

Study on Granular Dynamics in Vertically Vibrated Beds using Tracking Technique

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Abstract—The dynamics of granular motion have been studied in a vertically vibrated bed using Positron Emission Particle Tracking (PEPT), which allows the motion of a single tracer particle to be followed in a non-invasive way. Regardless of the surface behaviour, particles are found to travel in rotational movement in horizontal plane. Particle cycle frequency is found to increase strongly with increasing vibration amplitude. Particle dispersion also increased strongly with vibration amplitude. Horizontal dispersion is observed to always exceed vertical dispersion.

Index Terms—Granular Dynamics, Instabilities, Positron Emission Particle Tracking, Vertically vibrated beds

I. INTRODUCTION

GRANULAR materials are of great interest to researchers in both engineering and science communities and to industry. The importance of their study derives from the complex rheological character of granular materials and from their wide applications in industry. Granular material subjected to vertical vibration demonstrates unusual flow behaviour [1]-[3], and this has led to applications in particle drying, mixing and separation processes. The motion in such devices is usually characterized by a dimensionless acceleration, Γ , which is the ratio of the vibration acceleration to the gravitational acceleration or $\Gamma = 4\pi^2 f^2 A/g$, where f is the oscillation frequency, A is the oscillation amplitude and g is the acceleration due to gravity. When the dimensionless acceleration exceeds unity [4], the free surface of a vertically vibrated bed becomes unstable and exhibits a variety of complex phenomena such as formation of heaps [5]-[7], fluidisation [8]-[10], formation of standing wave patterns (including square, stripe and hexagon) [11]-[17] and arching [18]. These kinds of instability have attracted the interest of many researchers using both experimental [1]-[16] and numerical [19]-[28] approaches.

In order to design and operate vibrated bed devices, an in-depth understanding of their solids behaviour is

essential. This paper therefore aims to investigate the solids motion in a vertically vibrated bed at the level of single particle motion using the state-of-the-art motion-following technique—Positron Emission Particle Tracking (PEPT). PEPT is a non-invasive method derived from the more familiar and widely used medical technique—Positron Emission Tomography (PET). It has proved to be a powerful means of studying the behaviour of granular materials in a variety of processing devices such as fluidised beds [29]-[30], rotating drums [31]-[32], V-mixers [33], ploughshare mixers [34] and bladed mixers [35]-[36].

II. EXPERIMENTAL

A. Experimental Apparatus

The vibration test system in this paper has four major components: a Perspex vessel, a LDS Ling V408 vibrator, LDS 100E Power Amplifier and Lodestar function generator. The experimental set-up is shown schematically in Fig. 1. The vessel used in this paper has a square base with dimensions of 13.7 cm x 13.7 cm and is of 9.5 cm height. The whole vessel was made from Perspex of 5 mm thickness to permit direct observation. It was directly mounted on the vibrator (Ling Dynamic Systems Ltd., Royston, Herts, UK), and was subjected to vertical oscillation driven by sinusoidal signals created by the function generator through a Power Amplifier (Ling Dynamic Systems Ltd., Royston, Herts, UK).

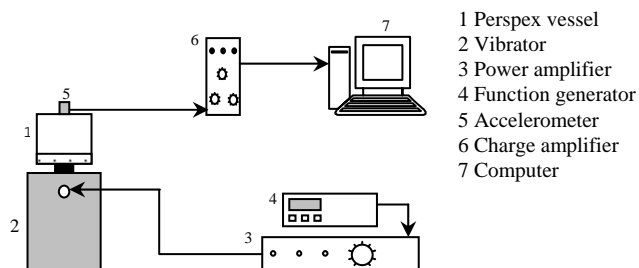


Fig. 1. Schematic diagram of the experimental apparatus.

