Multidisciplinary Structural Optimization Considering Uncertainty

William Nadir[*]
Massachusetts Institute of Technology, Cambridge, MA 02139

This paper proposes structural optimization considering uncertainty. Loading and inclination uncertainties are considered in this project. The methodology of structural optimization considering uncertainty results in structural designs which perform adequately when exposed to a range of uncertain loading and inclination conditions. The performance metrics used in this study are structural assembly time and maximum stress. The improvement of these metrics can be achieved by size optimization for truss structure elements exposed to uncertain conditions. The advantages and disadvantages of design for uncertainty are discussed.

Nomenclature

C	=	AWJ cutting speed estimation constant
C_{man}	=	Total manufacturing cost, [$]
C_{mat}	=	Material cost, [$]
$C_{machining}$	=	Machining cost, [$]
d_m	=	Mixing tube diameter of the AWJ cutting machine, [in]
d_o	=	AWJ cutter orifice diameter, [in]
E	=	AWJ cutter error limit
f	=	Objective function
f_a	=	Abrasive factor for abrasive used in AWJ cutter
h	=	Thickness of material machined by AWJ, [in]
m	=	Truss element mass, [kg]
M_a	=	AWJ abrasive flow rate, [lb/min]
n	=	Number of truss structure elements
N_m	=	Machinability number
OC	=	Overhead cost for machine shop, [$/hr]
P_w	=	AWJ water pressure, [ksi]
q	=	AWJ cutting quality
$t_{assembly}$	=	Structural assembly time, [min]
t_{joint}	=	Structural joint assembly time, [min]
$t_{machine}$	=	AWJ total machining time, [min]
$t_{transport}$	=	Truss element transport time, [min]
t_{unload}	=	Truss element unload time, [min]
u_{max}	=	AWJ maximum linear cutting speed approximation, [in/min]
$u_{transport}$	=	Truss element transport speed, [in/min]
x_i	=	Cross-sectional area of i^{th} truss element, [cm^2]
x^*	=	"Optimal" design vector
X	=	Design vector of truss element cross-sectional areas
x_{LB}	=	Lower bound of design variable side constraint
x_{UB}	=	Upper bound of design variable side constraint
α	=	Objective function weighting factor
δ_y	=	Vertical nodal deflection [cm]
σ	=	Stress [MPa]

Introduction

Typically, structural design optimization is performed by considering simply the structural performance of the design in the optimization process for one set of requirements. Conventional structural performance metrics considered are stress, mass, deformation, or natural frequencies. Another important aspect to be considered in structural optimization is uncertainty. Robust design or reliability-based design tries to make the system insensitive to uncertainties. In this work, a design optimization framework is proposed that deals with uncertainties. The goal is to design a structure such that it can deal with uncertainties well. The incorporation of the consideration of uncertainty into structural design can also lead to significant benefits such as reduced assembly time, reduced manufacturing cost, and reduced maximum stress.

An overview depicting the procedure used to produce an optimal structural design considering uncertainty introduced in this paper is shown in Figure 1. This illustrative example is of a bridge subject to two types of uncertainties: loading uncertainty and inclination uncertainty. The solution to be obtained is a single structural design that can perform well for all considered uncertainty cases.

[*] S.M. Candidate, Department of Aeronautics and Astronautics, AIAA Student Member.

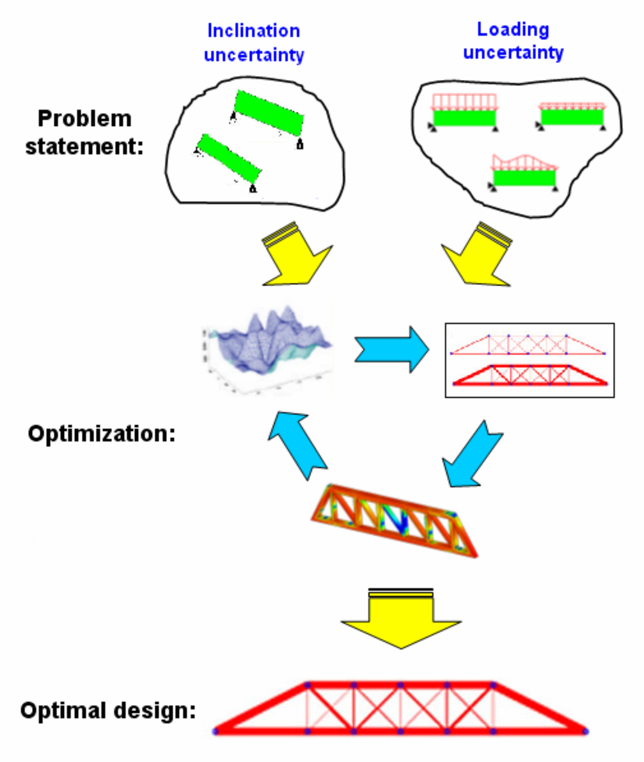

Figure 1 Optimization procedure

Uncertainty Definition

Uncertainty is defined in this project to be various loading and inclination conditions. Examples of these uncertainties are shown in Figure 1.

The motivation for considering uncertainty in the structural design process is to allow for the design of a structure which can accommodate more than one uncertainty scenario. Rather than design a different bridge to handle each potential uncertainty condition, a structure designed considering uncertainty could potentially be used for several different applications which have different loading and inclination conditions.

In the example illustrated in Figure 1, the goal is to design a bridge truss structure while considering several uncertainty conditions. We consider two important metrics to represent the performance of the structural design: structural assembly time and maximum stress. The single performance metric being minimized for each structure is a weighted sum of the two performance metrics previously mentioned. The end result of this design optimization is a set of "optimal" bridge structural designs that perform well for all uncertainty cases considered.

