# Multidisciplinary System Design Optimization (MSDO)

# Optimization of a Hybrid Satellite Constellation System

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# Outline

- Introduction
  - Satellite constellation design
- Simulation
  - Modeling
  - Benchmarking
- Optimization
  - Single objective
    - Gradient based
    - Heuristic: Simulated Annealing
  - Multi-objective
- Conclusions and Future Research

# Motivation/Background

Past attempts at mobile satellite communication systems have failed as there has been an inability to match user demand with the provided capacity in a cost-efficient manner (e.g. Iridium & Globalstar)

Two main assumptions:

- Circular orbits and a common altitude for all the satellites in the constellation
- Uniform distribution of customer demand around the globe

*Given a non-uniform market model, can the incorporation of elliptical orbits with repeated ground tracks expand the cost-performance trade space favorably?*

Aspects of the satellite constellation design problem previously researched:

-T Kashitani (MEng Thesis, 2002, MIT)

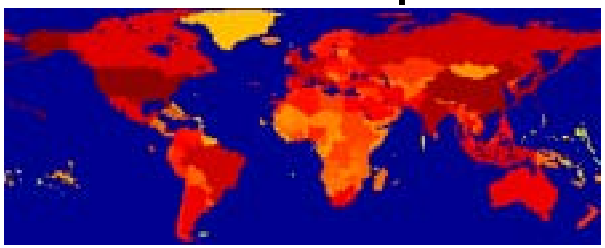
-M. Parker (MEng Thesis, 2001, MIT)

-O. de Weck and D. Chang (AIAA 2002-1866)

# Market Distribution Estimation



GNP PPP Map



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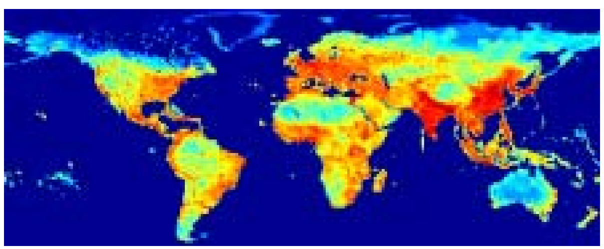
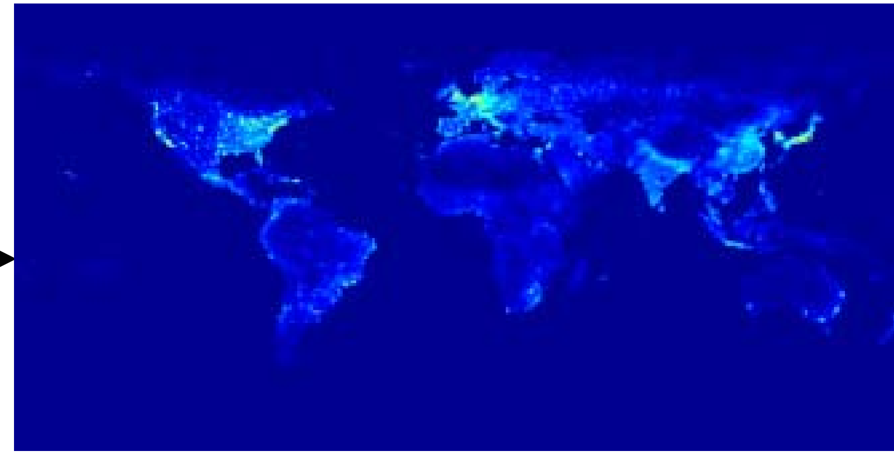
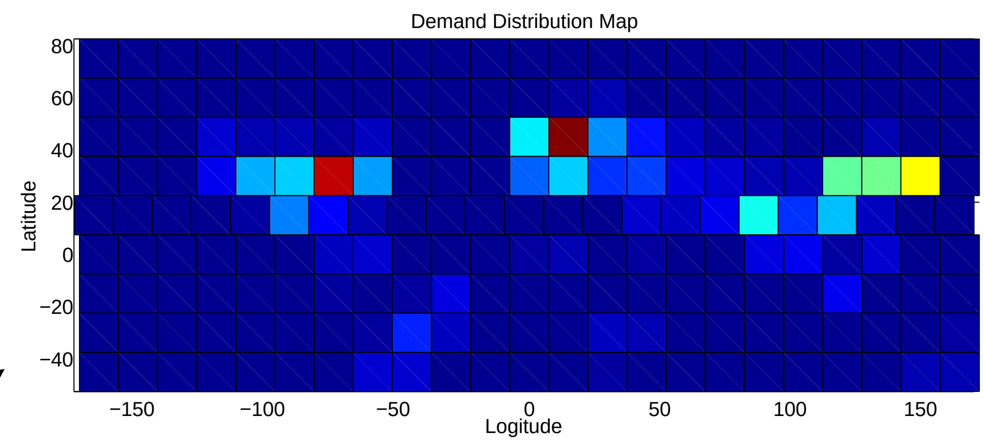
Population Map

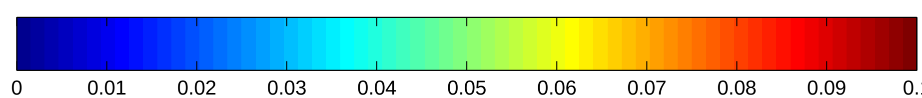


Market Distribution Map






**Reduced Resolution for Simulation**



# Problem Formulation

• A circular LEO satellite backbone constellation designed to provide minimum capacity global communication coverage,
• An elliptical (Molniya) satellite constellation engineered to meet high-capacity demand at strategic locations around the globe (in particular, the United States, Europe and East Asia).