The vibration amplitude was measured by an accelerometer (Brüel & Kjær, Nærum, Denmark) directly mounted on the top of the vessel. The accelerometer was connected to a charge amplifier and computer for data logging. The present experiments used spherical glass

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beads with density of 2500 kg/m^3 and with diameter of 0.8 mm (standard deviation of 5%) as the granular material. The vessel was filled with 400 g of glass beads and was vibrated at 18 Hz with an amplitude range $1.8\text{-}3.9 \text{ mm}$, corresponding to the dimensionless acceleration range, $\Gamma = 2.4$ to 5.1 .

Typically, three kinds of surface behaviour can be observed during experiments: 1) formation of heaps, where particles were avalanching against one of the sidewalls or one corner of the container; as shown in Fig. 2a; 2) surface waves, where planar stripes were formed on the bed surface; as shown in Fig. 2b; and 3) double kink, where arching can be clearly seen from both sidewalls; as shown in Figs. 2c and 2d. In this paper, the dominant motion depends on the vibration amplitude. Four vibration amplitude were investigated: case A: 1.8 mm , in which heap formation was observed; case B: 2.8 mm , which showed a surface wave pattern; case C: 3.5 mm , which also showed a surface wave pattern; case D: 3.9 mm , which was in arching state.

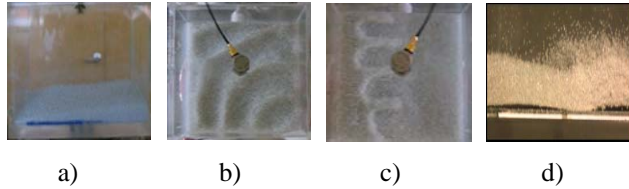
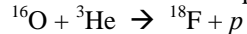


Fig. 2. Surface behavior: a) heap formation in vertical plane; b) surface waves in horizontal plane; c) double kink in horizontal plane; and d) arching in vertical plane.

B. Positron Emission Particle Tracking (PEPT)

Positron Emission Particle Tracking (PEPT) is a non-invasive technique which enables the position and 3D motion of a single tracer particle to be recorded in milliseconds. It has been developed at the Positron Imaging Centre at the University of Birmingham since 1987 [37]. A detailed description of this technique can be found in the literature [32],[37]. Before each PEPT experiment, a particle randomly taken from the bulk was irradiated by bombardment with a beam of ^3He ion from a cyclotron. Some of the oxygen atoms on the particle surface are converted to the isotope ^{18}F via the following reaction:



Alternatively, the radionuclide can be produced via adsorption of irradiated water onto the particle surface.

The isotope ^{18}F decays via positron emission with a half-life of 110 min . The vessel, which contains both bulk material and the single tracer particle, is placed between two planar detectors (Adac Laboratories, California, USA). As each positron is emitted from the tracer surface, it rapidly annihilates with a neighbouring electron, producing two “back-to-back” γ -rays. Each γ -ray has an energy of 511 keV , the electron’s mass-equivalent energy. Due to momentum conservation, the γ -rays are very close to collinear, traveling in opposite directions. This pair of γ -

rays is very penetrating and can be easily detected by the positron camera, resulting in the possibility of non-invasive tracking inside a wide range of processing units, including those with metal walls.

Each experiment lasted for approximately 45 minutes . The tracer trajectory (i.e. x , y and z co-ordinates vs. time) was recorded from the beginning of each experiment run for further computational analysis. The co-ordinate axes x and y are parallel to the plane of the positron camera in the horizontal and vertical directions respectively; z is perpendicular to the planar camera as presented in Fig. 3a. In order to ease the visualization of the PEPT data, the tracer trajectory is displayed in three orthogonal views (i.e. side view, end view and plan view) as defined in Figs. 3b-d. One of the main advantages of the PEPT technique is that it probes the bed motion at the single particle level, enabling information to be derived on, for example, cycle frequency and dispersion among other things.

