Vehicle Suspension Optimization

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Presentation Outline

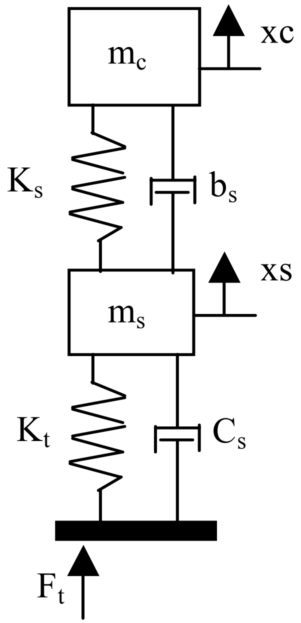
- Project introduction
- Simulation model description
- Trade space exploration
- Single objective optimization
- Sensitivity analysis
- Multi-objective optimization
- Summary
- Future work

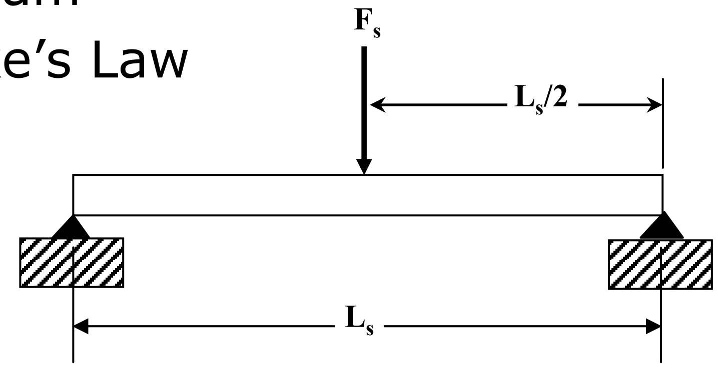
Project Introduction

- Ford F-350 rear suspension system
- Goal to maximize passenger "comfort"
- Objectives were vertical acceleration and settling time of the passenger cabin
- Modeled as 6 modules encompassing a range of disciplines

Simulation Modules

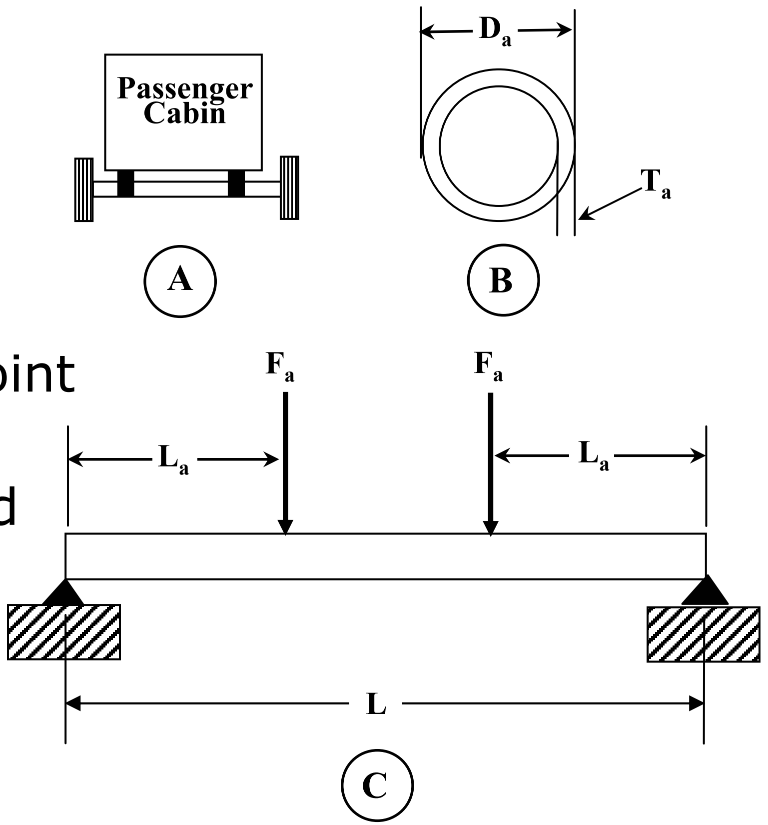
- Dynamics module
  - fourth order system spring, mass, damper system
  - road disturbance (F_t) modeled as a step input
- Leaf spring stiffness module
  - modeled as a simply supported steel beam
  - linear, obeys Hooke's Law