**Single Objective J:** **min** the lifecycle cost of the total hybrid satellite constellation sys.

**Constraints** :

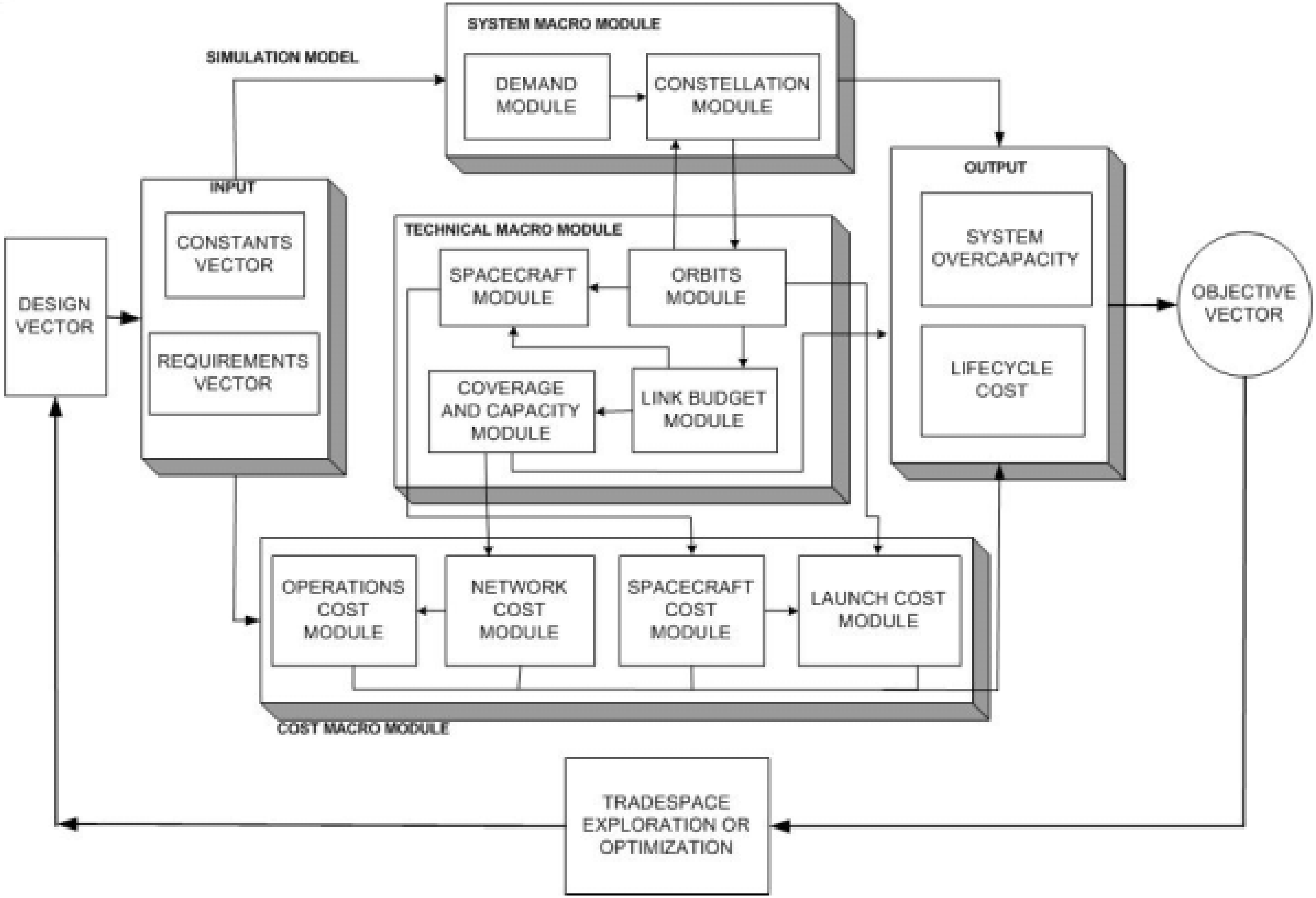
* the total lifecycle cost must be strictly positive
* the data rate market demand must be met at least 90% of the time
  - the satellites must service 100% of the users 90% of the time
  - data rate provided by the satellites >= to the demand
  - all satellites must be deployable from current launch vehicles

**Design Vector for Polar Backbone Constellation**:

<C [polar/walker], emin [deg], MA, ISL [0/1], h [km], Pt [W], DA [m]>

**Design Vector for Elliptical Constellation:** <T [day], e [-], Np [-], Pt [W], Da [m]>

# Simulation Model



# Tradespace Exploration



- An orthogonal array was implemented for the elliptical constellation DOE
- The recommended initial start point for the numerical optimization of the elliptical constellation is

  $Xo_{init}$ =[ T=1/6,e=0.6,NP=4,Pt=500,DA=3]$^T$
- In order to analyze the tradespace of the Polar constellation backbone, a full factorial search was conducted, the Pareto front of non dominated solutions was then defined
- The lowest cost Polar constellation was found to have the following design vector values

  X = [C=polar,emin=5 deg,MA=QPSK,ISL=1, h=2000,Pt=0.25,DA=0.5]$^T$

| Factor | Level | Effect |
|---|---|---|
| T | 4 | -207.3 |
| T | 6 | 159.8 |
| T | 12 | 131.13 |
| T | 24 | -95.5 |
| E | 0 | 53.8 |
| E | 0.2 | -13.98 |
| E | 0.4 | 217.93 |
| E | 0.6 | -515.55 |
| NP | 1 | -262.0 |
| NP | 2 | -36.85 |
| NP | 3 | 717.57 |
| NP | 4 | -319.13 |
| Pt | 500 | -975.78 |
| Pt | 1000 | -849.5 |
| Pt | 5000 | 532.03 |
| Pt | 10000 | 1441.1 |
| DA | 1.5 | 315.8 |
| DA | 2.0 | 25.15 |
| DA | 2.5 | 166.25 |
| DA | 3.0 | -571.0 |

# Code Validation

## LEO BACKBONE :

- Simulation created by de Weck and Chang (2002)
  - Code benchmarked against a number of existing satellite systems
    - Outputs within 20% of the benchmark's values
- Slight modifications made to suit the broadband market demand
  - # of subscribers, required data rate per user, avg. monthly usage etc…

## CODE VALIDATION:

- Orbit and constellation calculations
  - Validated by plotting and visually confirming orbits

# Elliptical Benchmarking

## ELLIPTICAL CONSTELLATION :

- Simulation benchmarked against Ellipso
- Ellipso
  - Elliptical satellite constellation system proposed to the FCC in 1990
  - (T = 24, NP = 4, phasing of planes = 90 degrees apart)
- System benchmarked on modular basis

- Ellipso didn't use the same demand model, thus a constraint benchmark process was not conducted.

| System / Module | | Ellipso | Simulation | Units |
|---|---|---|---|---|
| **Link Budget** | Antenna Gain | 12 | 11.93 | [dBi] |
| | EIRP | 27 | 24.93 | [dBW] |
| | Data Rate | 2.2 | 1.08 | [Mbps] |
| **Spacecraft** | Sat Mass | 68 | 98.68 | [Kg] |
| | Sat Volume | 0.0008 | 0.810 | $m^3$ |
| **Lifecycle Cost** | | 249.6 | 290.9 | [YR2002 $M] |

# Gradient-Based Optimization

- Sequential Quadratic Programming (SQP)
  - Simplification => number of planes integer
- Objective: minimize lifecycle cost

Initial guess:

| | |
|---|---|
| Period (T): | 0.5 day |
| Eccentricity (e): | 0.01 |
| # Planes (NP): | 4 |
| Transmitter Power (Pt): | 4000 W |
| Antenna Diameter (DA): | 3 m |

J: $6280.5999 M

Optimal:

| | |
|---|---|
| Period (T): | 0.7 day |
| Eccentricity (e): | 0 |
| # Planes (NP): | 4 |
| Transmitter Power (Pt): | 3999.7 W |
| Antenna Diameter (DA): | 1.76 m |

