Solution of eup cont'd

$$\underline{K}\,\underline{\phi} = \lambda\,\underset{n\times n}{\underline{M}}\,\underline{\phi} \qquad (1)$$

Inverse iteration

$$\underline{K}\,\bar{\underline{x}}_{k+1} = \underline{M}\,\underline{x}_k \qquad (2)$$

Assume
$$\underline{x}_1 = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$$

$$\underline{x}_{k+1} = \frac{\bar{\underline{x}}_{k+1}}{\left(\bar{\underline{x}}_{k+1}^T\,\underline{M}\,\bar{\underline{x}}_{k+1}\right)^{1/2}} \qquad (3)$$

Then $\underline{x}_{k+1}^T\,\underline{M}\,\underline{x}_{k+1} = 1$

Then if
$$\underline{x}_1^T\,\underline{M}\,\underline{\phi}_1 \neq 0$$

$\underline{x}_k \Rightarrow \underline{\phi}_1$ conv. rate is λ_1/λ_2 $(\lambda_1 < \lambda_2)$

Let's say, we have obtained ϕ_1, $\lambda_1 = \underline{\phi}_1^T\underline{K}\,\underline{\phi}_1$

Go now for λ_2, ϕ_2

Use (2), (3) but start with

$$\underline{x}_1 = \underline{\tilde{x}}_1 - \alpha \underline{\phi}_1 \qquad (4)$$

where we pick $\tilde{x}_1 = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}$ and calculate α

such that

$$\underline{x}_1^T \underline{M} \underline{\phi}_1 = 0$$

Hence

$$\underline{\phi}_1^T \underline{M} \underline{x}_1 = \underline{\phi}_1^T \underline{M} \underline{\tilde{x}}_1 - \alpha \underbrace{\phi_1^T M \phi_1}_{1} \equiv 0$$

$$\alpha = \underline{\phi}_1^T \underline{M} \underline{\tilde{x}}_1 \qquad (5)$$

Gram – Schmidt process, now we converge

to ϕ_2. $\lambda_2 = \underline{\phi}_2^T \underline{K} \underline{\phi}_2$

For $\underline{\phi}_2$: use (4) with

use $i=1,2$ $\Big\}$ $x_1 = \underline{\tilde{x}}_1 - \alpha \underline{\phi}_1 - \beta \underline{\phi}_2$ with $x_1^T M \phi_1 = 0$

$x_1^T M \phi_2 = 0$

$\phi_1^T M \underline{\tilde{x}}_1 = \alpha$

$\phi_1^T M \phi_2 = 0$

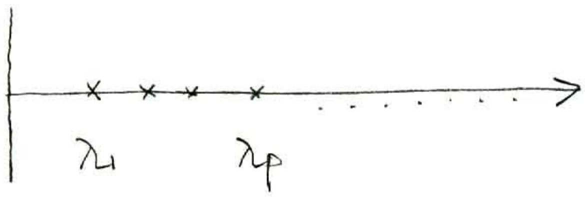
$\phi_2^T M \underline{\tilde{x}}_1 = \beta$

$\phi_1^T M \phi_1 = 0$

$\phi_2^T M \phi_2 = 0$

Subspace Iteration : pick $\underset{m \times p}{X_1}$ then for

$k = 1, 2, \ldots$



$$\underline{K}\,\underline{\bar{X}}_{k+1} = \underline{M}\,\underline{X}_k \qquad (a)$$

$$\underline{\bar{X}}^T_{k+1}\,\underline{K}\,\underline{\bar{X}}_{k+1} = \underset{p \times p}{\underline{K}_{k+1}} \qquad (b)$$

$$\underline{\bar{X}}^T_{k+1}\,\underline{M}\,\underline{\bar{X}}_{k+1} = \underset{p \times p}{\underline{M}_{k+1}} \qquad (c)$$

Calculate eigenvalue / vector approx

$$\underline{K}_{k+1}\,\underline{z} = \lambda\,\underline{M}_{k+1}\,\underline{z} \qquad (d)$$

Sol. of (d)

$$\underline{K}_{k+1}\,\underline{Q}_{k+1} = \underline{M}_{k+1}\,\underline{Q}_{k+1}\,\underline{\Lambda}_{k+1} \qquad (e)$$

where, $\underset{p \times p}{\underline{Q}_{k+1}}$ stores the eigenvectors of (d)

and $\underset{\text{diagonal}}{\underline{\Lambda}_{k+1}} = \begin{bmatrix} \times & & & & \\ & \times & & & \\ & & \times & & \\ & & & \times & \\ & & & & \times \end{bmatrix}$ stores the eigenvalues of (d)