Simulation Modules

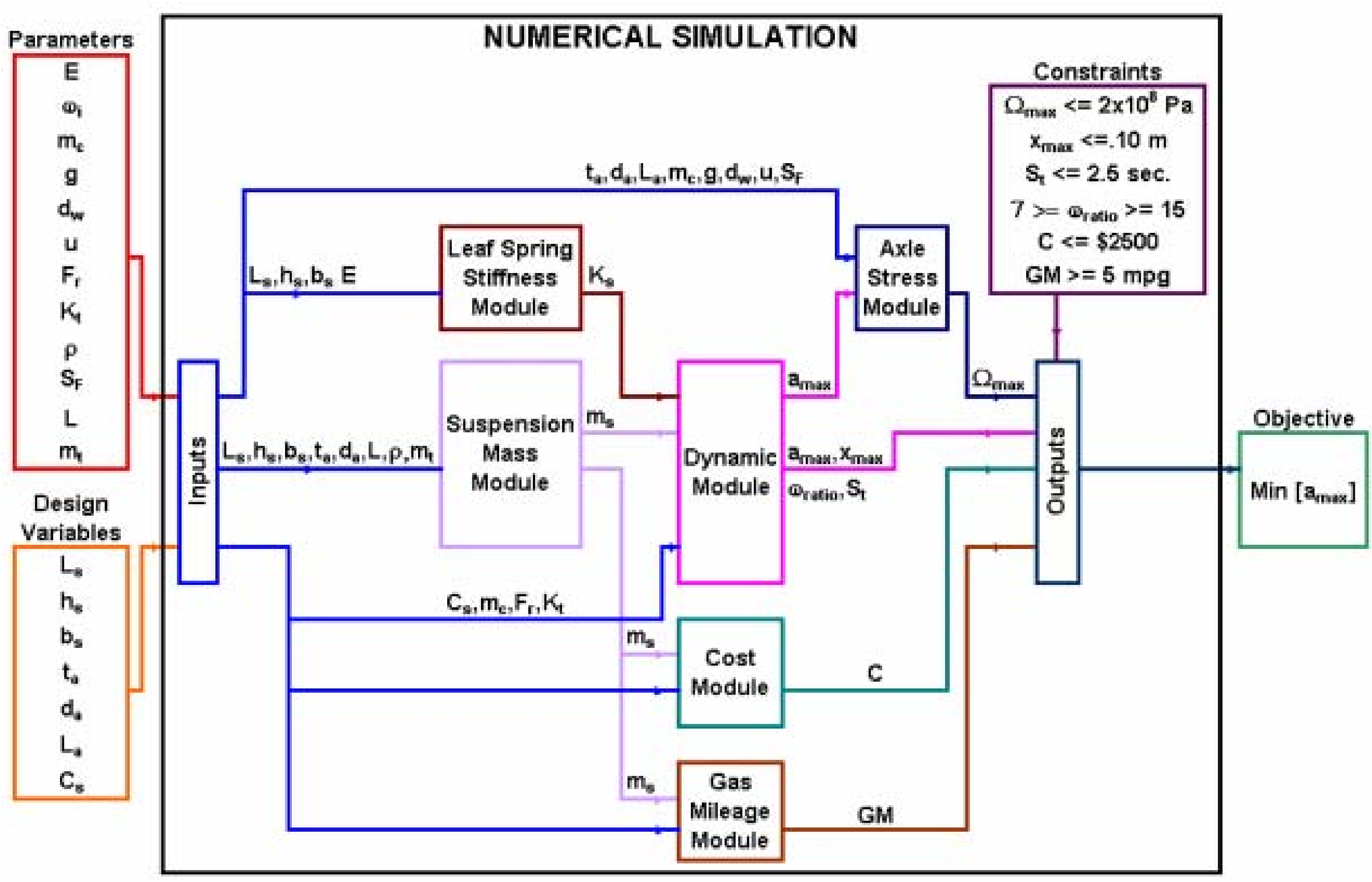
- Axle stress module
  - axle modeled as a simply supported beam
  - passenger cabin is modeled as two point loads on the axle
  - bending, shear and torsion stress are calculated



Suspension Modules

- Suspension mass module
  - mass obtained from a proprietary CAD model
  - composed of the leaf spring, shock absorber and hollow shaft
- Cost module
  - priced to compare with suspension parts available on the market
- Gas mileage module
  - heavily depends on mass of suspension

Numerical Simulation



Tradespace Exploration & Scaling

- Explored design space with L_{18} orthogonal array
- Only one point was feasible
- Scaled by computing Hessian

$$X_o = \begin{bmatrix} L_s \\ h_s \\ b_s \\ t_a \\ d_a \\ L_a \\ C_s \end{bmatrix} = \begin{bmatrix} 1.00 \\ .005 \\ .100 \\ .010 \\ .125 \\ .6 \\ 1500 \end{bmatrix}$$

Before Scaling

$$X_{os} = \begin{bmatrix} L_s \\ h_s \\ b_s \\ t_a \\ d_a \\ L_a \\ C_s \end{bmatrix} = \begin{bmatrix} 1.00 \\ 0.5 \\ 1.0 \\ 1.0 \\ 1.25 \\ .6 \\ 1.5 \end{bmatrix}$$