J*: $6187.8559 M

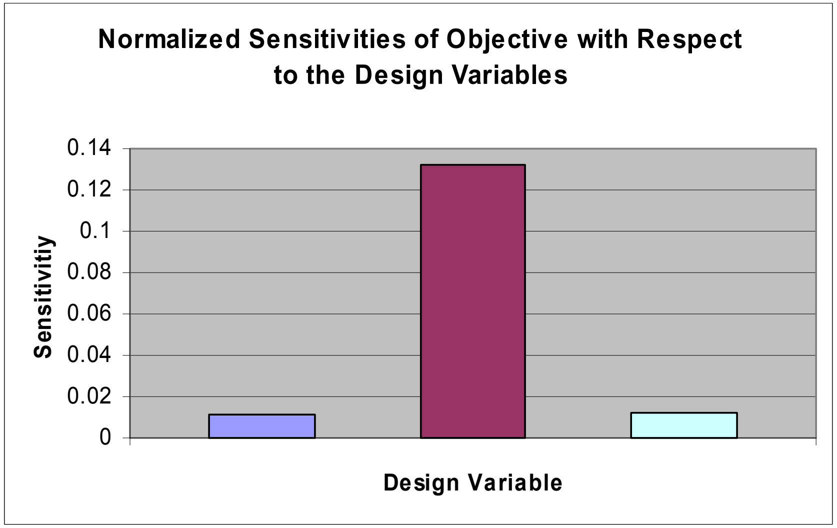
# Sensitivity Analysis

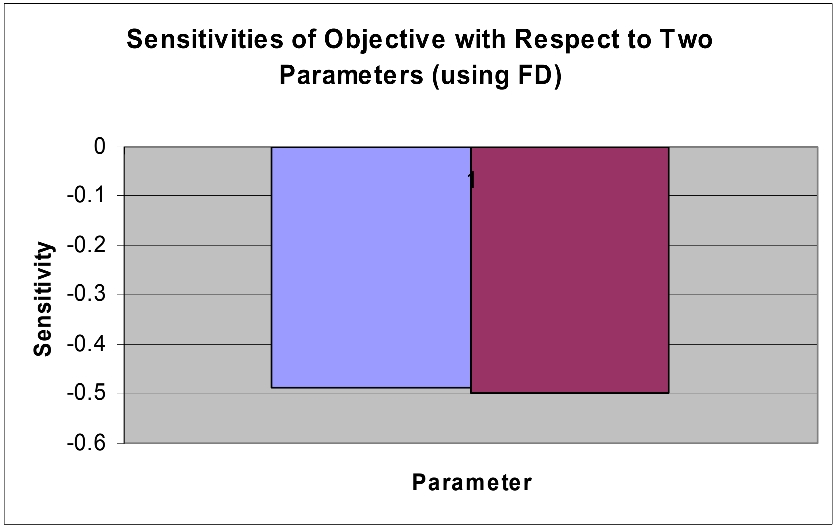
Optimal Design, x*:

| | |
|---|---|
| Period (T): | 0.7 day |
| Eccentricity (e): | 0 |
| # Planes (NP): | 4 |
| Transmitter Power (Pt): | 3999.7 W |
| Antenna Diameter (DA): | 1.76 m |

Parameters:

Data Rate: 1000 kbps
Step Size: 10 kbps
# Subscribers: 1000 users
Step Size: 10 users



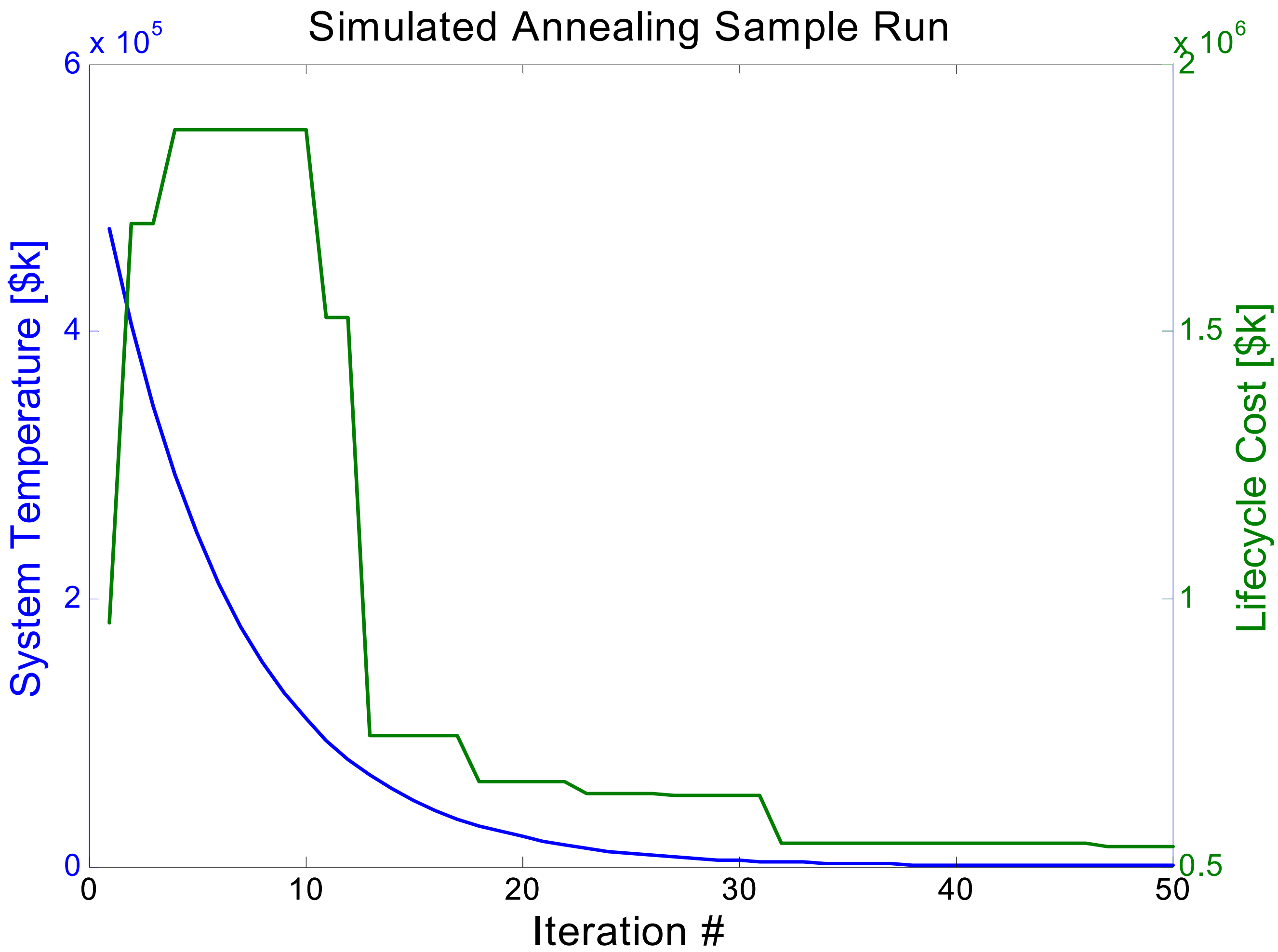



# Heuristic Optimization

- Simulated annealing was used
- Quite sensitive to cooling schedule and starting conditions
- Not very repeatable
  - Low confidence that global optimum was reached
- Total computational cost high
- Abandoned in favor of full-factorial evaluation of the tradespace for the multi-objective case
  - Possibly gain insight into key trends

