Concurrent Trajectory and Vehicle Optimization for an Orbit Transfer

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May 5, 2004

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Motivation

- Traditionally, orbit transfers are optimized with respect to a selected vehicle
- Competing objectives of initial mass and time of flight are weighted by the preference of the customer
  - High priority → minimize time of flight
  - Low priority → minimize initial mass
- Vehicle selection is as important as the trajectory
  - The choice of vehicle drastically impacts both performance and cost
  - Differing priorities would impact vehicle choice
- Evaluate both the trajectory and vehicle selection for different preferences to show ‘optimal’ orbit transfer configurations

Single Objective Optimization Problem Formulation

- Consider a co-planar orbit transfer from low Earth orbit (LEO) to geosynchronous Earth orbit (GEO) using a two-stage chemical rocket
- Minimize the initial mass of the system for a given payload mass
- Define three design variables
  - Transfer angle (ν): Angle between first and second burn
  - Specific impulse of first engine (Isp_1)
  - Specific impulse of second engine (Isp_2)
- Parameters
  - Payload mass (m_p) = 1000 kg
  - Structural factor (α) = 0.1 for both engines
  - Initial radius (r_0) = 6628 km (250 km altitude)
  - Final radius (r_f) = 42378 km (36000 km altitude)
- Define two disciplinary models
  - Orbit transfer calculation: Assumes first burn is tangent to initial orbit
    - Input: the transfer angle, and the initial and final radii
    - Output: ΔV of each burn and time of flight
  - Rocket equation: Assumes each burn is impulsive
    - Input: the ΔV of each burn, the specific impulse of each engine, the structural factor, and the payload mass
    - Output: the initial mass

Mathematical Problem Formulation

- Minimize J(x) = m_i
- Subject to the disciplinary model equations
  - Orbit transfer Equations
  - Rocket equation
- And subject to the variable bounds
  - 135 deg <=ν<=180 deg
  - 300 sec<=Isp_1,Isp_2 <=450 sec

Rocket Equation for Two stages

$$\frac{m_p}{m_i} = \Pi_{i=1}^{2}(1+\alpha_i)\exp\left(\frac{-\Delta V_i}{I_{sp_i} g_0}\right) - \alpha_i$$

