Quiz 2

6 pages of notes.

Including today's lecture

Solution of eqns – Gauss elimination

Skyline

↓

Sparse solver (ADINA)

*

Gauss elimination + Optimization.

many zeros (as many)

- difficult point : first equation which eq. should be considered next !!!
- Establish array

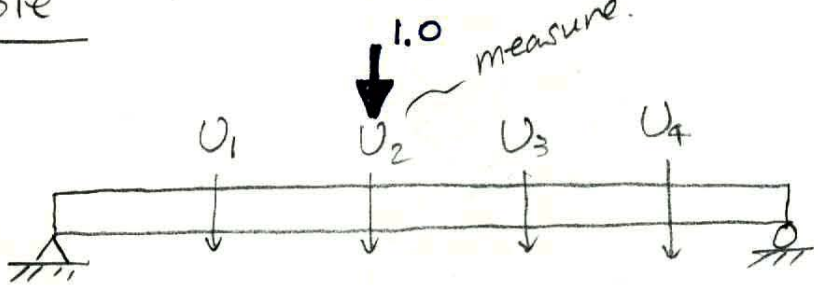
1/10 ~ 1/100

1/2 million → few minutes

pc enhancement.

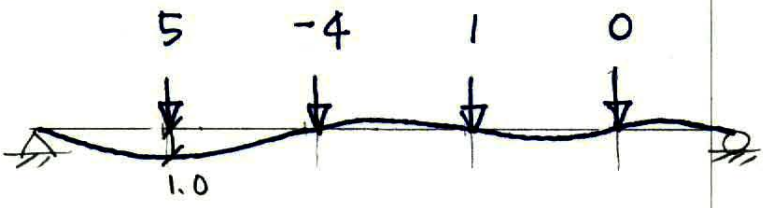
2 decades : 1) mixed method. } **
2) Sparse solver.

Example



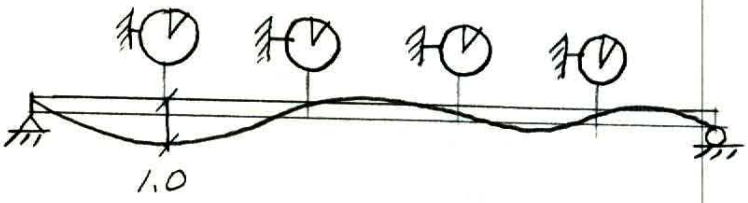
$$\begin{bmatrix} 5 & -4 & 1 & 0 \\ -4 & 6 & -4 & 1 \\ 1 & -4 & 6 & -4 \\ 0 & 1 & -4 & 5 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \quad (1)$$