# Sample Simulated Annealing Run





# Multi-Objective Optimization

- Minimum cost design tend not to have the possibility for future growth
- Try to simultaneously:
  - Minimize Lifecycle Cost (LCC)
  - Maximize Time Averaged Over Capacity

```
If % market served > min market share
      Over capacity = …
            Total capacity - Market served
Else
      Over capacity = 0
End
```

- Min market share chosen to be 90%

# Full Factorial Tradespace

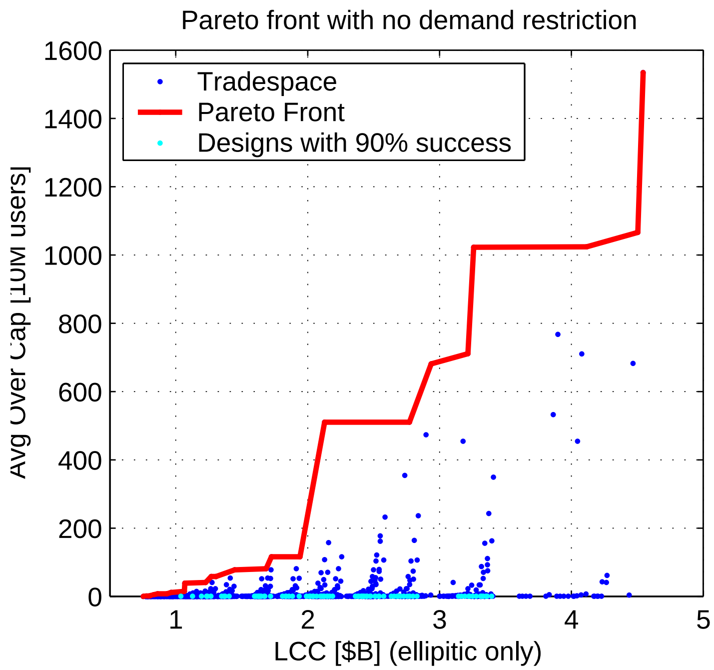
- 1280 designs evaluated
- Interesting trends revealed

| Factor | Levels | Units |
|---|---|---|
| T | 1,1/2,1/3,1/4,1/5 | [days] |
| e | 0.001, 0.1, 0.3 0.4 | [-] |
| NP | 2, 3, 4, 6 | [-] |
| Pt | 1, 2, 4, 6 | [kW] |
| DA | 1.5, 2, 2.5, 3 | [m] |