After Scaling

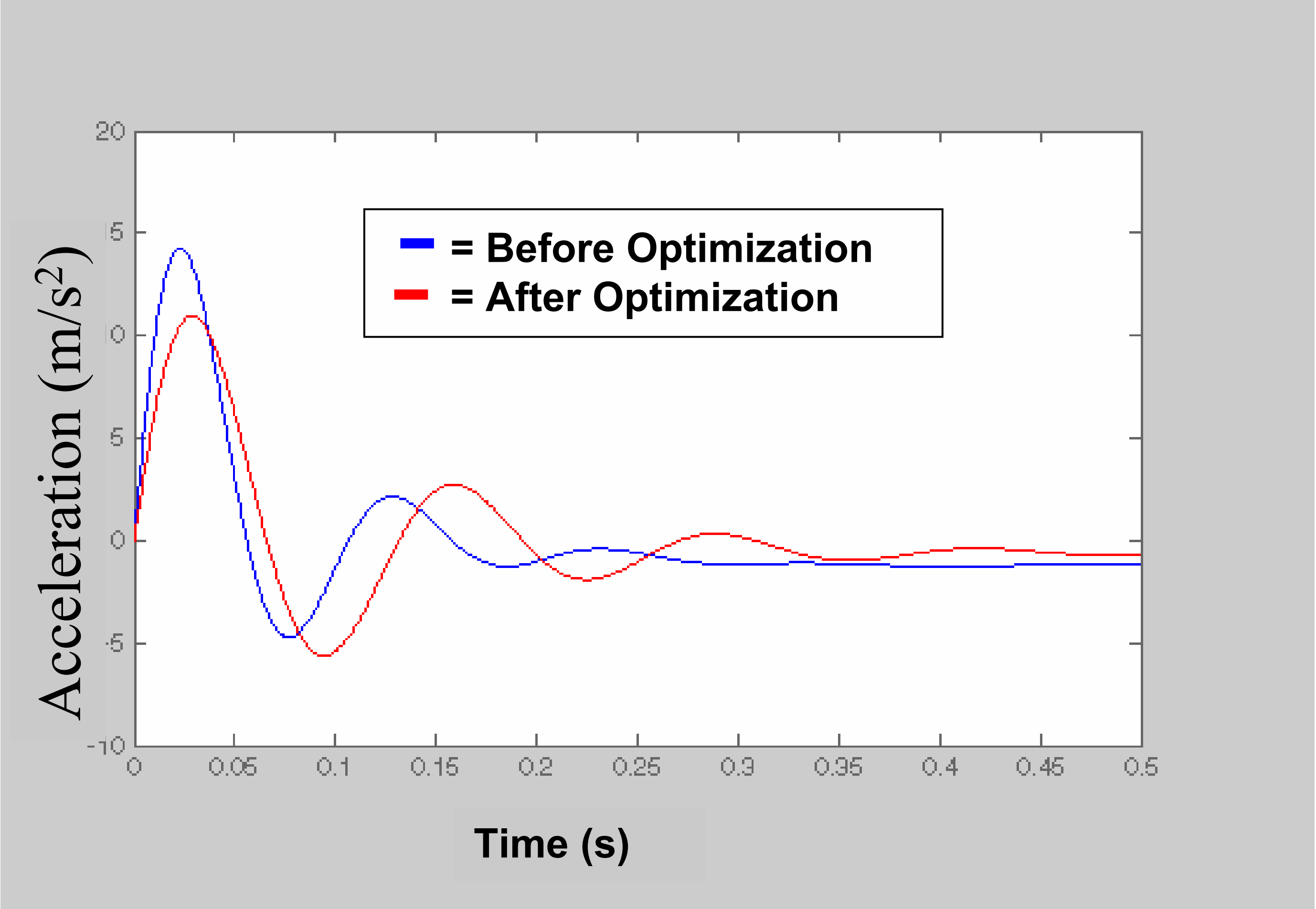
Suspension Optimization

- Gradient-based optimization
  - SQP gradient search method used on scaled and un-scaled design vectors
- Design vector before and after optimization
  - 23% decrease in a_m
  - t_a and d_a limited by bounds
  - L_a remains unchang

Run Description	a_{max} (m/s^2)	Elapsed Time (s)
Unscaled	10.954	16.174
Scaled	10.954	6.079

Scaling	L_s	h_s	b_s	t_a	d_a	L_a	Cs	$J(x)=a_{max}$
Before	1.00	0.500	1.00	1.0	1.25	0.60	1.50	**14.290**
After	1.38	0.51	1.22	1.50	1.50	0.60	1.32	**10.954**

Suspension Optimization – SQP Results



Suspension Optimization

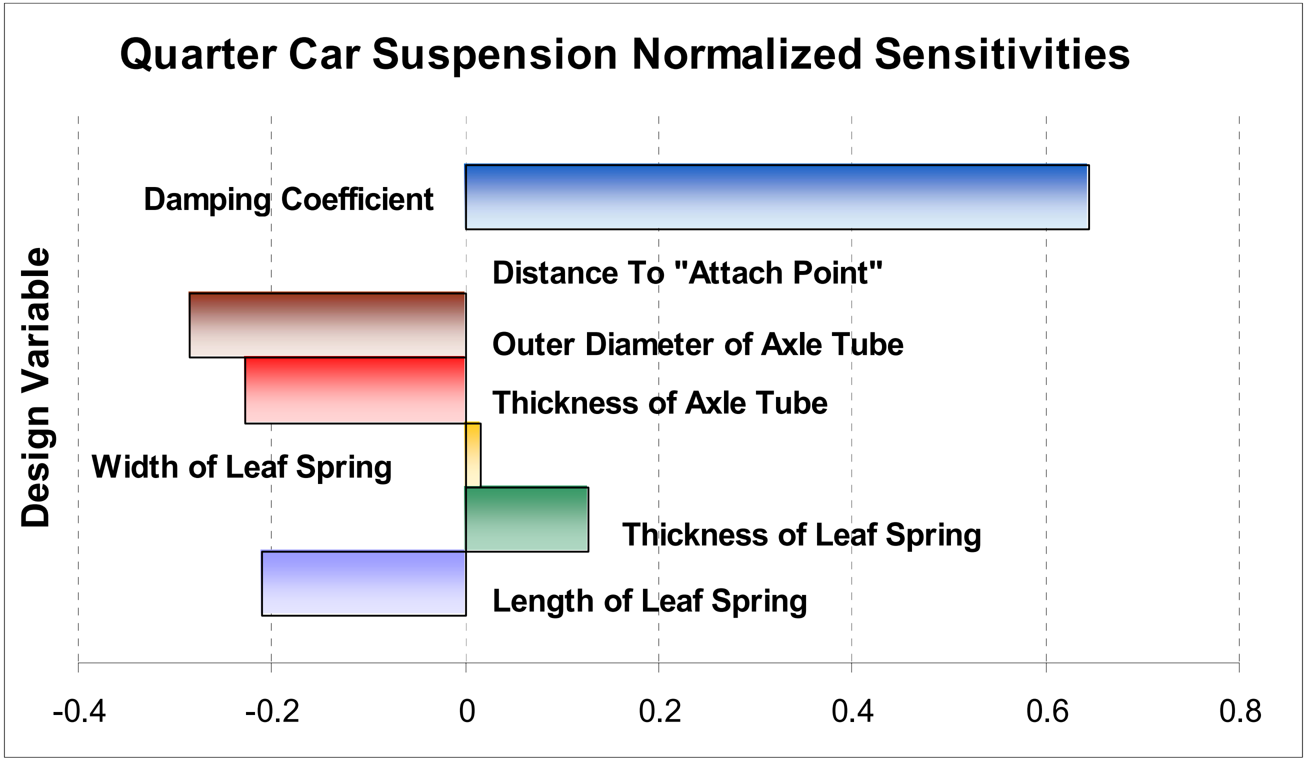
- GA and SA Used
- Provided similar results
- Great disparity in run times (70-140 times longer!)

