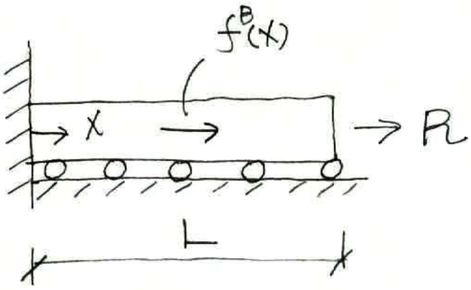
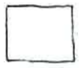
2.093

Formulation of the FEM (Section 3.3.4, 4.2)

◎ Simple example :



· Cross section A 

· Young's modulus E

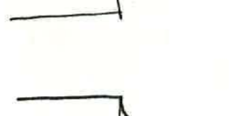
f^B = body force = ax (a: const)
(per unit length)

prob: Caculate "response" of the bar
↖ displ., stress, strain

* physical problem → mathematical model

Mathematical Model : can be simple or complicated.

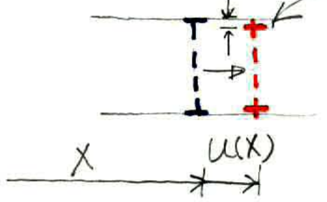
We point out : 1) Assumption of load application

Ass. 

When we decrease width stress goes to ∞

Actual 

2) Kinematics



Cross section will shrink.

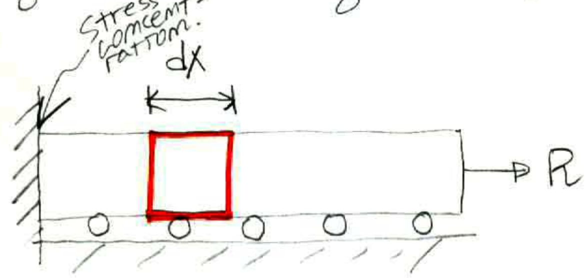
We assume $u(x)$ to be small

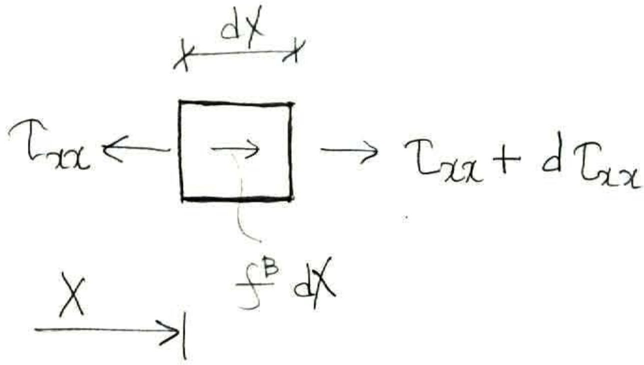
strain $\frac{du}{dx} = \varepsilon_{xx} \ll 1$ (~ 0.001)

- As long as we assume this kinematics, the difference of load application will not affect.

3) Stress - Strain Law $\tau_{xx} = E\,\varepsilon_{xx} \rightarrow d\tau_{xx} = E\,d\varepsilon_{xx}$

4) Equilibrium eqn. (Take in the original configuration)





X

$$A \cdot d\tau_{xx} + f^B dX = 0$$

$$A \frac{d\tau_{xx}}{dX} + f^B = 0$$

$$EA \frac{d^2u}{dx^2} + f^B = 0$$

Mathematical Model

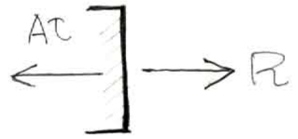
not at ∂V

$EAu'' + f^B = 0$ in V $\left\{ \text{volume of bar.} \right.$ (1)

At the ends

Essential b.c $\quad u(x=0)=0$ —— (a) $\quad$ **(2)**

At the end $X=L$



X·X direction.

$$A\tau\Big|_{X=L} = R \;\rightarrow\; EA\frac{du}{dx}\Big|_{x=L} = R \quad \text{—— (b)}$$

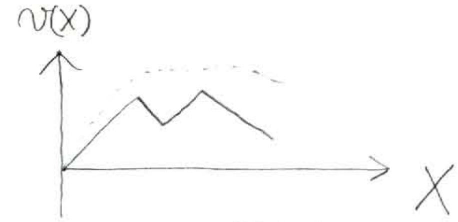
Solution

$$u(X) = \frac{-(aX^3/6) + (R + \frac{1}{2}aL^2)X}{EA}$$

FE Solution

$$(EAu'' + f^B)\,v = 0 \quad \text{(3)}$$

v = smooth displ. (continuous displ.)



with $v(0)=0$

v: any continuous displ. but zero at the left hand side

$$\int_0^L (EAu'' + f^B)\,v\,dX = 0 \quad \text{(4)}$$

⊙ frequently we denote v as δu

↑ virtual, not real.

Concentrate on this term

$$EAu''v = (EAu'v)' - (EAu'v')$$

hence

$$\int_0^L EAu''v\,dX = \int_0^L (EAu'v)'\,dX - \int_0^L (EAu'v')\,dX$$

$$\int_0^L EAu''v\,dX = EAu'v\Big|_0^L - \int_0^L EAu'v'\,dX$$

$$-\int_0^L EAu'v'\,dX + (EAu'v)\Big|_0^L + \int_0^L f^B v\,dX = 0 \qquad (5)$$

Impose b.c (2) a. & b.

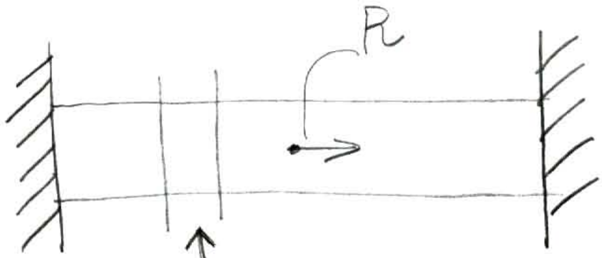
Consider $(EAu'v)\Big|_0^L = Rv\Big|_{X=L}$ for arbitrary ☆

$$\underbrace{\int_0^L EAu'v'\,dX}_{\text{Internal virtual work}} = \int_0^L f^B v\,dX + Rv\Big|_{X=L}$$

Principle of Virtual displacements (Virtual work)

※ We relieve the condition. (derivatives)

Ex)



Eq (1) is applicable.

• Our FEM can solve with linear ftn.