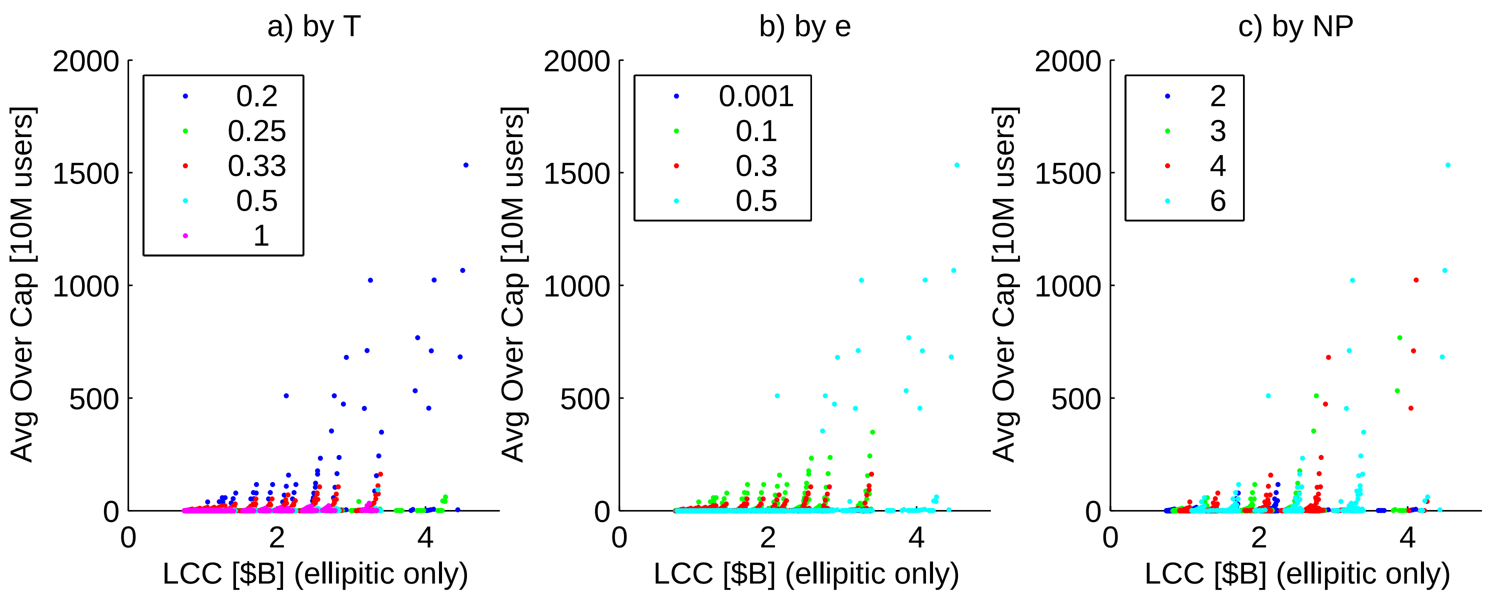
# Unrestricted Pareto Front

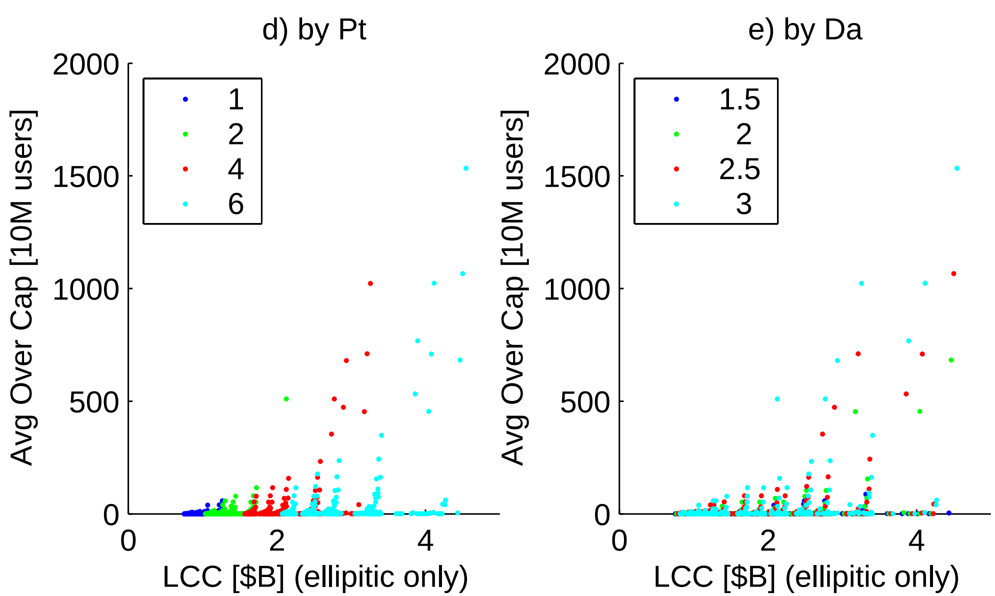


- Very high average over capacity
- Seems counterintuitive that high success does not yield high average over capacity
- Look at the design trade to find an explanation