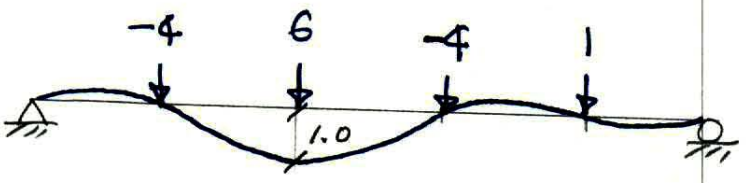
① ② ③ ④



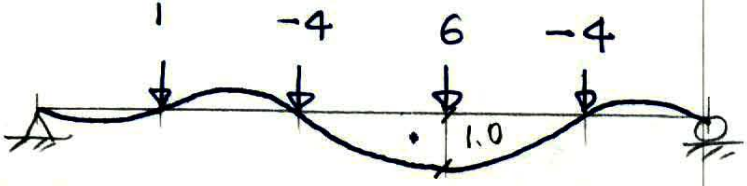
①

In the lab

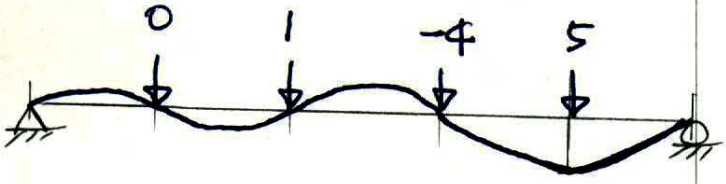




②



③



④

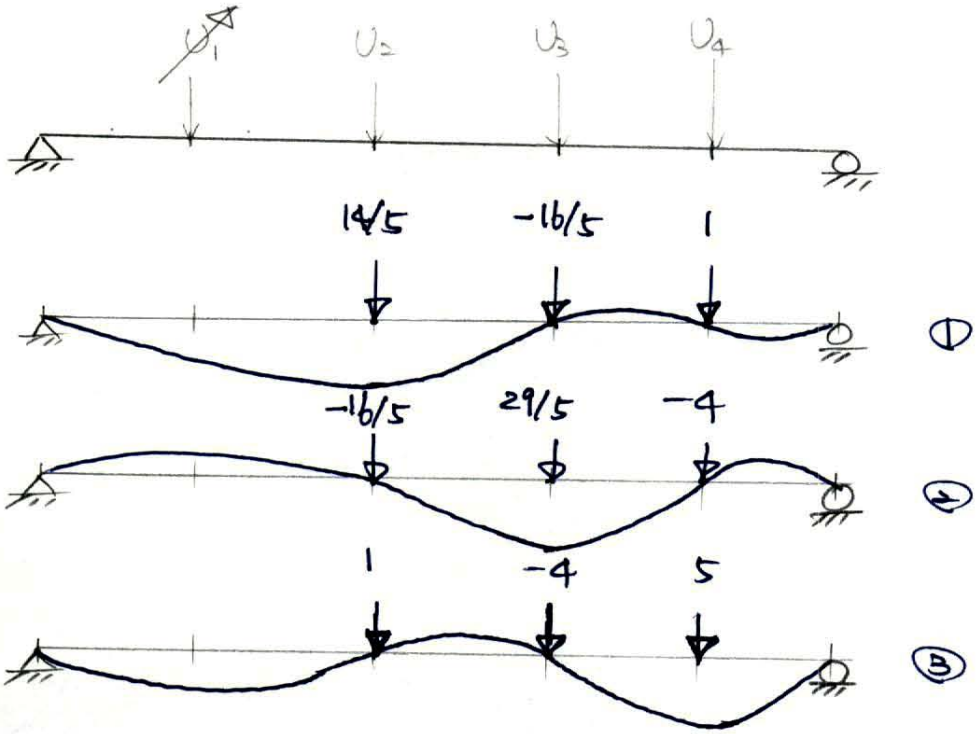
1st step of Gauss Elimination.

$$\begin{bmatrix} 5 & -4 & 1 & 0 \\ 0 & 14/5 & -16/5 & 1 \\ 0 & -16/5 & 29/5 & -4 \\ 0 & 1 & -4 & 5 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \leftarrow \underline{R}$$

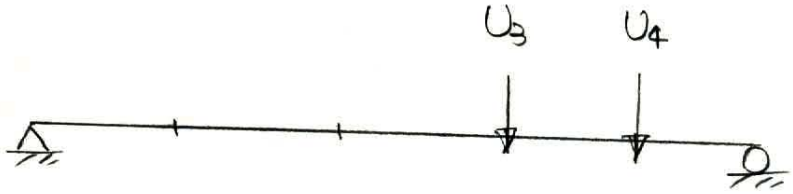
① ② ③

We used $5U_1 - 4U_2 + U_3 + 0 \cdot U_4 = 0$ to eliminate U_1 as a dof to the system.

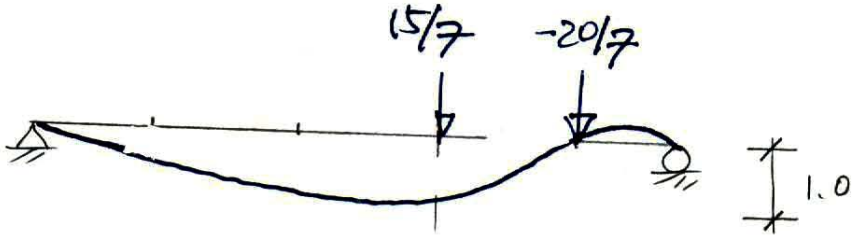
Note: What we "do" to the $\underline{K}$ matrix is indep. of $\underline{R}$.