Run Type	L_s	h_s	b_s	t_a	d_a	L_a	Cs	$J(x)=a_{max}$
SQP	1.38	0.51	1.22	1.50	1.5	0.60	1.32	**10.954**
SA	1.49	0.62	0.90	1.50	1.48	0.50	1.32	**11.040**
GA	1.47	0.61	0.93	1.50	1.50	0.57	1.32	**11.038**

Run Type	Ω_{max} (MPa)	GM (mpg)	S_t (s)	ω_{ratio}	C ($)	xc_{max} (m)	Elapsed Time (s)
SQP	58.10	5.00	2.50	15.00	2070.00	0.092	**6**
SA	50.21	5.23	2.49	14.80	2067.16	0.086	**420**
GA	56.12	5.08	2.49	14.55	2069.19	0.083	**840**

Sensitivity Analysis

- Sensitivity of objective function to design variables



Sensitivity Analysis

- Sensitivity of design variables with respect to parameters
  - looked at: (i) material density, (ii) mass of car, (iii) tire stiffness

	Design Variables						
Parameter	$\Delta L_s/\Delta p$	$\Delta b_s/\Delta p$	$\Delta h_s/\Delta p$	$\Delta t_a/\Delta p$	$\Delta d_a/\Delta p$	$\Delta L_a/\Delta p$	$\Delta C_s/\Delta p$
ρ (Material Density)	-6.20E-04	-2.40E-06	-1.00E-06	-2.00E-07	-1.00E-05	0.0	0.0
m_c (Mass of Car)	2.00E-04	6.00E-07	0.0	0.0	0.0	0.0	7.80E-04
K_t (Tire Stiffness)	1.50E-06	6.00E-09	0.0	0.0	0.0	0.0	0.0

Sensitivity Analysis

- Sensitivity of X* with respect to parameters
- 3 active constraints at X*
- Changing max S_t from 2.5 to 5.0 changes J(X*) from **10.95 m/s² to 6.87 m/s²**

Constraint	λ
GM (gas mileage)	1.223
S_t (settling time)	7.069
ω_{ratio} (natural frequency ratio)	1.551