Orbit Equations for One Tangent Burn

$$v_0 = \sqrt{\frac{\mu}{r_0}}$$

$$v_f = \sqrt{\frac{\mu}{r_f}}$$

$$e_T = \frac{\frac{r_0}{r_f - 1}}{\cos\nu - \frac{r_0}{r_f}}$$

$$a_t = \frac{r_0}{1 - e_T}$$

$$v_{TA} = \sqrt{\mu\left(\frac{2}{r_0} - \frac{1}{a_T}\right)}$$

$$v_{TB} = \sqrt{\mu\left(\frac{2}{r_f} - \frac{1}{a_T}\right)}$$

$$\phi_T = \tan^{-1}\left(\frac{e_T \sin\nu}{1 + e_T\cos\nu}\right)$$

$$\Delta V_A = \|v_{TA} - v_0\|$$

$$\Delta V_B = \sqrt{v_{TB}^2 + v_f^2 - v_{TB} v_f \cos\phi_T}$$

$$\Delta V_{Total} = \Delta V_A + \Delta V_B$$

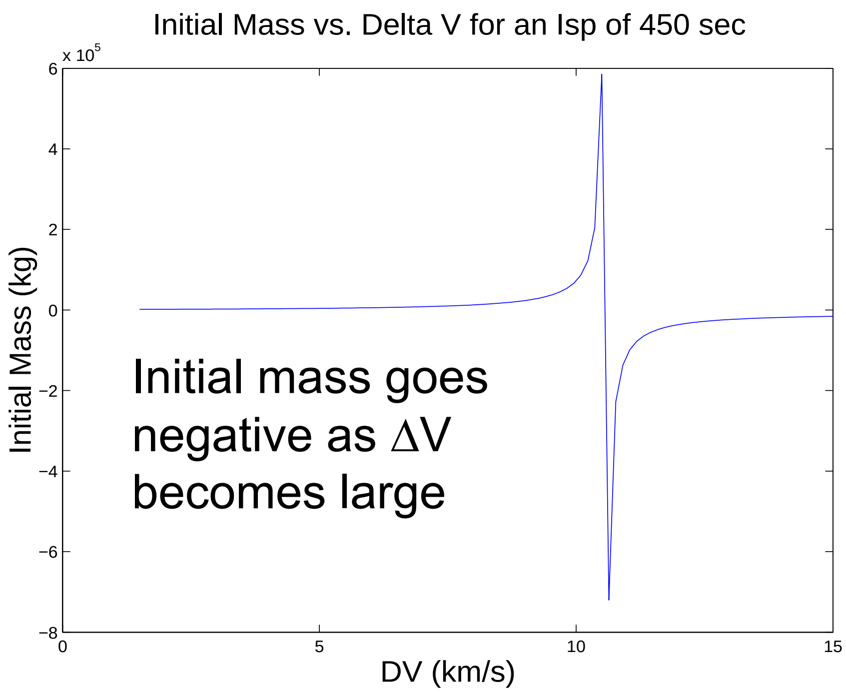
$$\cos E = \frac{e_T + \cos\nu}{1 + e_T\cos\nu}$$

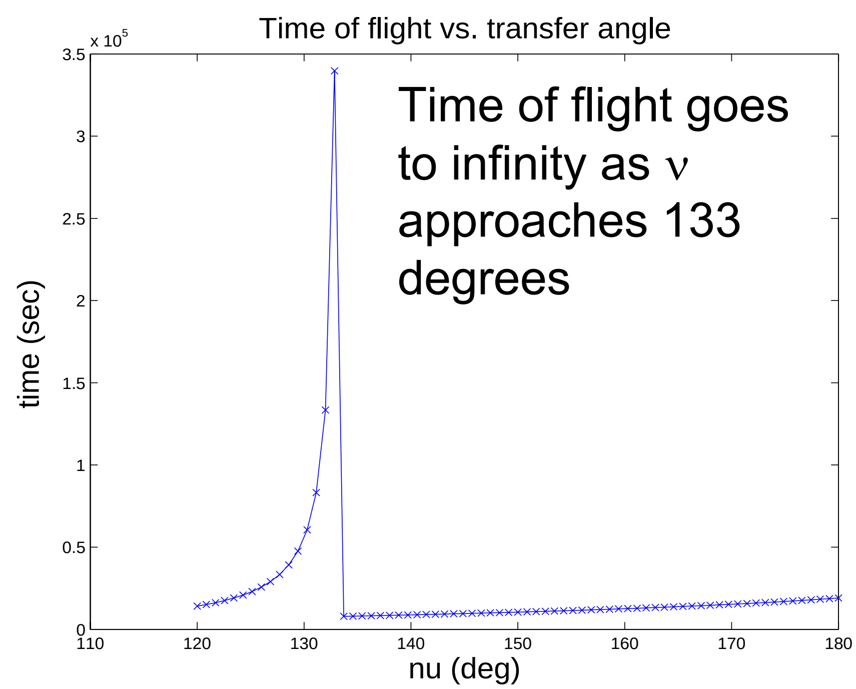
$$\sin E = \sqrt{1 - e_T^2}\frac{\sin\nu}{1 + e_T\cos\nu}$$