Assembly time is chosen to be one of the main performance metrics because the bridge design in this example is assumed to be for military use. The military is primarily concerned about protecting the lives of its soldiers and successfully defeating enemies in battle. These goals of the military are aided by designing military structures that can be assembled on the battlefield as quickly as possible. The faster the assembly time, the less vulnerable the soldiers are to the enemy. This time savings may also improve the fighting effectiveness of the troops. Although manufacturing cost is important, the military is willing to accept higher costs in order to save the lives of troops as well as improve the fighting effectiveness of the military. Therefore, manufacturing cost is considered in the problem as a constraint. This is discussed in the Problem Statement section.

An example of a military bridge designed considering uncertainty is the Medium Girder Bridge, used by the militaries of many countries around the world. This bridge can be assembled quickly using manual labor for various spans and various vehicle sizes from a jeep to a tank. A picture of this bridge design is shown in Figure 2.

Figure 2 Medium Girder Bridge (Swiss Army)

Multidisciplinary Problem

This problem deals with several disciplines. The first discipline used in this problem is structures. This discipline is used in a finite element analysis module.

Two more disciplines are used in this problem. These disciplines are from industrial engineering. One discipline is cost modeling and the other is assembly time estimation. While no specific governing equations exist for these industrial engineering disciplines, these are distinct disciplines nonetheless.

Problem Statement

In this section, a multidisciplinary, multiobjective optimization (MO) problem statement is presented. This is followed by a description of the optimization methods used to facilitate the solution of the MO problem.

$$\min(\max[f_{[1]}(X)\ f_{[2]}(X) \cdots f_{[n_{unc}]}(X)]) \tag{1}$$

where

$$f_{[i]}(X) = \alpha t_{assembly_i} + (1-\alpha)\sigma_{\max_i} \tag{2}$$

$$X \equiv \{x_1, x_2, \ldots, x_n\} \tag{3}$$

$$\alpha \in [0,1] \tag{4}$$

subject to

$$\delta_{y_{\max}} \le \delta_{y_c} \tag{4}$$

$$C_{man} \le C_{man_c} \tag{5}$$

$$\sigma_{\max} \le \sigma_{\max_c} \tag{6}$$

with

$$x_{LB} \le x_i \le x_{UB} \quad (i = 1, \ldots, n) \tag{7}$$

where $f_{[i]}(X)$ is the objective function for the i^{th} uncertainty case, X is the design vector composed of cross-sectional areas of each truss structure element, α is a weighting factor, t_{assy} is the assembly time of the structure, σ_{max} is the maximum stress in the truss structure, δ_y is the maximum vertical nodal deflection in the truss structure, C_{man} is the total estimated manufacturing cost of the structure, and x_{LB} and x_{UB} are the side constraints for the design vector variables. In addition, n_{unc} is the total number of uncertainty cases considered and n is the total number of truss elements being optimized in the truss structure.

Theory

Optimization Method

The optimal structural design for the considered loading and inclination uncertainties will be determined using an optimization approach visualized in a flow chart in Figure 3.

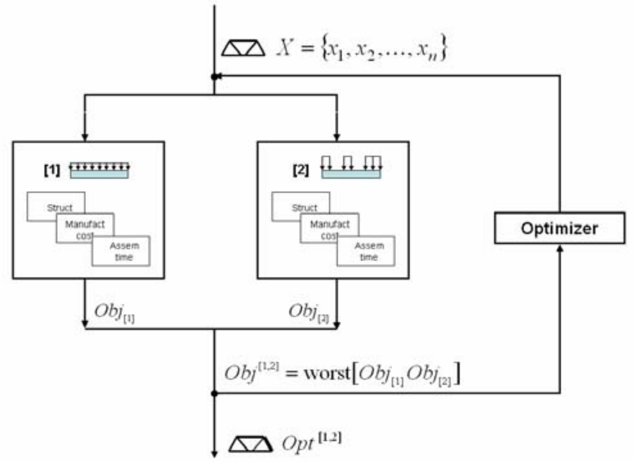

Figure 3 Design optimization considering uncertainty

A gradient-based optimization algorithm is used to perform the optimization. MATLAB function *fmincon.m*, a sequential quadratic programming-based optimizer, is used. The relative ease with which *fmincon.m* was incorporated with the system model modules, also written in MATLAB, made the algorithm a suitable choice for this problem. Another reason for the selection of a gradient-based optimization algorithm was the fact that all of the design variables are continuous.

Assembly Time Estimation

Bridge assembly time is estimated by considering unloading time, transporting time, and assembly time at each node for the entire bridge structure. These three assembly time factors are functions of mass, the number of structural elements, and the length of the structural elements. However, for any one specific design example, assembly time estimation is essentially a function of mass.

The equations used to estimate assembly time for unloading and joint assembly are shown below. In addition, the transport speed estimation equations are shown.

For truss element masses less than or equal to 3.0 kg:

$$t_{\text{unload}} = .33m_i + 10 \tag{8}$$

$$t_{\text{joint}} = .5m_i + 15 \tag{9}$$

$$u_{\text{transport}} = 1.60 \text{ m/s} \tag{10}$$

For truss element masses greater than 3.0 kg:

$$t_{\text{unload}} = .33m_i + 10 \tag{11}$$

$$t_{\text{joint}} = .5m_i + 15 \tag{12}$$

$$u_{\text{transport}} = 1.60m_i^{-.23} + .2 \tag{13}$$

where t_{unload} is the time required to unload the bridge structural members from a vehicle, t_{joint} is the time required to assemble the members together at each joint, $u_{transport}$ is the speed the members are transported from the unload point to the required location in the bridge assembly area, and m_i is the mass of truss structure element *i*.

Currently, the equations shown above are low fidelity estimates of the times required to perform the assembly tasks required for the bridge structure. The numbers used are based on real engineering work time measurement data.[1]

In order to estimate the transport time to move the structural elements from the unloading point to their respective assembly points, a transport speed is estimated for each element based on its weight and the transport distance is calculated. The figure and equations below define how this estimation is performed.