Multi-Objective Optimization

- Passenger comfort also depends on settling time, $\mathbf{S_t}$

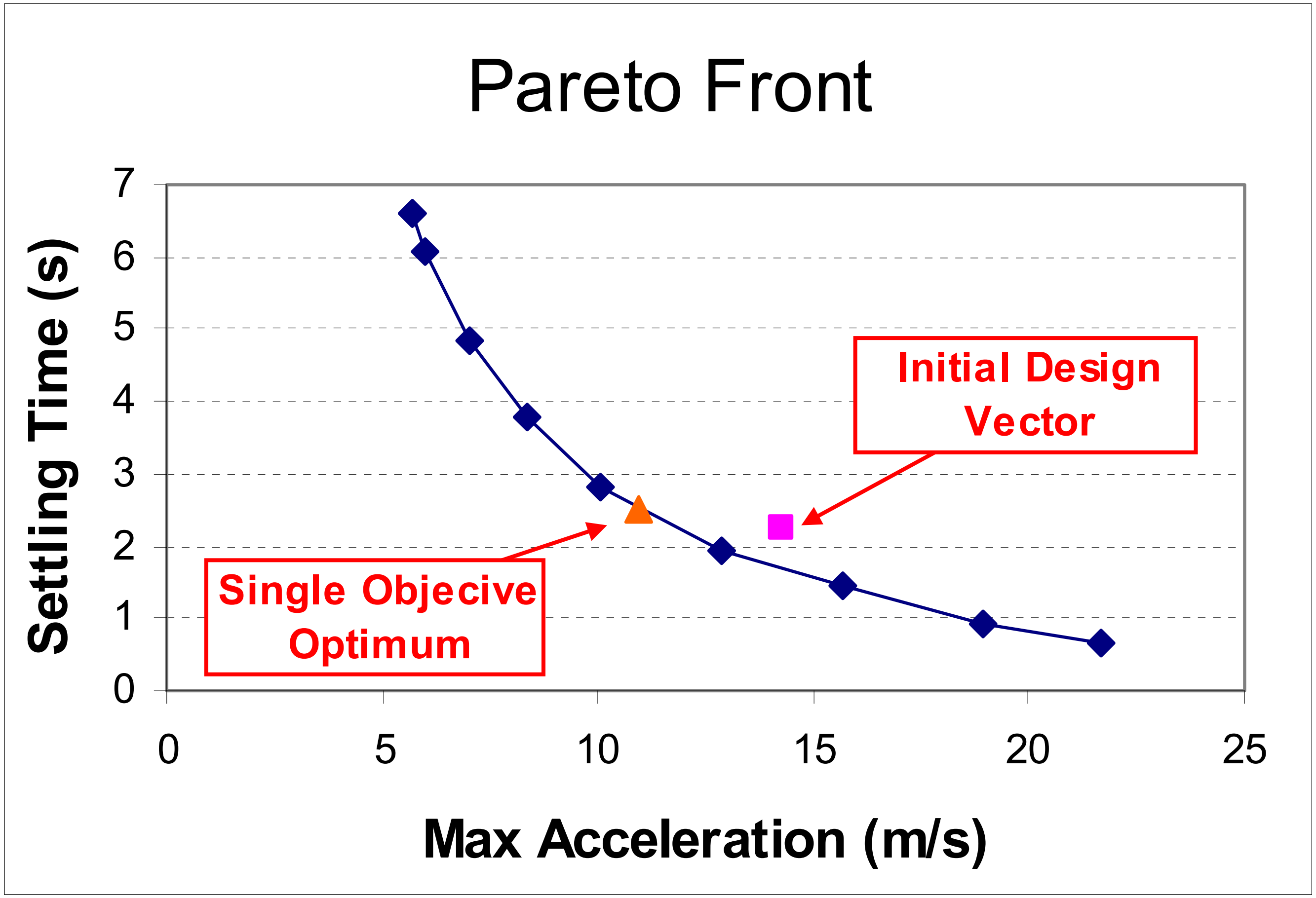
$$\min\left[J_i(x)\right]$$

$$where\, J_i(x) = (1 - w_i) * (a_{\max}) + w_i * (S_t)$$

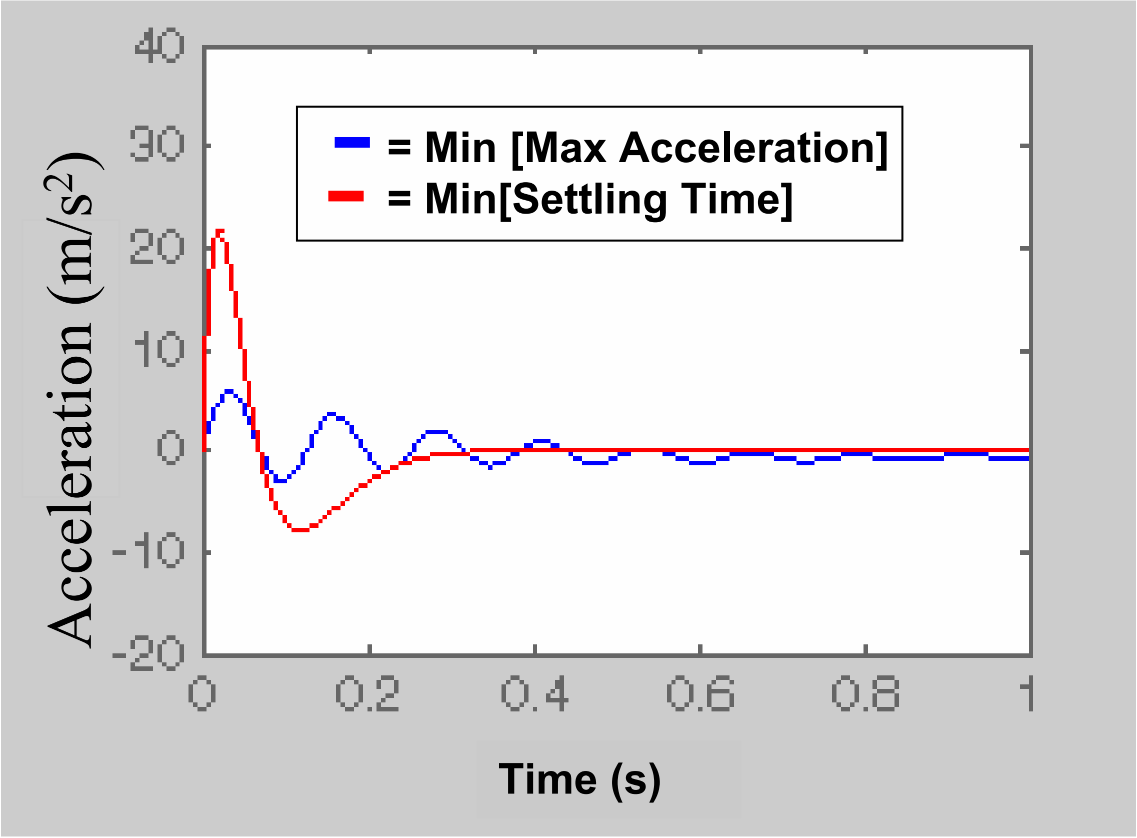
$$0 < w_i < 1$$

$$w_{i+1} - w_i = 0.1$$

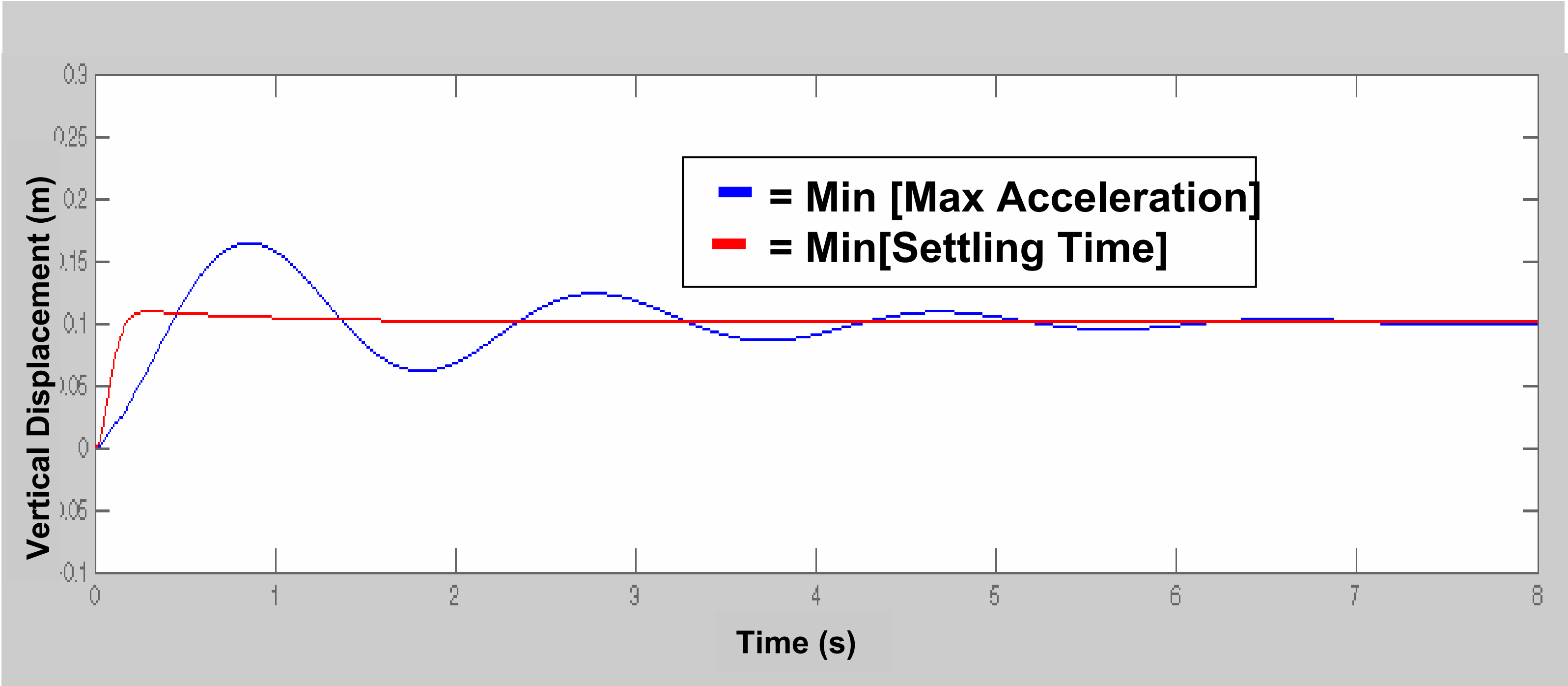
Multi-Objective Optimization



Multi-Objective Optimization



Multi-Objective Optimization



Summary

- Simulation models representative but simple
- Orthogonal array found a good starting point
- SQP performed much better than both heuristic optimization algorithms used
- a_{max} most sensitive to the damping coefficient
- Constraints such as S_t, bounds, very important
- S_t and a_{max} are competing objectives both related to passenger comfort

Future Work

- Increase the fidelity and accuracy of the modules
- Look at multi-objective problems across different disciplines (ex. cost or weight vs. comfort)
- Integration of simulation within the DOME framework

References

