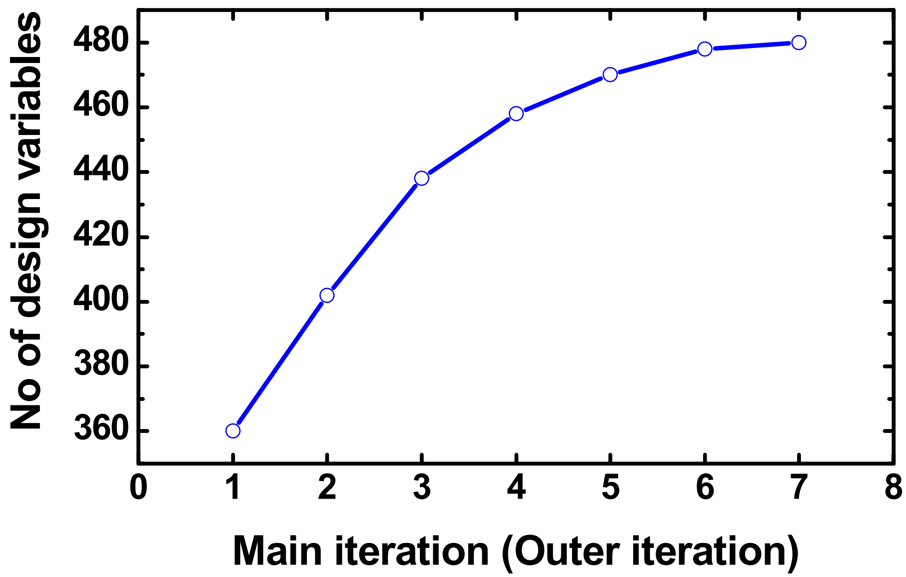


Number of
design variables
(pixel)
≤ 30×16 = 480

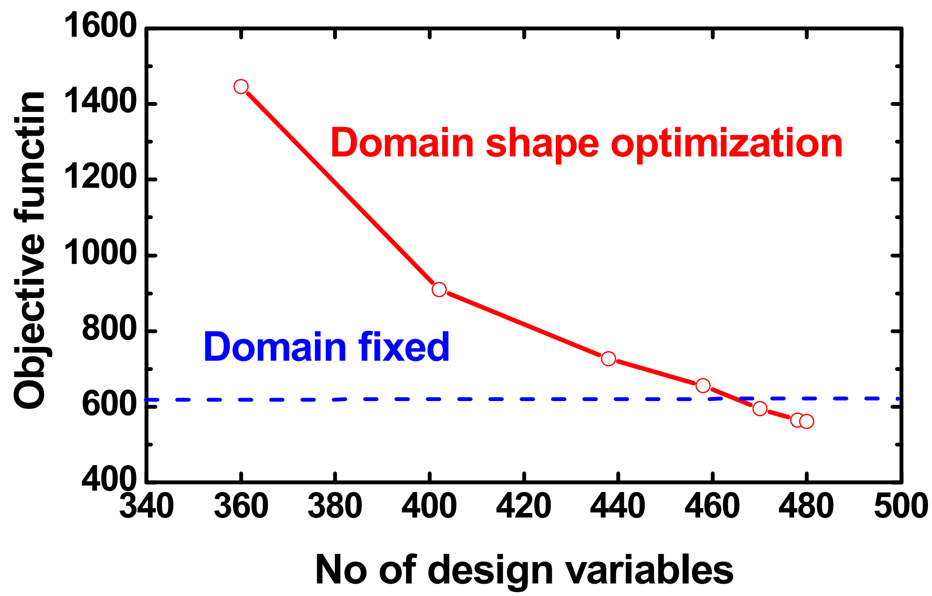


Topology Optimization Results

Initial design domain size: 30×12,   Final domain area: 480 (30×16)