Recall

$$\underline{K}\,\underline{\phi}_i = \lambda_i \underline{M}\,\underline{\phi}_i \qquad i=1,\cdots n$$

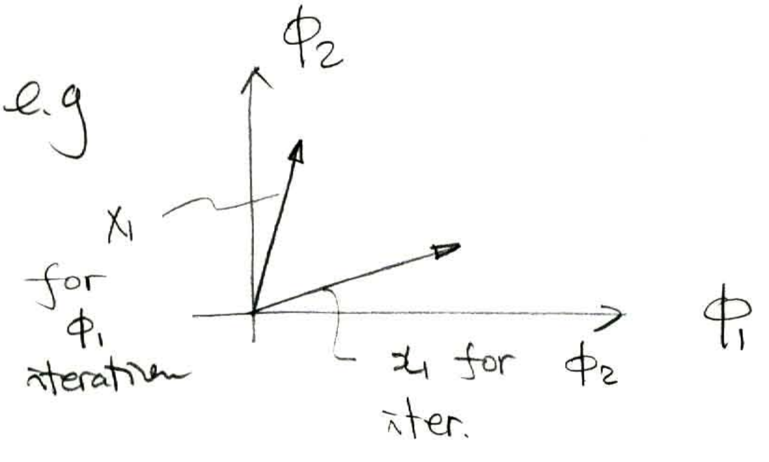
can be written as $\underline{K}\,\underline{\Phi} = \underline{M}\,\underline{\Phi}\,\underline{\Lambda}$, $\underline{\Phi} = [\underline{\phi}_1 \cdots \underline{\phi}_n]$

$$\underline{X}_{k+1} = \underline{\bar{X}}_{k+1}\,\underline{Q}_{k+1} \qquad (f)$$

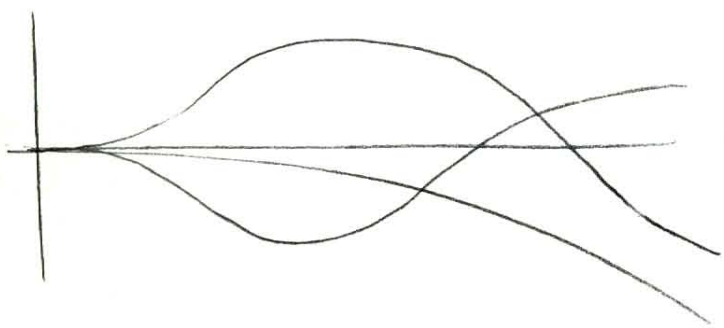
Properties

(I) If $\underline{X}_1$ spans the space of $\phi_1, \cdots \phi_p$ then the iteration converges in one step.

e.g



Hence construct $\underline{X}_1$ wisely!



$$\begin{bmatrix} 0 & 0 & \cdots \\ 0 & 0 & \\ 0 & 0 & \\ 1 & 0 & \\ 0 & 0 & \\ 0 & 0 & \\ 0 & 0 & \\ 0 & 1 & \\ 0 & \cdot & \end{bmatrix}$$

corresp to largest m_{ii}/k_{ii} value

II) Convergence is assured provided $\underline{X}_1$ is not M orthogonal to $[\phi_1 \cdots \phi_p]$

The vectors in X_1 must have components of $\phi_1, \cdots \phi_p$. and then, if we order the vectors in $\underline{X}_{k+1}$ s.t

$$\underline{X}_{k+1} = \Big[\ \ \Big|\ \ \Big|\ \ \Big|\ \ \Big]_{n \times p}$$

$\underline{x}^{i}_{k+1} \rightarrow \underline{\phi}_i$ at the rate λ_i/λ_{p+1}

corresp to smallest eigenv

corresp to next smallest evp.

ex $\left(\lambda_{30}/\lambda_{40+1}\right)$

Convergence will be good.

Sturm sequence check

Property of interest :

Consider $\underline{K} - \mu \underline{M} = \underline{L}\,\underline{D}\,\underline{L}^T$

μ is picked

then the number of negative elements in D is equal to the number of eigenvalues of $\underline{K}\,\underline{\phi} = \lambda \underline{M}\,\underline{\phi}$ smaller than μ

$p(\lambda)$

$\det(K - \lambda M)$