- **[1] Hyniova, Katerina, A. Stribrsky, and J. Honcu, “Fuzzy Control of Mechanical Vibrating Systems”, *Proceedings of International Carpathian Control Conference*, 1993.**
- **[2] Norton, Robert L., *Machine Design: An Integrated Approach*, New Jersey, Prentice-Hall, 2000.**
- **[3] Wong, Jo Yung, *Theory of Ground Vehicles,* New York, John Wiley, 2001.**

Questions???

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Appendix-Orthogonal Array

	Factors							
Factor Levels	Exp. No.	L_s	h_s	h_s	t_a	d_a	La	C_s
	1	1	0.005	0.05	0.003	0.075	0.5	500
	2	1	0.005	0.76	0.007	0.103	0.6	750
	3	1	0.005	0.1	0.01	0.125	0.7	1000
	4	1	0.009	0.05	0.003	0.103	0.6	1000
	5	1	0.009	0.076	0.007	0.125	0.7	500
	6	1	0.009	0.1	0.01	0.075	0.5	750
	7	1	0.015	0.05	0.007	0.075	0.7	750
	8	1	0.015	0.076	0.01	0.103	0.5	1000
	9	1	0.015	0.1	0.003	0.125	0.6	500
	10	1.25	0.005	0.05	0.01	0.125	0.6	750
	11	1.25	0.005	0.076	0.003	0.075	0.7	1000
	12	1.25	0.005	0.1	0.007	0.103	0.5	500
	13	1.25	0.009	0.05	0.007	0.125	0.5	1000
	14	1.25	0.009	0.076	0.01	0.075	0.6	500
	15	1.25	0.009	0.1	0.003	0.103	0.7	500
	16	1.25	0.015	0.05	0.01	0.103	0.7	750
	17	1.25	0.015	0.076	0.003	0.125	0.5	750
	18	1.25	0.015	0.1	0.007	0.075	0.6	1000

