* Solution of $\underline{M}\,\ddot{\underline{u}} + \underline{C}\,\dot{\underline{u}} + \underline{K}\,\underline{U} = \underline{R}$

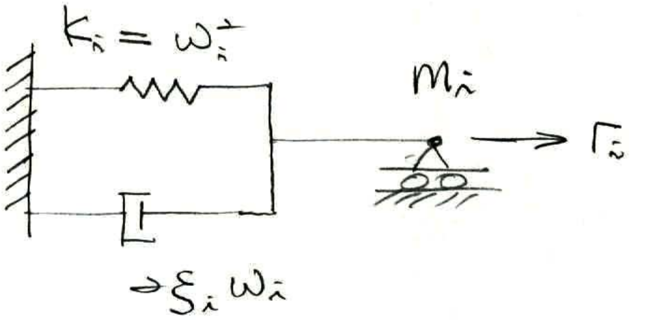
$$\left.\begin{array}{c} \underline{M}\,\ddot{\underline{u}} + \underline{C}\,\dot{\underline{u}} + \underline{K}\,\underline{U} = \underline{R} \\ {}^{0}\underline{u},\ {}^{0}\dot{\underline{u}} \end{array}\right\} \quad (1)$$

<u>mode superposition</u>

$$u(t) = \sum_{i=1}^{n} \underline{\phi}_i\, x_i \qquad (2)$$

$$0 \le \omega_1 \le \omega_2, \cdots \le \omega_n$$
$$\phi_1 \quad \phi_2 \quad \cdots \quad \phi_n$$

$$\left.\begin{array}{ll} \ddot{x}_i + 2\xi_i \omega_i \dot{x}_i + \omega_i^2 x_i = r_i & \\ i = 1, \cdots n & {}^{0}x_i,\ {}^{0}\dot{x}_i \end{array}\right\} \quad (3)$$

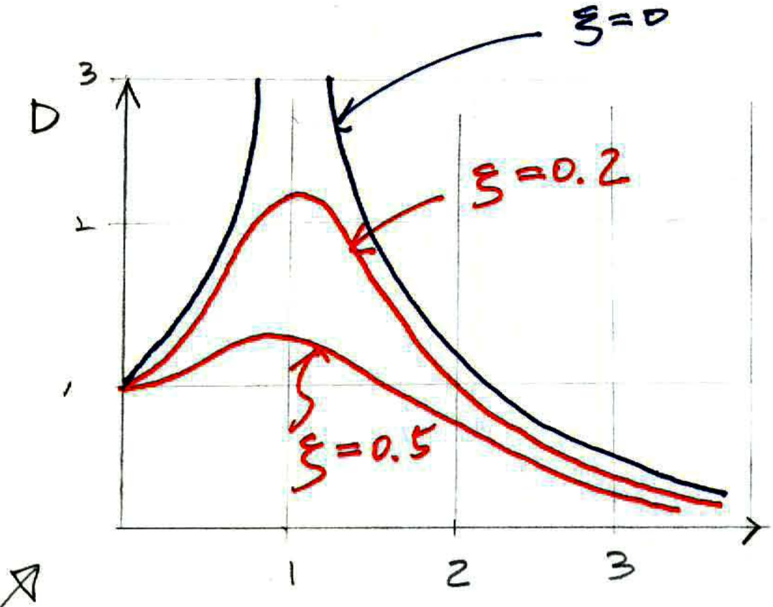


<u>Cases</u>

I) ${}^{0}\underline{u},\ {}^{0}\dot{\underline{u}} \sim \underline{\phi}_i$

II) $\underline{R} \sim \underline{M}\,\underline{\phi}_i$

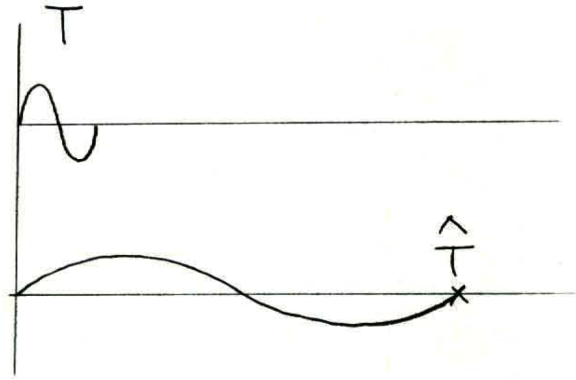
III) frequency content of $\underline{R}$



$$\frac{\hat{\omega}}{\omega} = \frac{T}{\hat{T}}$$

$$D = \text{dyn loadfactor} = \frac{\text{max dyn response}}{\text{static resp}}$$