$$E = \tan^{-1}\left(\frac{\sin E}{\cos E}\right)$$

$$time = \sqrt{\left(\frac{a_T^3}{\mu}\right)}(2k\pi + E - e_T\sin E)$$

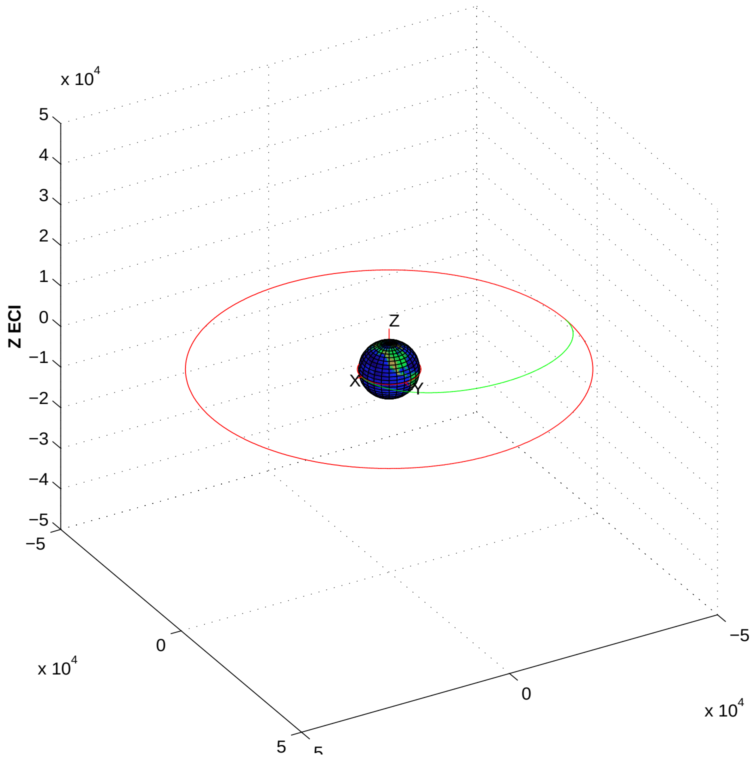
Transfer Angle Bounds





Model Verification

- Minimum initial mass solution is a Hohmann transfer
  - Transfer angle = 180 deg
  - Specific Impulse = 450 sec
  - Time of flight is half the transfer orbital period
- Using a SQP method, from any initial guess, model is verified
- Initial mass = 2721.2 kg
- Time of flight = 19086 sec
  (5.3 hours)



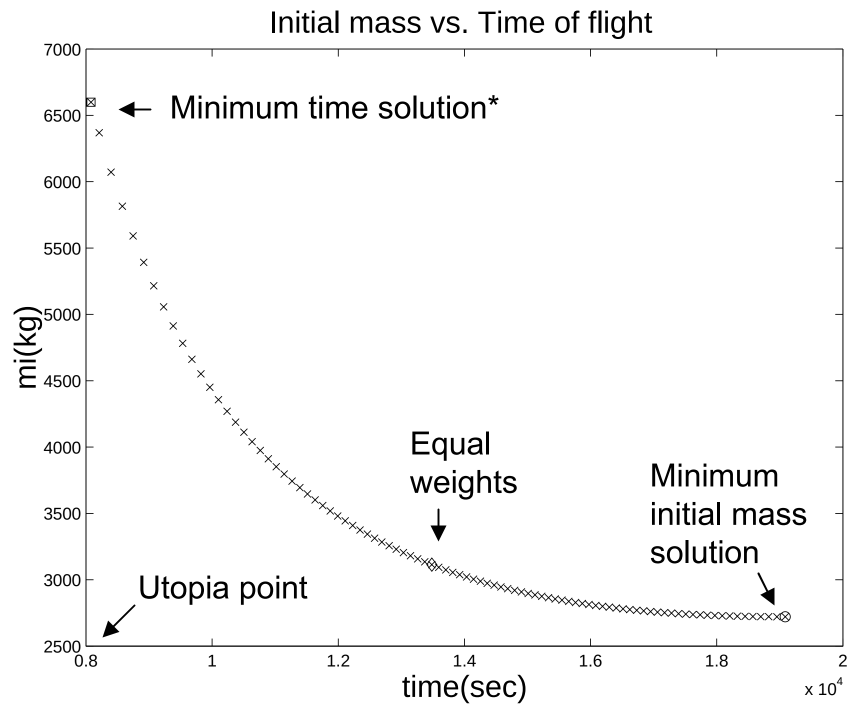
Multi-Objective Optimization

- The two objective to be minimized are initial mass and time of flight
- Scale each objective to be non-dimensional and approximately the same order of magnitude
  - Scale factor for mass is the payload mass (1000 kg)
  - Scale factor for time of flight is period of initial orbit (5370sec)