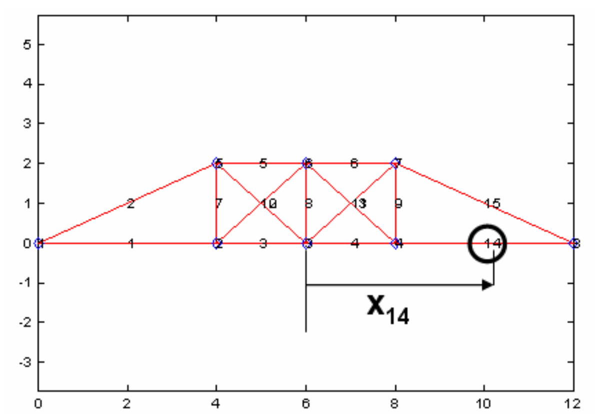

Figure 4 Transport distance example

$$t_{transport_i} = \frac{u_{\mathrm{transport}_i}}{x_i} \tag{14}$$

where x_i is the transport distance for the i^{th} bridge element.

Figure 4 illustrates how the transport distance for each element is estimated. First, the unload point is assumed to be at the center of the bridge assembly area. Next, the center of mass of the final desired location of the bridge element is located and the distance from that point to the center of the bridge is determined. Finally, Equation 14 is used to determine the transport time for each element.

The total assembly time for the bridge is determined from Equation 15

$$t_{\mathrm{assembly}} = t_{\mathrm{unload}} + t_{\mathrm{transport}} + t_{\mathrm{joint}} \tag{15}$$

Manufacturing Cost Estimation

The manufacturing method used to estimate manufacturing cost for the bridge structural components is abrasive water jet (AWJ) cutting. This manufacturing method uses a powerful jet of a mixture of water and abrasive and a sophisticated control system combined with Computer-Aided Machining (CAM) software. This allows for accurate movement of the cutting nozzle. The end result is a machined part with possible tolerances ranging from ±0.001 to ±0.005 inches. It is possible for AWJ cutting machines to cut a wide range of materials including metals, plastics, and composites.[2]

The inputs to this AWJ manufacturing cost estimation module include the design vector variables and parameters such as element lengths, cross-sectional areas, material properties, and material thickness. The output of this module is the manufacturing cost of each bridge structural element.

Based on the material thickness and material properties, a maximum cutting speed is determined for the AWJ cutter. An important assumption made in this module is that the cutting speed of the waterjet cutter is constant throughout the cutting operation. In reality, the cutting speed of waterjet will slow if any sharp corners or curves with small arc radii lie in the cutting path. A visualization for a generic truss element to be machined using the AWJ process is shown in Figure 5.

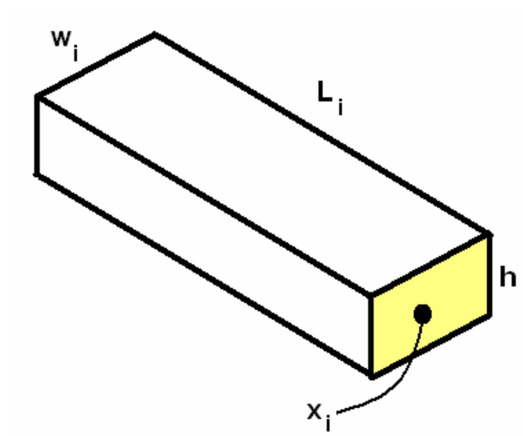

Figure 5 Example truss element to be machined using AWJ

where L_i is the length of element *i*, w_i is the width of element *i*, *h* is the user-defined material thickness, and x_i is the cross-sectional area of element *i*.

The important factors used in determining the manufacturing cost are the cutting length, P_i, the maximum linear cutting speed, u_{max}, the overhead cost associated with using the AWJ cutting machine, OC, the cross-sectional areas of each element, x_i, and the material thickness, h. The equations for the first three of these factors are detailed below in Equations 16, 17, and 18.

$$P_i = 2L_i + 2w_i \tag{16}$$

$$u_{\max} = \left(\frac{f_a N_m P_w^{1.594} d_o^{1.374} M_a^{0.343}}{C q h d_m^{0.618}} \right)^{1.15} \quad \text{[mm/min]} \tag{17}$$

$$OC = \$75 / hr \tag{18}$$

where f_a is an abrasive factor, N_m is the machinability number of the material being machined, P_w is the water pressure, d_o is the orifice diameter, M_a is the abrasive flow rate, q is the user-specified cutting quality, h is the material thickness, d_m is the mixing tube diameter, and C is a system constant that varies depending on whether metric or Imperial units are used.[3]

Total manufacturing cost is estimated using Equation 19.

$$C_{man} = \sum_{i=1}^{n_{elements}} \left(\frac{P_i}{u_{\max}} * OC \right) \tag{19}$$

In order to validate this module, a simple truss structure is created and manufacturing cost results from the cost estimation module are compared to hand calculations. The truss structure used to perform this validation is shown in Figure 6. Note that the numbers near the elements will be used to define the manufacturing cost associated with those truss elements.