Number of design variables



Objective function history

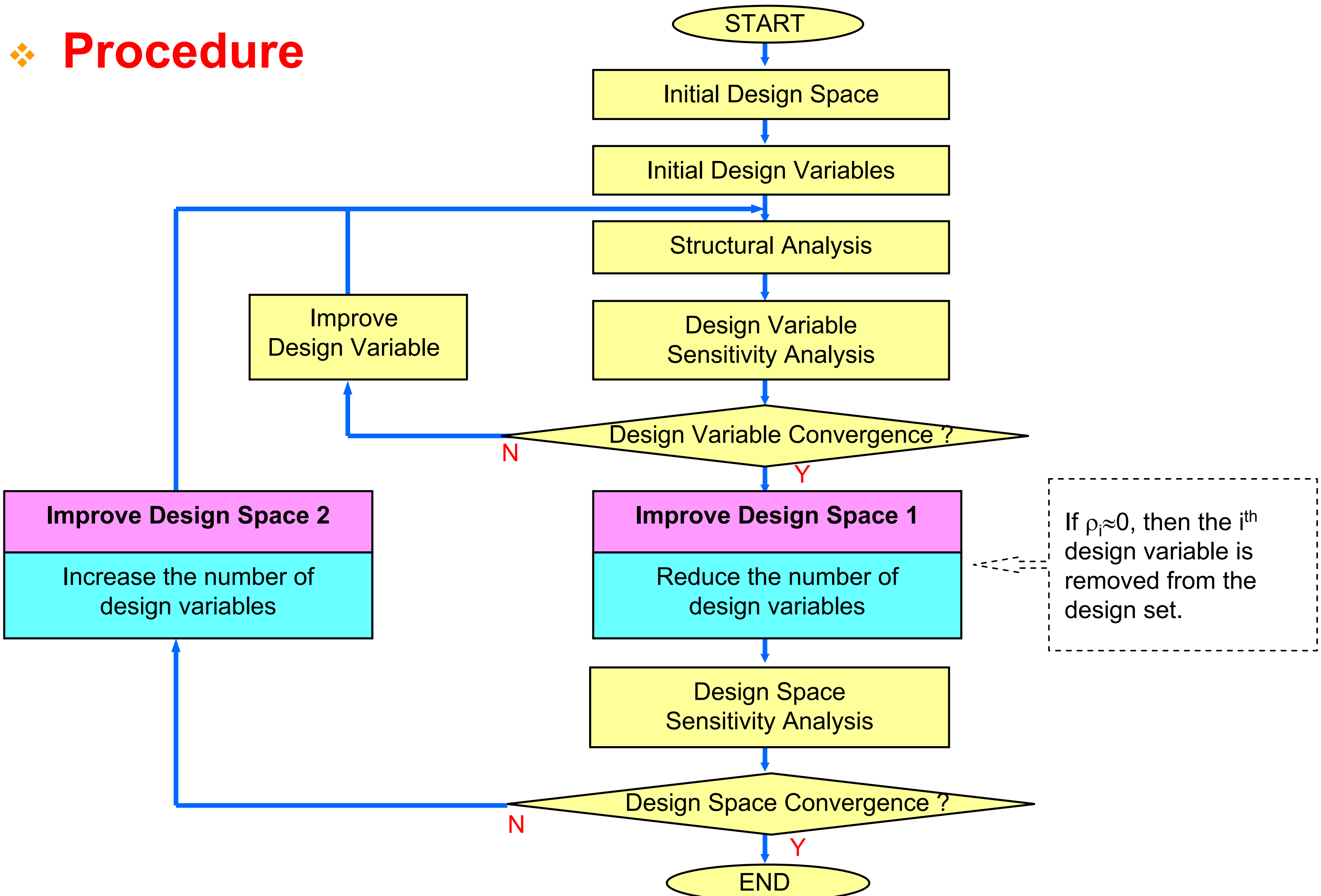
Young's modulus: 210×10^9 N/m^2

Poisson's ratio: 0.3

Maximum thickness: 0.012 m, Minimum thickness: 0.005 m

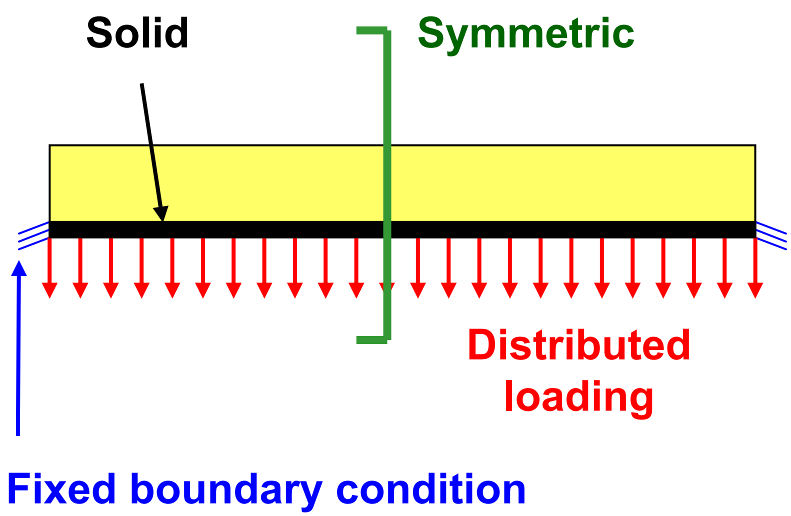
Design Variable Addition & Reduction

- **Procedure**



Design Variable Addition & Reduction

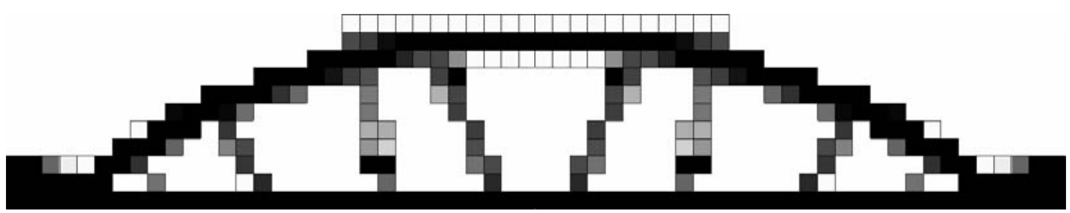