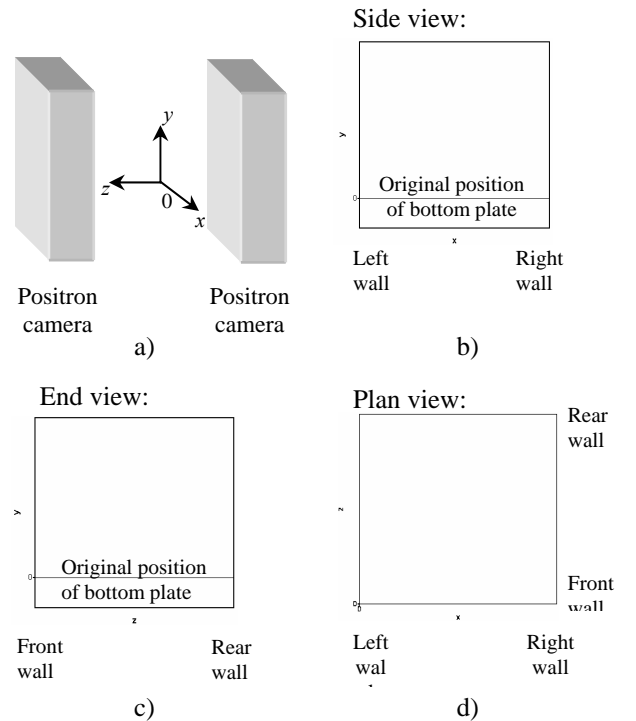


Fig. 3. a) Tracer Cartesian co-ordinates with respect to the PEPT camera; b) Side view of the bed; c) End view of the bed; and d) Plan view of the bed.

III. RESULTS AND DISCUSSION

A. Particle Motion

Fig. 4 shows plots of tracer vertical y co-ordinates over a time period of 1 s for cases A (heap formation) and B (surface waves). In both cases, in the region close to the bottom plate of the container, the tracer moved at a frequency which is the same as that of the system, i.e. 18 Hz ; as shown in Figs. 4a and 4c. In case A, the frequency of movement is the same at the surface (as shown in Fig.

4b), but in case B movement at the bed surface is at half the frequency of the system, i.e. 9 Hz (as shown in Fig. 4d). This pattern of the particle moving at the system frequency near the bottom plate region and at half this frequency at the surface was also observed for cases C and D. In summary, when the bed shows heap formation, its motion frequency will be the same as the excitation frequency throughout the bed; otherwise, if the bed shows surface waves or arches, the motion at the surface is at half the excitation frequency.

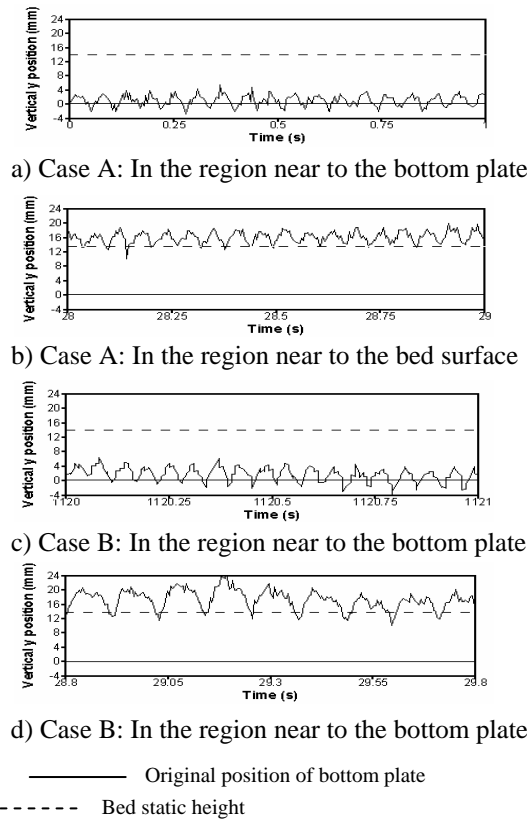


Fig. 4. Plots of tracer vertical y co-ordinates in regions near to the bottom plate and the bed surface for cases A (heap formation) and B (surface waves).

