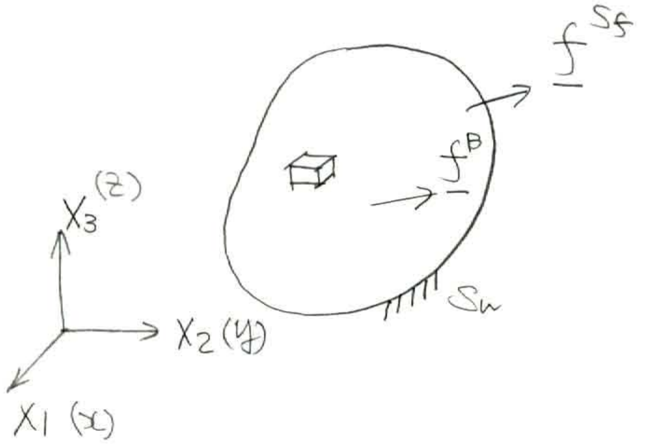
Note:

Project : compare with analytical solution

FEM formulation



Diff form

(1) $\tau_{ij,j} + f_i^B = 0$ in V
(for $i=1,2,3$)

(2) $\tau_{ij} n_j = f_i^{S_f}$ on S_f

(3) $u_i|_{S_u} = 0$

Var. formulation

$$\int_V \overline{\underline{\varepsilon}}^T \underline{\tau}\, dV = \int_V \overline{\underline{u}}^T \underline{f}^B\, dV + \int_{S_f} \overline{\underline{u}}^{S_f T} \underline{f}^{S_f}\, dS_f \qquad (4)$$

in any Cartesian stationary coordinate system.

$$\sum_m \int_{V^{(m)}} \overline{\underline{\varepsilon}}^{(m)T} \underline{\tau}\, dV^{(m)} = \sum_m \int_{V^{(m)}} \overline{\underline{u}}^{(m)T} \underline{f}^{B(m)}\, dV^{(m)}$$

$$+ \sum_m \int_{S_f^{(m)}} \overline{\underline{u}}^{S_f T} \underline{f}^{S_f(m)}\, dS_f^{(m)} \qquad (5)$$

for any convenient stationary coordinate system.

$$\boxed{\begin{array}{l}\text{Assumption :}\\ \quad \underline{u}^{(m)} = \underline{H}^{(m)} \underline{\hat{u}}\end{array}} \qquad (6)$$

$$\underline{\varepsilon}^{(m)} = \underline{B}^{(m)} \underline{\hat{u}} \qquad (7)$$

$$\tau^{(m)} = C^{(m)} \varepsilon^{(m)} \qquad (8)$$

let $S_u = 0$

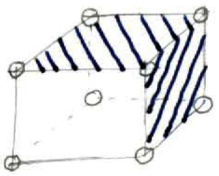
& We use (6) & (7) also for the virtual quantities.

$$\sum_m \left[\int_{V^{(m)}} (\underline{B}^{(m)} \underline{\bar{\hat{u}}})^T \underline{C}^{(m)} \underline{B}^{(m)} \underline{\hat{u}} \, dV^{(m)} \right] = \sum_m \left[\int_{V^{(m)}} (\underline{H}^{(m)} \underline{\bar{\hat{u}}})^T f^{B(m)} dV^{(m)} \right]$$

$$\circledast \rightarrow + \sum_m \left[\int_{S^{(m)}} (H^{S(m)} \underline{\bar{\hat{u}}})^T \underline{f}^{S(m)} dS^{(m)} \right] \qquad (9)$$

From (6) : $\underline{u}^{S(m)} = \underline{H}^{S(m)} \underline{\hat{u}}$; $\underline{H}^{S(m)} = H^{(m)}\big|_S$

Note :



"$\circledast$" means sum over all externally loaded f areas of el (m)

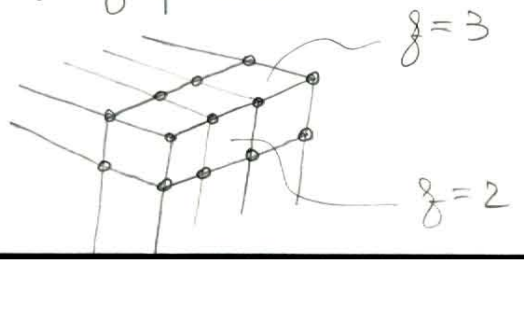
meaning

The last term in (8) is

$$\sum_m \sum_{i=1,..q} \int_{S_i^{(m)}} (H^{S_i(m)} \underline{\bar{\hat{u}}})^T \underline{f}^{S_i(m)} dS_i^{(m)}$$

q = no of ext surfaces loaded for element m.

ex) if pressure.