- **Bridge problem**



$$\textit{Minimize} \quad \int_{\Gamma} F^i z^i d\Gamma,$$

$$\textit{Subject to} \quad \int_{\Omega} \rho(x) d\Omega \le M_o,$$

$$0 \le \rho(x) \le 1$$



Mass constraints: 35%

Design Variable Addition & Reduction

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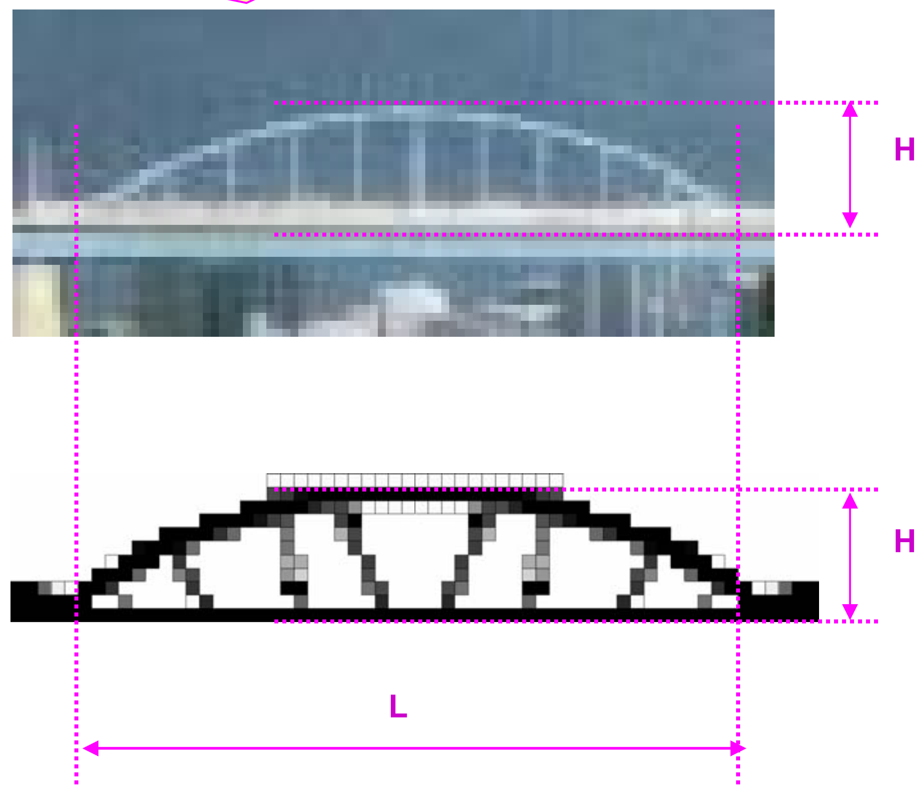
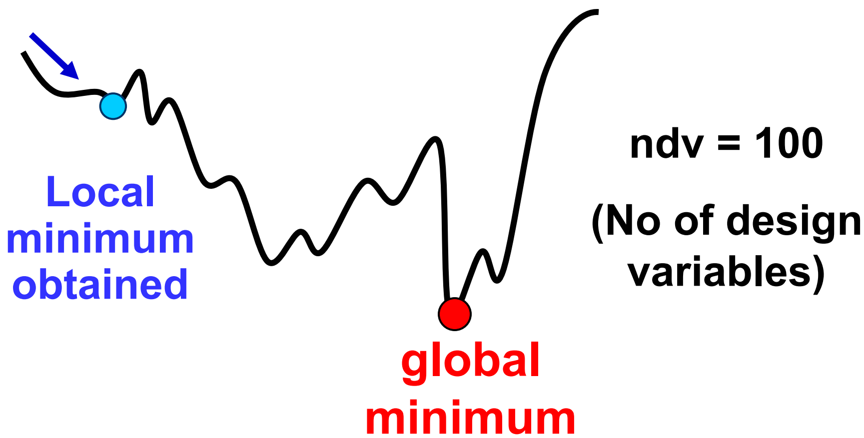
❖ DongJak Bridge