When the dimensionless acceleration exceeds unity, the granular material will form heaps which avalanche against one of the bed sidewalls or one corner of the bed [4],[6],[7],[11]. Thereafter, a variety of surface patterns such as standing waves and arching, appear as dimensionless acceleration increases. The reasons for the granular motion in vertically vibrated beds are still not well understood. This study therefore makes a first attempt to explain these instabilities at the level of single particle motion. Fig. 5 shows samples of the tracer trajectory over certain periods of time for case A (heap formation): a) $t = 667-786$ s; and b) $t = 60-290$ s. From this figure, it is evident that vertical vibration applied to the granular bed induces particle movement not only in vertical direction, but also in the horizontal direction. The occurrence of horizontal motion in vertically oscillated beds has also been observed by Melo et al. [16] and Deng and Wang

[17]. A cycle begins when the tracer is traveling upward parallel to the heap surface ($P_1 \rightarrow P_2$; as shown in end view in Fig. 5a), and then slumps down the bed ($P_3 \rightarrow P_4$; as shown in end view in Fig. 5a). When it approaches the rear wall region, it quickly moves downward to the bottom plate ($P_4 \rightarrow P_5$; as shown in end view in Fig. 5a) and starts another cycle again.

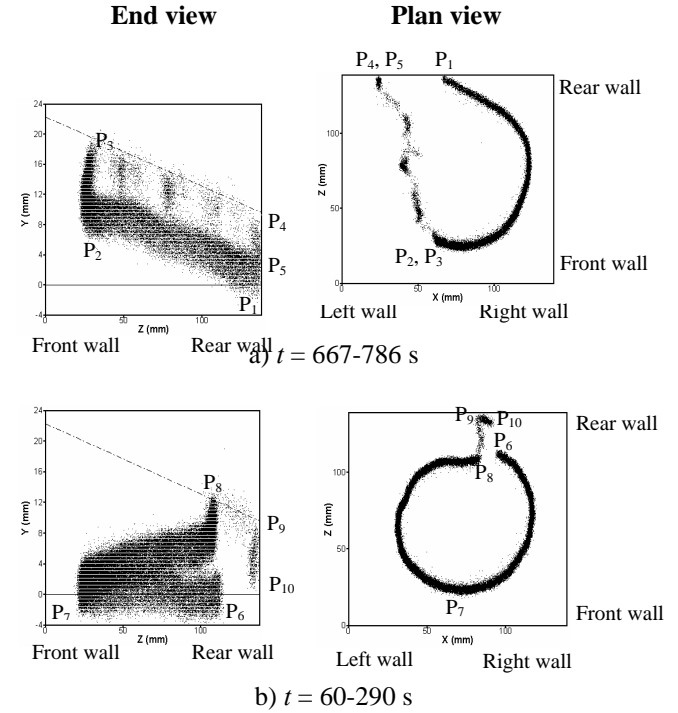


Fig. 5. Tracer trajectory over certain time periods for case A (heap formation): a) $t = 667-786$ s; and b) $t = 60-290$ s.

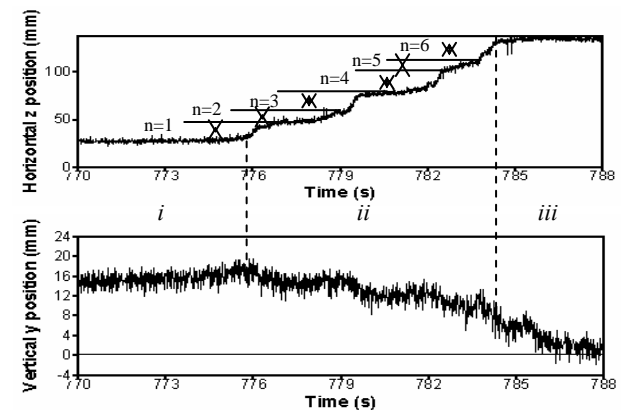


Fig. 6. Occurrence of successive avalanches over a time period of 770-788s in case A (heap formation).