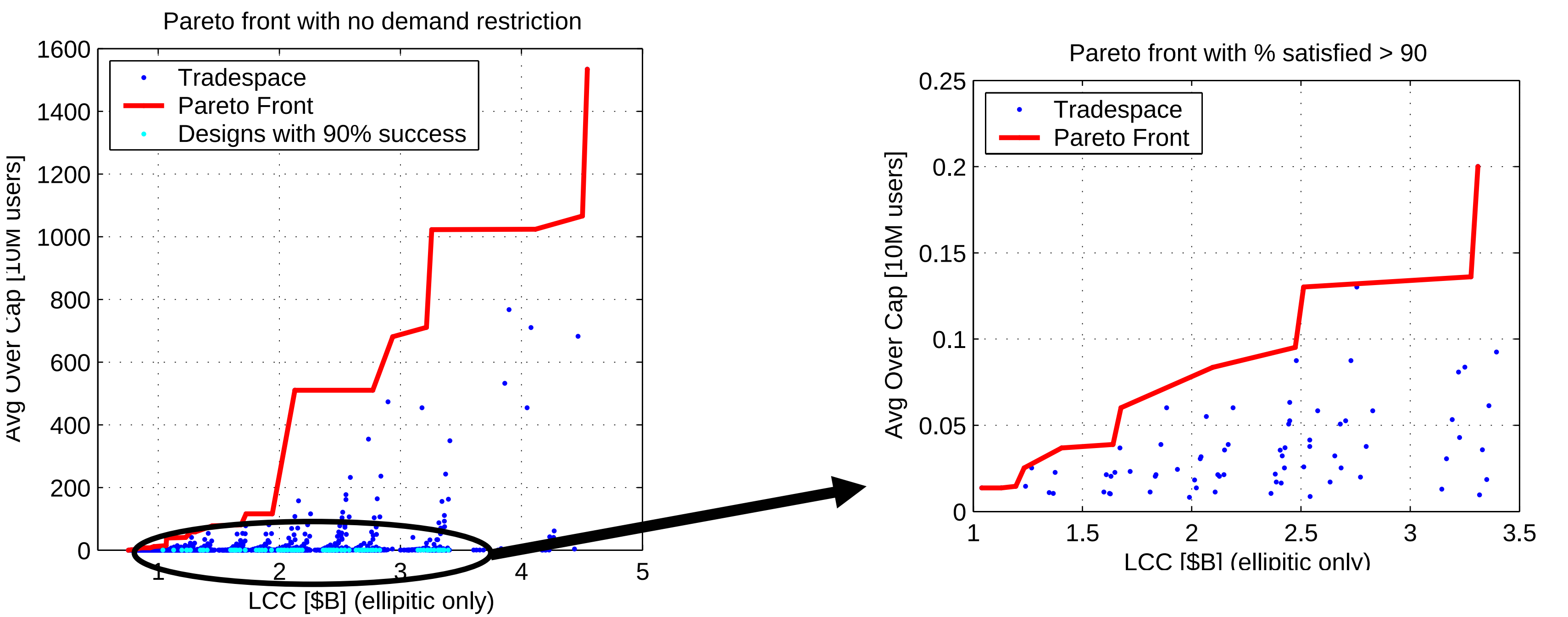
# Unrestricted Tradespace



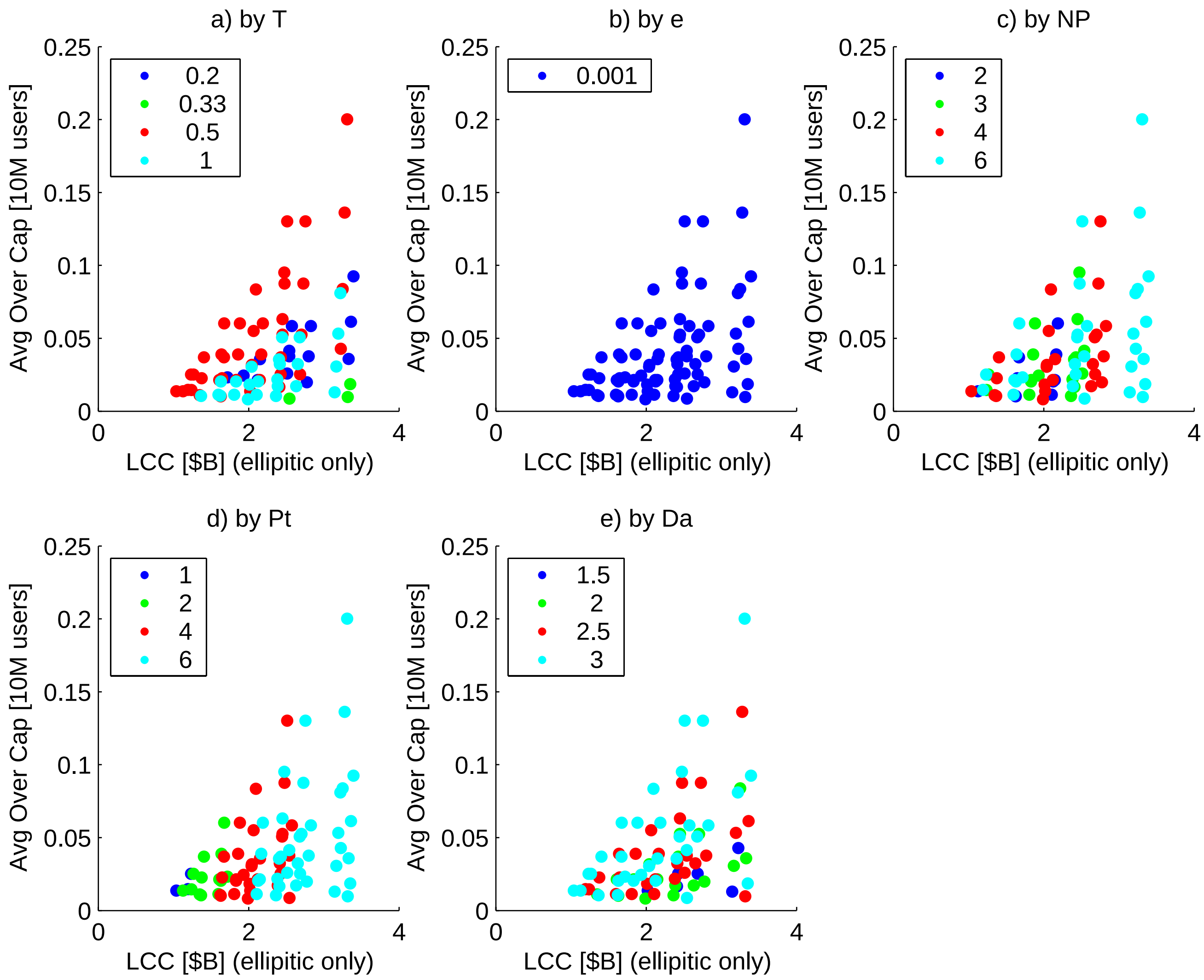

- All high AOC designs have high eccentricity and short period
- Many satellites per planes
  - Very high system capacity

# Restricted Pareto Front




- Much smaller AOC when demand constraint is enforced
- Again explore the tradespace by coloring by DV values

