Modeling for dynamic analysis, cont'd

We set up

$$\underset{\sim}{M}\ddot{\underset{\sim}{U}} + \underset{\sim}{C}\dot{\underset{\sim}{U}} + \underbrace{\underset{\sim}{K}\underset{\sim}{U} = \underset{\sim}{R}}_{\text{static}} \; ; \quad {}^0\underset{\sim}{u} \quad {}^0\dot{\underset{\sim}{u}} \qquad (1)$$

Solve eqn (1)

a) – Direct integration — explicit (CDM) / implicit (TR)

b) – Mode superposition.

We note that the following factors will decide what elements to use, what mesh to use, what technique for solution to use.

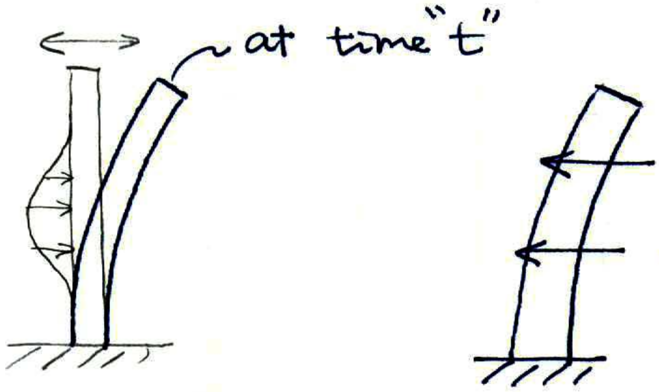
factors (i) ${}^0\underset{\sim}{u}$ ${}^0\dot{\underset{\sim}{u}}$

(ii) $\underset{\sim}{R}$ spatial distribution

(iii) frequency content in $\underset{\sim}{R}$

(iv) statics must be "captured by the mesh"

e.g.



at time "t"

$\underline{K}\underline{U} = \underline{\tilde{R}}$

$\overbrace{M\ddot{U},\ C\dot{U},\ R}^{\tilde{R}}$ all ftn of "t"

Damping

We assume Rayleigh damping (first terms of a Caughey series)

$$\underline{C} = \alpha \underline{M} + \beta \underline{K}$$

α, β constants   textbook.

$$\underline{\phi}_i^T \underline{C} \underline{\phi}_i = \alpha \overbrace{\underline{\phi}_i^T \underline{M} \underline{\phi}_i}^{1} + \beta \overbrace{\underline{\phi}_i^T \underline{K} \underline{\phi}_i}^{\omega_i^2}$$

in modal damping $\quad \phi_i C \phi_i \equiv 2\xi_i \omega_i$

$$2\xi_i \omega_i = \alpha + \beta \omega_i^2$$

If we use the modal damping at two ω_i's, we can establish two equations from which we can solve α, β

eg. $\xi_1, \omega_1 \to \xi_1 \omega_1 = \alpha + \beta \omega_1^2$

$\xi_2, \omega_2 \to \xi_2 \omega_2 = \alpha + \beta \omega_2^2$ $\Big\} \to \alpha, \beta$.

{e.g at 10 Hz, we want ξ_1}

In practice, we may say at $\tilde{\omega}_1$, we want $\tilde{\xi}_1$, at $\tilde{\omega}_2$ we want $\tilde{\xi}_2$

$\to$ determine α, β

The actual damping at all frequency is then

$$\xi_i = \underbrace{\frac{\alpha}{2\omega_i}}_{\substack{\text{mass proportional} \\ \text{low freq.}}} + \underbrace{\frac{\beta}{2}\omega_i}_{\substack{\text{stiffness prop.} \\ \text{high freq.}}}$$

In mode sup.
every ξ_i can be chosen.

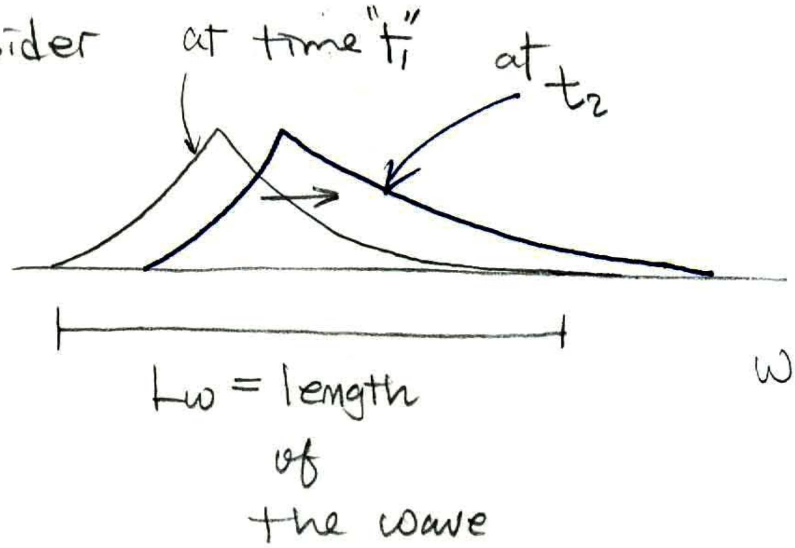
Aside

Consider. $\underline{\Phi}^T \underline{C}\, \underline{\Phi} = \begin{bmatrix} 2\xi_1\omega_1 & & \\ & \ddots & \\ & & 2\xi_n\omega_n \end{bmatrix}$

In explicit, plasticity will work for damping.