    $J_1 = m_i/m_p$

    $J_2 = \text{time}/P_0$
- Use weighted sum approach

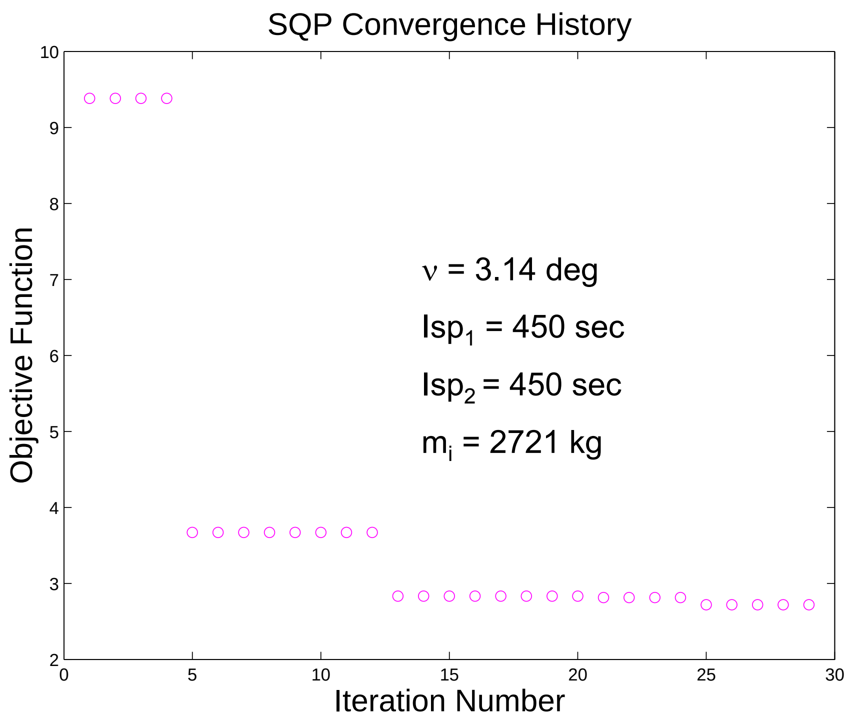
    $J = \lambda J_1 + (1-\lambda)J_2$

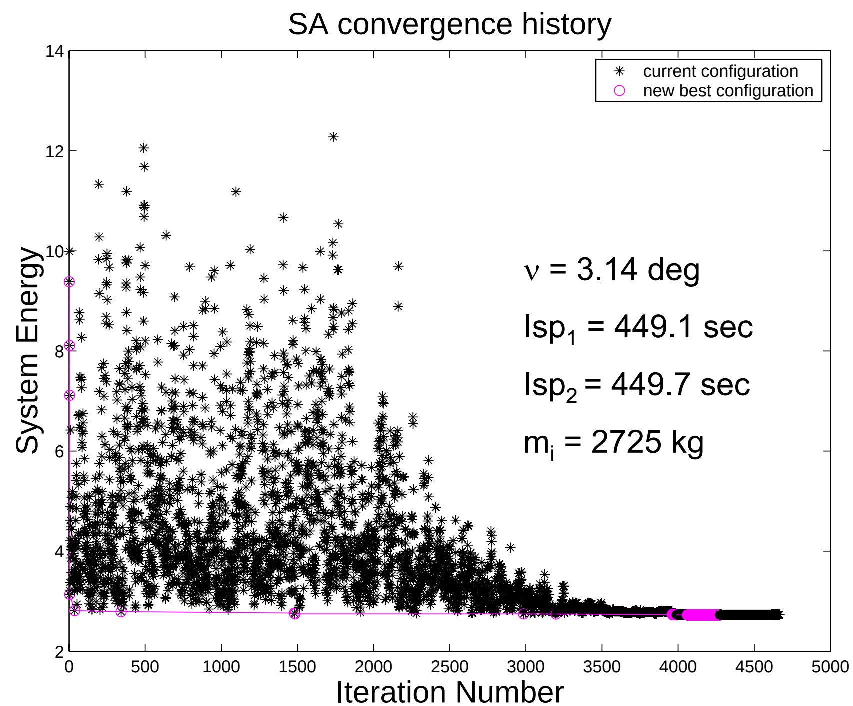


* Indicates that the solution is independent of engine selection

Minimum time of flight solution:
ν = 135 deg
Time of flight = 8078.5 sec (2.2 hrs)
Initial mass = 6598 kg

Comparison of SQP and SA Convergence





Convergence time for SQP = 0.32 sec

SQP parameters: objective function tolerance = 10^{-7}

Convergence time for SA = 33.48 sec

SA parameters: exponential cooling schedule,

dT = 0.75, neq = 50, nfrozen = 40

Sensitivity Analysis