Several successive avalanches can be observed along the heap surface ($P_3 \rightarrow P_4$; as shown in end view in Fig. 5a). Fig. 6 shows the occurrence of successive avalanches along the heap surface over a time period 770-788 s for case A (heap formation), where i is the upward movement

region, *ii* is the avalanching region and *iii* is the downward movement region. Apparently, the tracer is carried from the top region to the bottom region of the bed surface by six successive avalanches. These experimental phenomena agree very well with Evesque and Rajchenbach's [6] work. They have reported that the instability of the heap formation is caused by two different granular mechanisms: 1) convective upward transport; and 2) flow of avalanches on the free surface.

However, the tracer does not always follow the motion as described previously. There are times when the tracer upward movement is not exactly parallel to the heap surface ($P_6 \rightarrow P_7$; as shown in end view in Fig. 5b). When approaching the near front wall region, the tracer travels upward again, but in the direction opposite to the heap surface ($P_7 \rightarrow P_8$; as shown in end view in Fig. 5b). The tracer is then captured in the slumping down region when it approaches the heap surface region ($P_8 \rightarrow P_9$; as shown in end view in Fig. 5b). Apart from that, it is interesting to note that the tracer always moves in a circular trajectory in plan view (as shown in plan view in Figs. 5a and 5b) when traveling from the bottom region to the top region of the bed in the direction either parallel ($P_1 \rightarrow P_2$; as shown in end view in Fig. 5a) or non-parallel ($P_6 \rightarrow P_8$; as shown in end view in Fig. 5b) to the heap surface.

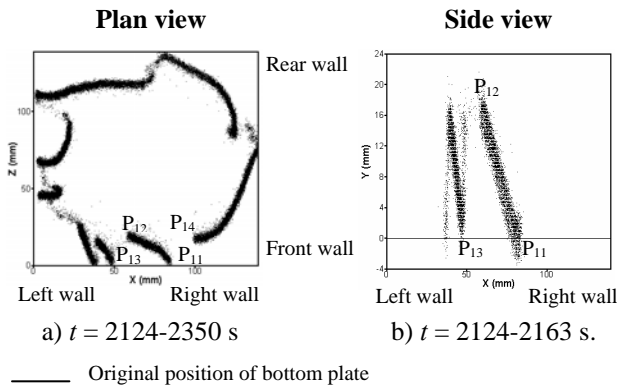


Fig. 7. Tracer trajectory over certain time periods for case A (heap formation): a) Plan view: $t = 2124-2350$ s ; and b) Side view: $t = 2124-2163$ s.

On the other hand, it is observed that the tracer does not move in a circular trajectory when it is in the near wall regions ($P_{11} \rightarrow P_{14}$; as shown in plan view in Fig. 7a). It rather travels up (as presented by more tortuous trajectory, for example, $P_{11} \rightarrow P_{12}$; as shown in side view in Fig. 7b) and down (as presented by less tortuous trajectory, for example, $P_{12} \rightarrow P_{13}$; as shown in side view in Fig. 7b) randomly in the near wall regions. The occurrence of this non-circular rotational movement (in plan view) suggests that the fluctuation induced by the sidewalls (through vertical vibration) has significant influence on the particle movement in the near sidewall regions. As reported by Aoki and Akiyama [19], the fluctuation motion first occurs at the boundary and then spreads over the entire bed through dissipative particle-wall and interparticle collisions. In other words, the energy dissipation at the

boundary and within the granular material (through dissipative collision) should balance the vibration energy applied to the granular material (through particle fluctuating motion at the boundary).