From (9) dim ?

$$\bar{\hat{\underline{u}}}^T \left\{ \sum_m \left[\int_{V^{(m)}} \underline{B}^{(m)T} \underline{C}^{(m)} \underline{B}^{(m)} \, dV^{(m)} \right] \right\} \hat{\underline{u}} = \bar{\hat{\underline{u}}}^T \left\{ \sum_m \left[\int_{V^{(m)}} \underline{H}^{(m)T} \underline{f}^{B(m)} \, dV^{(m)} \right] + \sum_m \left[\int_{S^{(m)}} \underline{H}^{S(m)T} \underline{f}^{S(m)} \, dS^{(m)} \right] \right\} \quad (10)$$

$\underline{I}$

Now virtual displ. patterns :

$$\bar{\hat{\underline{u}}}^T = [1 \; 0 \; 0 \cdots\cdots 0]$$

then

$$\bar{\hat{\underline{u}}}^T = [0 \; 1 \; 0 \cdots\cdots 0]$$

$\vdots$

finally $\bar{\underline{\hat{u}}}^T = [0 \; 0 \cdots\cdots 1]$

this means we apply (10) as many times as possible to generate linearly indep. eqns for $\hat{\underline{u}}$

Total No of eq $= N$

We obtain

(11) $\boxed{\underline{K} \, \hat{\underline{U}} = \underline{R}}$; $\underline{K} = \sum_m \underline{K}^{(m)}$; $\underline{K}^{(m)} = \int_{V^{(m)}} \underline{B}^{(m)T} \underline{C} \, \underline{B} \, dV^{(m)}$

Symm

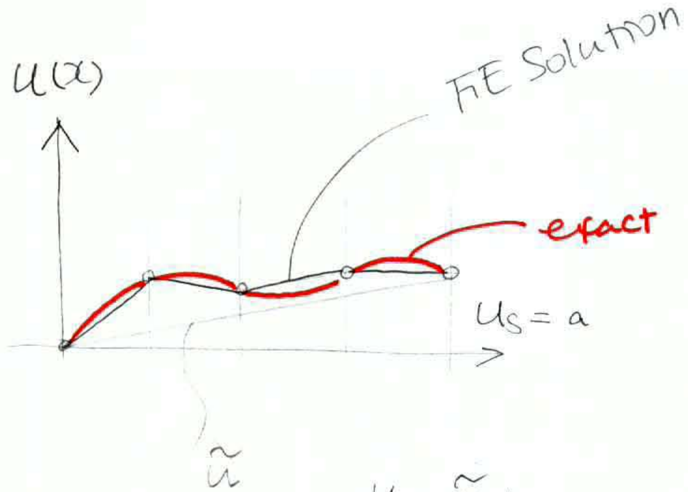
$$\underline{R} = \underline{R}_B + \underline{R}_S$$

$$\underline{R}_B = \sum_m R_B^{(m)} \; ; \; \underline{R}_S = \sum_m R_S^{(m)}$$

$$R_B^{(m)} = \int_{V^{(m)}} \underline{H}^{(m)T} \underline{f}^{B(m)} \, dV^{(m)} \; ; \; R_S^{(m)} = \int_{S^{(m)}} \underline{H}^{S(m)T} \underline{f}^{S(m)} \, dS^{(m)}$$

So far, we neglect b.c's

Now put in displ. b.c in solving (11)



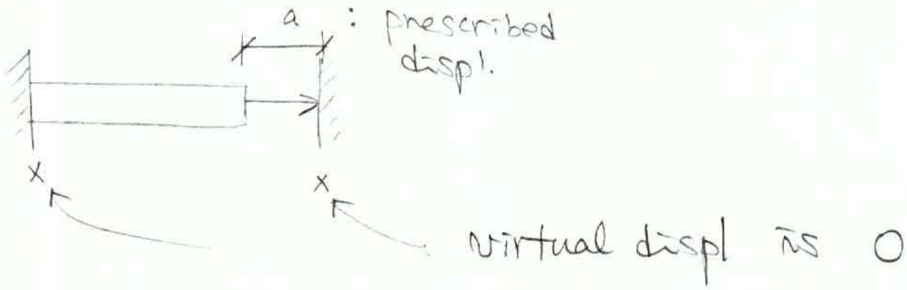
For $\mathcal{H}$ to be a space

$h_1 \in \mathcal{H}$

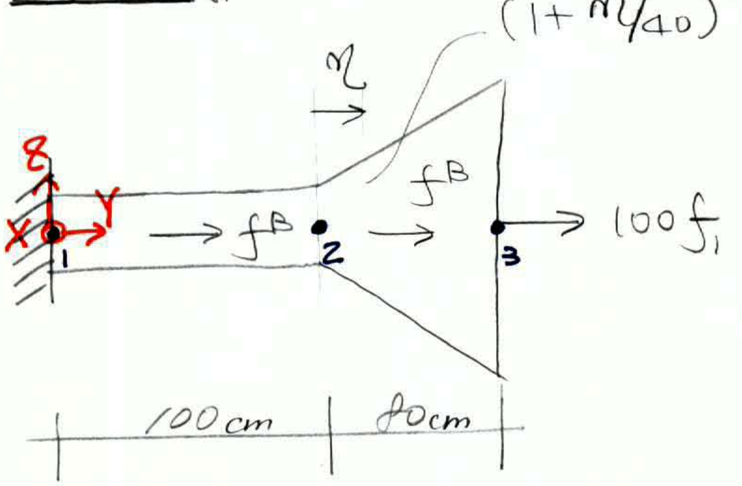
$h_2 \in \mathcal{H}$

$\rightarrow \alpha h_1 + \beta h_2 \in \mathcal{H}$

$u = \tilde{u} + u_{assumpt}$



Ex 4.5 (p166)



E, (ρ = mass density)

$f^B_Y \big|_{0 \text{ to } 100\text{cm}} = f_2(t)$

$f^B_Y \big|_{100 \text{ to } 180\text{cm}} = 0.1\, f_2(t)$