- Sensitivity to design variables
  - Minimum initial mass is most sensitive to specific impulse of first engine:

$$\nabla J(\vec{x^*}) = \begin{bmatrix} -3.3310^{-8} kg/rad \\ -1.4610^{-3} kg/sec \\ -8.3810^{-4} kg/sec \end{bmatrix}$$

$$\overline{\nabla J}(\vec{x^*}) = \begin{bmatrix} -3.810^{-11} \\ -2.4110^{-4} \\ -1.3810^{-4} \end{bmatrix}$$

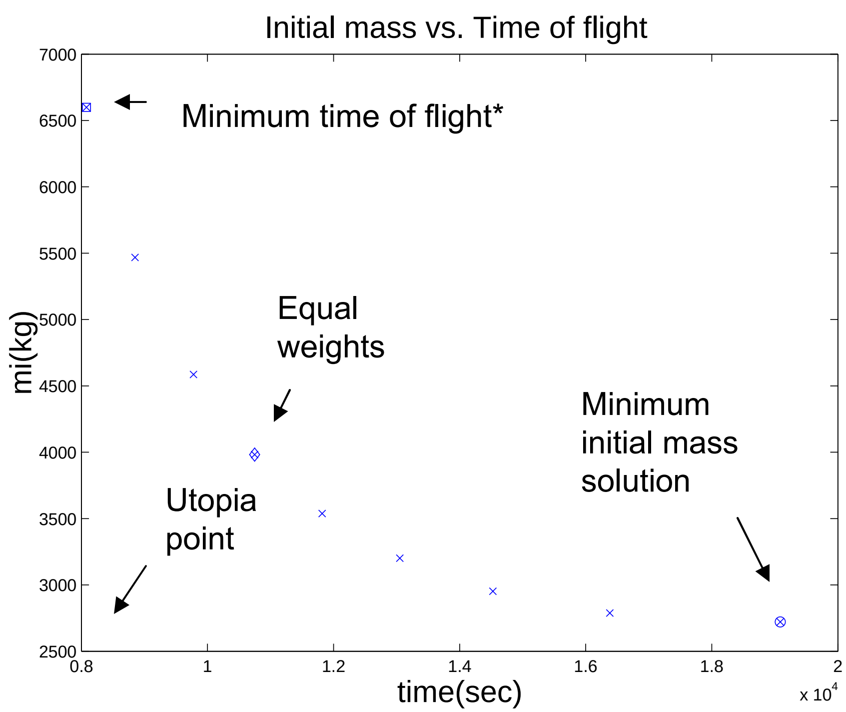
  - Minimum time of flight is only sensitive to transfer angle

$$\nabla J(\vec{x^*}) = \begin{bmatrix} 1.519 kg/rad \\ 0 kg/sec \\ 0 kg/sec \end{bmatrix}$$

$$\overline{\nabla J}(\vec{x^*}) = \begin{bmatrix} 5.4210^{-4} \\ 0 \\ 0 \end{bmatrix}$$