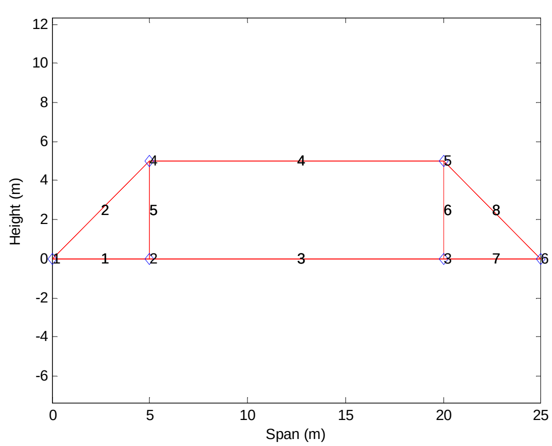

Figure 6 Truss structure design used to validate manufacturing cost module

In order to estimate the manufacturing cost, the cross-sectional areas for all of the truss structure elements are assumed to be equal to 100 cm^2, the material thickness is assumed to be 1 cm, and the structure material is selected to be A36 steel. Using these inputs, the manufacturing cost of each element was estimated using the manufacturing cost estimation module. These results are shown in Table 1.

Table 1 Manufacturing cost estimation module results

Element	Manufacturing Cost ($)
1	135.97
2	182.91
3	362.59
4	362.59
5	135.97
6	135.97
7	135.97
8	182.91
Total	*1634.90*

Compared to hand calculations, the manufacturing cost numbers in Table 1 were identical. This shows that the manufacturing cost model is performing as expected.

Structural Analysis Module

This module uses Integrated Modeling of Optical Systems (IMOS) software to perform structural analysis of the truss structure. Internal element forces, stresses, and node deflections are determined using this module. The inputs to this module include matrices defining all elements of the structure, material properties, and design variables. The design variables are the cross-sectional areas of each truss element. The outputs of the module include the internal element forces, stresses, and node deflections of the truss structure.

To validate this module, a simple truss structure is created and the results of the module are compared to hand calculations in order to determine the internal forces of the truss elements. The truss structure used to perform this validation is shown below in Figure 7 including the forces applied to the structure. Note that the numbers near the elements and nodes will be used to define the stresses and deflections in those truss components.

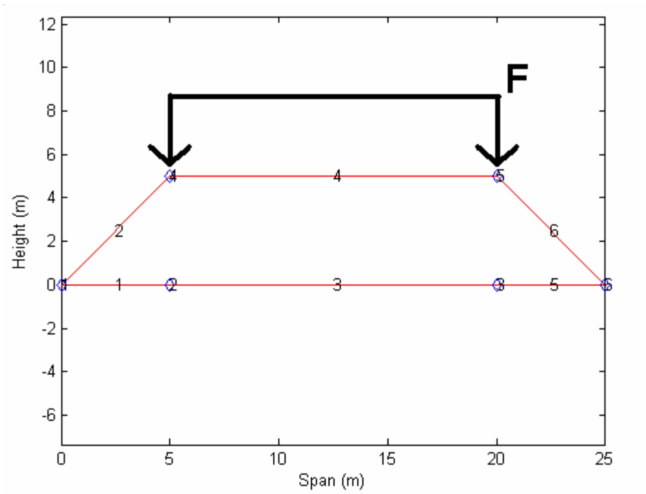

Figure 7 Structural analysis validation test case

For the validation of the module, a value of *F*, a load equally-applied to nodes 4 and 5, is chosen to be 100,000 N. The internal element forces calculated from the structural analysis module are shown below.

Table 2 Structural analysis module test results

Truss Element	Internal Force (N)
1	50000
2	-70711
3	50000
4	-50000
5	50000
6	-70711

Hand calculations were performed to validate the results shown in Table 2. The resulting internal forces from the hand calculations were identical to the values in the table above.

Results

Uncertainty Conditions Considered

This section will compare the performance of bridge designs considering uncertainty to the performances of designs in which uncertainty was not considered. Four uncertainty conditions were considered for this example. The four uncertainty conditions are illustrated in Figure 8.

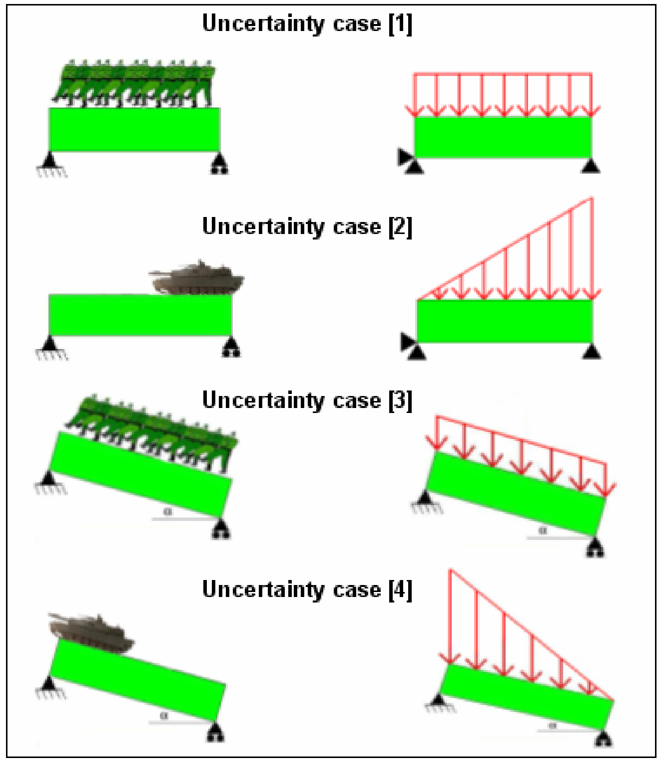

Figure 8 Uncertainty cases considered

Uncertainty case [1] is a level bridge with soldiers marching across. It is assumed that soldiers are marching in formation 50 cm apart and six columns deep. Each soldier plus gear is assumed to weigh 102 kg. Uncertainty case [2] is also a level bridge with an M1A2 tank[4] on the right-hand side of the bridge. This tank weighs approximately 63,500 kg. Uncertainty case [3] is the same as [1] except the bridge is inclined at an angle of 15 degrees. Uncertainty case [4] is similar to [2] except for a 15 degree inclination. Uncertainty conditions [2] and [4] also differ in the location of the tank on the bridge.