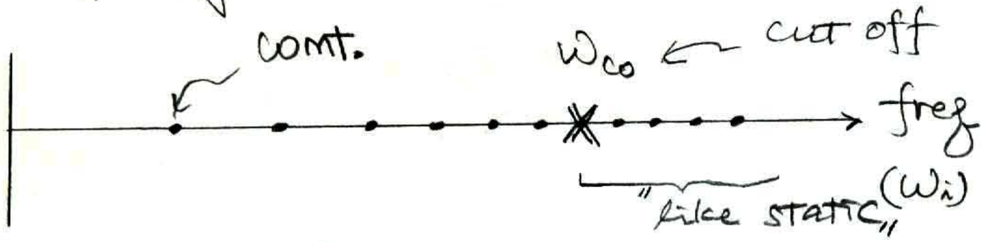
$$\ddot{x} + 2\xi\omega\dot{x} + \omega^2 x = \sin\hat{\omega}t$$



$$T << \hat{T}$$

Dynamic modeling

Cont. system

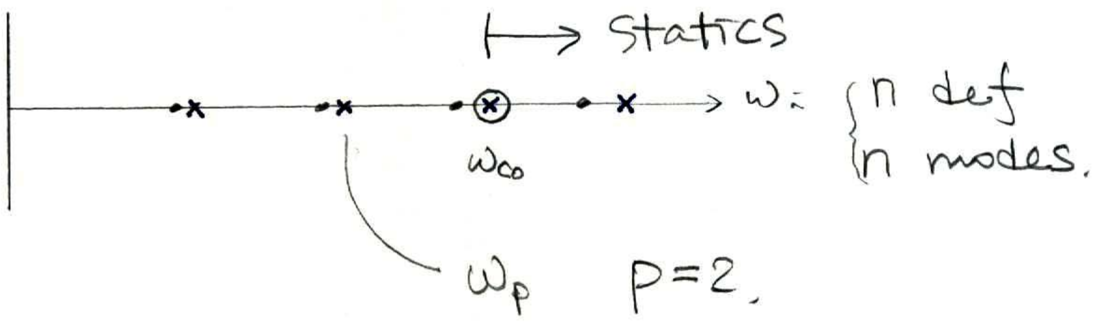


ω_u = upper limit of frequency contained in load vector

$$4\omega_u = \omega_{co}$$

Aside If we use consistent mass matrix

We have $\omega_i\big|_{FEA} \geq \omega_i\big|_{continuum}$



In eqn (2), we can use

"No longer n"

$$U(t) = \sum_{i=1}^{p} \underline{\Phi}_i x_i \qquad (4)$$

Static correction

Calculate
$$\Delta R = R - \sum_{i=1}^{p} r_i (\underline{M}\, \underline{\Phi}_i) \qquad (5)$$

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and $\underline{K}\, \Delta \underline{U}(t) = \Delta \underline{R}(t) \qquad (6)$

and

$$U_a \equiv \underset{\text{approx}}{\underline{U}(t)} = \sum_{i=1}^{p} \underline{\phi}_i x_i(t) + \Delta \underline{U}(t) \qquad (7)$$

Namely, consider $\Delta R(t)$ in a mode superposition analysis.

$$\ddot{x}_j + 2\xi_j \omega_j \dot{x}_j + \omega_j^2 x_j = \phi_j^T (\Delta R) \quad (8)$$

$$j = 1, \cdots n$$

Look at

$$\phi_j^T (\Delta R) = \underline{\phi}_j^T \underline{R} - r_j = 0 \qquad j = 1, 2, \cdots p$$

Conclusion

$$r_j = 0 \quad \text{for } j = 1, \cdots p$$

$$r_j = \underline{\phi}_j^T \underline{R} \quad \text{for } j = p+1, \cdots n$$

but in the $\omega_{p+1}, \cdots \omega_n$ frequencies we have at most static response and that is picked up by eq. 6.

Error measure

$$\frac{\| \underline{M}\,\ddot{\underline{u}}_a + \underline{C}\,\underline{u}_a + \underline{K}\,\underline{u}_a - \underline{R} \|_2}{\| \underline{R} \|_2} \leq \varepsilon_p \sim 0.01$$

$$\| \underline{u} \|_2 = \sqrt{\sum_{i=1}^{n} (u_i^2)} \qquad \text{Euclidian norm}$$

* In direct implicit integration, we use

$$\Delta t = \frac{T_p}{20}$$

does correction automatically.

* In direct explicit integration.

$$\Delta t = \frac{T_n}{\pi} (0.99)$$