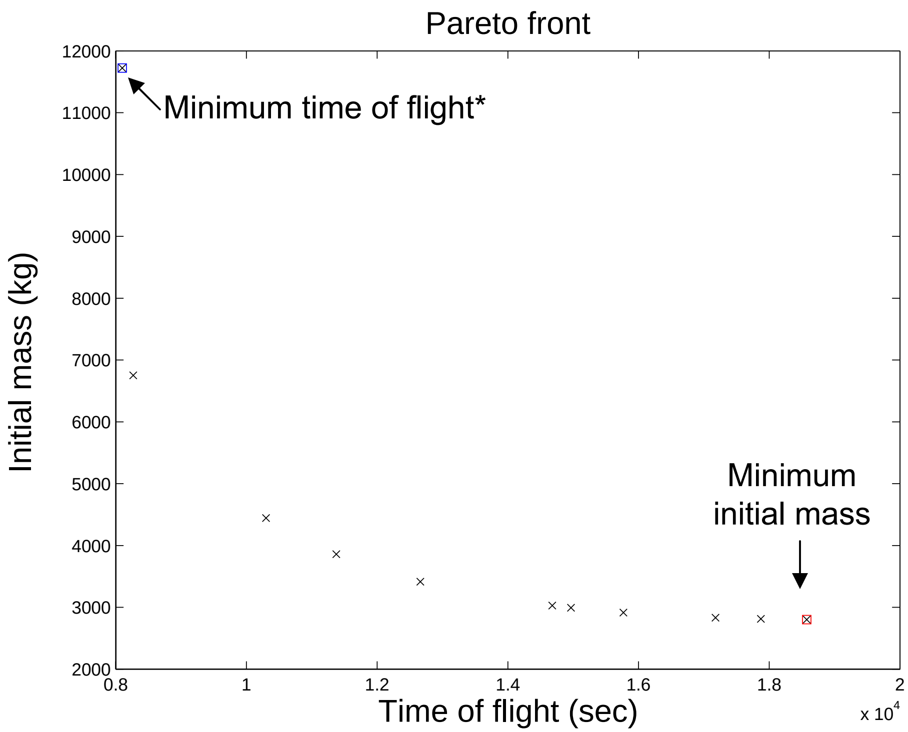
- Sensitivity to Scale Factors

As the scale factor for initial mass increases, the magnitude of J_1 decreases



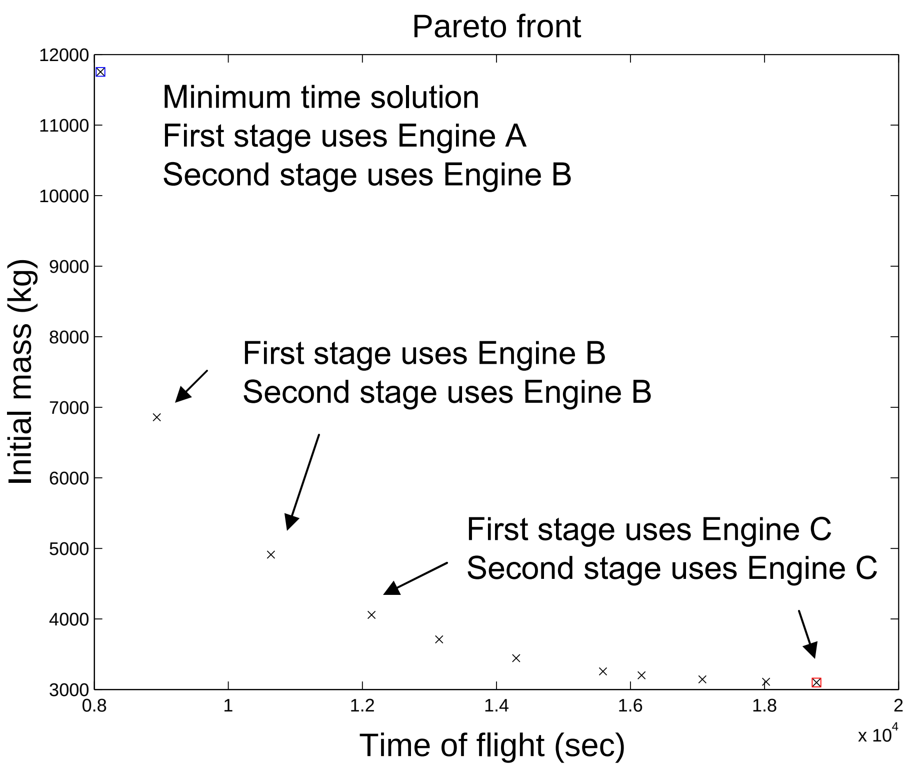
Vehicle Selection

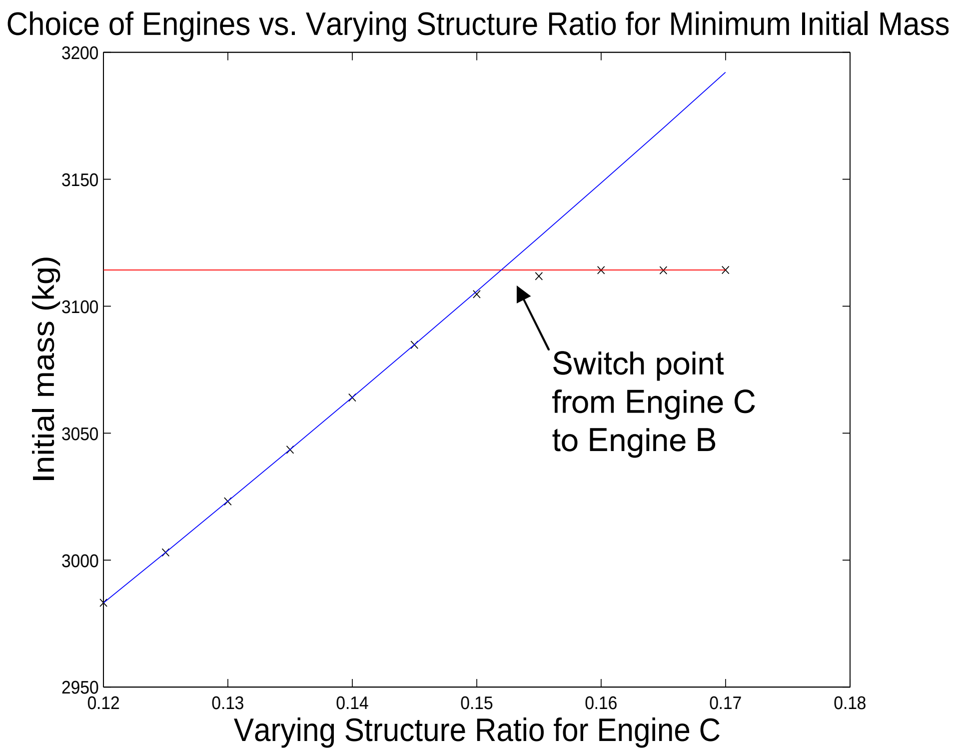
- Define three design variables
  - Transfer angle (ν)
  - Engine for first and second burns
- Select one of three different engines for each burn
  - Engine A: α = 0.08, Isp = 300 sec
  - Engine B: α = 0.10, Isp = 400 sec
  - Engine C: α = 0.12, Isp = 450 sec
- Use SA, determine Pareto front
  - Engine C is chosen for both engines and for each set of weights, except λ = 0



* Indicates that the solution is independent of engine selection

Sensitivity to Engine Change





Engine C: α = 0.148, Isp = 425 sec

Conclusions and Future Work

- Global Optimality?
  - Single objective optimization
  - Multi objective optimization
  - Discrete engine selection
- Expand the model to include different types of engines
  - Allow for low thrust, high impulse engines
  - Modify model to include constant thrust trajectories
- Analyze more complicated problems
  - Allow for out of plane maneuvers
  - Examine different origin/destination pairs
- Examine the relationship between scaling factors and the Pareto front