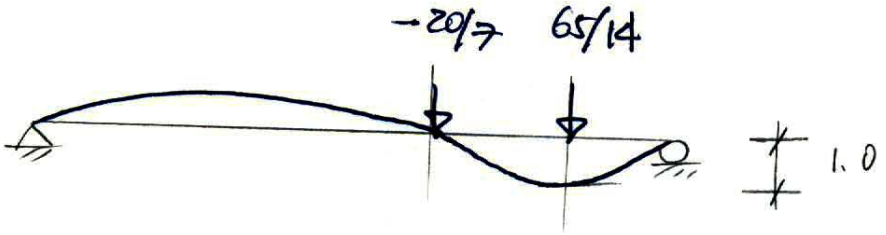


2nd Step



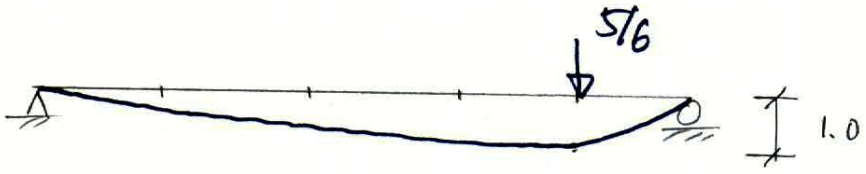
$$\begin{bmatrix} 5 & -4 & 1 & 0 \\ 0 & 14/5 & -16/5 & 1 \\ 0 & 0 & 15/7 & -20/7 \\ 0 & 0 & -20/7 & 65/14 \end{bmatrix} \begin{pmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 8/7 \\ -5/14 \end{pmatrix}$$

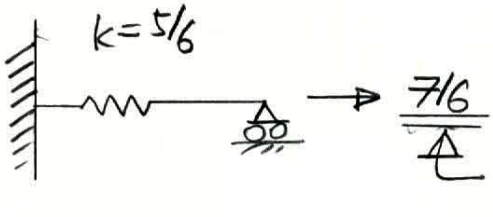




Finally

$$\begin{bmatrix} 5 & -4 & 1 & 0 \\ 0 & 14/5 & -16/5 & 1 \\ 0 & 0 & 15/7 & -20/7 \\ 0 & 0 & 0 & 5/6 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 8/7 \\ 7/6 \end{bmatrix}.$$





$$U_4 = \frac{7/6}{5/6}$$

Observations

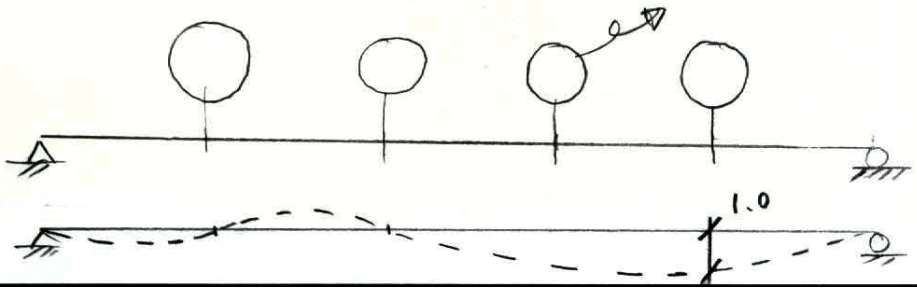
1) The matrices remain symmetric. → Saving space.

2) The diagonal element can only decrease

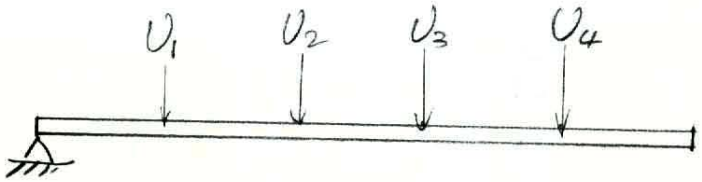
(constraints are removed, so the necessary force decreases)

3) For stable structure, they must remain positive.

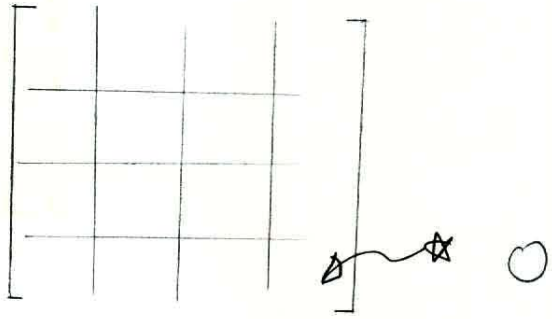
4) We can do Gauss el. in any order we desire and the above still holds



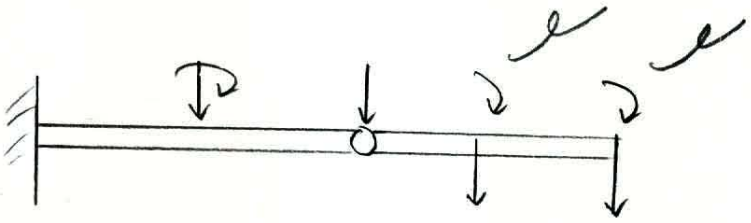
ex



$\underline{K} =$



ex



then zero pivot.

$k_{ij} \sim 10^{6}$

$k_{ii} \sim 10^{-2}$

considered as 0

- Skyline
- Wave front
- Cholesky
- Substructuring
- Sparse solution.

- $\underline{L}\underline{D}\underline{L}^{T}$ decomp.