Appendix-Orthogonal Array Results

	Contraints						
Exp. No.	**a_{max}**	**Ω_{max} <= 2x10^8**	**x_{max} <=.10**	**S_t <= 2.5**	**7 >= ω_{ratio} >= 15**	**C <= 2500**	**GM >= 5**
1	8.2437	726	0.0642	6.584	26.6723	2400	13.741
2	20.274	344	0.0935	4.4252	5.9585	1571.1	2.277
3	11.345	139	0.0565	3.3433	13.3952	1897.8	6.7949
4	16.034	614	0.0746	3.5345	11.7878	2324.6	12.6983
5	15.145	218.32	0.0908	7.3653	8.2948	2025.9	8.5669
6	18.221	407	0.0892	5.0845	7.9984	2075.7	9.2562
7	19.989	809	0.0908	5.214	8.1531	2154.9	10.3532
8	24.113	247	0.0934	4.2343	6.4628	1848.5	6.114
9	29.034	614	0.0981	8.9994	7.1143	2049.5	8.8932
10	8.5092	104.55	0.042	4.3556	25.7039	1929.7	7.2365
11	13.288	1240	0.0401	3.2805	28.8288	2361.6	13.2112
12	7.2964	166.81	0.0646	6.5866	20.6171	2111.7	9.7541
13	11.709	135.98	0.0539	3.3296	15.372	2027.4	8.5881
14	10.174	205.14	0.0686	4.5009	12.0323	1916.5	7.0537
15	9.866	547.5	0.0806	6.8347	12.6016	2171.3	10.5788
16	14.854	252.25	0.0849	4.8564	7.9553	1899.6	6.8205
17	18.608	377.35	0.0896	5.1149	7.937	2076.6	9.2817
18	21.29	725.42	0.09	4.0129	6.9451	1931.5	7.2613

Appendix-SA Start Parameters

Run #	# of Designs	# of Convergence Checks	$\Delta\varepsilon$	Rate of parameter annealing	Rate of cost annealing	Rate of parameter quenching	Rate of cost quenching
SA 1	1000	5	1.00E-05	1	1	1	1
SA 2	1000	5	**1.00E-02**	1	1	1	1
SA 3	1000	5	1.00E-05	1	1	1	5
SA 4	1000	5	1.00E-05	1	0.1	1	1

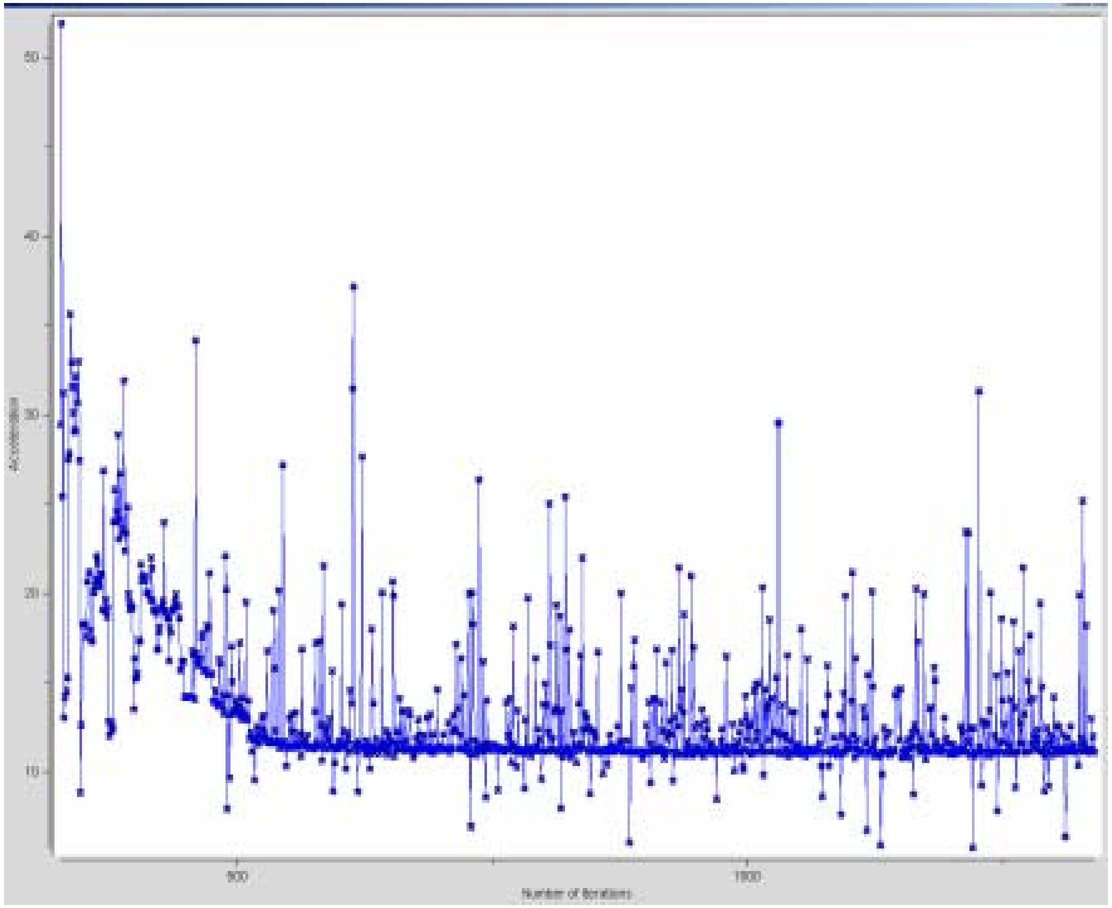
Appendix-SA Run Results

Run #	Ls	hs	bs	ta	Da	La	Cs	J(x)
SA 1	1.498	0.701	0.640	1.495	1.49	0.580	1.319	**11.123**
SA 2	1.493	0.620	0.898	1.497	1.484	0.500	1.32	**11.040**
SA 3	1.347	0.682	0.562	1.471	1.4954	0.635	1.321	**11.313**
SA 4	1.279	0.655	0.741	1.348	1.3541	0.618	1.333	**12.119**
Gradient	1.380	0.511	1.216	1.500	1.5	0.600	1.317	**10.954**
GA	1.474	0.610	0.933	1.500	1.500	0.574	1.322	**11.038**

