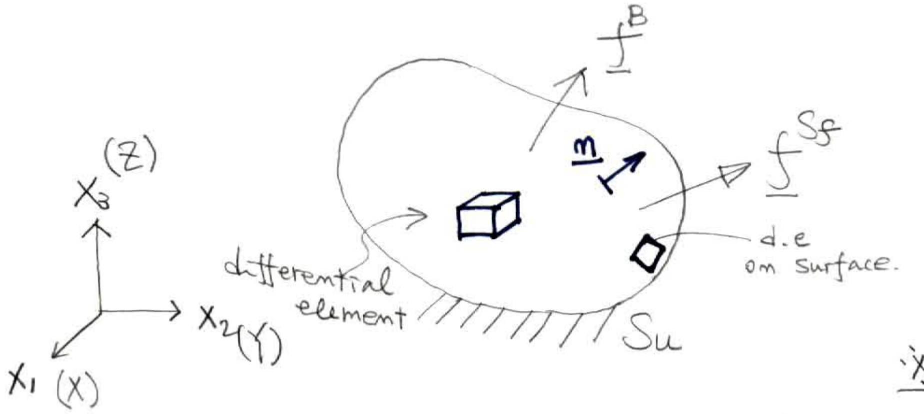
FEM Formulation (Cont'd)



$S_u \cup S_f = S$

$S_u \cap S_f = \phi$

can be refer to original coordinate

※ Infinitesimal strain

Diffential formulation.

Assume : Cartesian Coord.

(1) $\tau_{ij,j} + f_i^B = 0$ $\quad i = 1, 2, 3$ (Sum over j)

(2) $\tau_{ij} n_j \big|_{S_f} = f_i^{S_f}$ $\quad i = 1, 2, 3$ (〃)

(3) $u_i \big|_{S_u} = 0$ $\quad i = 1, 2, 3$

Note : in static $S_u = \phi$: Non-Unique sol.
in dynamic 〃 : Stable because of inertia

P. V. Work (displacements) (Ex 4.2 textbook)

(1) & (2)
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$$\int_V \bar{\underline{\varepsilon}}^T \underline{\tau} \, dV = \int_V \bar{\underline{u}}^T \underline{f}^B \, dV + \int_{S_f} \bar{u}^{S_f^T} \underline{f}^{S_f} \, dS_f \quad (4)$$

given, or known.

$\underline{\tau}^T = [\, \tau_{xx} \ \tau_{yy} \ \tau_{zz} \ \tau_{xy} \ \tau_{yz} \ \tau_{zx} \,]$

$\underline{\varepsilon}^T = [\, \varepsilon_{xx} \ \varepsilon_{yy} \ \varepsilon_{zz} \ \gamma_{xy} \ \gamma_{yz} \ \gamma_{zx} \,]$

$\underline{\tau} = \underline{C} \ \underline{\varepsilon}$ $\quad (5)$

$\varepsilon_{xx} = \frac{\partial u}{\partial x}$

$\gamma_{zx} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}$

$\bar{\varepsilon}_{xx} = \frac{\partial \bar{u}}{\partial x}$

$$\bar{\underline{u}}(X,Y,Z) = \begin{bmatrix} \bar{u} \\ \bar{v} \\ \bar{w} \end{bmatrix} \text{ ftn of } (x,y,z). \qquad \left.\begin{pmatrix} \bar{u} \\ \bar{v} \\ \bar{w} \end{pmatrix}\right|_{S_u} = 0$$

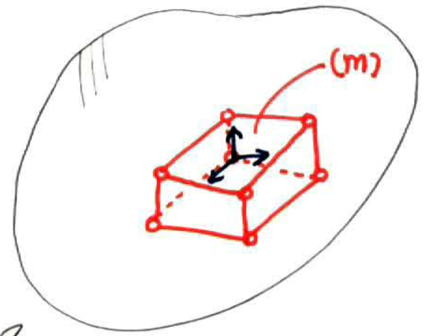
<u>Note</u>: $\bar{\underline{\varepsilon}}^T \underline{\tau} = \bar{\varepsilon}_{xx}\tau_{xx} + \cdots \bar{\gamma}_{zx}\tau_{zx}$