Design not Considering Uncertainty

Not considering uncertainty results in the creation of a "point design." For a "point design" to perform well, it is desired that the point design structure only experiences the uncertainty condition for which it has been designed. Less severe uncertainty conditions are acceptable as well. However, there is a chance that an unexpected uncertainty condition may cause a "point design" structure to fail. This idea is visualized in Figure 9.

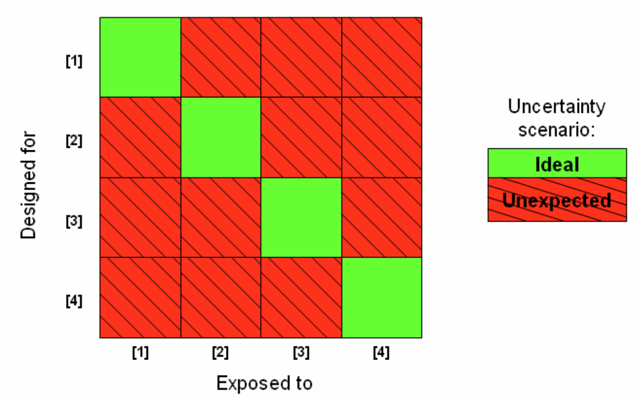

Figure 9 Problem with not considering uncertainty in design

In the above figure, an uncertainty scenario experienced by designs not considering uncertainty would need to lie along the main diagonal to ensure they are exposed only to the uncertainty conditions designed for. However, if any of these point designs experience uncertainty conditions not considered during the design process, the resulting scenario would lie off the diagonal. There is a serious risk of structural failure associated with this off-diagonal region of the grid in Figure 9. There is a chance, however, that an uncertainty condition not considered may be more benign than the uncertainty condition designed for. This would result in favorable performance for an off-diagonal region in the above figure.

The requirement that point designs remain on the main diagonal is a major drawback for this method of structural design. These designs can not cope with many unexpected loading conditions.

Results illustrating the penalty associated with structural design not considering uncertainty are shown in Figure 10.

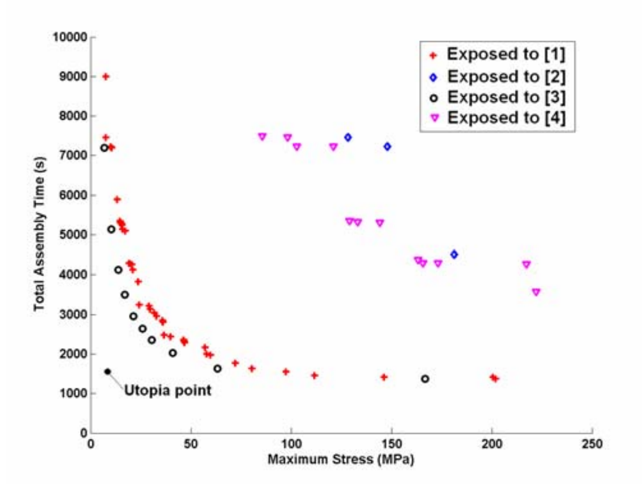

Figure 10 Results not considering uncertainty for structures designed for [3]

For the loading cases considered, case [3] is the most benign. Therefore, this point design will perform poorly when exposed to the other three unexpected loading conditions. The most severe uncertainty cases, [2] and [4], can be seen in diamonds and triangles, respectively, in the figure.

Consider Figure 10. The performance of a point design for uncertainty case [3] is shown to perform poorly for all uncertainty cases not considered during the design process. The performances of the design exposed to uncertain conditions lie farther from the utopia point than the "point design" structure for uncertainty condition [3]. In particular, the performance for the structure when exposed to unexpected uncertainty cases [2] and [4] is very poor.

Design Considering Uncertainty

Benefits and penalties result from considering all uncertainties in the design process. The main benefit is that the resulting structural design will be able to accommodate all uncertainty conditions considered in the design process. The benefit from designing for uncertainty is show in Figure 11.

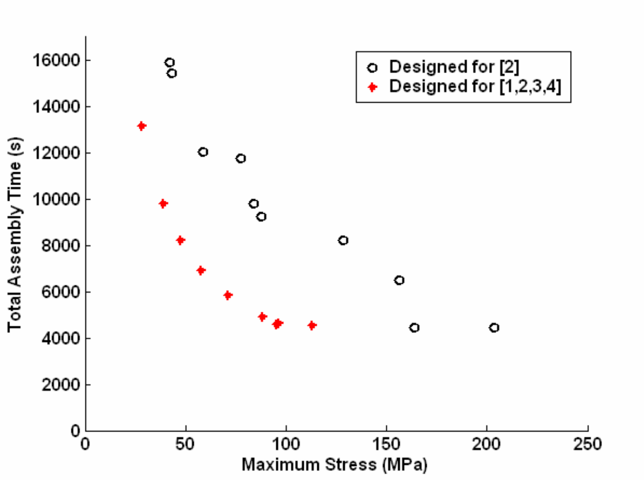

Figure 11 Result of designing for uncertainty for structures exposed to uncertainty [4]

The previous figure illustrates the performance benefit of designing for uncertainty. The performance of a structure designed considering uncertainty is shown to have better performance over a structure exposed to the same yet unexpected uncertainty condition.

A penalty associated with considering all uncertainties is the potential for the "optimal" structural design considering uncertainty to be "over-designed" for a subset of the uncertainty conditions. This will occur because the structure will need to be able to perform adequately for the worst case uncertainty condition considered. For structural design optimization, this results in a mass penalty. This penalty is illustrated in Figure 12.