Appendix-SA Run Results

Run #	Max Stress (Mpa)	Mileage (mpg)	Settling Time (seconds)	Wn Ratio	Cost (Dollars)	Max Displacement (meters)	Elapsed Time (seconds)
SA 1	58.60	5.56	2.490	14.759	2062.89	0.0831	**390**
SA 2	50.21	5.23	2.490	14.803	2067.16	0.0863	**420**
SA 3	63.79	6.07	2.499	14.291	2055.03	0.0781	**120**
SA 4	85.90	8.15	2.492	13.118	2027.00	0.0381	**50**
Gradient	58.10	5.00	2.500	15.000	2070.00	0.0917	**6**
GA	56.12	5.08	2.494	14.550	2069.19	0.0826	**840**

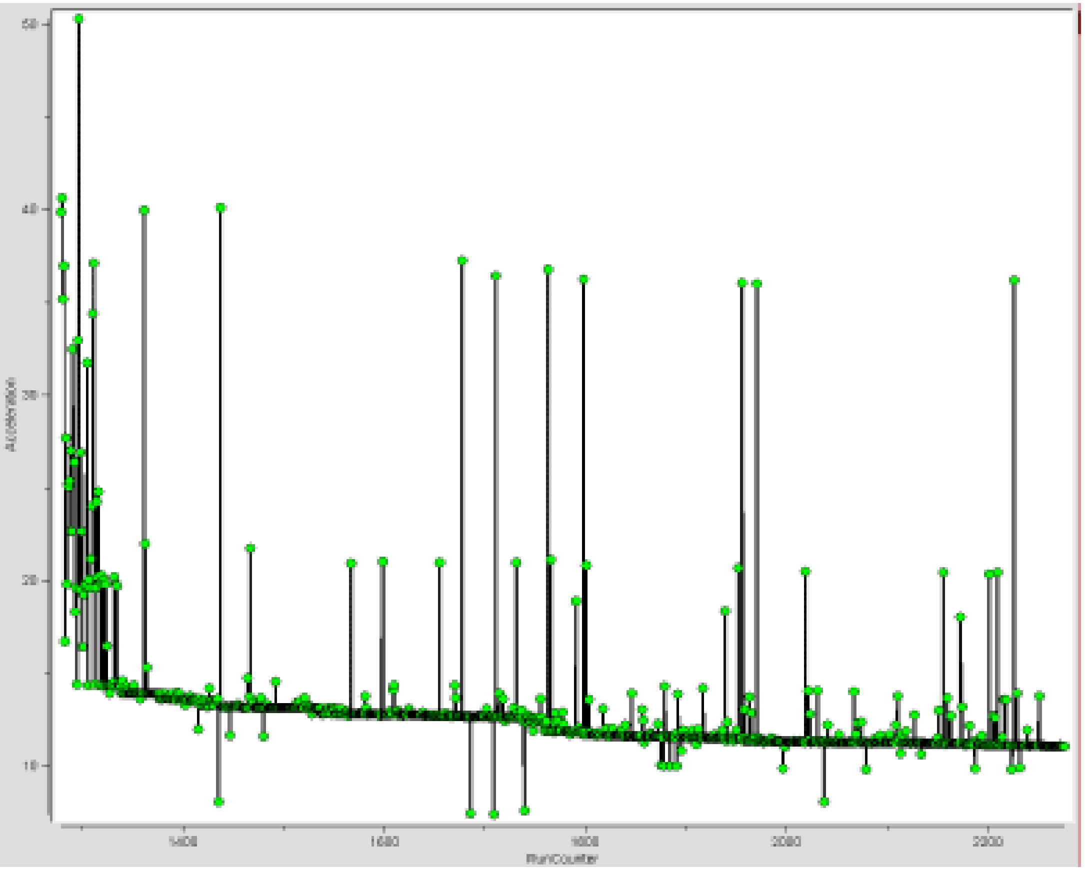
Appendix-SA History



Appendix-GA Start Parameters

GA Algorithm Parameters	Values
Subpopulation Size	20
Number of Populations	1
Generations	50
Gene Bits	32
Crossover	1
Rate of Mutation	.01
Rate of Migration	.5
Interval of MIgration	5
Elite Size	1
Relative Tournament Size	.5

Appendix-GA History



Appendix-Multiobjective Results

Weighting Factor	MaxAxcel	Settling Time	Ls	hs	bs	ta	da	La	**Cs**
0.01	5.6963	6.5832	1.496	0.551	1.234	1.500	1.463	0.500	**0.500**
0.1	5.6963	6.5832	1.495	0.551	1.236	1.500	1.463	0.500	**0.500**
0.2	5.6963	6.5832	1.494	0.550	1.239	1.490	1.470	0.500	**0.500**
0.3	5.6963	6.5832	1.495	0.550	1.240	1.485	1.475	0.500	**0.500**
0.4	6.0051	6.0885	1.485	0.552	1.202	1.486	1.482	0.500	**0.541**
0.5	7.0341	4.8315	1.463	0.549	1.168	1.490	1.489	0.500	**0.681**
0.6	8.335	3.7711	1.407	0.532	1.145	1.500	1.500	0.500	**0.873**
0.7	10.0878	2.8375	1.405	0.531	1.149	1.500	1.500	0.500	**1.160**
0.8	12.8668	1.9311	1.434	0.545	1.125	1.496	1.498	0.500	**1.705**
0.85	15.651	1.4508	1.499	0.567	1.145	1.494	1.478	0.500	**2.269**
0.9	18.9568	0.9241	1.500	0.564	1.163	1.463	1.500	0.500	**3.562**
0.99	21.6838	0.6583	1.453	0.565	1.051	1.498	1.500	0.500	**5.000**