$\bar{\underline{u}}^T \underline{f}^B = \bar{u} f^B_x + \cdots \bar{w} f^B_z$

$$\int \bar{\underline{\varepsilon}}^T \underline{C}\, \underline{\varepsilon}\, dV = \int_V \bar{\underline{u}}^T \underline{f}^B dV + \int \bar{\underline{u}}^{S_f T} \underline{f}^{S_f} dS_f \qquad (6)$$

We want the unknown displacements

$$\begin{bmatrix} u\,(X,Y,Z) \\ v\,(X,Y,Z) \\ w\,(X,Y,Z) \end{bmatrix}$$

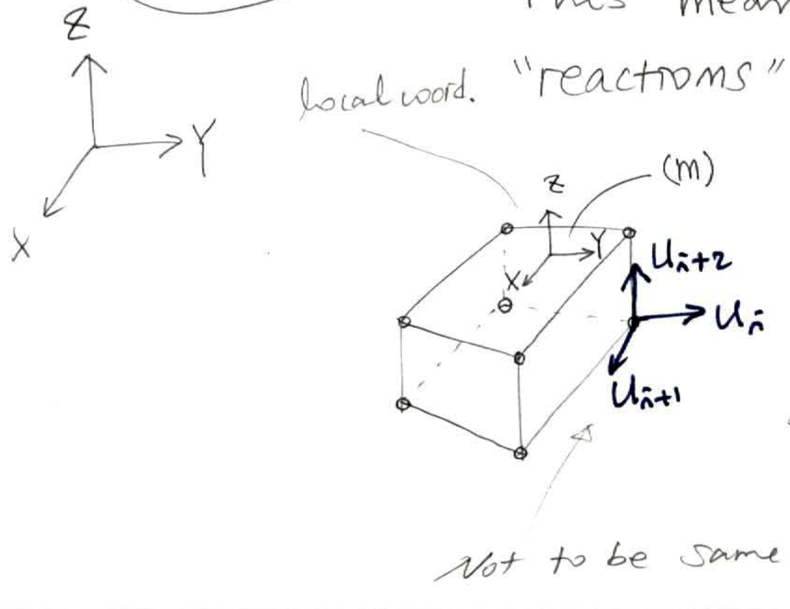


For the solution let's "remove" S_u

(i.e, assume $S_u = 0$)

$S \equiv S_f$

This means, we assume that "reactions" are known.



$$\underline{u}^{(m)} = \begin{bmatrix} u\,(X,Y,Z) \\ v\,(X,Y,Z) \\ w\,(X,Y,Z) \end{bmatrix}^{(m)}$$

3 unknown nodal displ per node.

(7) $\boxed{\underline{u}^{(m)} = \underline{H}^{(m)} \underline{\hat{u}}}$; $\underline{\hat{u}}^T = [u_1 \cdots\cdots u_{3N}]$