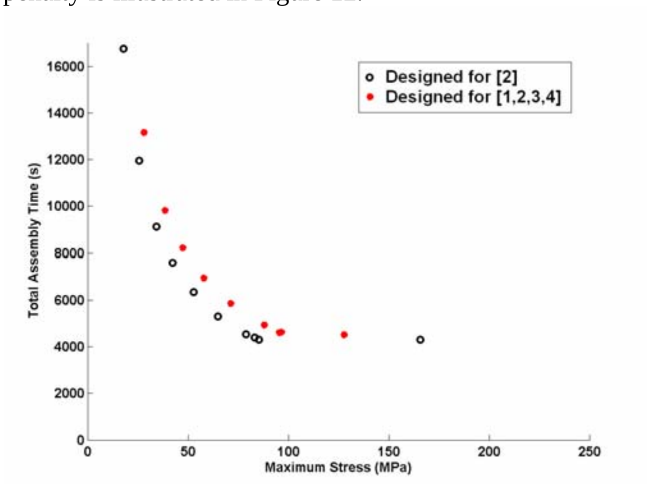

Figure 12 Result of designing for uncertainty for structures exposed to uncertainty [2]

This "over-designed" penalty occurs when the structure designed considering uncertainty is exposed to a condition more benign than the worst case uncertainty condition. When compared to the design considering uncertainty, the point design has a performance advantage over the design considering uncertainty. This performance advantage is a lower structural mass.

A visualization of the how designs resulting from optimization considering uncertainty vary across the Pareto frontier is shown in Figure 13.

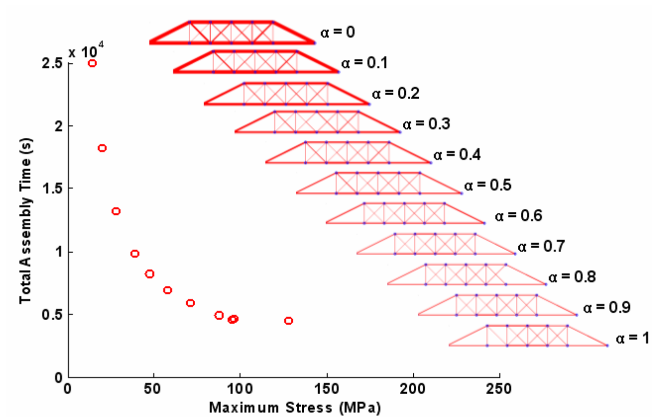

Figure 13 Structural designs considering uncertainty along Pareto frontier exposed to uncertainty [2]

The figure above shows a trend in the structural designs as the weighting factor in the objective function is increased from zero to one. At a weighting factor value of zero, only maximum stress is considered. This results in an "optimal" structural design with elements of large cross-sectional areas. However, as the weighting factor is increased, maximum stress is considered less and assembly time is weighted more heavily. As expected, the observed trend shown as assembly time is weighted more heavily is the reduction in the cross-sectional areas of the truss structure elements. This is due to the fact that assembly time is essentially a function of mass. At a weighting factor of one, the cross-sectional areas of the truss structure will be designed to be as small as possible as long as the design satisfies all the imposed constraints.

Sensitivity Analysis

Sensitivity analysis of "optimal" designs both considering and not considering uncertainty was performed. The goal was to understand how the cross-sectional areas of the truss structure individually affect the performance of the structure as a whole.

This sensitivity analysis was performed by examining the change in the value of the objective function from the perturbation of each design variable at the "optimal" design of each case considered. A forward difference finite difference method was used for sensitivity analysis.

The gradient of the objective function with respect to each design variable was determined. The gradient terms were then normalized using the value of the objective function and the value of the design variable being investigated at the "optimal" design point. This is shown in Equation 20. The equations used to determine the sensitivity of the objective function to the design variables are shown below.

$$\nabla\bar{f} = \frac{x^*}{f(x^*)}\nabla f \tag{20}$$

where x^* is the design variable at the "optimal" design point and ∇f is the gradient of the objective function. The sensitivities were normalized once again by dividing the sensitivities by the absolute value of the maximum sensitivity determined.

First, the sensitivities of the design variables was performed for a structure designed not considering uncertainty. This structure was designed for exposure only to uncertainty case [2]. The resulting "optimal" structure and sensitivities are shown below.

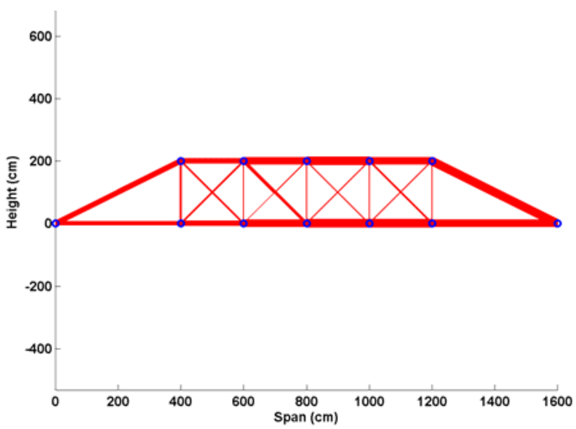

Figure 14 "Optimal" structural design considering uncertainty [2]