Wave propagation

Consider



wave speed C

$$C = f_n(E, \nu, \rho)$$

time it takes for the wave to travel past a fixed point $= t_w$

$$C = \frac{L_w}{t_w}$$

Choose $\quad \Delta t = \dfrac{t_w}{n} \quad$ (n to be chosen)

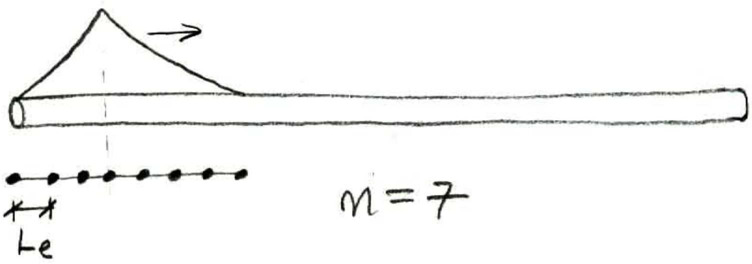
at one time step wave will go through one el. if L_e is element size

then use $\quad L_e =$ (effective) length of an element $= C \cdot \Delta t \quad$ (A)

Note $\quad L_w = C\, t_w = C \cdot (\Delta t \cdot n) = L_e \cdot n$

This way we use n elements to represent the wave length. L_w

In 1d



We use low-order element

1d : 2 node
2d : 4 node
3d : 8 node

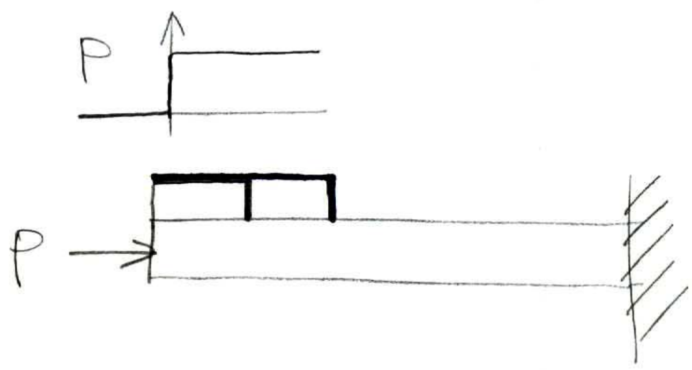
use smallest distance between nodes.

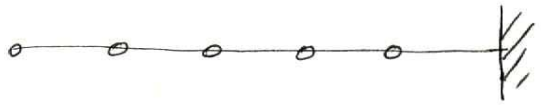
* Don't use 2-d element for 1-d prob.

Considering the 1D problem we have with(A) that $\Delta t = \Delta t_{cr}$ using CDM.

$$\Delta t = \frac{L_e}{C}$$

Ex



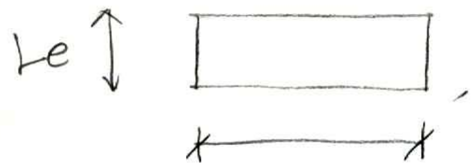


CDM
lumped M
$\underline{C} = 0$

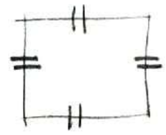
Smaller time step will be worse

☆ best : one element travel at a step.

Even with very coarse mesh. $\left(\Delta t = \frac{L_e}{C}\right)$
you will get exact value.



, so use square element!



Rayleigh quotient

defined as $\rho(\underline{\phi}) = \dfrac{\underline{\phi}^T \underline{K}\, \underline{\phi}}{\underline{\phi}^T \underline{M}\, \underline{\phi}}$

pick a ϕ and calculate $\rho(\phi)$

Properties

I) Assume $\underline{\Phi} \underline{M} \underline{\Phi} = 1$ (means we "Scaled" $\underline{\Phi}$)

Then $\rho(\Phi)$ = twice the strain energy in the system which subjected to $\underline{\Phi}$

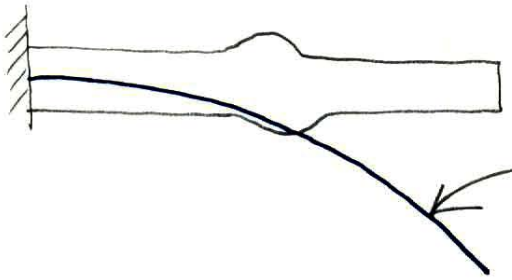
strain energy $= \frac{1}{2} \underline{\Phi}^T \underline{K} \underline{\Phi}$

II) If $\underline{\Phi} = \underline{\Phi}_n$ then $\rho(\underline{\Phi}) = \omega_n^2$

III) If $\underline{\Phi} = \underline{\Phi}_n + \varepsilon \underline{v}$, $\varepsilon \ll 1$

then $\rho(\Phi) = \omega_n^2 + O(\varepsilon^2)$

can be −, +



ϕ_1

$\neq$

Φ to get lowest frequency.

e.g. to estimate ω_1^2

IV) $\omega_1^2 \le \rho(\Phi) \le \omega_n^2$