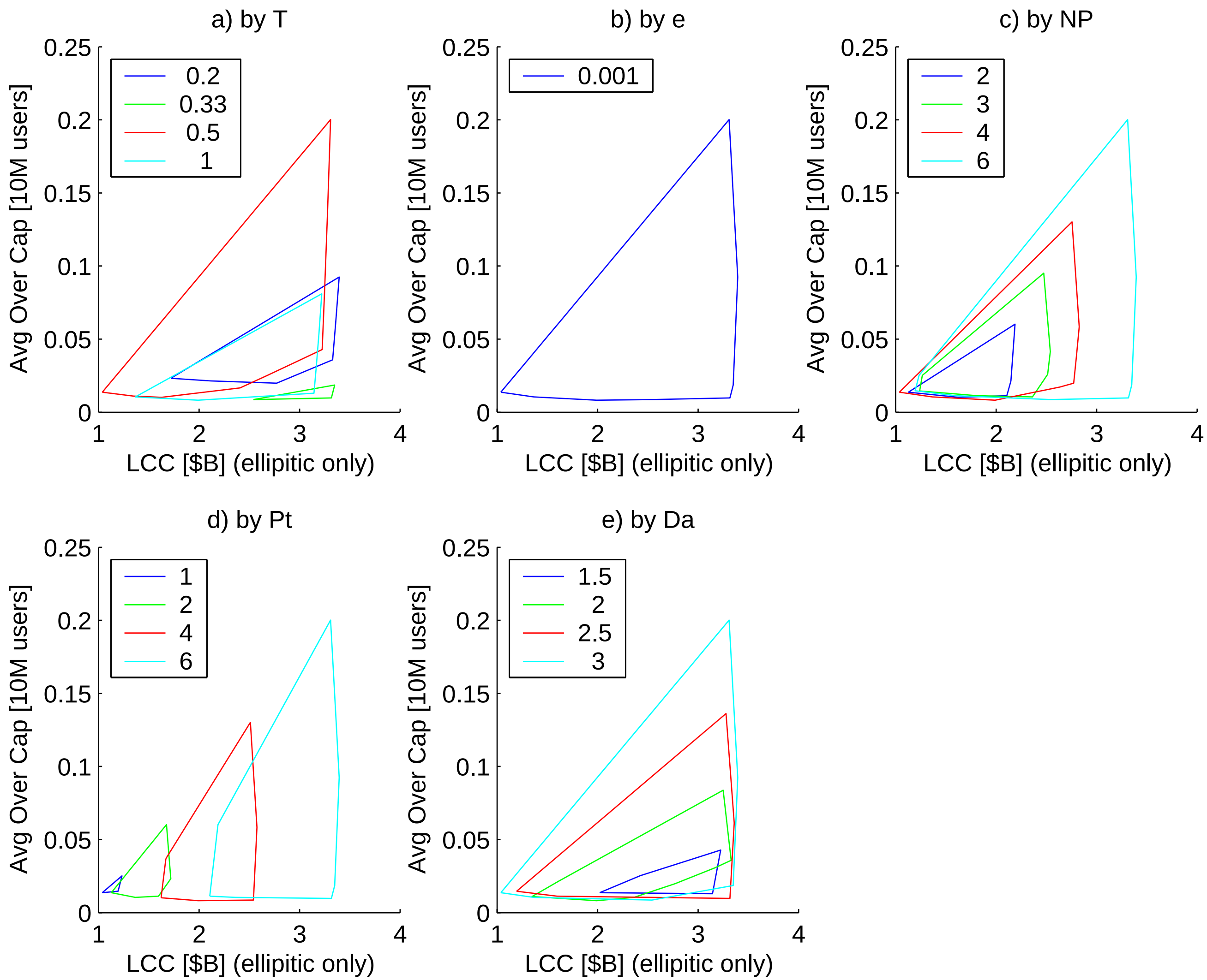
# Restricted Tradespace



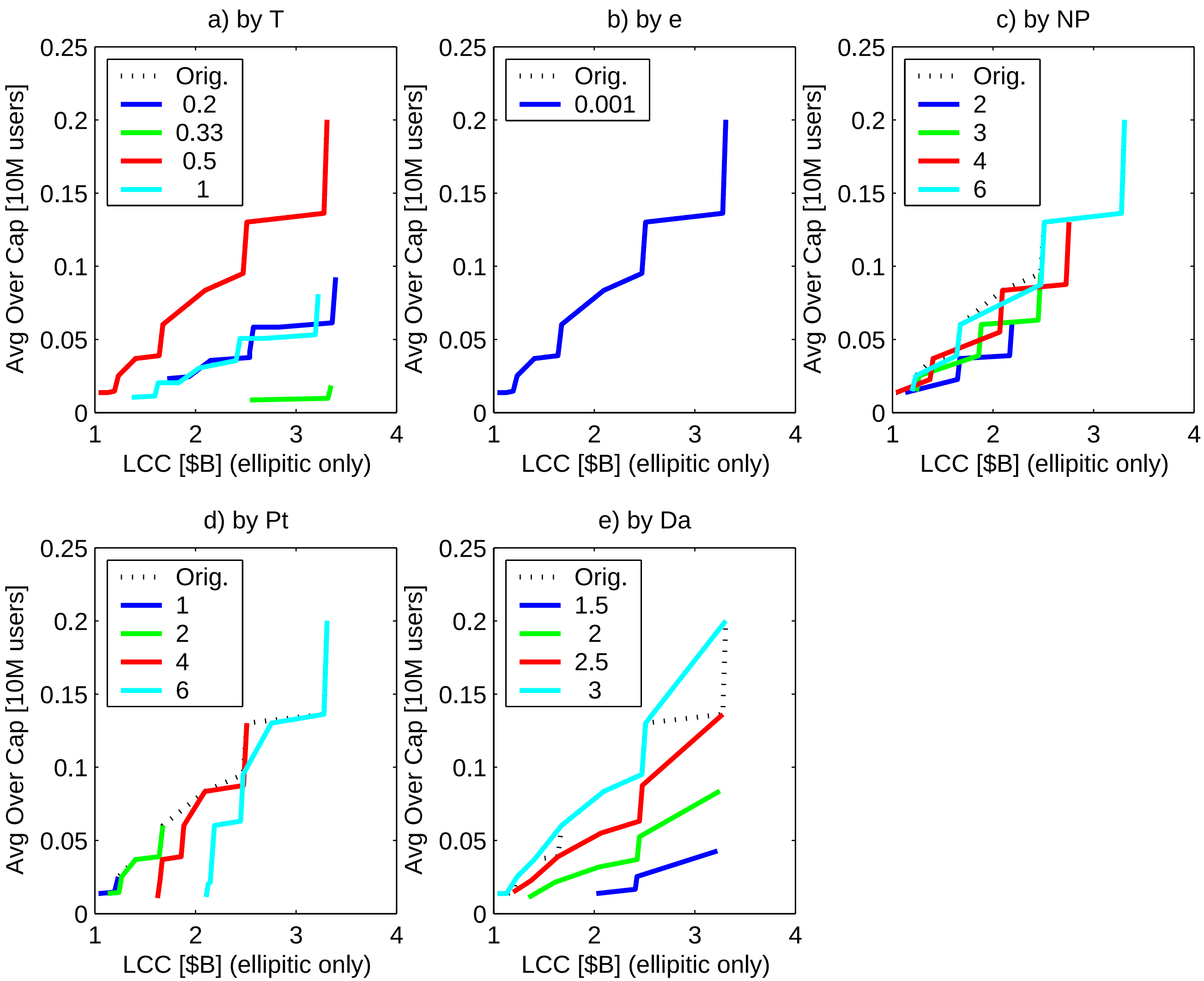
# Some Useful Visualizations

- Convex Hulls
  - Smallest convex polygon that contains all points in the tradespace that have a design variable at a particular value
  - Determines regions that are 'closed off' when a design choice is made
- Conditional Pareto Fronts
  - Pareto optimal set of points given that a particular design choice has been made
  - When compared to the unconditioned front, can determine key characteristics of designs on sections of the Pareto front

# Convex Hulls



# Conditional Pareto Fronts



# Conclusions and Future Work

- Historic mismatch between capacity and demand
- Hybrid constellations
  - First provide baseline service
  - Then supplement backbone to cover high demand
  - Allows for staged deployment that adjusts to an unpredictable market
- Pareto analysis
  - ½ day period, ~0 eccentricity
  - Transmitter power key to location on Pareto front
  - Number of planes, antenna gain not as important

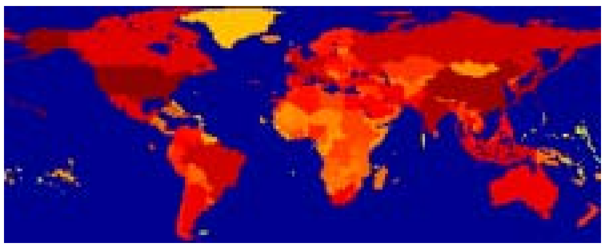
# Future Work

- Coding for radiation shielding due to van Allen belts
  - Current CER for satellite hardening is taken as 2-5% increment in cost
  - Can compute hardening needed using NASA model – need to translate hardening requirement into cost increment
- Model hand-off problem
  - Transfer of a ‘call’ from one satellite to another
  - Not addressed in current simulation
  - Key component of interconnected network satellite simulations
- Increase the fidelity of the simulation modules with less simplifying assumptions
- Increase fidelity of cost module
  - Include table of available motors for the apogee and geo transfer orbit kick motors