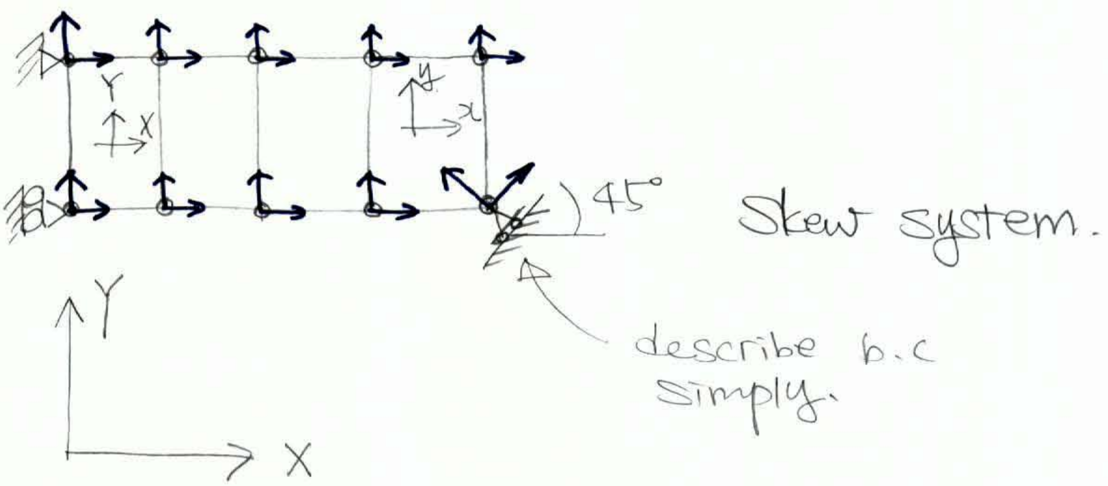
local coord system.

$\underline{H}^{(m)} = fn(x, y, z)$

= interpolation matrix

N = no of total Nodes of the Mesh

Ex)



Use (6) & (7)

$$\sum_m \int_{V^{(m)}} \underline{\bar{\varepsilon}}^{(m)^T} \underline{C}^{(m)} \underline{\varepsilon}^{(m)} dV^{(m)} = \sum_m \int_{V^{(m)}} \underline{\bar{U}}^{(m)^T} \underline{f}^{B^{(m)}} dV^{(m)}$$

$$+ \sum_m \int_{S_f^{(m)}} \underline{\bar{U}}^{S_f^{(m)^T}} \underline{f}^{S_f^{(m)}} dS_f^{(m)} \qquad (8)$$

Note : $\underline{\bar{u}}^{(m)} = \underline{H}^{(m)} \underline{\bar{\hat{u}}}$

all from (7)

$\underline{\varepsilon}^{(m)} = \underline{B}^{(m)} \underline{\hat{u}}$

$\underline{\bar{\varepsilon}}^{(m)} = \underline{B}^{(m)} \underline{\bar{\hat{u}}}$

$B^{(m)}$ is obtained from $\underline{H}^{(m)}$ in el. coord system.

From (8): for the whole volume.

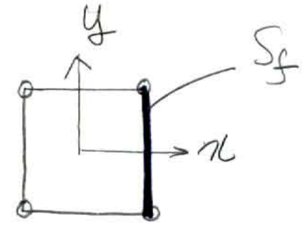
$$\sum_m \left[\int \left(\underline{B}^{(m)} \, \bar{\hat{\underline{U}}} \right)^T \underline{C}^{(m)} \, \underline{B}^{(m)} \, \hat{\underline{U}} \, dV^{(m)} \right] = \tag{9}$$

$$\sum_m \left[\int_{V^{(m)}} \left(H^{(m)} \, \bar{\hat{\underline{U}}} \right)^T \underline{f}^B \, dV^{(m)} \right] + \sum_m \left[\int_{S_f^{(m)}} \left(H^{(m)} \bar{\hat{\underline{U}}} \right)^T \Big|_{S_f^{(m)}} \underline{f}^{S_f^{(m)}} \, dS_f^{(m)} \right]$$

$$\underline{H}^{(m)} \, \bar{\hat{\underline{U}}} \Big|_{S_f^{(m)}} \equiv H^{(m)} \Big|_{S_f^{(m)}} \bar{\hat{\underline{U}}}$$

$$\equiv H^{S(m)} \, \bar{\hat{\underline{U}}}$$

e.g



$$\underline{H}^{(m)} \rightarrow \underline{H}^{S(m)}$$

Aside $(\underline{A}\underline{C})^T = \underline{C}^T \underline{A}^T$

Next lecture we obtain

$$\underline{K} \, \hat{\underline{U}} = \underline{R}$$