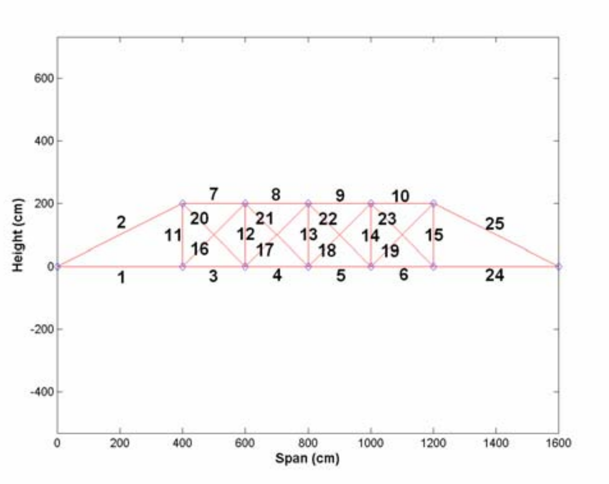

Figure 15 Truss structure element number assignment

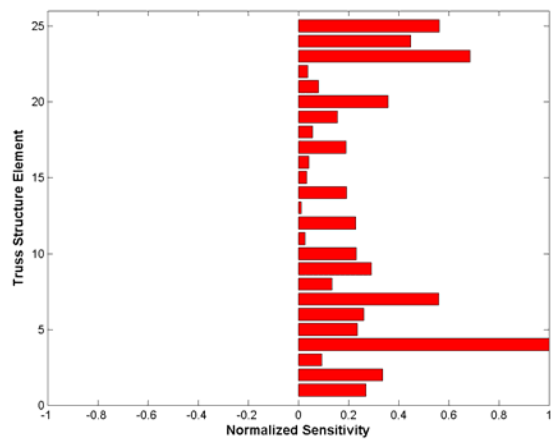

Figure 16 Normalized sensitivities (% of maximum) for truss structure element cross-sectional areas

For a "point design" considering only uncertainty [2], the resulting structure in Figure 14 and the normalized sensitivities in Figure 16 were determined. The sensitivity results make sense because first of all, all the sensitivities are positive. Since the design variables were perturbed using a forward difference method, this means that at the "optimal" design point, an increase in the cross-sectional areas of every truss element would result in a greater objective function. This means that performance would be reduced if any truss element were increased in cross-sectional areas.

Another interesting observation from the sensitivity analysis results is that the elements on the right-hand side of the structure affect the performance greater than the elements on the opposite side.

The sensitivities also make sense because the objective function is much less sensitive to the perturbation of many the internal members of truss. This is likely because many these elements of the truss structure are much less massive than the other structural members. This means the assembly time for these elements is less and the stresses of these elements may be less as well. This means that the overall objective function may not be significantly affected by a perturbation at any one of these internal structural members.

On the other hand, the structural members along the outside of the truss are the most sensitive because they experience the greatest loading.

Sensitivity analysis was also performed for a point design for uncertainty case [4].

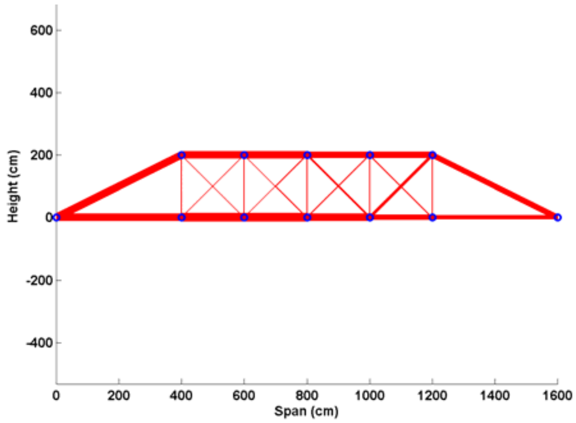

Figure 17 "Optimal" structural design considering uncertainty [4]

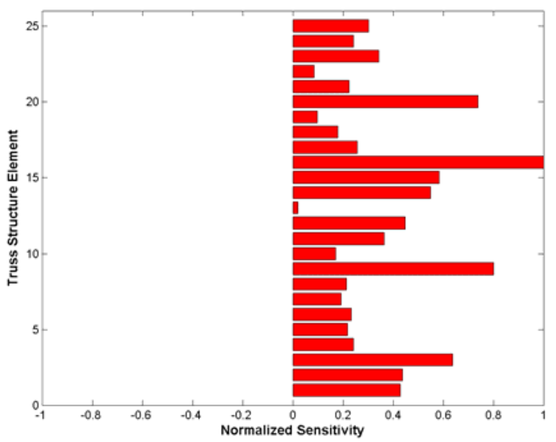

Figure 18 Normalized sensitivities (% of maximum) for truss structure element cross-sectional areas

It is interesting to observe that the objective function is most sensitive to a few internal members of the structure for this design.

A good comparison to the sensitivity analysis for the structure designed for [2] is the fact that now that the loading is concentrated on the left-hand side of the bridge structure, the elements on the left-hand side of the bridge are now more sensitive to the objective function.

This sensitivity of the objective function to the perturbation in the design variables of a structure designed considering uncertainty is also interesting to observe. This data is shown in the next two figures.

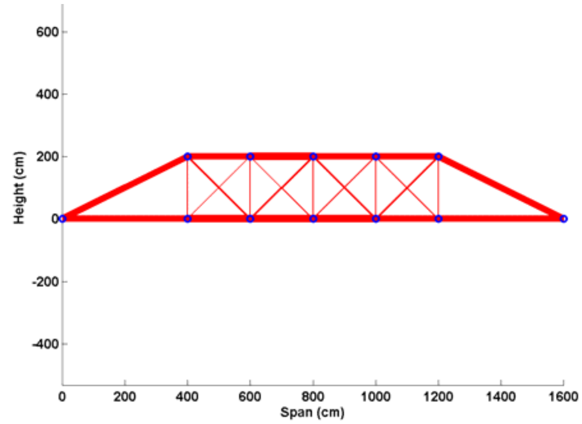

Figure 19 "Optimal" structural design considering uncertainty

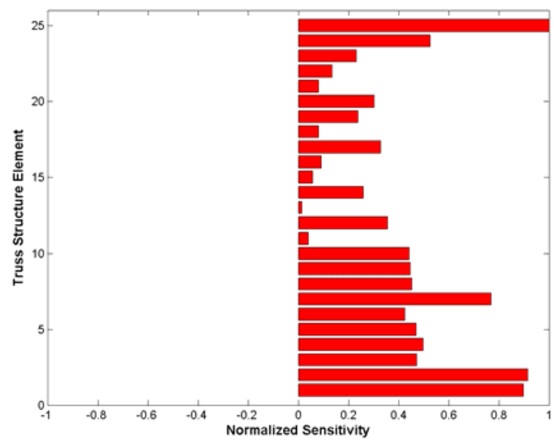

Figure 20 Normalized sensitivities (% of maximum) for truss structure element cross-sectional areas

The sensitivities shown in Figure 20 contain some similarities and differences from the results for the point designs for uncertainties [2] and [4].