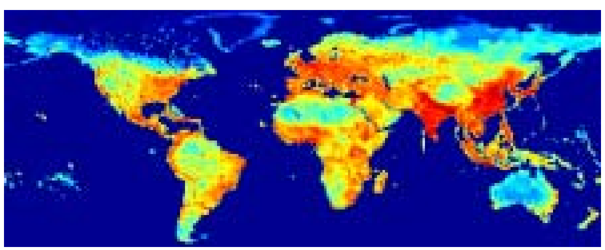
# Backup Slides

# Demand Distribution Map

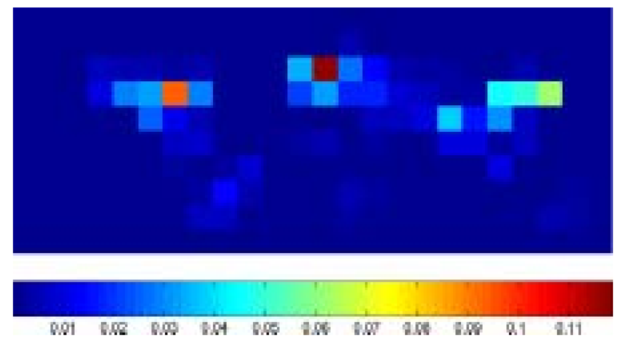
GNP-PPP



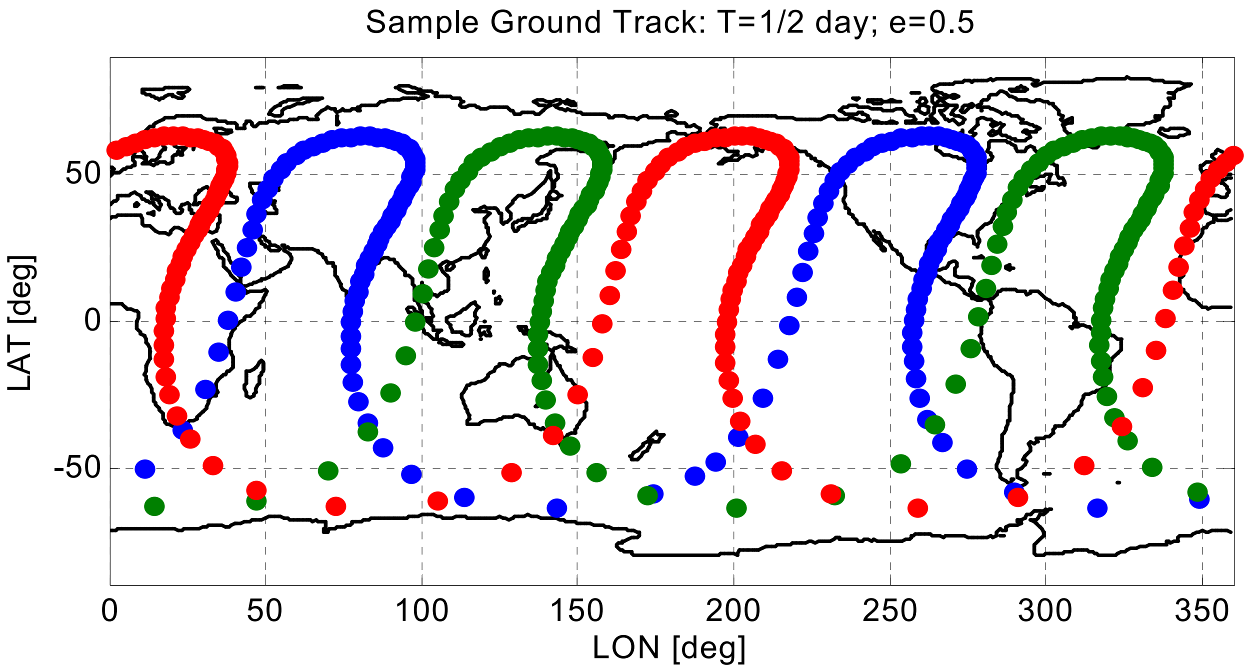
Population



Demand



# Example Ground Tracks



# Sensitivity Analysis: Design Variables

- Compute Gradient

$$\nabla J = \begin{bmatrix} \frac{\partial J}{\partial T} \\ \frac{\partial J}{\partial \varepsilon} \\ \frac{\partial J}{\partial NP} \\ \frac{\partial J}{\partial Pt} \\ \frac{\partial J}{\partial DA} \end{bmatrix} = \begin{bmatrix} -102.1317 \\ 114.5666 \\ 204.0848 \\ 0.3328 \\ 40.5873 \end{bmatrix}$$

- Normalize

$$\nabla Jnormalized = \frac{x^*}{J(x^*)} \nabla J = \begin{bmatrix} \left(\frac{0.7}{6187.8559}\right) * -102.1317 \\ \left(\frac{0}{6187.8559}\right) * 114.5666 \\ \left(\frac{4}{6187.8559}\right) * 204.0848 \\ \left(\frac{3999.7}{6187.8559}\right) * 0.3328 \\ \left(\frac{1.8}{6187.8559}\right) * 40.5873 \end{bmatrix} = \begin{bmatrix} 0.0116 \\ 0 \\ 0.1319 \\ 0.0002 \\ 0.0118 \end{bmatrix}$$

# Sensitivity Analysis: Parameters

- Basic Equation
  - Finite Differencing

$$\frac{\Delta J}{\Delta p} = \frac{J(p^o + \Delta p) - J(p^o)}{\Delta p}$$

- Data Rate
  - Step Size: 10 kbps

$$\frac{\Delta J}{\Delta p} = \frac{J(p^o + \Delta p) - J(p^o)}{\Delta p} = \frac{2003.884M\$ - 2008.7703M\$}{10} = -0.48863$$

- # Subscribers
  - Step Size: 10 users

$$\frac{\Delta J}{\Delta p} = \frac{J(p^o + \Delta p) - J(p^o)}{\Delta p} = \frac{2003.7966M\$ - 2008.7703M\$}{10} = -0.49737$$

# Simulated Annealing Tuning (I)



| Nature of Tuning Implemented | J* [$M] | x* [T, e, NP ,Pt, DA]$^T$ | Improvement from optimal SA cost of 5389 [$M]? |
|---|---|---|---|
| **1.** Geometric progression cooling schedule with a 15% decrease per iteration | $5753.4 (50 runs) | [1/7, 0.01, 2, 2918.23, 2.33]$^T$ | No, optimal cost increased by $364 million dollars |
| **2.** Geometric progression cooling schedule with a 25% decrease per iteration | $5427.9 (50 runs) | [1/7, 0.01, 3, 1581.72, 2.23]$^T$ | No, optimal cost increased by $39 million dollars |
| **3.** Stepwise reduction cooling schedule with a 25% reduction per iteration | $6278.7 (50 runs) | [1/2, 0.01, 4, 4000, 3]$^T$ | No, optimal cost and design vector remained the values they were before optimization |
| **4.** Geometric progression cooling schedule with a 15% decrease per iteration but with the added constraint that the result of each iteration has to be better than the one preceding it. | $5800.1 (41 runs) | [1/2, 0.01, 3, 3256.08, 2.17]$^T$ | No, optimal cost increased by $411 million dollars |

# Simulated Annealing Tuning (II)

| **Nature of Tuning Implemented** | **J* [$M]** | **x* $[T, e, NP, Pt, DA]^T$** | **Improvement from optimal SA cost of 5389 [$M]?** |
|---|---|---|---|
| **5.** Initial Temperature is doubled (i.e., initial temperature changed from 6278.7 [$M] to 12557.4 [$M] | $6278.7 (50 runs) | $[1/2, 0.01, 4, 4000, 3]^T$ | No, optimal cost and design vector remained the values they were before optimization |
| **6.** Initial Temperature is halved. (i.e., initial temp changed from 6278.7 [$M] to 3139.4 [$M] | $5622.7 (50 runs) | $[1/2, 0.01, 2, 3658.08, 2.3]^T$ | No, optimal cost increased by $234 million dollars |
| **7.** Initial design vector is altered such that $x_0 = [1, 0, 3, 3000, 3]^T$ | $5719.1 (50 runs) | $[1, 0, 3, 3000, 3]^T$ | No, optimal cost increased by $330 million dollars |
| **8.** Initial design vector was altered such that $x_0 = [0.25, 0.5, 5, 3000, 3]^T$ | Failed to find a feasible solution | --- | ---- |