To summarize, the tracer particle in a bed with the heap formation surface pattern can travel from the bottom region to the top region of the bed in three ways as follows:

- 1) It travels upward in the direction parallel to the heap surface and then slumps down the surface by successive avalanches (as shown in end view in Fig. 5a);
- 2) It travels upward in the direction non-parallel to the heap surface and slumps down the surface when approaching the heap surface (as shown in end view in Fig 5b); and
- 3) It travels up and down randomly in the near sidewall region (as shown in side view in Fig. 7b).

In contrast to heap formation, the surface topography in the cases of surface waves (i.e. cases B and C) or arches (i.e. case D) is constantly changing and it is not possible to locate the particle relative to the surface. In these cases, it is suggested that further PEPT experiments would be performed in conjunction with a high-speed camera facility. In general, particles in beds with surface waves and arching (i.e. cases B-D) were found to travel in rotational movement in non-circular trajectories (in plan view) throughout the system; for example, rotational movement in non-circular trajectory for case C (surface waves, $P_{15} \rightarrow P_{16}$); as shown in plan view in Fig. 8.

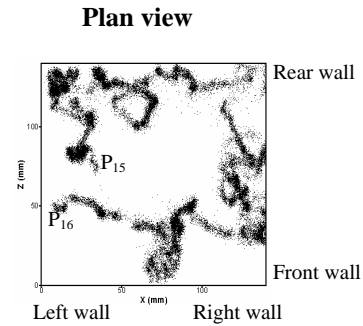


Fig. 8. Tracer trajectory in plan view over certain time period 1000-1320 s for cases C (surface waves).

B. Particle Cycle Frequency

Granular mixing behaviour is of importance in design and performance of equipment for industrial applications, but little information on solids mixing is available for vertically vibrated system. The cycle frequency can be a useful indicator in assessing solids motion in any equipment of interest [30]. The time required by the tracer to travel between parts of the bed to be measured can be extracted from the PEPT data. In this paper, a cycle time is defined as the time during which the tracer travels from the lower part of the bed (25% of the expanded bed height) to the upper part of the bed (75% of the expanded bed height), and eventually goes back to the lower part of the bed again. The cycle frequency is the inverse of the cycle

time and the average cycle frequency for the entire experiment can then be calculated. Fig. 9 presents effect of vibration amplitude on the cycle frequency qualitatively. From this figure, it is apparent that the cycle frequency increases strongly with the dimensionless acceleration. This is most likely particles receive more vibration energy to travel up and down the vessel at higher amplitudes.

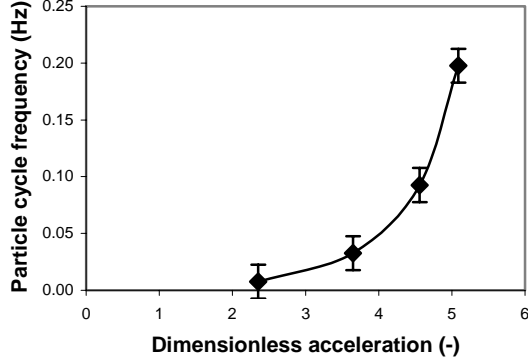


Fig. 9. Effect of vibration amplitude on the particle cycle frequency.

C. Particle Dispersion

A useful indicator of granular mixing behaviour is dispersion. Conventionally, particle concentration is used to examine the quality of solids mixing in a wide range of processing units. Since this type of concentration information cannot be obtained from PEPT measurement, Martin [38] and Kuo [39] have developed techniques to calculate dispersion coefficients from PEPT data and used this to quantify mixing. A volume element within the system is selected. Each time the tracer's PEPT trajectory passes through this element, it is treated as an independent starting point. "Final locations" are then found after a certain predefined travelling time. If necessary, interpolation is used to calculate the tracer final position. For each volume element, a set of short sections of trajectory, all starting from the same point but finishing at different end points (x_i, y_i, z_i) are obtained. A dispersion index for the volume element is defined based on the variance of these end positions from the average end position:

$$\sigma = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2 + (y_i - \bar{y})^2 + (z_i - \bar{z})^2}{n}}$$

where σ is the position standard deviation (mm), n is the total number of traces, x_i , y_i and z_i are i th end point co-ordinates (mm) and \bar{x} , \bar{y} and \bar{z} are the averages of final point co-ordinates. The greater the dispersion, the more varied the end points and the greater the value of σ . According to Martin [38], the length of time period between the tracer starting point and its end point should permit the tracer to travel 1/3 of the smallest bed linear dimension with its velocity averaged over the entire

system. Following Martin's approach, the time period for the studies reported here is set at 773 ms. The dispersion index has been analysed in each volume element with size of 25 mm (width) x 7 mm (height) and 25 mm (depth).

The average value of dispersion index can be determined by averaging the total index value for all volume elements over the entire system. The effect of vibration amplitude on the particle average dispersion index are summarized in Fig. 10. The average dispersion index increases strongly with increase in dimensionless acceleration. When this dispersion is separated into vertical and horizontal components (as shown in Fig. 11), it is clear that the horizontal dispersion always exceeds vertical dispersion. This is most probably because the bed width (i.e. 137 mm) is much larger than the layer height, and hence the tracer is likely to travel longer distance in horizontal direction than in vertical direction.

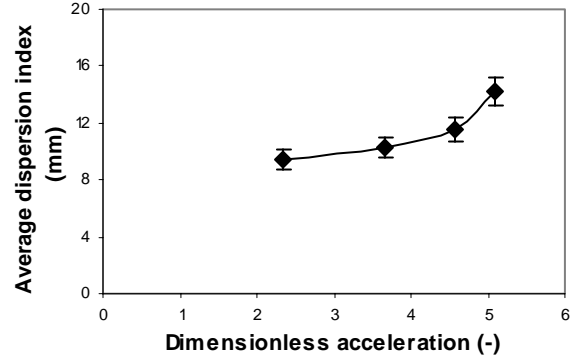


Fig. 10. Effect of vibration amplitude on the particle average dispersion index.

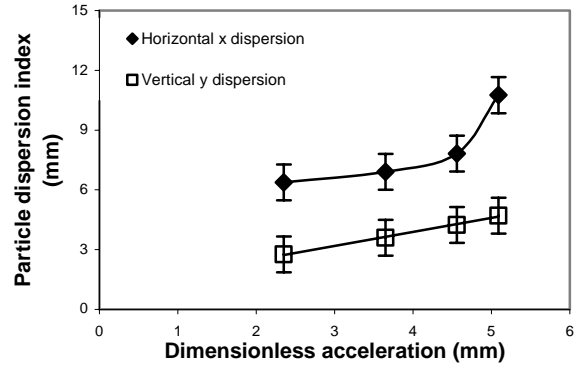


Fig. 11. Effect of vibration amplitude on both the horizontal x dispersion and vertical y dispersion.

IV. CONCLUSION

Positron Emission Particle Tracking has been used to examine the quality of solids mixing behaviour in vertically vibrated beds with heap formation, surface waves and arching. The investigated solids motion features include velocity profile, cycle frequency and solids dispersion. In beds with surface waves and arching state

the particles move up and down at the excitation frequency in the region near to the bottom plate, but at half this value at the bed surface. There is a need to extend this work to study the location and nature of this change in the frequency of particle motion. Conversely, for beds with heap formation, the motion frequency is at the system frequency over the entire bed. For beds which exhibit heap formation, successive avalanches can be found along the heap surface, carrying particles from the top region to the bottom region of the bed surface.

Regardless of the surface behaviour, particles tend to have a rotational movement in plan view. Vibration amplitude is found to have significant influence on the solids mixing behaviour. Cycle frequency and dispersion index are observed to increase strongly with increasing vibration amplitude. Such understanding should provide routes to improving the quality of granular mixing behaviour in vertically vibrated beds.

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