First, it is interesting that the sensitivities of the elements on each end of the bridge are more even for the case considering all uncertainty conditions. This makes sense because this design has taken into account designs [2] and [4] in the design process.

A trend which is somewhat consistent with the sensitivity analysis for the point designs is the fact that the objective function is less sensitive to the internal truss members than the outer truss members. This may still be due to less loading of the internal truss elements.

There may be another explanation for the differences in sensitivities between the internal and outer truss structure elements. The outer elements tend to have the largest cross-sectional areas the largest sensitivity magnitudes. This trend is likely related to the fact that the elements with the smallest cross-sectional areas contribute less to assembly time and are likely the elements experiencing the lowest stress levels. Therefore, it is reasonable that these elements would affect the objective function less than the structural elements with larger cross-sectional areas.

Convergence History

The performance of the gradient-based optimization algorithm at converging to a solution has important implications for computation time. A visualization of the convergence history provides some insight into how the algorithm performs for this problem.

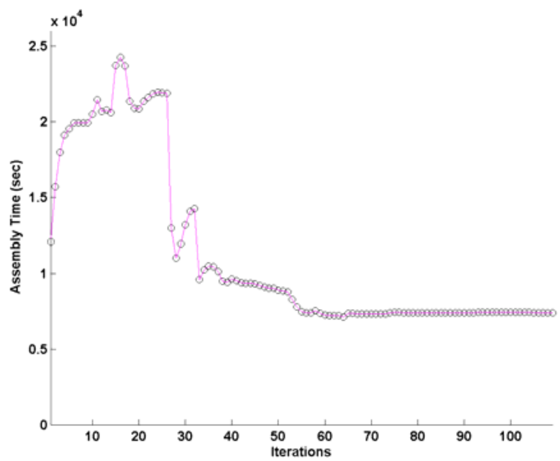

Figure 21a: Assembly time convergence history

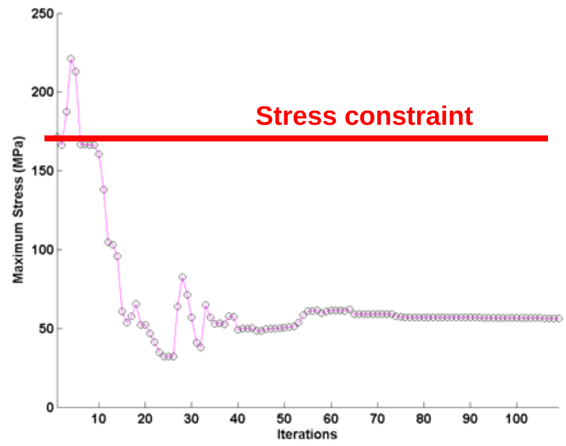

Figure 21b: Maximum stress convergence history

Figure 21 Convergence history for $\alpha = 0.5$, $x_0 = [56 ... 56]$

Figure 21, although for a specific weighting factor and initial design vector, is representative of the algorithm convergence performance for the problem in general. While the number of iterations does depend on the weighting factor and initial guess, the ability of the algorithm to converge to "good" feasible solutions most of the time is of importance. It is important to mention that the above optimization trial took 6.7 minutes and required 4472 objective function evaluations.

An important observation from Figure 21 is the ability of the algorithm to start or enter an infeasible region of the design space and still recover and converge to a "optimal" solution. In Figure 21b, the first ten iterations are on or violate the stress constraint. However, the successive iterations satisfy the constraint and are minimized to produce a desirable solution.

Summary

By performing structural design optimization considering uncertainty, performance metrics of assembly time and maximum stress have been minimized for a structure which can accommodate all uncertainty conditions considered *a priori* in the design process.

Pareto fronts have been created which are composed of a set of "optimal" designs considering uncertainty. The performances of these "optimal" designs considering have been compared to point designs and the benefits and penalties of such a design methodology have been discussed. While the structural design considering uncertainty can accommodate all loading cases considered, the "optimal" structural design considering uncertainty may be over-designed for a subset of the uncertainty cases considered.

Designing a structure for uncertainty reduces assembly time and manufacturing cost while the potential exists for incurring a mass penalty due to the *a priori* consideration of a set of uncertainty conditions.

Future work dealing with structural design optimization for flexibility will build upon what is presented in this paper. Reconfigurability, modularity, and extensibility will be considered in the structural design optimization process. An improvement over designing for uncertainty will be presented by considering flexibility in the design process.

References

[1] Karger, D. and Bayha, F., 1966, *Engineered Work Measurement: The principles, techniques, and data of Methods-time Measurement, modern Time and Motion Study, and related Applications Engineering data*, The Industrial Press, New York, NY.

[2] Olsen, C., 1994-2003, "Frequently Asked Questions," Waterjet Web Reference Website, http://www.waterjets.org/waterjet_faq.html.

[3] Zeng, J. and Kim, T., 1993, "Parameter Prediction and Cost Analysis in Abrasive Waterjet Cutting Operations," *Proceedings of the 7th American Water Jet Conference*, Seattle, Washington, August 28-31, pp. 175-189.

[4] General Dynamics Website, "Abrams Main Battle Tank," http://www.gdls.com/programs/abrams.html, General Dynamics Land Systems, 2004.