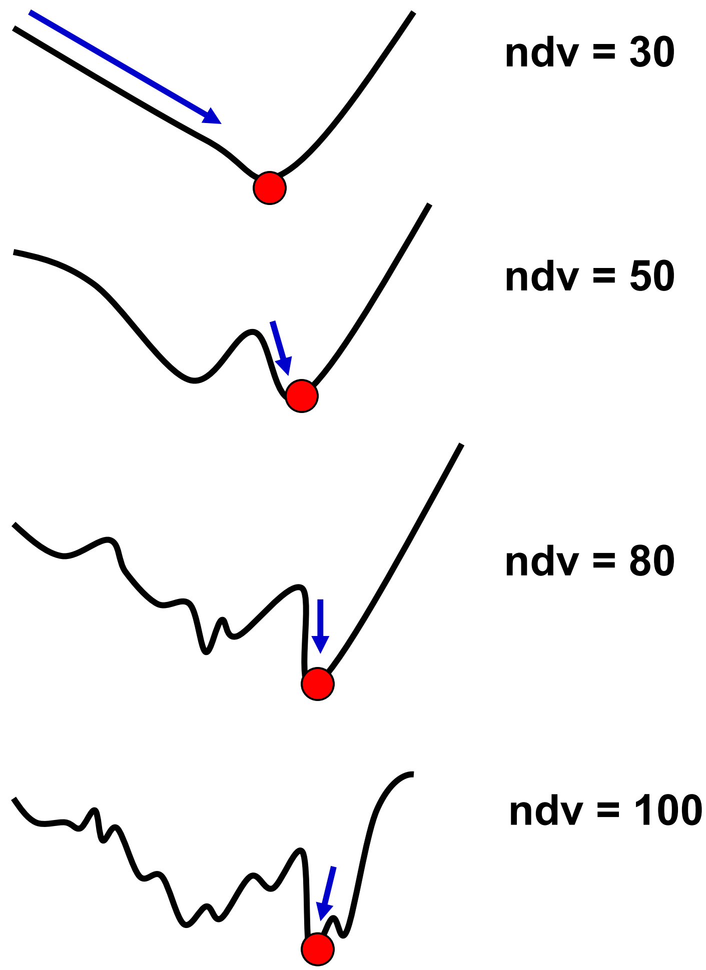


DSO - discussion

* To find the minimum value

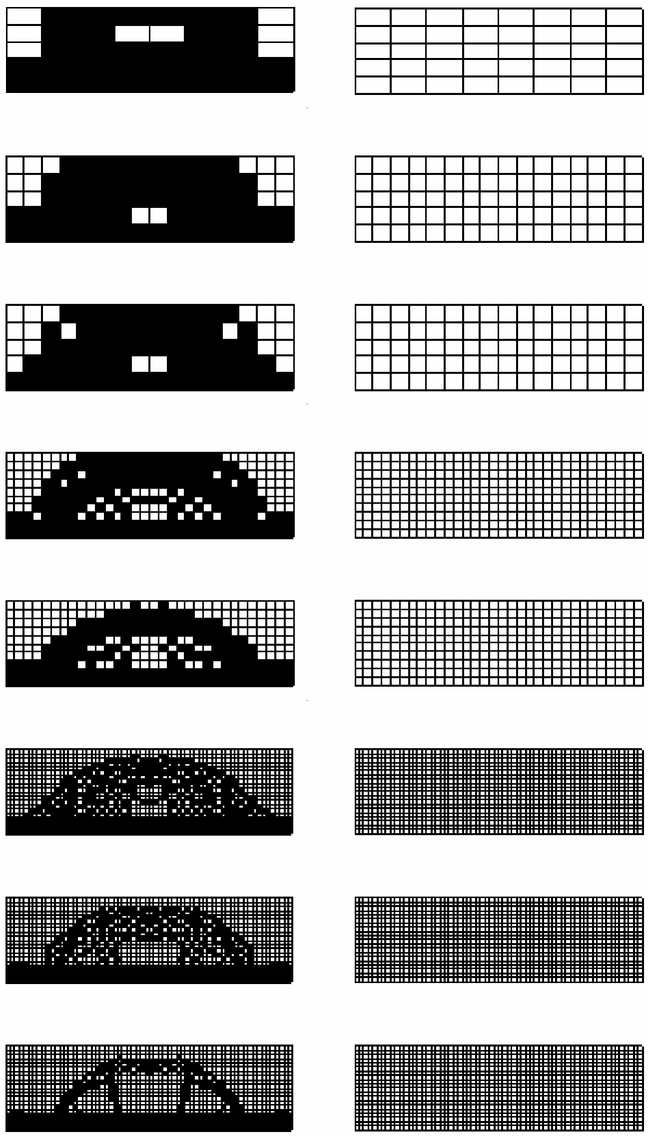


* Design space optimization

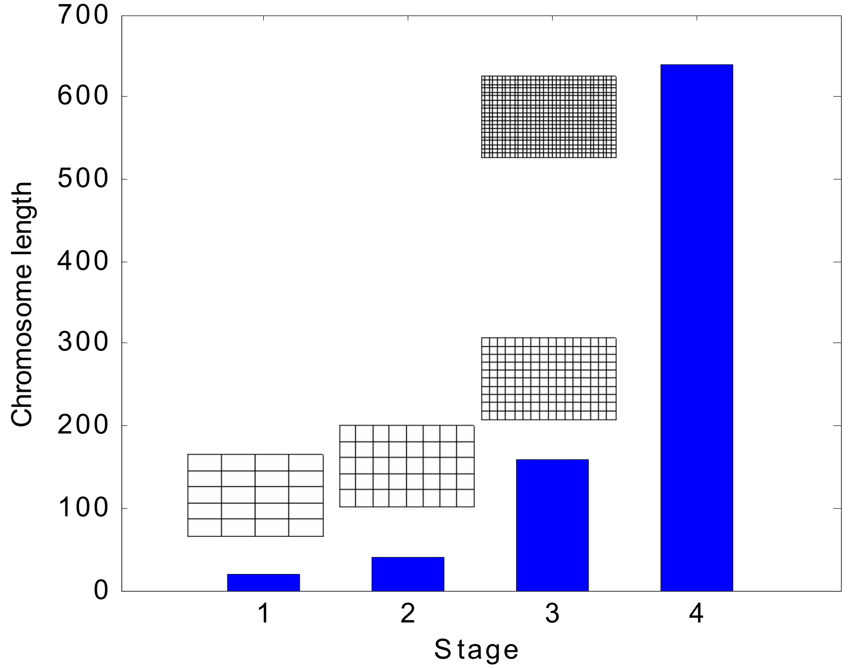


* Optimization based on sensitivity analysis

DSO based on GA



Chromosome length changes as generations progress



Summary

I. Structural Optimization

- Size optimization
- Shape optimization
- Topology optimization

II. Integrated Structural Optimization

- Integration of size and shape optimization

III. Design Space Optimization

- The number of design variables is considered as a design variable
- Effect of adding new design variables is determined at the pivot phase

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